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Abstract: We present the results of an amplitude analysis of $\bar{p}p \rightarrow \pi^-\pi^+$, using differential cross sections and new polarisation results in the c.m. energy region between 2020 and 2580 MeV. There is strong evidence for resonances with spin 3, 4 and 5 at mass values of 2150, 2310 and 2480 MeV/c^2 with widths between 200 and 280 MeV and quantum numbers $J^{PC}\ I^G = 3^{--}\ 1^+$, $4^{++}\ 0^+$, $5^{--}\ 1^+$.

Differential cross sections for the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ in the centre of mass range 2020 to 2580 MeV/c^2 have been available for some time[1]. We recently reported[2] measurements of polarisation for this process between 2080 and 2500 MeV/c^2. By combining these two sets of data, which consist of cross-sections at 20 momenta and polarisations at 11 momenta, we have been able to perform an amplitude analysis of the reaction over the complete energy range. The data when binned in intervals of 0.04 in cos θ^* have a typical accuracy of 10% per point; the complete data set comprises approximately 2000 points. The results are consistent with the formation of three resonances with spins of 3, 4 and 5 at masses of 2150, 2310 and 2480 MeV/c^2 in the $\bar{p}p \rightarrow \pi^-\pi^+$ system.

In this reaction the initial $\bar{p}p$ state is constrained to have spin equal to one (see ref. [1]), and the scattering can be expressed in terms of two amplitudes, F_{++} and F_{+-}; these refer respectively to helicity non-flip and helicity flip contributions, where

$$F_{++} = \frac{1}{2k} \sum_J (J + \tfrac{1}{2})\ f^J_{++}\ P_J\ (\cos\theta^*)$$

$$F_{+-} = \frac{1}{2k} \sum_J \frac{(J + \tfrac{1}{2})}{\sqrt{J\ (J + 1)}}\ f^J_{+-}\ P^1_J\ (\cos\theta^*)$$

with k being the centre of mass momentum of the $\bar{p}p$ system, and P_J, P^1_J the Legendre polynomials. The differential cross section and polarisation are then defined as

$$\frac{d\sigma}{d\Omega} = |F_{++}|^2 + |F_{+-}|^2$$

and $P = 2\ \mathrm{Im}\ (F^*_{++}\ F_{+-})\ /\ \{|F_{++}|^2 + |F_{+-}|^2\}$

In order to estimate the maximum value required for J (the angular momentum of the $\pi\pi$ system) the data, at each energy, were fitted to a conventional Legendre polynomial series. These fits indicated that only terms up to J = 5 were significant, even at the highest energy[1, 2]. By factorising the fit to the differential cross section at each energy it is straightforward to obtain its zeros as a function of cos $\theta^* \equiv z$. These will in general be complex, and appear in conjugate pairs:

$$\frac{d\sigma}{d\Omega} = A \prod_i (z - \lambda_i)\ (z - \lambda_i^*)$$

where λ_i, λ_i^* are the roots. If the differential cross section were dominated by a single J-state then the values of the roots would be the same as those of the equation:

$$J\ (J + 1)\ |f_{++}^J|^2\ [P_J\ (z)]^2 + |f_{+-}^J|^2\ [P_J^1\ (z)]^2 = 0$$

A detailed discussion of the significance of cross section zeros is given in references [3] and [4].

By comparing the zeros of the data with those of pure J-states we conclude that there are three dominant partial wave contributions to the differential cross sections, each in a separate energy region[4]:

(i) $2020 < W < 2220$ MeV/c^2 $\quad$ J = 3 with $f_{+-}^3 > f_{++}^3$

(ii) $2270 < W < 2390$ MeV/c^2 $\quad$ J = 4 with $f_{++}^4 > f_{+-}^4$

(iii) $2400 < W < 2580$ MeV/c^2 $\quad$ J = 5 with $f_{+-}^5 > f_{++}^5$

where W is the centre of mass energy. The positions of the real zeros of polarisation support this interpretation.

To provide initial quantitative information for the overall analysis we then made the simplest assumption - that these states obeyed Breit-Wigner parametrisations - and inserted estimates of mass, width and relative coupling resulting from the analysis of $d\sigma/d\Omega$ zeros. The subsequent fitting to the complete data set then proceeded in three basic steps:

(1) In general, for each set of polarisation measurements there are two nearby energies at which differential cross sections are available. Such combined data sets, each spanning only a small energy interval, were used to obtain initial estimates of the $J = 0$, 1 and 2 waves in an energy independent analysis. The Breit-Wigner parametrisations of $J = 3$, 4 and 5 were not allowed to vary in these χ^2 minimisation fits. The results suggested that the low J-states were smoothly varying in magnitude and approximately constant in phase over the complete energy region.

(2) Having starting values, as a function of energy, for the smaller $J = 0$, 1, 2 contributions the energy independent analysis was repeated, allowing in each fit a background term for the $J = 3$, 4 and 5 states, in addition to the fixed Breit-Wigner forms. This provided new estimates of all the waves in each energy interval. The major helicity amplitudes of the upper three spin states, i.e. f^3_{+-}, f^4_{++}, f^5_{+-}, were each still consistent with pure resonance forms, though with somewhat different masses and widths. These amplitude data were each fitted to find better estimates of the $J = 3$, 4 and 5 Breit-Wigner parameters. An investigation of the widths showed that no significant variation with energy was required by the data. The resonant amplitudes were subsequently parametrised as:

$$f_{+\pm} = \frac{e^{i\phi_J}\,\gamma_{+\pm}\,\Gamma\sqrt{s_o}}{(s - s_o) + i\,\Gamma\sqrt{s_o}}$$

where s is the square of the centre of mass energy, s_o the square of the resonance mass, Γ the total width and ϕ_J the overall phase factor. To be consistent with a pure resonance model the partial couplings γ_{++}, γ_{+-} were maintained relatively real for a given value of J.

(3) In the final stage the J = 0, 1 and 2 waves were each parametrised as smooth interpolated energy-dependent forms in both magnitude and phase. In the final χ^2 minimisation these were each allowed one overall, energy-independent scaling factor and the parameters of the Breit-Wigner amplitudes were all treated as variables.

The resulting fit has a χ^2 contribution per data point of 1.65; this should be compared with the typical value of 1.35 for the χ^2 per degree of freedom obtained for the conventional fits to $d\sigma/d\Omega$ and $P\,d\sigma/d\Omega$ at a given energy with Legendre series. It should be noted that although this energy-dependent analysis only used the information from zeros to provide starting values for the dominant waves, the final fits to the complete data set are perfectly consistent with this initial hypothesis; the procedure is thus internally self-consistent. The parameters for the Breit-Wigner amplitudes are given in Table 1, and the fractional contributions of all the spin states to the total channel cross section as a function of momentum are displayed in Table 2. The resonances have estimated uncertainties of $\sim \pm 30$ MeV/c^2 and $\sim \pm 25$ MeV/c^2 in their masses and widths respectively. A sample of fits to the data is shown in Fig. 1.

We have investigated the implications of alternative solutions by studying the zeros of the transversity amplitudes[5]. Energy independent fits of Legendre polynomial series to $d\sigma/d\Omega$ and $P\ d\sigma/d\Omega$ allow the zeros of the polarised cross sections to be obtained by factorisation. From these the ambiguous solutions are constructed, and it is clear that all solutions require large f^3_{+-}, f^4_{++}, f^5_{+-} contributions in the energy regions where our detailed analysis finds them consistent with resonance forms. This contrasts with the analyses of πN two-body channels, where ambiguous solutions frequently lead to varying interpretations of the underlying physics. However, this difference is easily explained, as the absence of diffractive processes in the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ allows the cross section to show the explicit structure of dominant waves, whereas in πN scattering this is normally only the case near the backward direction. Ambiguity does nonetheless exist in the smaller waves, and the present data are able to accommodate several alternative smooth solutions for the $J = 0$, 1 and 2 states.

With the differential cross section data alone[1], Donnachie and Thomas[6], have conjectured the existence of resonances with $J = 3$, 4 and 5, using a model that also incorporated a constituent interchange background. In addition, Hyams et al.[7], using a one pion exchange approximation to study $\pi^-p \rightarrow p\bar{p}n$, have extracted information on the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ in the mass range 1.95 to 3.0 GeV/c^2. They obtained a satisfactory fit by allowing resonances in all waves. Finally, Nicholson et al.[8] invoked $J = 3$ and $J = 5$ resonances to fit their folded differential cross section data in the $\bar{p}p \rightarrow \pi^-\pi^+$ channel. The results of these fits are compared to our results on a spin versus mass squared plot in Fig. 2.

A final state $\pi\pi$ of spin J can be accessed from two initial $\bar{p}p$ angular momentum states: $L = J \pm 1$. From the couplings of the

Breit-Wigner amplitudes it is straightforward to see that our results indicate the $L = J + 1$ contribution is significantly greater than that of the $L = J - 1$ state for all three resonances. It is also interesting to note that the simple angular momentum relation $L = kr$ indicates for $L = J + 1$ a values for the interaction radius, r, very close to an inverse pion mass. Again this result is true, to within a few percent, for all three states, and may indicate that the triplet $\bar{p}p$ system has an enhanced interaction for an internal separation of $\sim 1.4 - 1.5$ fm. This is also the characteristic interaction radius seen in $\bar{p}p$ elastic scattering[9].

Conclusion

The new polarisation data have allowed a detailed energy dependent analysis of the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ to be done in the mass range from 2020 to 2580 MeV/c^2. Using Breit-Wigner parametrisations for the dominant waves and incorporating the previously available differential cross sections we have obtained a satisfactory fit to the data that strongly supports the existence of three meson resonances in this region.

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Figure Captions

Figure 1 : Sample fits to differential cross section data (in μb/sr) and polarisations. Labels indicate the laboratory momentum of the incident $\bar{p}$. Data are available for $d\sigma/d\Omega$ in the range 0.79 to 2.43 GeV/c and for polarisation between 1.0 and 2.2 GeV/c.

Figure 2 : A plot of spin J, versus mass squared, M^2, for the mesons. Established states are shown on the leading trajectory. Results from the present and previous fits to the process $\bar{p}p \rightarrow \pi^-\pi^+$ are compared. Horizontal bars indicate error estimates on M^2, where available.

Table Captions

Table 1 : Parameters of the Breit-Wigner resonances for the $J = 3, 4$ and 5 amplitudes.

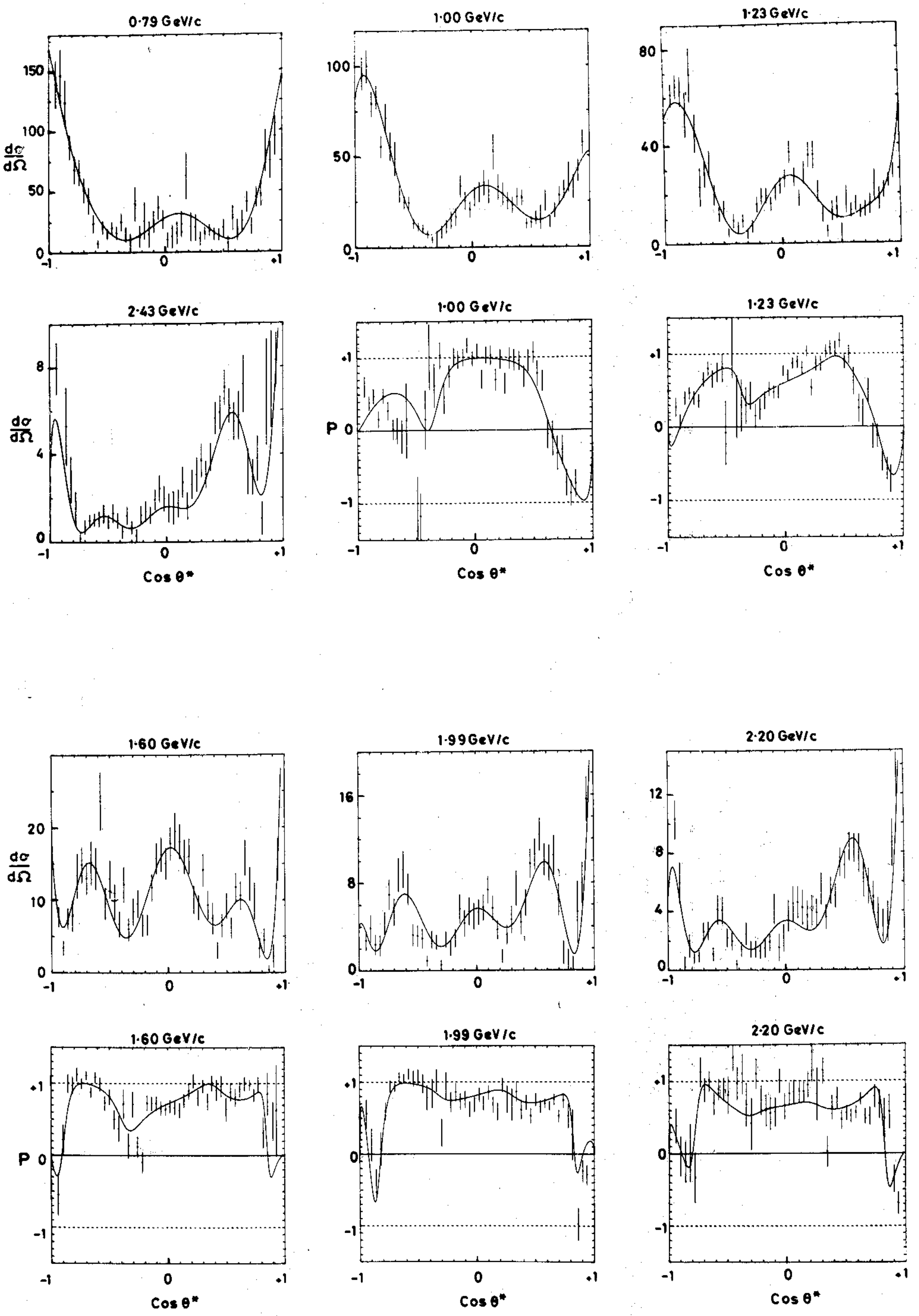
Table 2 : Fractional contribution to the channel cross section of the different spin states as a function of the laboratory momentum of the incident $\bar{p}$.

Table 1

J (Spin)	Mass (MeV/c^2)	Width, Γ (MeV)	Phase ϕ (radian)	γ_{++}	γ_{+-}
3	2150	200	0	-0.073	0.153
4	2310	210	-0.85	-0.083	0.041
5	2480	280	2.36	0.035	-0.068

Table 2

Laboratory Momentum (GeV/c)	J = 0	J = 1	J = 2	J = 3	J = 4	J = 5
0.8	0.13	0.28	0.16	0.36	0.04	0.03
1.0	0.10	0.10	0.18	0.54	0.05	0.03
1.2	0.04	0.05	0.05	0.74	0.09	0.03
1.4	0.03	0.05	0.04	0.61	0.20	0.07
1.6	0.02	0.05	0.04	0.37	0.40	0.12
1.8	0.02	0.08	0.04	0.23	0.39	0.24
2.0	0.01	0.12	0.05	0.16	0.23	0.43
2.2	0.01	0.13	0.06	0.12	0.14	0.54
2.4	0.02	0.16	0.09	0.11	0.10	0.52



0·79 GeV/c
1·00 GeV/c
1·23 GeV/c
2·43 GeV/c
1·00 GeV/c
1·23 GeV/c
1·60 GeV/c
1·99 GeV/c
2·20 GeV/c
1·60 GeV/c
1·99 GeV/c
2·20 GeV/c
dσ/dΩ
P
Cos θ*

FIG. 1

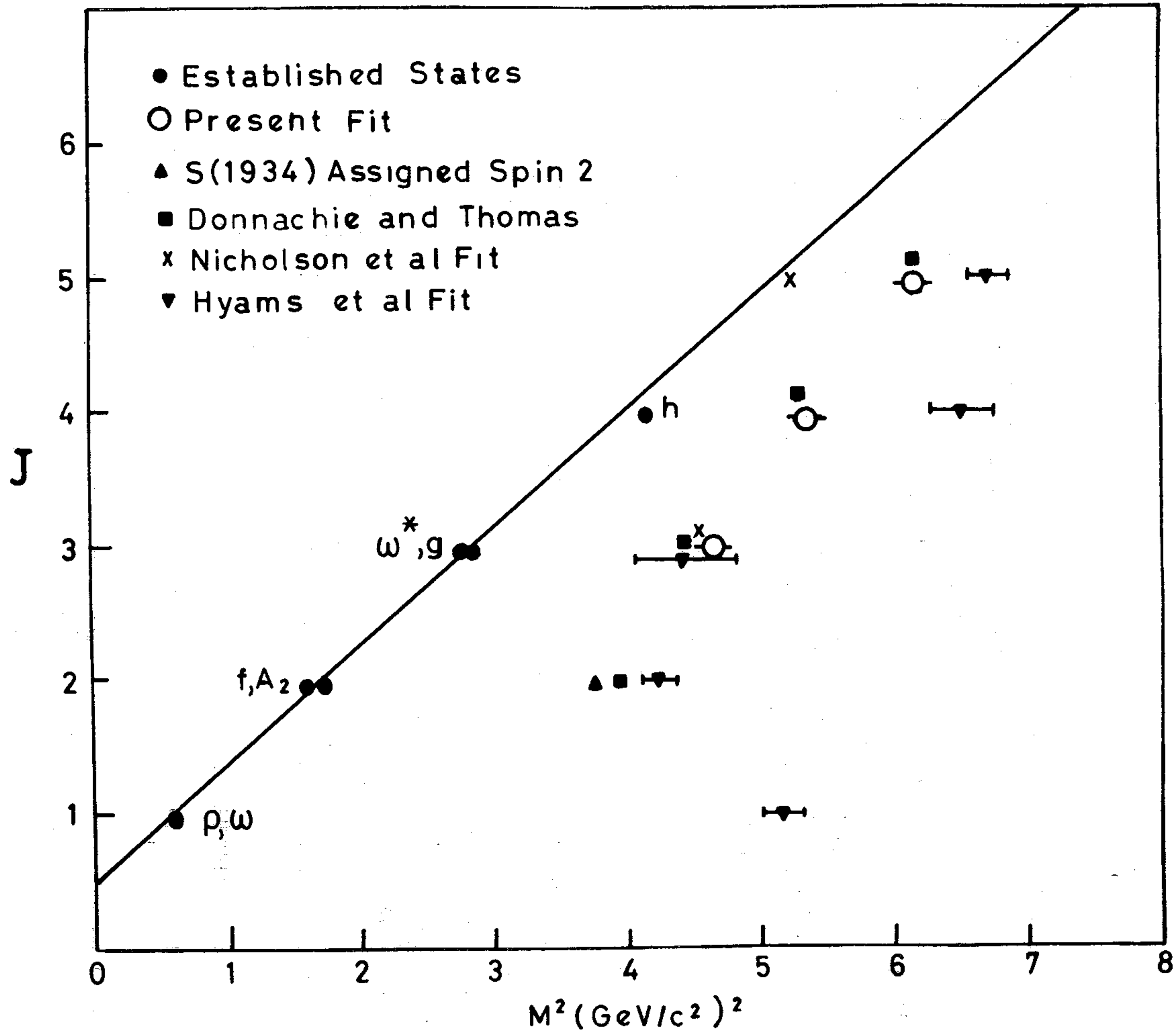

Established States
Present Fit
S(1934) Assigned Spin 2
Donnachie and Thomas
Nicholson et al Fit
Hyams et al Fit
ρ,ω
f,A₂
ω*,g
h
J
M²(GeV/c²)²
0
1
2
3
4
5
6
7
8

FIG. 2