

# The $B \rightarrow \pi$ form factor from light-cone sum rules in soft-collinear effective theory\*

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Recently, we have derived light-cone sum rules for exclusive  $B$ -meson decays into light energetic hadrons from correlation functions within soft-collinear effective theory [1]. In these sum rules the short-distance scale refers to “hard-collinear” interactions with virtualities of order  $\Lambda_{\text{QCD}} m_b$ . Hard scales (related to virtualities of order  $m_b^2$ ) are integrated out and enter via external coefficient functions in the sum rule. Soft dynamics is encoded in light-cone distribution amplitudes for the  $B$ -meson, which describe both the factorizable and non-factorizable contributions to exclusive  $B$ -meson decay amplitudes. Factorization of the correlation function has been verified to one-loop accuracy. Thus, a systematic separation of hard, hard-collinear, and soft dynamics in the heavy-quark limit is possible.

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The QCD factorization theorems for exclusive, energetic  $B$ -decays, first proposed in [2], identify short-distance effects that can be systematically calculated in perturbation theory. Non-perturbative effects are parametrized in terms of a few universal functions such as form factors and light-cone distribution amplitudes, on which our information is rather restricted. Moreover, phenomenological applications are limited by the insufficient information on (power-suppressed) non-factorizable terms. These restrictions should be tackled by the large data sets available by current and future  $B$ - physics experiments on the experimental side, and by the development of suitable non-perturbative methods on the theory side.

The quantum field theoretical framework corresponding to the QCD factorization theorems is the soft collinear effective theory (SCET) [3, 4]. In contrast to the well-known heavy-quark effective theory (HQET), the recently proposed SCET does not correspond to a local operator expansion. While HQET is only applicable to  $B$  decays, when the energy transfer to light hadrons is small, for example to  $B \rightarrow D$  transitions at small recoil to the  $D$  meson, it is not applicable, when some of the outgoing, light particles have momenta of order  $m_b$ ; then one faces a multiscale problem: a)  $\Lambda = \text{few} \times \Lambda_{\text{QCD}}$ , the *soft* scale set by the typical energies and momenta of the light degrees of freedom in the hadronic bound states; b)  $m_b$ , the *hard* scale set by the heavy- $b$ -quark mass; c) the hard-collinear scale  $\mu_{\text{hc}} = \sqrt{m_b \Lambda}$  appears via interactions between soft and energetic modes in the initial and final states. The dynamics of hard and hard-collinear modes can be described perturbatively in the heavy-quark limit  $m_b \rightarrow \infty$ . The separation of the two perturbative scales from the non-perturbative hadronic dynamics is determined by the small expansion parameter  $\lambda = \sqrt{\Lambda/m_b}$ . On a technical level the implementation of power counting in  $\lambda$  for fields and operators in SCET corresponds directly to the well-known method of regions for Feynman diagrams [5].

A simple example where the above considerations apply is given by the  $B \rightarrow \pi$  transitions form factor. In the large recoil-energy limit the heavy-to-light form factors obey relations [6] that are broken by radiative and power corrections. Each form factor can be decomposed into two basic contributions, one piece that factorizes into a perturbatively calculable coefficient function  $T_i$  and light-cone distribution amplitudes  $\phi_B$  and  $\phi_\pi$  for heavy and light mesons, respectively, and a second contribution, where the hard-collinear interactions are *not* factorizable, leaving one universal “soft” form factor  $\xi_\pi$  [7]:

$$\langle \pi | \bar{\psi} \Gamma_i b | B \rangle = C_i(E, \mu_I) \xi_\pi(\mu_I, E) + T_i(E, u, \omega, \mu_{\text{II}}) \otimes \phi_+^B(\omega, \mu_{\text{II}}) \otimes \phi_\pi(u, \mu_{\text{II}}) + \text{subleading terms}.$$

Here  $C_i$  is a short-distance function arising from integrating out hard modes, and  $T_i$  contains hard and hard-collinear dynamics related to spectator scattering. Consequently  $\mu_I$  is a factorization scale below  $m_b$ , while  $\mu_{\text{II}}$  is a factorization scale below  $\mu_{\text{hc}}$ . Both functions can be computed as perturbative series in  $\alpha_s$  (the effective theories for the two short-distance regimes are known as SCET<sub>I</sub> and SCET<sub>II</sub>). We have shown in [1] that light-cone sum rules can be formulated for the *soft* (i.e. non-factorizable) part of the form factor *within* SCET<sub>I</sub>. In the following we briefly discuss the basic idea using the tree-level construction in an exemplary mode. For the derivation of all further non-trivial results, the phenomenological application, and also for the general notation used in this short letter, we guide the reader to the original publication [1].

Within SCET<sub>I</sub> the non-factorizable (i.e. end-point-sensitive) part of the  $B \rightarrow \pi$  form factor in

the heavy-quark limit is described by the current operator

$$J_0(0) = \bar{\xi}_{\text{hc}}(0)W_{\text{hc}}(0)Y_s^\dagger(0)h_v(0), \quad \langle \pi(p') | J_0(0) | B(m_B v) \rangle = (n_+ p') \xi_\pi(n_+ p', \mu_I), \quad (1)$$

where  $\xi_{\text{hc}}$  is the ‘‘good’’ light-cone component of the light-quark spinor with  $\not{n}_- \xi_{\text{hc}} = 0$ , and  $W_{\text{hc}}$  and  $Y_s^\dagger$  are hard-collinear and soft Wilson lines. Finally,  $h_v$  is the usual HQET field. The heavy quark is nearly on-shell in the end-point region. In SCET<sub>I</sub>, this is reflected by the fact that hard subprocesses (virtualities of order  $m_b^2$ ) are already integrated out and appear in coefficient functions multiplying  $J_0$ , which can be determined from the matching of the corresponding QCD current on SCET<sub>I</sub>. Therefore we will *not* introduce an interpolating current for the  $B$  meson as in the usual light-cone sum-rule approach. Instead, the short-distance (off-shell) modes in SCET<sub>I</sub> are the hard-collinear quark and gluon fields, and therefore the sum rules should be derived from a dispersive analysis of the correlation function

$$\Pi(p') = i \int d^4x e^{ip'x} \langle 0 | T [J_\pi(x) J_0(0)] | B(p_B) \rangle, \quad (2)$$

where  $p_B^\mu = m_B v^\mu$ , and the interpolating current  $J_\pi$  for a pion in the effective theory is chosen as

$$J_\pi(x) = -i \bar{\xi}_{\text{hc}}(x) \not{n}_+ \gamma_5 \xi_{\text{hc}}(x) - i (\bar{\xi}_{\text{hc}} W_{\text{hc}}(x) \not{n}_+ \gamma_5 Y_s^\dagger q_s(x) + h.c.). \quad (3)$$

with  $\langle 0 | J_\pi | \pi(p') \rangle = (n_+ p') f_\pi$ . Here we denoted soft and hard-collinear quark fields in SCET as  $q_s$  and  $\xi_{\text{hc}}$ , respectively. Notice that soft-collinear interactions require a multipole expansion of soft fields, which is always understood implicitly. We also point out that the effective theory SCET<sub>I</sub> contains SCET<sub>II</sub> as its infrared limit (i.e. when the virtuality of the hard-collinear modes is lowered to order  $\Lambda^2$ ). For this reason, the hard-collinear fields that define the interpolating current  $J_\pi$  also contain the collinear configurations that show up as hadronic bound states (see also the discussion in [8]).

In the following we will consider a reference frame where  $p'_\perp = v_\perp = 0$  and  $n_+ v = n_- v = 1$ . In this frame the two independent kinematic variables are  $(n_+ p') \simeq 2E_\pi = O(m_b)$  and  $0 > (n_- p') = O(\Lambda)$  with  $|n_- p'| \gg m_\pi^2 / (n_+ p')$ . The dispersive analysis will be performed with respect to  $(n_- p')$  for fixed values of  $(n_+ p')$ . As with all QCD sum-rule calculations, the procedure consists in writing the correlator in two different ways. On the hadronic side, one can write

$$\Pi^{\text{HAD}}(n_- p') = \Pi(n_- p') \Big|_{\text{res.}} + \Pi(n_- p') \Big|_{\text{cont.}}, \quad (4)$$

where the first term represents the contribution of the pion, while the second takes into account the role of higher states and continuum above an effective threshold  $\omega_s = O(\Lambda^2 / n_+ p')$ . The former can be rewritten as

$$\Pi(n_- p') \Big|_{\text{res.}} = \frac{\langle 0 | J_\pi | \pi(p') \rangle \langle \pi(p') | J_0 | B(p_B) \rangle}{m_\pi^2 - p'^2} = \frac{(n_+ p')^2 \xi_\pi(n_+ p') f_\pi}{m_\pi^2 - p'^2}. \quad (5)$$

On the SCET side of the sum rule, the tree-level result (see Fig. 1(a)), for the correlation function involves one hard-collinear quark propagator, which reads

$$S_F^{\text{hc}} = \frac{i}{n_- p' - \omega + i\eta} \frac{\not{n}_-}{2}, \quad (6)$$

where  $\omega = n_- k$ , and  $k^\mu$  is the momentum of the soft light quark that will end up as the spectator quark in the  $B$  meson. The propagator is always off shell and always induces light-like separations as long as  $|n_- p'| \sim \Lambda$ . This leads to matrix elements of operators that are formulated only in terms of soft fields, which are separated along the light cone and thus define light-cone distribution amplitudes for the  $B$  meson in HQET [9]. The final result already has the form of a dispersion integral in the variable  $n_- p'$  :

$$\Pi(n_- p') = f_B m_B \int_0^\infty d\omega \frac{\phi_-^B(\omega)}{\omega - n_- p' - i\eta}, \quad (7)$$

where the  $B$  light-cone distribution amplitude enters through

$$\langle 0 | \bar{q}(x_-) Y_s \gamma_5 \frac{\not{n}_+ \not{n}_-}{2} Y_s^+ h_v(0) | B(p_B) \rangle = i f_B m_B \int d\omega' e^{-i\omega' \frac{n_+ x}{2}} \phi_-^B(\omega'). \quad (8)$$

The result for the SCET side of the sum rule shows that the considered correlation function in the (unphysical) Euclidean region factorizes into a perturbatively calculable hard-collinear kernel and a soft light-cone wave function for the  $B$  meson, where the convolution variable  $\omega$  represents the light-cone momentum of the spectator quark in the  $B$  meson.

Finally,  $\Pi(n_- p') \Big|_{\text{cont.}}$  on the hadronic side can be written again according to a dispersion relation. Moreover, assuming global quark-hadron duality, we identify the spectral density with its perturbative expression above some threshold  $\omega_s$ . The Borel transform with respect to the variable  $n_- p'$  introduces the Borel parameter  $\omega_M$  and reads in this case  $\hat{B}(\omega_M)[1/(\omega - n_- p')] = 1/\omega_M e^{-\omega/\omega_M}$ . As usual, the physical role of the Borel parameter is to enhance the contribution of the hadronic-resonance region, where the virtualities of internal propagators have to be smaller than the hard-collinear scale. Equating the two representations of the correlator, using global quark-hadron duality (and neglecting the pion mass), we obtain the final sum rule for the soft form factor at tree level (see also [10]):

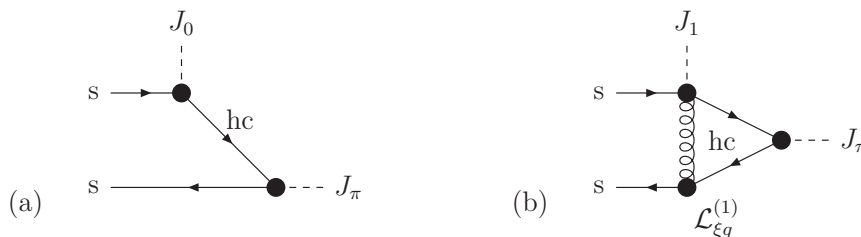
$$\xi_\pi(n_+ p') = \frac{f_B m_B}{f_\pi(n_+ p')} \int_0^{\omega_s} d\omega e^{-\omega/\omega_M} \phi_-^B(\omega). \quad (9)$$

The result for  $\xi_\pi$  has the correct scaling with  $\Lambda/m_B$  as obtained from the conventional sum rules or from the power counting in SCET. The resulting estimate for the soft form factor at maximum recoil,  $0.27_{-0.11}^{+0.09}$ , is compatible with other determinations. The uncertainty is dominated by the variation of the sum-rule parameters and by the product  $f_B \phi_-^B$ .

In [1], we have also reproduced the result for the factorizable form-factor contribution from hard-collinear spectator scattering [7]. For this purpose we have to consider the correlation function

$$\Pi_1(p') = i \int d^4 x e^{ip'x} \langle 0 | T [J_\pi(x) J_1(0)] | B(p_B) \rangle, \quad J_1 \equiv \bar{\xi}_{\text{hc}} g A_{\text{hc}}^\perp h_v, \quad (10)$$

where we used the light-cone gauge. The leading contribution is given by the diagram in Fig. 1(b) which involves the insertion of one interaction vertex from the order- $\lambda$  soft-collinear Lagrangian. We derived the remarkable feature of the SCET-sum-rule approach to the  $B \rightarrow \pi$  form factor that the ratio of factorizable and non-factorizable contributions is independent of the  $B$ -meson wave function to first approximation, and about 6%, which is in line with the power counting used in



**Figure 1:** (a) Leading contribution to the correlation function for the non-factorizable SCET current  $J_0$ . (b) Leading contribution to the sum rule for the factorizable SCET current  $J_1$ .

QCD factorization [2], but contradicts the assumptions of the so-called pQCD approach [11]. The non-trivial issues related to the factorization of the correlation function arise beyond tree level. We have shown that the  $\mathcal{O}(\alpha_s)$  short-distance radiative corrections from hard-collinear loops preserve factorization, i.e. the encountered IR divergences correspond to the renormalization of the  $B$ -meson distribution amplitude(s) in HQET (three-particle Fock states have been neglected so far). At this point our approach differs from the conventional sum rules formulated in QCD, where the separation of three different scales ( $m_b, \mu_{hc}, \Lambda$ ) is usually not attempted. However, in a recent article [14] Lee proposes to formulate sum rules within the effective field theory framework of SCET for the conventional set-up where the pion is represented by a collinear light-cone distribution amplitude and the  $B$  meson is interpolated by a current in HQET. We stress that in that case the radiative corrections would involve soft *and* collinear momentum regions; according to the general discussion in [8, 12, 13], these are not expected to factorize. Indeed the occurrence of end-point singularities that spoil factorization has been observed in [14]. Our choice of interpolating the pion instead avoids the problem of collinear end-point divergences. In this way the SCET-based sum rules generalize the ideas of QCD factorization to the non-factorizable parts of exclusive decay amplitudes, and allow for a systematic and consistent expansion in terms of  $1/m_b$  and  $\alpha_s$ , where the hadronic information is encoded in light-cone distribution amplitudes for the  $B$  meson and non-perturbative sum-rule parameters. At present, because of the limited information on the  $B$ -meson wave function, the theoretical uncertainties are larger than those found in QCD light-cone sum rules.

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