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# Status Report on the Experiments NA48/1 and NA48/2

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### 1 Introduction

Present document contains the latest results of the NA48/1 experiment on neutral kaon and hyperon decays, and the first results obtained in the NA48/2 experiment on charged kaon decays. These experiments were carried out in the different beams specially designed and constructed in the K12 beam line for each of those experiments [1], [2]. Both experiments used the NA48 apparatus [3] updated and optimized accordingly.

The NA48/1 experiment aimed to study of rare decays has accumulated data in 2002 run in the corresponding high intensity  $K_s$  and hyperon beams, which allowed to reach a sensitivity of  $10^{-9}$  in kaon decays.

It has been already reported on the first observation of  $K_s \to \pi^0 e^+ e^-$  [5] and  $K_s \to \pi^0 \mu^+ \mu^-$  [6] decays and the measurement of corresponding branching ratios. This allowed to estimate an indirect CP-violating contribution to the decays  $K_L \to \pi^0 l^+ l^-$  providing input to the determination of the imaginary part of the element  $V_{td}$  of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Many other decays of  $K_s$  and hyperons are observed and studied.

The main goal of the NA48/2 experiment is to search for direct CP-violation in  $K^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$  and  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$  decays, in order to achieve a precision in corresponding asymmetry parameter measurements at the level, which is at least one order of magnitude better than existing data. Another goal is to measure a basic parameter of the  $\chi PT$  theory, a pion scattering length, via precision study of  $K_{e4}$  decays. It has been shown as well that the pion scattering length could be measured in  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$  decays, thanks to the first observation of so called "cusp structure" in an invariant mass spectrum of two neutral pion subsystem.

In addition, a wide possibility to study of charged kaon leptonic, semileptonic and rare decays is provided by accumulated record statistics with well measured kinematic parameters.

In total around  $4 \cdot 10^9$  charged kaon decays have been accumulated in two runs of data taking in 2003 and 2004.

Now the collaboration is at the stage of intensive analysis of data accumulated in both experiments, NA48/1 and NA48/2.

# 2 Rare Decays in the NA48/1 Experiment

#### 2.1 $K_S \rightarrow \pi e \nu$

The time evolution of kaons decaying into  $\pi e \nu$  can be described by:

$$N(\pi^{\pm}e^{\mp}\nu)(t_s) \propto (e^{-\frac{t_s}{T_L}} + |\eta|^2 e^{-t_s}) \mp 2D |\eta| e^{\frac{1+T_L}{2T_L}t} \cos(\Delta Mt + \phi)$$
 (1)

where  $T_L = \Gamma_S/\Gamma_L = 578.0 \pm 5.0$ ,  $\Delta M = (m_L - m_S)/\Gamma_S = 0.4730 \pm 0.0015$ ,  $|\eta|$  and  $\phi$  are respectively a module and a phase of a ratio of corresponding decay amplitudes for  $K_S$  and  $K_L$ , and D is the dilution. Neglecting CP violation, the Branching ratio is related to  $\eta$  via:  $BR(K_S \to \pi e \nu) = |\eta|^2 \frac{\Gamma_L^{TOT}}{\Gamma_S^{TOT}} BR(K_L \to \pi e \nu)$ . About 234000 decays have been selected, of which about 221000 are  $K_L$  decays and 13000 are  $K_S$  decays, with a kaon visible energy between 70 and 130 GeV and an electron energy above 20 GeV (see Fig. 1). Using the Ginsberg recipe for the radiative corrections, a value for  $\eta$  is extracted by fitting the exponential time evolution of the  $K \to \pi e \nu$  decays and a value for the branching ratio is therefore obtained:

$$BR(K_S \to \pi e \nu) = (6.8 \pm 0.2_{stat} \pm 0.2_{syst})10^{-4}$$
.

This value is in agreement with the PDG value of  $(6.9 \pm 0.4)10^{-4}$  [4] and improves the uncertainty.

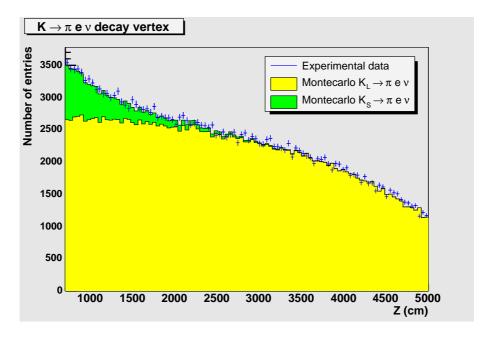


Fig. 1: Both, experimental and simulated event distributions over Z-coordinate of reconstructed decay vertexes

**2.2** 
$$K_S \to \pi^+ \pi^- \pi^0$$

The  $K_S \to \pi^+\pi^-\pi^0$  decay amplitude is dominated by two angular momentum components (l=0,1 where l is the angular momentum of the neutral pion with respect to the system of the charged pions). The CP conserving transition to the l=1 state can be measured through its interference with the dominant  $K_L \to \pi^+\pi^-\pi^0$  decay. Neglecting CP violation, the decay amplitude into 3 pions can be parametrized in terms of the Dalitz variables X and Y, where X is a measure of the difference of the energies of the two charged pions in the kaon's rest frame while Y is a measure of the energy of the neutral pion in the same frame. In NA48  $K_S$  and  $K_L$  are produced in equal amounts at the target, and the interference of the l=1 component of the  $K_S$  decays with the l=0 components of the dominant  $K_L$  decays can be described by the complex parameter  $\lambda$  in the distribution:

$$V(t) \approx \frac{2D(Re(\lambda)cos(\Delta mt) - Im(\lambda)sin(\Delta mt))e^{-t/2(1/\tau_S + 1/\tau_L)}}{e^{-t/\tau_L}},$$

where  $\Delta m$  is the mass difference between  $K_S$  and  $K_L$ ,  $\tau_S$  and  $\tau_L$  are the respective lifetimes, and D is the dilution. From 19 million  $\pi^+\pi^-\pi^0$  decays, an analysis of the  $3\pi$  Dalitz plot has been performed to measure the contribution of CP-conserving  $K_S$  decays to the total  $K^0 \to \pi^+\pi^-\pi^0$  rate. The result on the parameter  $\lambda$  of the CP conserving  $K_S \to \pi^+\pi^-\pi^0$  amplitude is:

$$Re(\lambda) = +0.038 \pm 0.008_{stat} \pm 0.006_{syst}$$

$$Im(\lambda) = -0.013 \pm 0.005_{stat} \pm 0.004_{syst}$$

From the real part of  $\lambda$ , the branching ratio for the  $K_S$  can be obtained:

$$BR(K_S \to \pi^+ \pi^- \pi^0) = (4.7^{+2.2}_{-1.7}(stat)^{+1.7}_{-1.5}(syst))10^{-7}.$$

The result agrees with  $\chi$ PT and with two other measurements with comparable uncertainties. Fig. 2 shows the distribution V(t) derived from the data, and the curve obtained using the fitted values of  $Re(\lambda)$  and  $Im(\lambda)$ ; a clear contribution from  $K_S$  is visible (the fit was performed in bins of energy, and not by fitting this histogram directly).

# 2.3 $\Xi^0$ and $\overline{\Xi^0}$ semileptonic decays

The study of hadron beta decays gives important information on the interplay between the weak interaction and hadron structure determined by the strong interaction. In this context the  $\Xi^0$  beta decays allow testing both of SU(3) symmetry, via its strong analogy with the well-known neutron beta decay, and of the quark mixing model, via extraction of  $V_{us}$  (the sine of the Cabibbo

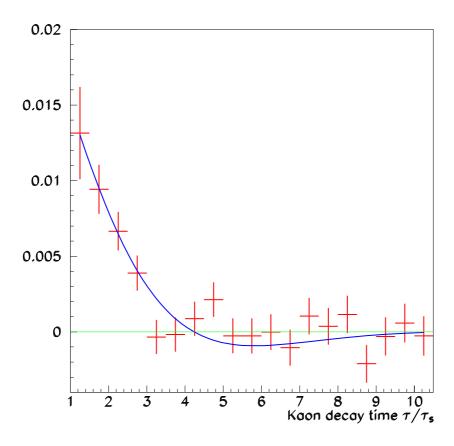


Fig. 2: Three pion reconstructed decays distributed over time of flight in the  $K_S$  lifetime units (crosses), and their approximation by the best fit result (a line).

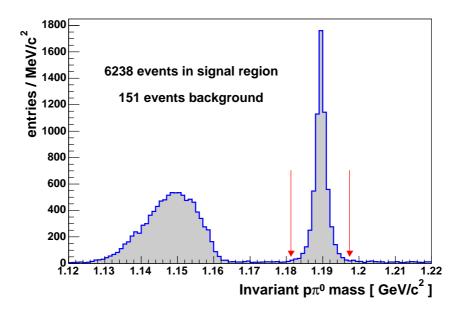


Fig. 3: Distribution of selected events with electron over  $p\pi^0$  invariant mass.

angle). NA48 collected a sample of 6316  $\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$  events (with the  $\Sigma^+$  decaying into  $p\pi^0$ ) with energy between 70 and 220 GeV and decay vertex between 5 m and 50 m from the final collimator (see Fig. 3).

The background is about 2%. A value for the branching ratio has been extracted:

$$BR(\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e) = (2.51 \pm 0.03_{stat} \pm 0.09_{syst})10^{-4},$$

where the systematic error is dominated by the trigger efficiency determination, the geometrical acceptance and the form factors. Including the dependence of form factors from the transfer momentum and radiative corrections, the following value for  $V_{us}$  can be extracted from the branching ratio measurement:

$$V_{us} = 0.208 \pm 0.006 ^{+0.030}_{-0.025} g_1/f_1.$$

The systematic error is largely dominated by the error on the ratio of the form factors  $g_1/f_1$  taken from PDG; the contribution to  $V_{us}$  error coming from the uncertainty on the branching ratio measurement is now comparable with the contribution coming from the uncertainty on the  $\Xi^0$  lifetime.

In the same data and fiducial volume, a sample of 515  $\overline{\Xi^0} \to \overline{\Sigma^+} e^+ \nu_e$  were also collected with a larger rate of background (see Fig. 4), from which a well consistent value for the branching ratio has been extracted:

$$BR(\overline{\Xi^0} \to \overline{\Sigma}^+ e^+ \nu_e) = (2.57 \pm 0.12(stat) ^{+0.10}_{-0.09}(syst)) \cdot 10^{-4}.$$

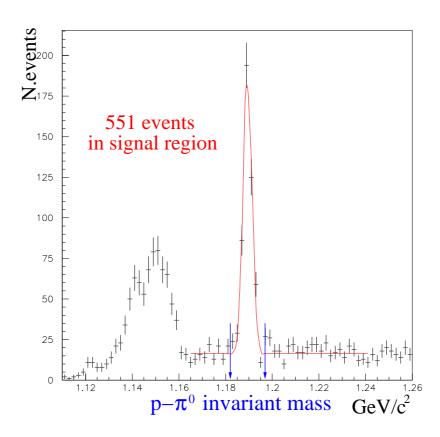


Fig. 4: Distribution of selected events with positron over  $\overline{p}\pi^0$  invariant mass.

Similar criteria were applied to select  $\Xi^0 \to \Sigma^+ \mu^- \bar{\nu_{\mu}}$  decays, except for the requirement applied on the muon detector. A sample of 102 events was selected, with a background of  $32 \pm 3.0$  (see Fig. 5), from which the measurement of the branching ratio was extracted:

$$BR(\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_{\mu}) = (2.2 \pm 0.3_{stat} \pm 0.2_{syst}) 10^{-6}.$$

This is the largest sample collected so far of muonic decays. The measured branching ratio is to be compared with the published value from KTeV of:  $BR(\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_{\mu}) = (4.7^{+2.0}_{-1.4}(stat) \pm 0.8(syst))10^{-6}$ .

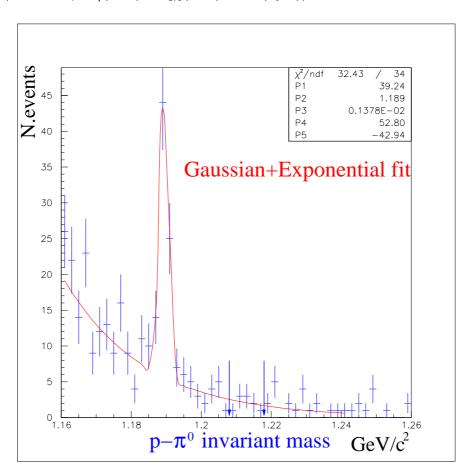


Fig. 5: Distribution of selected events with negative muon over  $p\pi^0$  invariant mass.

In this fiducial volume a large number of  $\Xi^0 \to \Lambda \pi^0$  and  $\overline{\Xi^0} \to \overline{\Lambda} \pi^0$  were recorded:  $N(\Xi^0) = (2.422 \pm 0.003_{stat} \pm 0.018_{syst}) \cdot 10^9$ ,  $N(\overline{\Xi^0}) = (2.254 \pm 0.012_{stat} \pm 0.017_{syst}) \cdot 10^8$  and mainly used for normalization purposes of the various branching ratio measurements. The ratio of their fluxes has been measured as a function of the energy (see Fig. 6), while an integrated value

is given by:

$$N(\overline{\Xi^0})/N(\Xi^0) = (9.31 \pm 0.05_{stat} \pm 0.04_{syst}) \cdot 10^{-2}.$$

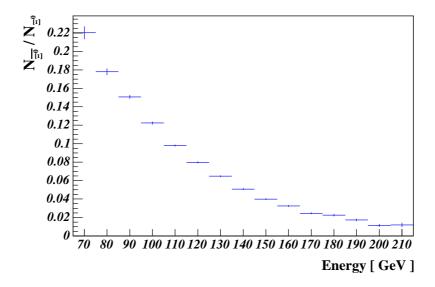


Fig. 6: The ratio of  $\overline{\Xi^0}$  and  $\Xi^0$  fluxes measured in different energy bins.

# 3 Search for Direct CP Violation in the NA48/2 Experiment

A study of the direct CP-violation is an important issue in order to search for physics beyond the Standard Model (SM). The direct CP-violation has been indicated for the first time by the NA31 experiment [7] and then undoubtedly observed in the experiments KTEV [8] and NA48 [9] via precise measurement of  $\varepsilon'/\varepsilon$  parameter in neutral kaon decays. A most promising observable of the direct CP-violation in charged kaon decays would be a charged asymmetry  $A_g = (g^+ - g^-)/(g^+ + g^-)$  of the slopes  $g^+$  and  $g^-$  describing, respectively, the linear dependence of the  $K^+$  and  $K^-$  three pion decay probabilities on the u kinematic variable of the Dalitz plots. The u variable is related to the energy  $(E^*_{odd})$  of the odd pion (the pion having the distinguished sign) in the kaon center of mass system.

The experiment NA48/2 is designed to measure the asymmetry parameters  $A_g^c$  and  $A_g^0$  in both three pion decay modes  $K^{\pm} \to \pi^+\pi^-\pi^{\pm}$  and  $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ , respectively, at the precision level of  $\sim 2 \cdot 10^{-4}$ , which is limited by statistics rather than systematics. An observation of the asymmetries at this level would be considered as an indication of a new physics evidence [10], [11].

## 3.1 Measurement of $A_g^c$

A preliminary result of the  $A_g^c$  measurement using more than 1.6 billion of  $K^\pm \to \pi^+\pi^-\pi^\pm$  decays accumulated in 2003 has been reported at CERN seminar and various conferences in 2005 [12]. This measurement is based on a comparison the u-distributions of  $K^+$  and  $K^-$  decays. The corresponding ratio of the u spectra  $N_{K^+}(u)/N_{K^-}(u)$  is proportional to  $(1+\Delta gu)$ , and  $A_g^c = \Delta g/2g$  is obtained with a linear fit of this ratio. In order to equalize acceptances for  $K^+$  and  $K^-$  decays the magnet polarities of both the beam line (achromats, focusing quadrupoles, muon sweeping, etc.) and spectrometer dipole were reversed during data taking respectively on a weekly basis and every day. Thus, a week cycle represents a super-sample which is treated in the analysis as an independent data unit. Four such super-samples (SS0–3) have been collected in the period of 2003.

Each super-sample contains four  $K^+ \to \pi^+ \pi^- \pi^+$  and four  $K^- \to \pi^+ \pi^- \pi^-$  samples with different combination of achromat and spectrometer magnet polarities. The ratio R(u) is obtained as a product of four  $N_{K^+}(u)/N_{K^-}(u)$  ratios:

$$R(u) = R_{US}R_{UJ}R_{DS}R_{DJ} \approx \overline{R}(1 + 4\Delta gu), \tag{2}$$

where the index U represents a configuration in which  $K^+$  beam runs through the upper beam path in the achromats, and the index D represents the configuration in which  $K^+$  beam runs through the lower path. The index S represents the spectrometer magnet polarity corresponding to decay products having the same charge as the corresponding beam deflected to the right (toward the Saleve mountain), and the index J represents the magnet polarity corresponding to the decay products deflected to the left (toward the Jura mountain). A linear fit to Eq. 2 results in two parameters, normalization  $\overline{R}$  and  $\Delta g$  directly related to  $A_g^c$ . This method allows a three-fold cancellation of systematic biases:

- beam line local biases cancel between  $K^+$  and  $K^-$  samples in which the beam follows the same path;
- local detector biases cancel between  $K^+$  and  $K^-$  samples deflected toward the same parts of the detector;
- as a consequence of simultaneous beams, global time-variable biases cancel between  $K^+$  and  $K^-$  samples.

Further reduction of systematic biases especially due to the presence of stray permanent magnetic fields (the Earth's field, vacuum tank magnetization, etc.) is obtained by keeping azimuthal symmetry in the acceptance. Using this method the result remains sensitive only to time variation of asymmetries in experimental conditions which have a characteristic time smaller than corresponding field-alternation period. Moreover, the measurement does not need a Monte Carlo simulation of the acceptance. Nevertheless, a detailed GEANT-based Monte Carlo simulation has been developed as a tool for systematic studies. This Monte Carlo simulation includes full geometry and material description, simulation of time variable local drift chamber inefficiencies and time variation of the beam geometry and drift chamber alignment.

Invariant mass spectra of the reconstructed  $K^+ \to \pi^+\pi^+\pi^-$  and  $K^- \to \pi^+\pi^-\pi^-$  (figs.7 (a) and (b)), which are the dominant three track decays, indicate a negligible background level. The invariant mass resolution is 1.7 MeV/c<sup>2</sup>. The tails of the invariant mass distribution are dominated by events in which one of the three pion decayed and the spectrometer reconstructed the track of the resulting muon. The corresponding systematic uncertainty limit for  $\Delta g$  was obtained by Monte Carlo simulation.

A fine calibration of the spectrometer magnetic field has been done by adjusting the global momentum scale for each day sample separately equalizing the corresponding average value of reconstructed  $K^+$  and  $K^-$  masses to the PDG value. Small time variation of the drift chambers (DCH) alignment ( $\sim 4\mu m/day$ ) was detected in 2003, and corresponding time-dependent corrections have been introduced with relative precision of about  $10^{-5}$ .

To reduce the acceptance asymmetry for positive and negative kaon decays caused by the different relative beam positions and to minimize any time

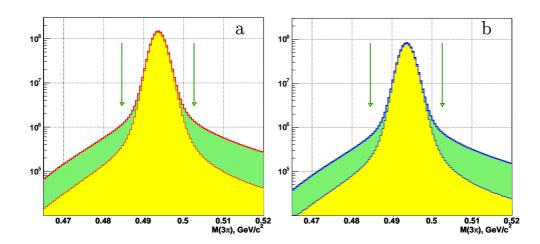


Fig. 7: Spectra of reconstructed  $K^+$  and  $K^-$  masses. Green areas correspond to events with a muon identified by muon veto system.

variation of such differences, the selection criteria include radial cuts around the measured beam positions. These positions are defined by the centers of gravity (COG) for positive and negative kaon decay products separately and were evaluated in short time periods. In addition to the time variation of the beam position, also the dependence of the beam position on the kaon momentum ( $\sim \pm 1mm$  in horizontal and  $\sim \pm 10mm$  in vertical directions) are taken into account. In this way the  $K^+$  and  $K^-$  acceptances cancel entirely, and no Monte Carlo calculations are needed to correct for their difference.

The trigger has been considered as a potential source of systematic bias as well. Inefficiencies of different trigger components are studied and measured using control samples of low bias trigger collected along with the main triggers. the corresponding measured uncertainty is fully dominated by the statistics of the control samples.

No essential systematics have been found which require the corrections. The following limits on the systematic uncertainties on  $\Delta g$  (in units  $10^{-4}$ ) were obtained, part of those having statistical nature:

- acceptance and beam geometry  $\leq 0.5$ ;
- spectrometer alignment  $\leq 0.1$ ;
- magnetic field  $\leq 0.1$ ;
- pion decay  $\leq 0.4$ ;
- calculation of u and fitting  $\leq 0.5$ ;
- pile-up  $\leq 0.3$ .

In total an upper limit of  $0.9 \cdot 10^{-4}$  is considered as the systematic uncertainty. In addition the same level of trigger uncertainty has been estimated for two levels of trigger.

The obtained preliminary result is based on three independent analyses, which agree within uncorrelated uncertainties. The result is calculated separately for each of the four super-samples and then combined taking into account the correlated systematic uncertainties. All four measurements are compatible with each other with  $\chi^2/ndf = 3.2/3$  (fig. 8 (a)). As a systematic check, the quadruple ratio (Eq. 2) rearranged so that instead of four  $K^+/K^-$  ratios, four ratios of samples in which even pions deflected to the left in the spectrometer magnet, are presented in fig. 8 (b). In this case, the physical quantity  $\Delta g$  cancels, and the result is expected to be equal to zero in the absence of any residual left-right detector asymmetries. Similarly, the asymmetry of the two beam parts are presented in fig. 8 (c). The corresponding asymmetries, which cancel at first order, show that the cancellation of systematic biases due to residual time variable imperfections in the apparatus is at the level of few  $10^{-4}$  and therefore second order effects are negligible. Moreover, the comparison with Monte Carlo simulations shows a good agreement.

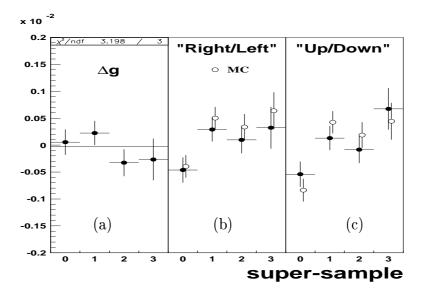


Fig. 8: Asymmetries in super-samples: (a) – the result  $\Delta g$ , (b) – right-left asymmetry, (c) – achromat up-down asymmetry. Blank dots correspond to MC simulation.

The obtained preliminary result from all four super-samples is

$$\Delta g = (-0.2 \pm 1.0_{stat} \pm 0.9_{stat(trig)} \pm 0.9_{syst}) \cdot 10^{-4},$$

which could be converted to the asymmetry using the PDG value of the slope:

$$A_g^c = (0.5 \pm 2.4_{stat} \pm 2.1_{stat(trig)} \pm 2.1_{syst}) \cdot 10^{-4} = (0.5 \pm 3.8) \cdot 10^{-4}.$$

This result is compatible with no CP violation and with SM predictions [11], [13], [14], [15], but has more than an order of magnitude better precision than similar previous measurements [16], [17]. Further improvements are expected in future.

# 3.2 First result of the $A_q^0$ measurement

 $K^\pm \to \pi^\pm \pi^0 \pi^0$  decays are selected by a first-level trigger requiring a signal in a scintillator hodoscope of a charged track in coincidence with energy deposit in the LKr calorimeter consistent with at least two photons, and excluding  $K^\pm \to \pi^\pm \pi^0$  decays implementing a proper limit on corresponding invariant mass rough calculation by a second level trigger. To reconstruct these decays both Lkr calorimeter and magnetic spectrometer were used. The calorimeter allows to identify two neutral pions by proper pairing of four reconstructed gammas requiring that both of two gamma pairs have the closest vertexes calculated under assumption of  $\pi^0 \to \gamma \gamma$  decays. The corresponding spectrum of product invariant mass selected under proper geometry and kinematic restrictions shows clear signals of kaon with a negligible background level, less than 1 % (fig.9(a) and (b)). This background is mainly due to the wrong photon pairing ( $\sim 0.25\%$ ) and  $\pi^\pm \to \mu^\pm$  decays. In total,  $31 \cdot 10^6~K^+$  decays and  $17 \cdot 10^6~K^-$  decays were selected for the asymmetry measurement.

For calculation of u only information from the LKr was used:  $u=(M_{00}^2-M_K^2/3)/m_{\pi^+}^2-1/3-2\cdot(m_{\pi^0}^2/m_{\pi^+}^2)/3$ , where  $M_{00}$  is the invariant mass of two  $\pi^0$ 's. To measure the asymmetry parameter  $A_g^0$  similar technique has been implemented as for the measurement of  $A_g^c$ . Only one charged track (instead of three) selection has been corrected to balance acceptance symmetry. For each super-sample the quadruple ratio like in "charged" mode was evaluated. The fit function, however, is different due to the non negligible value of  $g_0=0.638$ :

$$R(u) = N \cdot (1 + \Delta g_0 \cdot u / (1 + g_0 \cdot u)).$$

This nonlinear fit is more sensitive to the events with small u, where is a good resolution due to the chosen way of u-calculation. The fits for each of three data-sets, shown on fig.10, have good  $\chi^2$ . The value of  $\Delta g_0$  for each super-sample is plotted on fig.11a with the statistical error only, for experiments data and simulated ones (MC). On fig.11(b) and (c) left-right and up-down asymmetries of the apparatus are shown, which are canceled in the quadruple ratio. The result shows good stability in kaon momentum bins (fig.12(a)), in Z-coordinate of the reconstructed vertexes (fig.12(b)), in

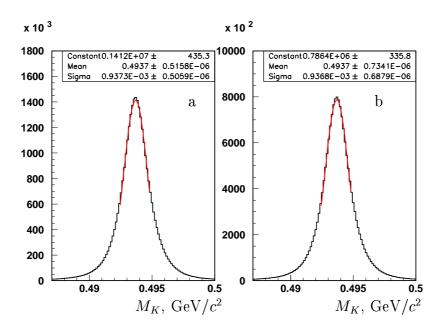


Fig. 9: Mass of reconstructed  $K^+$  (a) and  $K^-$  (b)

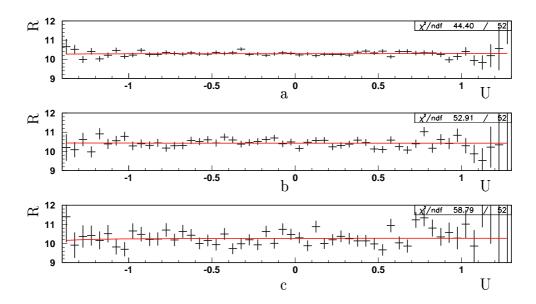


Fig. 10: Quality of the fit of the quaduple ratio for super-sample SS0 (a), SS12 combined (b) and SS3 (c)

|v|-variable (Dalitz plot parameter complementary to u) (fig.12(c)) and in time (fig.12(d)). The weighted average of  $\Delta g_0$  for three super-samples gives the value  $\Delta g_0 = (2.2 \pm 2.2_{stat}) \cdot 10^{-4}$ . The main systematic sources and their

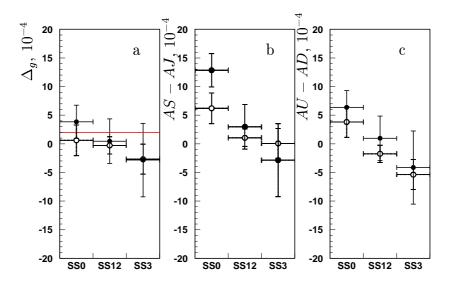


Fig. 11:  $\Delta_g$  (a), left-right (b) and up-down (c) asymmetry of the aparatus for each supersample; black dots - experimental data, open circles - MC.

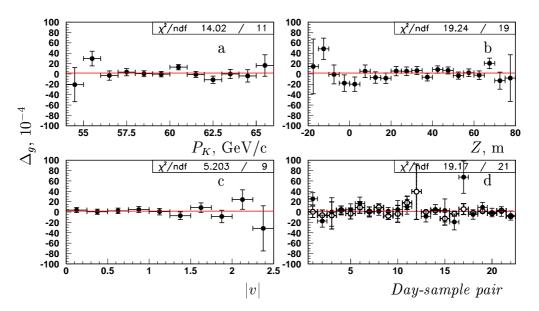


Fig. 12:  $\Delta_g$  as a function of kaon momentum (a), Z coordinate of the vertex (b), |v| (c) and day-sample pair (c) (black dots - experimental data; open circles - MC)

uncertainty estimations (in units  $10^{-4}$ ) are the following:

- acceptance and beam geometry  $\leq 1.1$ ;
- alignment  $\leq 0.1$ ;
- momentum scale  $\leq 0.1$ ;
- u calculation and fitting method  $\leq 0.4$ ;
- accidentals  $\leq 1.0$ ;
- trigger L1  $\leq 1.5$ :
- trigger  $L2 \le 0.4$ :
- Lkr nonlinearity corrections  $\leq 0.4$ .

The biggest part of the systematics error  $(1.5 \cdot 10^{-4})$  comes from L1 trigger inefficiency and has a purely statistical nature due to the limited statistics for the control sample to measure this inefficiency. The L2 trigger was simulated properly with large statistics. and the corresponding systematic uncertainty of  $0.4 \cdot 10^{-4}$  has been achieved. The two second biggest sources of systematics are acceptance and beam geometry effects  $(1.1 \cdot 10^{-4})$ , and possible accidentals  $(1.0 \cdot 10^{-4})$ , but there is still room for improvement.

In addition, the external  $0.3 \cdot 10^{-4}$  systematics error comes from the uncertainty of the parameter  $g_0$  used in the fit function. Taking into account all above mentioned uncertainties the preliminary result for asymmetry measurement in the "neutral" mode is:

$$A_g^0 = (1.7 \pm 1.7_{stat} \pm 1.7_{syst} \pm 0.2_{ext}) \cdot 10^{-4}.$$

This result do not indicate to a CP-violation at the precision level of  $3 \cdot 10^{-4}$ , which is one order of magnitude better than other experiments [18], [19]. More precise result will be obtained using larger statistics accumulated in 2004.

# 4 First Observation of a Cusp-like Structure in the $\pi^0\pi^0$ Invariant Mass Distribution from $K^\pm \to \pi^\pm\pi^0\pi^0$ Decay and Determination of the $\pi\pi$ Scattering Lengths

The invariant mass of  $\pi^0\pi^0$  subsystem  $(M_{00})$  in  $K^\pm \to \pi^\pm\pi^0\pi^0$  decays has been studied to search for possible anomaly at the threshold region:  $M_{00} = 2 \cdot m_{\pi^\pm}$ . Fig.13(a) shows the  $M_{00}$  distribution, without any acceptance correction, for  $\sim 23 \cdot 10^6$  events selected from SS1-3 similarly to the ones for the asymmetry measurement (see section 3.2). Fig.13(b) shows the same distribution in the region close to the threshold  $M^2(\pi^0\pi^0) = 4m_{\pi^\pm}^2 = 0.0779(MeV/c^2)^2$ . The slope change at the threshold is clearly visible. Such an anomaly has not been observed in previous experiments.

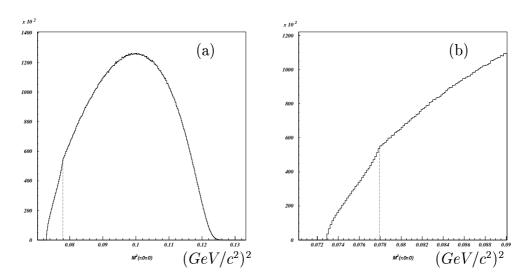


Fig. 13: The  $M^2(\pi^0\pi^0)$  distribution for the subsystem of the reconstructed  $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$  decays: (a) – in the full kinematic region; (b) – in the region around the threshold; the value corresponding to  $4m_{\pi^{\pm}}^2$  is indicated by vertical line

The sudden change of slope observed in this plot suggests the presence of a threshold "cusp" effect from the decay  $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$  contributing to the  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$  amplitude through the charge exchange reaction  $\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}$ . The presence of a cusp at  $M_{00}^{2} = (2 \cdot m_{\pi^{\pm}})^{2}$  in  $\pi^{0}\pi^{0}$  elastic scattering due to the effect of virtual  $\pi^{+}\pi^{-}$  loops has been discussed first by Meissner et al. [20]. A possibility to measure  $\pi - \pi$  scattering length in tau-decays has been predicted even earlier by L. B. Okun [21]. For the case of  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$  decay Cabibbo [22] has proposed a simple re-scattering

model describing the decay amplitude as the sum of two terms representing "unperturbed amplitude" and contribution from the  $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$  decay amplitude through  $\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}$  charge exchange. The latter contribution is proportional to  $a_{x}$ , the S-wave  $\pi^{+}\pi^{-}$  charge exchange scattering length. In the limit of exact isospin symmetry  $a_{x} = (a_{0} - a_{2})/3$ , where  $a_{0}$  and  $a_{2}$  are the  $\pi\pi$  scattering lengths in the I=0 and I=2 states, respectively.

In this simple re-scattering model [22] there is only one additional parameter  $a_x m_+$ . A fit to the  $M_{00}^2$  distribution in the region  $0.074 \leq M_{00}^2 \leq$  $0.097(GeV/c^2)^2$  using  $a_x m^+$ , as a free parameter gives  $\chi^2 = 420.1$  for 148 degree of freedom. The quality of this fit is illustrated in Fig. 14(a) which displays the difference between data and best fit normalized to the data value  $(\Delta)$  as a function of  $M_{00}$ . Recently Cabibbo and Isidori [23] have proposed a more complete model, that takes into account all re-scattering processes in two-loop approximation. In the limit of exact isospin symmetry these five S-wave scattering lengths can be expressed as linear combinations of  $a_0$  and  $a_2$ . An isospin breaking corrections have been applied to this model, which are expressed as a function of one parameter  $\epsilon = (m_+^2 - m_0^2)/m_+^2 = 0.065$ [24]. The fit by this model (fig. 14(b)) allows to extract four parameters:  $(a_0 - a_2)m_+$ ,  $a_2m_+$ ,  $g_0$  (the slope),  $h^+$  (a quadratic term in the Dalitz plot distribution over u variable) and one parameter for normalization. It leads to  $\chi^2 = 154.8$  for 146 degrees of freedom. The better fit shown in Fig.14(c)  $(\chi^2 = 149.1/145d.f.)$  is obtained by adding to the model a term describing the expected formation of a  $\pi^+\pi^-$  atom ("pionium") decaying into  $\pi^0\pi^0$  at  $M_{00}=2m_{+}$ . The fit value for the rate of  $K^{\pm}\to\pi^{\pm}+pionium$  decays is  $(1.61\pm0.66)\cdot10^{-5}$ , which is in agreement with the predicted value  $\sim0.8\cdot10^{-5}$ 

The model used do not include radiative corrections, which are particularly important near  $M_{00} = 2m_+$ , and contribute to the formation of the pionium. For this reason a group of seven consecutive bins centered at  $M_{00} = 2m_+$  have been excluded from the final fit shown in fig. 14(d)  $(\chi^2/d.f. = 145.5/139)$ .

Table 1 lists the best fit values of the parameters, as obtained by two independent analysis which use different event selection criteria and different Monte Carlo simulations to take into account acceptance and resolution effects.

Study of systematics shows that no corrections are needed. However, the corresponding uncertainty were taken into account. For measured  $(a_0 - a_2)m_+$  parameter the major ones are the following:

- acceptance calculation  $\pm 0.001$ ;
- trigger efficiency  $\pm 0.001$ ;
- fit interval  $\pm 0.0025$ ;

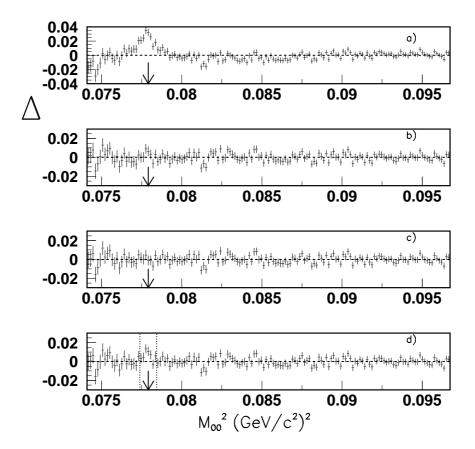


Fig. 14: Difference between data and best fits, normalized to the data value, in different versions of the model.

- $\pi \gamma$  minimum distance at the LKr  $\pm 0.002$ ;
- LKr resolution and nonlinearity corrections  $\pm 0.001$ .

In total, the systematic uncertainty is  $\pm 0.004$ . The corresponding uncertainty for  $a_2m_+$  has been obtained as  $\pm 0.014$ . Taking into account these estimations the following results are obtained:

$$(a_0 - a_2)m_+ = 0.268 \pm 0.010(stat) \pm 0.004(syst)$$

and

$$a_2 m_+ = -0.041 \pm 0.021(stat) \pm 0.014(syst).$$

A critical parameter of the models [22] and [23] is the ratio  $R=A_{++-}/A_{+00}$  between weak amplitudes of  $K^\pm\to\pi^\pm\pi^+\pi^-$  and  $K^\pm\to\pi^\pm\pi^0\pi^0$  decays. The

Table 1:

Parameter	Analysis A	Analysis B	Average value
$(a_0 - a_2)m_+$	$0.269 \pm 0.010$	$0.268 \pm 0.010$	$0.268 \pm 0.010$
$a_2m_+$	$-0.053 \pm 0.018$	$-0.030 \pm 0.022$	$-0.041 \pm 0.021$
$g_0$	$0.643 \pm 0.004$	$0.647 \pm 0.004$	$0.645 \pm 0.004$
$h^+$	$-0.055 \pm 0.010$	$-0.039 \pm 0.011$	$-0.047 \pm 0.012$

corresponding "external" uncertainty should be complemented by an additional theoretical error of  $\pm 5\%$  as the result of neglecting higher-order terms and radiative corrections.

It has been shown that analyticity and chiral symmetry provide a constraint between  $a_0$  and  $a_2$  [26], which implementing to the fit gives:

$$a_0 \cdot m_+ = 0.220 \pm 0.006(stat) \pm 0.004(syst) \pm 0.003(R) \pm 0.011(theor.)$$

and

$$(a_0 - a_2)m_+ = 0.264 \pm 0.006(stat) \pm 0.004(syst) \pm 0.003(R) \pm 0.013(theor.).$$

These values are in good agreement with the results on  $(a_0 - a_2) \cdot m_+$  obtained in the E865 BNL [27] and Dirac [28] experiments.

### 5 Rare Decays of Charged Kaons

The large statistics of charged kaon decays, collected by NA48/2 in the run periods of 2003 and 2004 allow for a wide variety of rare kaon decays to be studied. In most cases the statistics exceed those of previous experiments by one or even several orders of magnitude.

One of the major goals of NA48/2 is the precise extraction of the  $\pi\pi$  scattering length  $a_0^0$  using charged  $K_{e4}$  decays. However, also several other decays as  $K^{\pm} \to \pi^{\pm}\pi^0\gamma$ ,  $K^{\pm} \to \pi^{\pm} + \gamma\gamma$ ,  $K^{\pm} \to \pi^{\pm}l^+l^-$  are of strong theoretical interest.

For most of the rare kaon decay channels analyses are currently performed. It is planned to have several preliminary results for the winter conferences, using the data of super-samples SS1 – 3 accumulated in 2003.

### 5.1 $K_{e4}$ and $K_{u4}$ Decays

The strong interest in  $K_{e4}$  decays  $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu(\bar{\nu})$  is motivated by the possibility to precisely measure the S-wave  $\pi\pi$  scattering lengths  $a_{0}^{0}$  and  $a_{2}^{0}$  by a fit to the observed decay topologies. These scattering lengths are accurately predicted in the framework of Chiral Perturbation Theory, their measurement therefore is a crucial test of the validity of the theory.

The analysis of the  $K_{e4}$  decay on the super samples SS1-3 is far advanced. Around 350 000 events have been reconstructed with an almost negligible background contribution of less than 0.5% (Fig. 15). From this sample alone, the statistical precision on the determination of  $a_0^0$  is expected to be about  $\pm 0.01$ , similar to the result extracted from the analysis on the cusp-effect in  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ , and more precise than all previous measurements. A preliminary result is expected to be ready for the winter conferences 2006.

The final result will be based on the whole 2003 and 2004 data sample with roughly one million reconstructed  $K_{e4}$  decays. Together with the  $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$  cusp-analysis it is expected that the most precise determination of  $a_{0}^{0}$  will be obtained.

The comparison of analysis results obtained from  $K^+$  and  $K^-$  decays, recorded simulataneously but analyzed independently, will provide powerful tests of various symmetries (CPT, T, CP) as mentionned almost 40 years ago in [29]. The  $K_{e4}^-$  decay sample is the first one available in the world and amounts to about 55% of the  $K_{e4}^+$  sample due to the beam composition.

amounts to about 55% of the  $K_{e4}^+$  sample due to the beam composition. In addition,  $K_{e4}^{00}$  ( $K^{\pm} \to \pi^0 \pi^0 e^{\pm} \nu(\bar{\nu})$ ) and  $K_{\mu4}$  decays are investigated. While the statistics in these channels does not reach the level of  $K_{e4}^{+-}$  decays, their investigation serves as an important cross-check for the main analysis. For both  $K_{e4}^{00}$  and  $K_{\mu4}$  decays about one hundred times more events have been observed using SS1-3 super-samples, than in the best previous measurements.

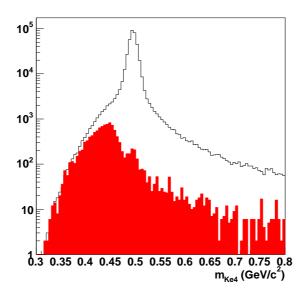


Fig. 15: Invariant mass of reconstructed  $K_{e4}$  ( $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu(\bar{\nu})$ ) decays in SS1-3. The background, determined from wrong-sign data events, is indicated in dark.

# 5.2 $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \gamma$ Decays

The decay  $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$  is dominated by inner bremsstrahlung. However, with enough statistics it is possible to access also the direct emission amplitude and the interference term between both. In particular the study of interference term is of high interest, as it might show a rather large CP violating asymmetry between  $K^{+}$  and  $K^{-}$  decays.

Previous experiments were only able to observe the direct emission amplitude. With the NA48/2 data sample, which for SS1–3 is already more than 10 times larger than all previously collected statistics, an observation of the interference term and investigation of possible CP violation in this decay are expected.

The selected events are practically background-free (Fig. 16). Also for this channel the data analysis is far advanced.

# 5.3 $K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$ Decays

Similar to the corresponding decay of the long-lived  $K_L$  meson into  $\pi^0 \gamma \gamma$ , which was measured previously by the NA48 collaboration, the decay  $K^{\pm} \to \pi^{\pm} \gamma \gamma$  obtains an importance from the insight into Chiral Perturbation Theory. In both these decays the  $\mathcal{O}(p^2)$  contributions are zero and  $\mathcal{O}(p^4)$  can

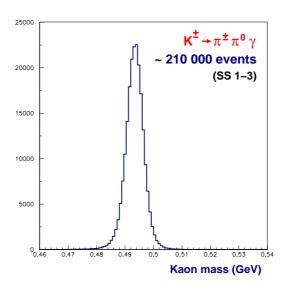


Fig. 16: Invariant mass of reconstructed  $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$  events in SS1-3.

be calculated, in the case of the charged decay depending of only one free parameter. Therefore, by measuring the decay  $K^{\pm} \to \pi^{\pm} \gamma \gamma$  the amount of  $\mathcal{O}(p^6)$  contributions can be determined.

During the data-taking, a tight trigger requirement on the invariant  $\gamma\gamma$  mass was set in order to efficiently suppress  $K^{\pm} \to \pi^{\pm}\pi^{0}$  decays. Nevertheless almost 2000  $K^{\pm} \to \pi^{\pm}\gamma\gamma$  events in SS1–3 have been found and reconstructed, which are almost a factor of 100 more than in previous experiments. Currently the form factor measurement is underway.

To access also the region of low  $\gamma\gamma$  invariant mass,  $K^{\pm} \to \pi^{\pm}\gamma e^{+}e^{-}$  decays are started to investigate, where one of the photons undergoes an internal conversion. This decay, which has so far not yet been observed, is not trigger-suppressed, however its statistics will be much smaller compared to our  $K^{\pm} \to \pi^{\pm}\gamma\gamma$  sample.

### 5.4 $K^{\pm} \rightarrow \pi^{\pm} l^+ l^-$ Decays

The decays  $K^{\pm} \to \pi^{\pm} e^{+} e^{-}$  and  $K^{\pm} \to \pi^{\pm} \mu^{+} \mu^{-}$  are flavour changing neutral currents, which proceed via penguin diagrams. In the 2003 data set, several thousand very clean events for both decays (Fig. 17) are seen. At the moment, form factor measurements are performed.

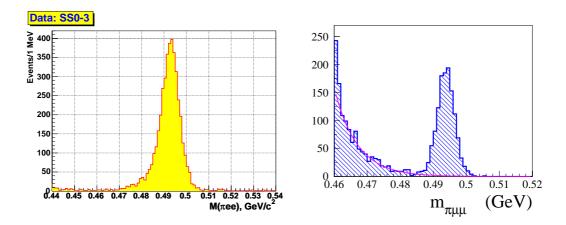


Fig. 17: Invariant mass of reconstructed  $K^{\pm} \to \pi^{\pm} e^{+} e^{-}$  (left) and  $K^{\pm} \to \pi^{\pm} \mu^{+} \mu^{-}$  (right) events.

### 6 Leptonic and Semileptonic Decays

New measurements of charged kaon semileptonic decays have been done using the data accumulated in 2003:  $K^{\pm} \to \pi^0 \mu^{\pm} \nu$  ( $K_{\mu 3}$ ) and  $K^{\pm} \to \pi^0 e^{\pm} \nu$  ( $K_{e3}$ ). Most previous measurements are quite old and based on relatively small data samples. New precise measurements of semileptonic decays are based on larger data samples, and the latest theoretical input – crucial to issues, like radiative corrections.

The main goals of this study are to extract: (1) the individual decay widths because they will allow one to determine the  $V_{us}$  parameter in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. A better value of  $V_{us}$  is needed in order to test the unitary condition of the CKM matrix using its first row; (2) the decay width ratio  $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$  which is a unique function of the slope parameters of the form factors. This ratio provides a consistency check between the measurements made from the form factors and the partial decay widths. This check is important at the moment as large differences in the slope parameters are observed between neutral and charged kaons. In addition, it will allow one to make further test of lepton universality.

The ratios were measured:

$$R_{Ke3/K2\pi} = \frac{\Gamma(K^{\pm} \to \pi^{0} e^{\pm} \nu)}{\Gamma(K^{\pm} \to \pi^{0} \pi^{\pm})}, \qquad R_{K\mu3/K2\pi} = \frac{\Gamma(K^{\pm} \to \pi^{0} \mu^{\pm} \nu)}{\Gamma(K^{\pm} \to \pi^{0} \pi^{\pm})}, \quad (3)$$

and

$$R_{K\mu3/Ke3} = \frac{\Gamma(K^{\pm} \to \pi^0 \mu^{\pm} \nu)}{\Gamma(K^{\pm} \to \pi^0 e^{\pm} \nu)}.$$
 (4)

as they should be insensitive to experimental systematic errors. In both the numerator and denominator, there are a charged track and at least two gamma's that are consistent with a  $\pi^0$ , thus leading to a partial cancellation in the acceptance uncertainties. Contributions from internal bremsstrahlung are included for all three decay modes. The reconstruction methods are the same in all cases, and the main differences are due to particle identification and acceptances.

It is obtained that

$$R_{Ke3/K2\pi} = 0.2505 \pm 0.0009 (stat) \pm 0.0012 (sys)$$

and

$$R_{K\mu3/K2\pi} = 0.1646 \pm 0.0006 (stat) \pm 0.0011 (sys).$$

Both values imply higher branching fractions for the corresponding semileptonic decays than the current world average given by the PDG. As a consequence, the  $V_{us}$  value extracted from these measurements will increase accordingly. In addition, more precise value is obtained for

$$R_{K\mu3/Ke3} = 0.657 \pm 0.003 (stat) \pm 0.003 (sys),$$

which is the most precise measurement and still not contradicts the theoretical prediction.

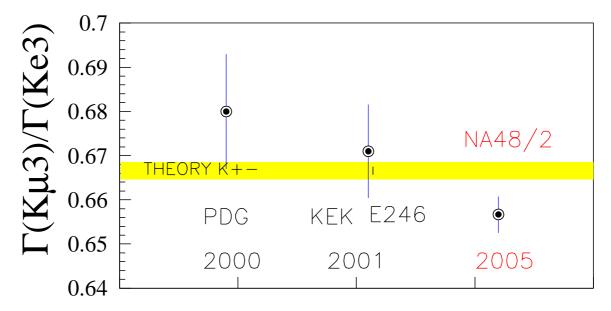


Fig. 18:  $R_{K\mu3/Ke3}$  results compared to the predictions assuming lepton universality.

New result on a ratio of leptonic branching ratios  $\Gamma(K_{e2})/\Gamma(K_{\mu 2})$  was obtained on the base of  $\sim 4000K^{\pm} \rightarrow e^{\pm}\nu$  and  $\sim 3.9 \cdot 10^{6} K^{\pm} \rightarrow \mu^{\pm}\nu$  decays

reconstructed from 2003 data, and one using a second data sample of similar size taken in 2004. These results, shown at the summer conferences are:

$$\Gamma(K_{e2})/\Gamma(K_{\mu 2}) = 2.416 \pm 0.043(stat) \pm 0.024(syst)$$

from the 2003 data analysis, and

$$\Gamma(K_{e2})/\Gamma(K_{\mu 2}) = 2.453 \pm 0.046(stat) \pm 0.026(syst)$$

obtained using the 2004 data.

Therefore, the accuracy of this ratio measurement is improved with respect to the PDG value,  $2.45 \pm 0.11$ . The standard model predicts  $2.472 \pm 0.001$ . The purpose of this measurement is to provide further test of lepton universality.

In addition, about 6 millions of  $K_{e3}$  events were recorded in 2004 to perform a precision measurement of the form factor and its corresponding radiative decay.

### 7 Plans on Data Processing

After the completion of reprocessing for both 2003 and 2004 data, a major effort was invested in 2005 by the collaboration in assessing the data quality and providing fine tuning of all detectors, including the beam settings.

High statistics Monte-Carlo simulations were run for the charged and neutral 3-pion analyses, shared between CERN and collaborating Institutes. All resulting files have been transferred to CERN and archived on CASTOR.

Continuous centralization of NA48 disk-storage resources, done in close interaction and help of IT, led to optimization for the needs of the ongoing analyses.

Even if we do not have any indication of such a need at the moment, we would like to keep the potential for another reprocessing of 2003 and 2004 data, in case of unexpected effects showing up during analysis. This would require an extra 30TB storage space on CASTOR. The final Monte-Carlo simulations will also require 5TB of CASTOR space.

Thanks to IT support, we have been able to setup a Virtual Organization for NA48 on the GRID (called NA48VO). It will be used primarily for data transfer between CERN and collaborating Institutes, exporting some data subsets and importing Monte-Carlo simulations produced in homelabs.

All these tasks require relevant resources to be allocated at CERN/IT and in the collaborating Institutions.

### 8 Summary

The experiments NA48/1 and NA48/2 have been prepared and carried out in 2002 - 2004. The accumulated record statistics provide possibility to high precision study of  $K_S$ , hyperons and charged kaons at a sensitivity not reachable in other experiments. This would allow to get new information on CP-violating processes, precise measurement of  $\chi$ PT theory parameters, check some parameters of the CKM quark mixing matrix, and search for new physics.

The data accumulated in 2004 run of NA48/2 experiment has been just reprocessed and corresponding analysis has started. New results are expected in 2006.

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