Ref.TH1612-CERN
CERN/D-Ph.II/PHYS 73-2

MULTIPLICITY CORRELATIONS BETWEEN NEUTRAL AND CHARGED PIONS FROM CHARGED MULTIPLICITY DISTRIBUTIONS

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A B S T R A C T

It is shown that for several simple possibilities of distributing charge between pions (the so-called π , ε and ρ models) the ($\pi^0 \pi^{ch}$) multiplicity correlations can be expressed directly in terms of the charged multiplicity distributions. Each of the models can then be tested experimentally, irrespectively of the unknown total multiplicity distribution.

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*) On leave of absence from the Institute for Theoretical Physics, Warsaw University, Warsaw.

Ref.TH.1612-CERN
24 January 1973

1. Multiplicity distributions belong to the most fundamental features of the high energy collisions. The relevant experimental information is contained at present in the charge multiplicity distributions and in the dependence of the average number of neutral pions, $E(n_0|n_-)$, on the number of simultaneously produced charged particles. The latter quantity expresses some features of the multiplicity correlations between neutral and charged pions. It has been measured in πp and pp collisions at several energies. Those data seem to indicate that the shape of $E(n_0|n_-)$ as a function of n_- is changing with energy from a flat behaviour at 12 and 19 GeV/c to an approximately linear rise at NAL and ISR energies [1].

The question of multiplicity correlations between neutral and charged particles has also been considered in the framework of theoretical models [for recent discussions see Refs. 2) and 3)]. Those models involve two basic ingredients: the shape of the <u>total</u> multiplicity distribution $P(N)$ and the way in which charge is distributed among particles. Partition into different charge states is often specified by a statistical assumption. Denoting the statistical factor by $p(n_+,n_-,n_0|N)$, the probability $p(n_+,n_-,n_0)$ for finding pions with given charges is the following

$$p(n_+,n_-,n_0) = P(N)\cdot p(n_+,n_-,n_0|N)$$

For given assumptions about charge partition, the $(\pi^0\pi^{ch})$ correlations have been studied using explicitly definite forms for the total multiplicity distribution $P(N)$. It is an obvious requirement that the distribution $P(N)$ used in such calculation should give, with the same assumptions about charge partition, correct charged multiplicity distributions. Only then, comparison of model calculations for $(\pi^0\pi^{ch})$ multiplicity correlations with experimental data provides a test of assumptions on charge partition. Since a correct guess for $P(N)$ is equivalent to a good model for particle production, this program encounters some difficulties.

We want to point out in this note that such an approach to the problem of $(\pi^0\pi^{ch})$ multiplicity correlations is indeed unnecessarily complicated, at least for several simple possibilities of charge partition. In the cases which we consider, charge is distributed among pions according to statistical assumptions about:

a) charge of single pions, b) pion pairs with isospin $I=0$, and c) pion pairs with isospin $I=1$ (they are respectively named π, ε and ρ models). For the above three models the $(\pi^0\pi^{ch})$ multiplicity correlations can be expressed directly in terms of the charged multiplicity distributions. Therefore, the assumptions about charge partition can be experimentally tested [comparing calculated and measured values of e.g., $E(n_0|n_-)$], irrespectively of the essentially unknown total multiplicity distribution.

Finally, we remark that our results could be also obtained by solving first the general problem of reconstructing the total multiplicity distribution, given the assumptions about charge partition, from the measured charged multiplicity distribution. In the three cases considered one can find the solutions of such a problem in a closed form and in general it is solvable numerically. In the following we concentrate, however, on the $(\pi^0\pi^{ch})$ multiplicity correlations and do not discuss further the total multiplicity distributions. Discussion of the latter requires more precise data for the charged multiplicity distributions than available at present. We first present a few formulae for the models which we have studied, we then discuss their predictions and compare them with the existing data.

2. As pointed out earlier the different models are specified by an explicit form of $p(n_+,n_-,n_0|N)$: in the π model a multinomial law in the numbers of particles in the three charged states is used (in order to test the importance of the charge conservation we discuss two versions of the π model: one with charge conserved only in the average and the other with the exact charge conservation). In the ε model a binomial law in the numbers of neutral pairs $\pi^+\pi^-$ and $\pi^0\pi^0$ is used. The ρ model is defined by a multinomial law in the numbers of pairs $\pi^-\pi^0$, $\pi^+\pi^0$ and $\pi^+\pi^-$. We can obtain general formulae for this model when charge conservation is valid in the average sense only. The over-all charge, when explicitly conserved, is assumed equal to zero, but this limitation does not affect the gross features obtained. We assume also that the average number of neutral pions is equal to half the average number of charged pions.

In subsequent formulae N represents the total multiplicity, n_+, n_- and n_0 are respectively the number of π^+'s, π^-'s and π^0's produced.

i) The π model with charge conserved only in the average is specified by the distribution

$$p(n_-, n_+, n_0) = P(N) \frac{N!}{n_-!\, n_+!\, n_0!} \left(\frac{1-q}{2}\right)^{n_-} \left(\frac{1-q}{2}\right)^{n_+} q^{n_0}$$

where $q = 1/3$ as follows from the requirement $E(n_+) = E(n_-) = E(n_0) = 1/3E(N)$.

The expectation values of factorial moments of the π^0 distribution for a given number n_c of charged particles are then the following *)**)

$$E\left[n_0(n_0-1)\dots(n_0-k)\,\middle|\,n_c\right] =$$

$$= \left(\frac{q}{1-q}\right)^{k+1} \frac{(n_c+k+1)!}{n_c!} \frac{p_c(n_c+k+1)}{p_c(n_c)}$$

In particular

$$E(n_0|n_c) = \tfrac{1}{2}(n_c+1)\frac{p_c(n_c+1)}{p_c(n_c)} \tag{1}$$

and

$$E\left[n_0(n_0-1)|n_c\right] = \tfrac{1}{4}(n_c+1)(n_c+2)\frac{p_c(n_c+2)}{p_c(n_c)} \tag{2}$$

ii) The π model with charge conserved is defined as follows:

$$p(n_0, n_- = k_- + \varepsilon) = \frac{P(N = n_0 + 2n_-)}{R(N,\varepsilon,\varepsilon)} \cdot \frac{N!}{n_-!\, n_-!\, n_0!} \left(\frac{1-q}{2}\right)^{n_-} \left(\frac{1-q}{2}\right)^{n_-} q^{n_0}$$

k_- integer, $0 \le \varepsilon < 1$.

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*) $p_c(n)$ and $p_-(n)$ will be used throughout as the distribution of charged and negative particles, respectively.

**) Notice that $E[n_0(n_0-1)\dots(n_0-k)|n_c]$ where $n_c = n_+ + n_-$ is different from $E[n_0(n_0-1)\dots(n_0-k)|n_-]$ as charge is not conserved. The latter quantities can be easily calculated as well.

[We define the probability $p(n_0,n_-)$ for integer and non-integer values of n_- as in the following we use it for $\varepsilon = \frac{1}{2}$.] The normalization function R is defined by

$$R(N,\alpha,\beta) = q^N \sum_{i=0}^{M} \frac{N!}{(i+\alpha)!\,(i+\beta)!\,(N-\alpha-\beta-2i)!} \left(\frac{1-q}{2q}\right)^{2i+\alpha+\beta} \qquad \alpha,\beta \geq 0 .$$

where M is the integer value of $[(N-\alpha-\beta)/2]$.

In the present case we have the following expression for the mean value of negative particles:

$$E(n_-) = \left(\frac{1-q}{2}\right) \sum_{N=2}^{\infty} N\,P(N)\, \frac{R(N-1,1,0)}{R(N,0,0)} .$$

We notice that the summation starts with $N=2$ and also the ratio of the two R functions is not equal to 1 but converges towards 1 as N increases. For these two reasons, $E(n_-)$ is not equal to $1/3E(N)$ for $q=1/3$. Nevertheless, for mean multiplicity $E(N)$ not too small [say, $E(N) \geq 5$] $q = 1/3$ is a good enough approximation for our needs [the value of q for which $E(n_-) = 1/3E(N)$ depends on the shape of $P(N)$].

The average values of factorial moments of the π^0 distribution for a given n_- are now the following:

$$E[n_0(n_0-1)\dots(n_0-k)\,|\,n_-] = \left(\frac{2q}{1-q}\right)^{k+1} \left[\frac{(n_- + \frac{1+k}{2})!}{n_-!}\right]^2 \frac{p_-(n_- + \frac{1+k}{2})}{p_-(n_-)} . \qquad (3)$$

For even values of k the unphysical charged multiplicities $p_-(n_-+(1+k)/2)$ appear in the above formula. They have, however, been defined in terms of the physical total multiplicity distribution and can be obtained by interpolation between physical charged multiplicities. Furthermore, for k even, formula (3) is not exact (although quite close to be) since we have used the approximation

$$\rho(N) = \frac{R(N,0,0)}{R(N,\frac{1}{2},\frac{1}{2})} \simeq 1 \quad (\rho(1)=.79\,,\ \rho(2)=1.18\,,\ \rho(3)=.97\dots)$$

[$E[n_o(n_o-1)\ldots(n_o-k)|n_-]$ depends only on those ratios $\varrho^{(N)}$ for which $N \geq 2n_- + \delta n_{-o}$] The formula (3) gives in particular *)

$$E(n_o|n_-) \simeq (n_- + 3/4)\,\frac{p_-(n_- + 1/2)}{p_-(n_-)} \tag{4}$$

and

$$E[n_o(n_o-1)|n_-] = (n_- + 1)^2\,\frac{p_-(n_- + 1)}{p_-(n_-)}\;. \tag{5}$$

iii) The ε model is defined by the distribution

$$p(n_-, n_o = 2k_o) = P(N = 2n_- + 2k_o)\,\frac{(n_- + k_o)!}{n_-!\;k_o!}\,(1-q)^{n_-}\,q^{k_o}$$

Here $q = 1/3$ according to the same argument as for case i). Then it follows that

$$E\left[\frac{n_o}{2}\left(\frac{n_o}{2} - 1\right)\ldots\left(\frac{n_o}{2} - k\right)\Big|\,n_-\right] =$$

$$= \left(\frac{q}{1-q}\right)^{k+1}\frac{(n_- + k + 1)!}{n_-!}\quad\frac{p_-(n_- + k + 1)}{p_-(n_-)}\;. \tag{6}$$

In particular, we obtain :

$$E(n_o|n_-) = (n_- + 1)\,\frac{p_-(n_- + 1)}{p_-(n_-)} \tag{7}$$

$$E[n_o(n_o-1)|n_-] = (n_- + 1)(n_- + 2)\frac{p_-(n_- + 2)}{p_-(n_-)} + E(n_o|n_-) \tag{8}$$

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*) For the ratio of factorials with half integer difference, one may use the approximation $[(x+\frac{1}{2})!/x!]^2 = x+3/4$ with small relative error (4.5% at $x = 0$, 1% at $x = 1$, 0.1% at $x = 5$).

iv) Denoting the number of $\pi^-\pi^0$, $\pi^+\pi^0$ and $\pi^+\pi^-$ pairs by n_1, n_2 and n_3, respectively, the ρ model (with charge conserved in average only) is specified by the distribution:

$$p(n_1,n_2,n_3) = P(2N = 2(n_1+n_2+n_3)) \frac{N!}{n_1!\,n_2!\,n_3!}\left(\frac{1-q}{2}\right)^{n_1}\left(\frac{1-q}{2}\right)^{n_2} q^{n_3}$$

where again $q = 1/3$. The mean values of factorial moments of the π^0 distribution for a fixed number n_c of charged particles are now the following:

$$E\left[n_0(n_0-1)\ldots(n_0-k)\,|\,n_c\right] =$$

$$= \sum_{t=0}^{n_c-1}\left(\frac{-q}{1-q}\right)^t (n_c-t)\,\frac{p_c(n_c-t)}{p_c(n_c)} \sum_{r=0}^{\alpha}(-1)^r \frac{(n_c+t-2r-1)!}{(n_c-k-2r-1)!\,r!\,(t-r)!} \qquad (9)$$

where $\alpha = \min(t, [(n_c-k-1)/2])$.

In particular we have

$$E(n_0\,|\,n_c) = n_c + \sum_{i=1}^{n_c-1}\left(\frac{-2q}{1-q}\right)^i (n_c-i)\,\frac{p_c(n_c-i)}{p_c(n_c)} \qquad (10)$$

and

$$E\left[n_0(n_0-1)\,|\,n_c\right] = n_c(n_c-1) + \sum_{i=1}^{n_c-1}\left(\frac{-2q}{1-q}\right)^i (n_c-i)\,\frac{p_c(n_c-i)}{p_c(n_c)}(i+1)\left(n_c-\frac{i+2}{2}\right). \qquad (11)$$

3. The predictions of formulae (4), (7) and (10) for $E(n_0|n_-)$ in pp collisions at 19 and 205 GeV/c obtained using measured charged multiplicity distributions (ε model) or their interpolation by means of the Czyzewski-Rybicki formula [4] (π and ρ models) are shown in Figs. 1 and 2. We notice that the predictions of all three models are close to each other and in agreement with experimental values *). In particular, all

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*) In the ε model our predictions at 19 GeV/c, although correct in shape, give too low values for $E(n_0|n_-)$. The ρ model predictions are shown only for 205 GeV/c. At 19 GeV/c they are in agreement with the data for $n_- = 1,2$, whereas for $n_- \geq 3$ they depend strongly on the uncertain high multiplicity tail of the Czyzewski-Rybicki formula [expressions (9) and (10) are alternating series).

three models reproduce correctly the variation with incoming energy of the shape of $E(n_o|n_-)$ as a function of n_-. This feature is related to the broadening of the charged multiplicity distributions when incoming energy is increasing [see Ref. 3) for qualitative arguments). When more precise results for charged multiplicity distributions are available it will be also interesting to study in detail the drop of $E(n_o|n_-)$ for large values of n_-.

We see that measurements of $E(n_o|n_-)$ are not a sensitive tool to discriminate between the π, ε and ρ models.

The role of the constraint following from charge conservation can be tested for the π model by comparing the predictions of formula (4) with those of formula (1). Both expressions give almost identical results indicating that in this case charge conservation is not an important constraint for the $(\pi^o\pi^{ch})$ correlations once charge is already conserved in the average sense. For the ρ model, we have reached the same conclusion numerically. We have checked that, for several plausible forms of the total multiplicity distribution $P(N)$, imposing exact charge conservation does not affect appreciably the results obtained when charge is conserved only in the average.

Searching for a clearer test of the three models discussed one is led to study higher multiplicity moments. The behaviour of the second moment or of the function $f_2 = E(n_o(n_o-1)|n_-) - E^2(n_o|n_-)$ is at present the most interesting one as it can be fairly soon accessible experimentally. In Fig. 3, we show the π, ε and ρ model predictions for the function f_2 following from the Czyzewski-Rybicki formula for charged multiplicity distributions *). We notice that the function f_2 is more sensitive to the assumed pairing properties of pions (in particular it is negative for the ρ model 3)). However, more precise measurements of topological cross-sections are necessary to get meaningful predictions for f_2.

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*) In principle, experimental values of topological cross-sections can be directly used in that calculation. Their experimental errors are, however, at present still too large to give a meaningful prediction for f_2. Therefore, we illustrate the behaviour of f_2 using the Czyzewski-Rybicki interpolation.

We conclude that, with the present experimental precision, all three models reproduce the data obtained for $E(n_0|n_-)$. The measurements of f_2 would allow a better discrimination between the models.

We are grateful to Drs. M. Jacob, P. Pirilä, Ph. Salin and M. Szeptycka for many valuable discussions.

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FIGURE CAPTIONS

Figure 1 :

Plotted is the average number of neutral pions, $E(n_o|n_-)$ per inelastic pp collision at 19 GeV/c as a function of the number of negative particles n_-. The data are compared with the predictions of the π and ε models.

Figure 2 :

Plotted is the average number of neutral pions, $E(n_o|n_-)$ per inelastic pp collision at 205 GeV/c as a function of the number of negative particles n_- The data are compared with the predictions of the π ε and ρ models.

Figure 3 :

Plotted are the predictions of the π , ε and ρ models for the correlation function between two neutral pions, $f_2 = E[n_o(n_o-1)|n_-] - E^2(n_o|n_-)$, as a function of n_- for pp collision at 205 GeV/c. The predictions of the ρ model are shown only for $n_- \leq 3$ as for higher values of n_- they depend strongly on the details of the high multiplicity tail of the Czyzewski-Rybicki formula.

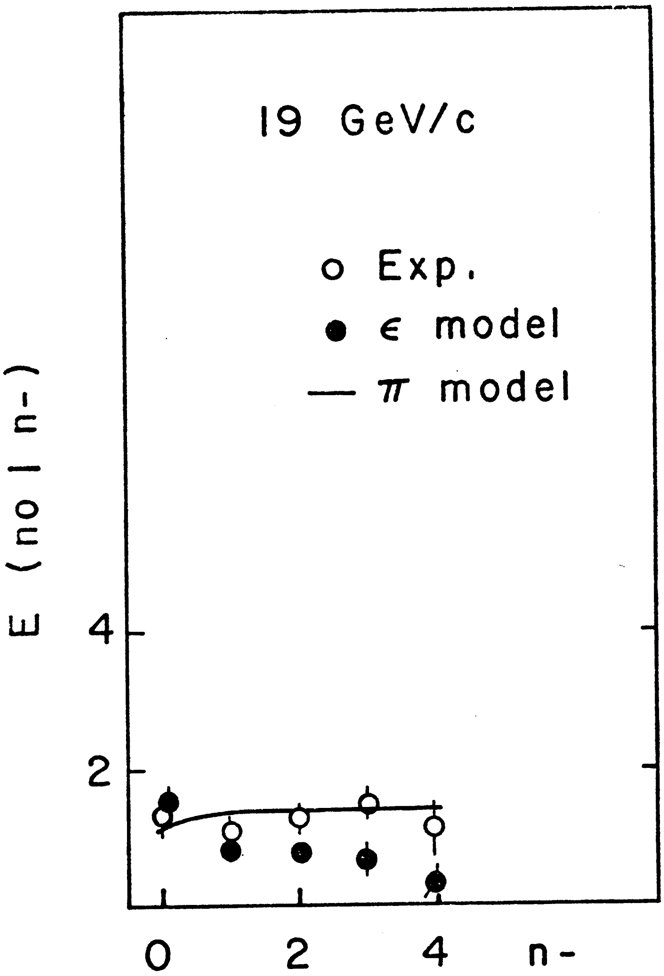

fig. 1

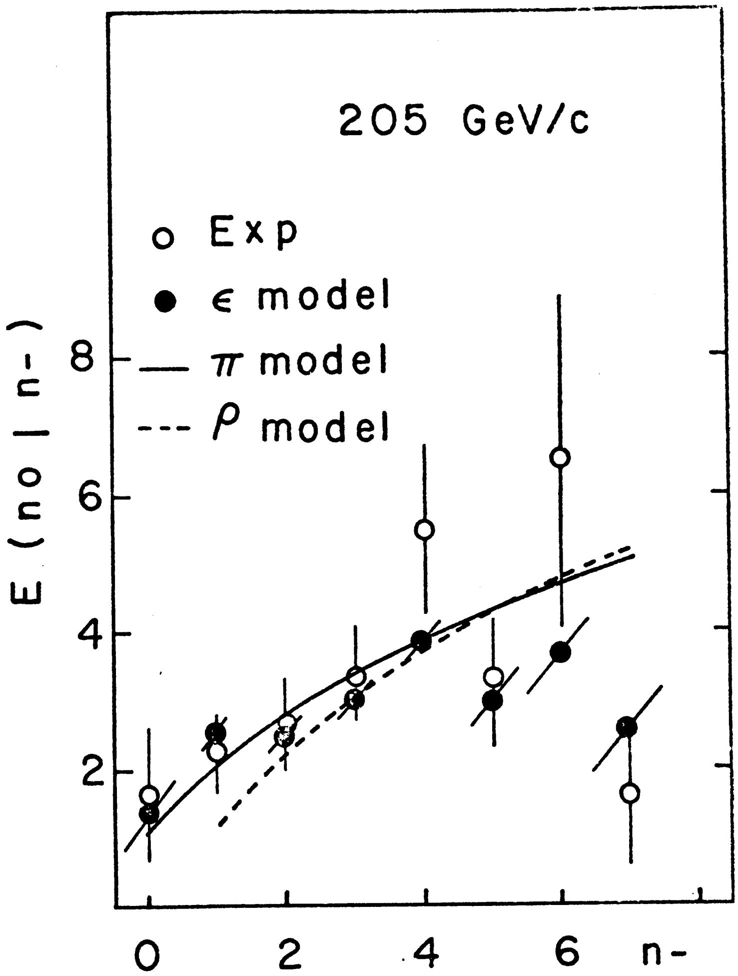

fig. 2

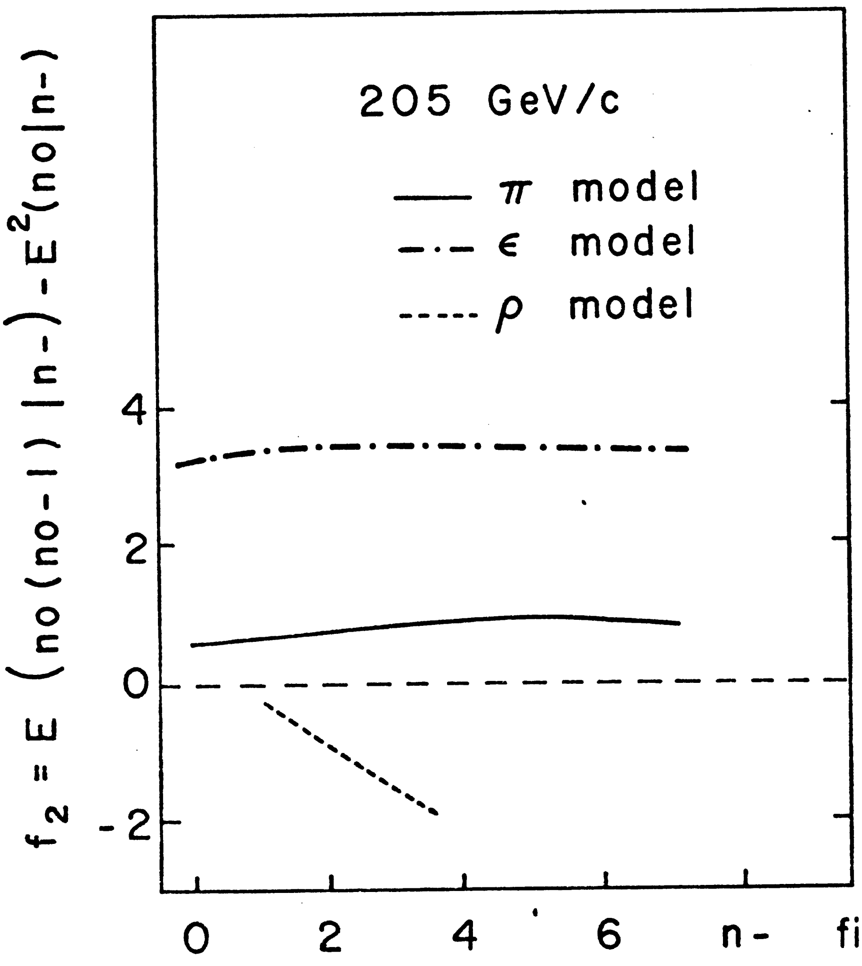

fig. 3