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DEEP INELASTIC SCATTERING, PARTONS AND STRONG INTERACTION IDEAS

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A B S T R A C T

The quark parton model as applied to deep inelastic lepton-hadron scattering is used as a laboratory model for the study of ideas encountered in strong interaction physics. First, two-component duality constraints are imposed on single particle inclusive deep inelastic distributions. In the target fragmentation region this involves a modification of the definition of the E functions introduced by Feynman. A duality relation connecting neutrino and electroproduction single particle inclusive distributions is obtained. For $\omega \to 1$ and $\omega \to \infty$ we predict in the target fragmentation region: an excess of π^+ over π^- production for proton targets and an excess of π^- over π^+ for neutron targets. For deuteron targets we find an excess of π^- over π^+ for $\omega \to 1$.

In our laboratory model we study the analogue of the problem of approach to Feynman scaling encountered in hadronic reactions and the related problem of exoticity criteria. We find that for large ω, in both fragmentation regions, Feynman scaling is reached from above. If ab is exotic, single particle inclusive distributions for any type of observed hadron are flat as a function of ω. For the production of fast hadrons in the fragmentation regions, even when ab is not exotic, cross-sections are flat when $ab\bar{c}$ is exotic.

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1. INTRODUCTION

The deep inelastic interactions of leptons with hadrons serve as a useful tool for probing the structure of the latter. The application of the parton model to such processes seems to be a particularly successful approach although this model has not been proven to be right. First, it had been applied to total deep inelastic scattering [1]. Later on, it had been extended [2),3] to the description of single particle inclusive distributions in the deep inelastic region and in e^+e^- annihilation into hadrons. Single particle inclusive distributions have been also treated in the light cone approach [4]. Some general consequences following from symmetry principles have been derived by Lipkin and Paschos [5] for the target fragmentation region.

To the extent that in the deep inelastic region, the photon, or weak current, can be looked upon as a hadron, this region can serve as a laboratory for testing ideas developed for purely hadronic reactions. In this spirit, the two-component duality [6] idea has been applied to total deep inelastic scattering both within the parton model [7], and without explicitly assuming it [8]. Such an approach naturally leads to the decomposition of the quark distribution within a hadron [9] into a valence component (identified with the resonance Regge contribution) and a sea component (background Pomeron contribution). This is achieved through the interpretation of quark parton diagrams as duality diagrams [10] as suggested by Harari [7]. The two-component duality idea has been also applied to single particle inclusive distributions in the current fragmentation region [11),12].

In this paper we extend the two-component duality idea to the target fragmentation region within the framework of the quark parton model as suggested by Feynman [2]. We further use the model, now constrained by duality, for the study of another aspect of hadron physics - the approach to Feynman scaling [1] and the related problem of exoticity criteria [13], both in the current and target fragmentation regions.

In hadronic physics one studies the inclusive distribution for $a + b \rightarrow c + \text{anything}$ by means of the six-point forward amplitude $a + b + \bar{c} \rightarrow a + b + \bar{c}$ [14]. The s dependence of the inclusive distribution is related to the quantum numbers of the missing mass channel $(ab\bar{c})$ or some of its subchannels $(ab,\ a\bar{c},\ b\bar{c})$. One looks for the exoticity criterion which guarantees a rapid approach to (or an early set in of) Feynman scaling [15] in the fragmentation region of a or b. This amounts to the fast disappearance of the s dependent term in the invariant single particle inclusive cross-section.

In the deep inelastic region single particle inclusive distributions obey Bjorken scaling [2)-4)]. Thus, the cross-section is energy independent (it only depends on ω). However, here one can ask a question which is the analogue of the problem of approach to Feynman scaling in hadron physics. Namely, which exoticity criterion is sufficient in order to eliminate the resonance (or valence quark) contribution to the cross-section? In the $\omega \to \infty$ limit this amounts to the absence of an $\omega^{-\frac{1}{2}}$ contribution. In this paper we try to answer this question within the framework of the dual quark parton model.

In Section 2 we describe the kinematics of the process under study, and the notation for the structure functions in the parton model. In Section 3 we introduce the two-component duality constraints. In Ref. 12) they have been developed for the current fragmentation region. Using the same approach we now extend them to the target fragmentation region. This naturally implies a modification of Feynman's [2] description of the structure functions in this region.

In Section 4 we discuss the problem of exoticity criteria in the quark parton model. We find that in order to eliminate the valence quark (or direct channel resonance) contribution one needs, in general: <u>ab exotic</u>. However, for the production of fast hadrons (fragments of the current or the target) the <u>exoticity of $ab\bar{c}$</u> is sufficient. Results of this approach are presented in Section 5.

We conclude in Section 6 with a few remarks concerning the approach presented here.

2. NOTATION

We consider the process $J_V(q) + N(p) \to h + \text{anything}$, as shown in Fig. 1. The virtual current (electromagnetic or weak) carries four-momentum q, p is the momentum of the nucleon target, and the detected hadron carries four-momentum h. Following Feynman [2], we use a frame in which the hadron moves in the +z direction, and the current, purely spacelike, moves in the -z direction. Thus,

$$p_\mu = \left(\sqrt{P^2+M^2}\,,\ 0\,,\ 0\,,\ P\right) \cong (P\,,0\,,0\,,P)$$

$$q_\mu = \left(0\,,\ 0\,,\ 0\,,\ -\sqrt{-q^2}\right) \tag{2.1}$$

$$h_\mu = \left(\sqrt{h_z^2+h_T^2+m_h^2}\,,\ \vec{h}_T\,,\ h_z\right) .$$

The Bjorken limit

$$q^2 \longrightarrow -\infty\ ,\quad p\cdot q \longrightarrow \infty\ ,\quad x = \frac{-q^2}{2p\cdot q}\ \text{fixed}\ , \tag{2.2}$$

implies in this frame

$$\sqrt{-q^2} = 2xP \tag{2.3}$$

.

The target fragmentation region is defined by

$$h_z = +uP\ ,\quad h_T\ \text{finite}\ ,\quad u = h\cdot q/q\cdot p\ \text{finite}\ . \tag{2.4}$$

That is, the hadron h carries a finite fraction $0 \leq u \leq 1$ of the nucleon momentum in the $+z$ direction and a bounded transverse momentum.

The current fragmentation region is defined by

$$h_z = -uP\ ,\quad h_T\ \text{finite} \tag{2.5}$$

.

That is, the hadron h moves in the direction of the photon $(-z)$ with longitudinal momentum which is a finite fraction of the photon momentum, and a bounded transverse momentum.

It has been suggested by Feynman [2)] that within the parton model this process occurs in two steps. In the first one, the photon is absorbed by a parton whose longitudinal momentum is a fraction x of the nucleon momentum. In the second step, this parton fragments into hadrons which are current fragments, and the rest of the partons materializes in the form of hadrons which are the target fragments. These two processes are assumed to occur independently on the average (namely, not in individual events). The two steps are depicted in Fig. 2.

In the frame defined by Eq. (2.1), the structure functions obtain a simple form. For example, in the current fragmentation region of electroproduction on a proton one has:

$$L^{ep}_{1,h}(x,z,h_T^2) = \frac{1}{2}\sum_i Q_i^2\, d_p^i(x)\, D_i^h(z,h_T^2) \tag{2.6}$$

$L^{ep}_{1,h}$ is the structure function for the inclusive electroproduction of a left moving hadron h on a proton. It is the analogue of $F_1^{ep}(x)$ in the total deep inelastic cross-section. The index i runs over all types of quarks and anti-quarks. Q_i is the charge of a parton of type i, $d_p^i(x)$ is the longitudinal momentum distribution of this parton (carrying a momentum fraction x of the proton momentum), and $D_i^h(z,h_T^2)$ is the function describing the fragmentation of this quark into h + anything. Here, the left moving hadron h carries a fraction z $(0 \leq z \leq 1)$ of the quark momentum and its transverse momentum is $\vec{h}_T$. In terms of Eq. (2.5):

$$u = z x \tag{2.7}$$

A simple calculation in the parton model shows that, in general, the factorized form of Eq. (2.6) [namely, $d(x)\cdot D(z,h_T^2)$] need not hold unless one integrates over $\vec{h}_T$, as has been found, for example, by Altarelli and Maiani [16]. However, such a factorized form may be still valid (e.g., if the cut-off in the transverse momentum distribution for a non-wee parton within the target nucleon does not depend on the longitudinal momentum of this parton; in view of the situation in multiparticle production, this is a rather reasonable assumption for small transverse momenta). The results we present in this paper depend on a factorizable form, but not on whether it is achieved with or without $\vec{h}_T$ integration.

In the target fragmentation region one similarly has

$$R^{ep}_{1,h}(x,r,h_T^2) = \frac{1}{2}\sum_i Q_i^2\, d_p^i(x)\, E^h_{p-i}(x,r,h_T^2) \tag{2.8}$$

Here E^h_{p-i} is the function describing the fragmentation of the right moving quarks (the proton minus the quark of type i which has been kicked out) into h + anything. The right moving hadron h carries a longitudinal momentum which is a fraction r $(0 \leq r \leq 1)$ of the remaining momentum [$(1-x)P$, see Fig. 2]. The E functions carry information about the type of the ejected quark and about its x value. In terms of Eq. (2.4) one has

$$u = r(1-x) \quad . \tag{2.9}$$

Isospin and charge conjugation invariance relate the D functions among themselves and the E functions for a proton target to those of a neutron target. Expressions similar to Eqs. (2.6) and (2.8) can be written for neutrino and anti-neutrino structure functions (for details see Feynman[2] and Gronau et al.[3]).

3. TWO-COMPONENT DUALITY IN THE PARTON MODEL

The introduction of two-component duality constraints at the total cross-section[7),8)] level yields the decomposition of the quark distributions within the nucleon[9)] into valence and sea contributions: $d^i = v^i + s^i$. Within a proton the $d^i(x)$ (we omit the subscript p, as we treat nucleon targets only) are decomposed according to:

$$\begin{aligned} d^u(x) &= v^u(x) + s(x) \\ d^d(x) &= v^d(x) + s(x) \\ d^{\bar{u}}(x) &= d^{\bar{d}}(x) = s(x) \\ d^s(x) &= d^{\bar{s}}(x) = s'(x) \quad . \end{aligned} \tag{3.1}$$

Here $s(x)$ [$s'(x)$] is the distribution function of quarks in the neutral sea [SU(2) singlet] and $v(x)$ is their distribution in the valence (or quantum number carrying) component. If the sea is an SU(3) singlet, $s(x) = s'(x)$.

The application of two-component duality constraints to single hadron inclusive distributions in the current fragmentation region yields[12)] the decomposition of the D functions into a sum of a valence term and a sea term. For example, for the production of pions one obtains:

$$D_u^{\pi^+} = V^{\pi} + S^{\pi}$$
$$D_d^{\pi^+} = S^{\pi} \qquad (3.2)$$
$$D_s^{\pi} = S'^{\pi}$$

In the duality diagram language (see Fig. 3) V^{π} is the contribution of diagrams in which the quark that has been hit by the current ends up as a valence quark of the produced pion, and S^{π} (S'^{π}) is the contribution of diagrams in which it ends up in the sea of the pion. In the production picture one may interpret this as follows: V^{π} is the average number of pions in that part of the produced hadrons which carries quantum numbers (valence component). S^{π} (S'^{π}) is the average number of pions in the neutral part (or sea component).

We now want to introduce two-component duality ideas in the description of the target fragmentation region. In this case, however, our task is more complicated due to the more complicated nature of the E functions. As mentioned in Section 2, E^{h}_{p-i} carries information about the type of quark i which has been kicked out by the photon and about its x value. In a dual picture we distinguish *) between valence quarks and sea quarks. Thus, and we shall later on see the significance, the E functions should "know" whether the ejected quark was a valence quark or a sea quark. To include this information we rewrite Eq. (2.8) in the following form:

$$R^{ep}_{1,h}(x,\tau,h_T^2) = \frac{1}{2}\sum_i Q_i^2 \Big(v^i(x)\, E^{h}_{p-i_v}(x,\tau,h_T^2) + s^i(x)\, E^{h}_{p-i_s}(x,\tau,h_T^2) \Big) \qquad (3.3)$$

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*) This does not mean that the quarks are labelled as valence and sea quarks. Such an effective distinction may be possible, for example, if in the wave function describing an N quark configuration within the nucleon the valence component is orthogonal to the sea component. The decomposition will become self evident when we study the duality diagram content (Fig. 4).

Here $E^{h}_{p-i_V}$ is the conditional fragmentation function of a proton into a hadron h after a parton of type i has been ejected from the valence component of the proton. $E^{h}_{p-i_S}$ is the same quantity when i has been ejected from the sea. Since the definition of the E functions is a statistical one, namely:

1) E^{h}_{p-i} is the average number of hadrons of type h emitted after a quark of type i has been ejected from anywhere within the proton;

2) $E^{h}_{p-i_V}$ the same as above under the condition that the quark is limited to the valence component;

3) $E^{h}_{p-i_S}$ the same, under the condition that the quark is limited to the sea component;

we have the following trivial identity:

$$d^i(x)\,E^{h}_{p-i}(x,\tau,h_T^2) = v^i(x)\,E^{h}_{p-i_v}(x,\tau,h_T^2) + s^i(x)\,E^{h}_{p-i_s}(x,\tau,h_T^2) \quad . \tag{3.4}$$

Thus, the proposed modification does not change the content of Eq. (2.8). It only describes this content in more detail.

By the decomposition (3.4) of $d^iE^{h}_{p-i}$ we include the full information obtained from two-component duality constraints at the four-point function (current target) level only (see Chaichian et al. [8]). We now want to impose the scheme at the six-point function level. The complication here arises due to the fact that now we have a configuration of quarks fragmenting into hadrons, as compared with a single quark in current fragmentation. As has been done in the latter case [12], we identify resonance Regge contributions with valence quark contributions and background Pomeron terms with the sea contribution. Thus, the E functions are now decomposed into a sum of a valence term and a sea term. We shall denote them by $\tilde{V}$ and $\tilde{S}$ respectively.

$$\begin{aligned} E^{h}_{p-i_v} &= \tilde{V}^{h}_{p-i_v} + \tilde{S}^{h}_{p-i_v} \\ E^{h}_{p-i_s} &= \tilde{V}^{h}_{p-i_s} + \tilde{S}^{h}_{p-i_s} \quad . \end{aligned} \tag{3.5}$$

Because of the physical meaning attributed to the E's in the parton model, all the $\tilde{V}$'s and $\tilde{S}$'s are positive quantities, since they describe average numbers of hadrons produced via different production mechanisms.

Inserting (3.5) in (3.4) we obtain four terms, namely,

$$d^i E^k_{p-i} = v^i \tilde{V}^k_{p-i_v} + v^i \tilde{S}^k_{p-i_v} + s^i \tilde{V}^k_{p-i_s} + s^i \tilde{S}^k_{p-i_s} \tag{3.6}$$

In terms of duality diagrams the four terms in Eq. (3.6) correspond to the graphs in Fig. 4 in the same order. In the parton model the current first hits a parton in the target. The over-all cross-section is an incoherent sum over the individual quark contributions. This is due to the separation into a left moving quark and right moving quark (see Fig. 2). As a result, in the pure valence component (Fig. 4a) one cannot have a duality diagram in which the produced hadron h is placed between the current and the target. Moreover, in the Mueller diagram, the two currents are always adjacent. The $s\tilde{V}$ term of Eq. (3.6) obtains contributions from the two diagrams of Fig. 4c.

In general, one cannot forbid the second diagram, whose contribution is not positive definite *). However, as mentioned above, in the dual quark parton model presented here the sum of these two contributions should be non-negative.

In the diagrams of Figs. 4c, 4d, one can also place the produced hadron between the target and the current on both sides of the diagram. However, the resulting ordering is equivalent to the original one. Thus, one has to take into account only one of the two.

For the production of baryons the analogues of each diagram in Fig. 4a and 4c are doubled. The diagrams in Fig. 4b and 4d have only one analogue each. Altogether there are eight graphs. The graphs corresponding to Fig. 4a, for example, are shown in Fig. 5. Notice that from the parton model point of view these two diagrams should be added, as they represent different production mechanisms.

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*) For a special approach in which this term has been dropped in the hadronic six-point function see Tye and Veneziano [17].

The new fragmentation functions, introduced by Eqs. (3.4) and (3.5) satisfy several relationships. For simplicity, let us consider the case of pion emission. All isospin relations satisfied by E^{π}_{N-i} are also satisfied by both $E^{\pi}_{N-i_v}$ and $E^{\pi}_{N-i_s}$. Furthermore, they are satisfied by $\tilde{V}^{\pi}_{N-i_v}$, $\tilde{V}^{\pi}_{N-i_s}$, $\tilde{S}^{\pi}_{N-i_v}$ and $\tilde{S}^{\pi}_{N-i_s}$ separately. Here N stands for a nucleon. The duality content of Eqs. (3.4) and (3.5), namely, that the sea is an SU(2) singlet, together with the valence structure of pions imply the following relations:

$$\tilde{S}^{\pi^+}_{N-u_v} = \tilde{S}^{\pi^-}_{N-u_v} = \tilde{S}^{\pi^+}_{N-d_v} = \tilde{S}^{\pi^-}_{N-d_v} \equiv \tilde{S}^{\pi}_{v} \quad , \tag{3.7}$$

$$\tilde{V}^{\pi^\pm}_{N-u_s} = \tilde{V}^{\pi^\pm}_{N-\bar{u}_s} = \tilde{V}^{\pi^\pm}_{N-d_s} = \tilde{V}^{\pi^\pm}_{N-\bar{d}_s} = \tilde{V}^{\pi^\pm}_{N-s_s} = \tilde{V}^{\pi^\pm}_{N-\bar{s}_s} \equiv \tilde{V}^{\pi^\pm}_{N;s} \quad , \tag{3.8}$$

$$\begin{aligned} &\tilde{S}^{\pi^+}_{N-u_s} = \tilde{S}^{\pi^-}_{N-u_s} = \tilde{S}^{\pi^+}_{N-\bar{u}_s} = \tilde{S}^{\pi^-}_{N-\bar{u}_s} = \tilde{S}^{\pi^+}_{N-d_s} = \tilde{S}^{\pi^-}_{N-d_s} = \\ &\tilde{S}^{\pi^+}_{N-\bar{d}_s} = \tilde{S}^{\pi^-}_{N-\bar{d}_s} = \tilde{S}^{\pi^+}_{N-s_s} = \tilde{S}^{\pi^-}_{N-s_s} = \tilde{S}^{\pi^+}_{N-\bar{s}_s} = \tilde{S}^{\pi^-}_{N-\bar{s}_s} \equiv \tilde{S}^{\pi}_{s} \quad . \end{aligned} \tag{3.9}$$

For the general case of an observed hadron h, using equations similar to Eqs. (3.7)-(3.9), one may rewrite Eq. (3.5) as follows:

$$\begin{aligned} E^{h(I_z)}_{p-i_v} &= \tilde{V}^{h(I_z)}_{p-i_v} + \tilde{S}^{h}_{v} \\ E^{h(I_z)}_{p-i_s} &= \tilde{V}^{h(I_z)}_{p;s} + \tilde{S}^{h}_{s} \quad . \end{aligned} \tag{3.10}$$

Where $h(I_z)$ is the member of an isospin multiplet h with a given I_z. Notice that only $\tilde{V}^{h(I_z)}_{p-i_v}$ is explicitly dependent on the type of quark hit by the current.

Equations (3.8) and (3.9) can be looked upon as a manifestation of a "nucleon + hole" fragmentation picture [18]. Let us emphasize that the equality between contributions related to strange quarks and those of non-strange quarks does not require an SU(3) singlet sea inside the target. This follows from the definition of the various $\tilde{V}$'s and $\tilde{S}$'s through conditional probabilities. The distinction between the different types of quarks hit by the current has been already taken into account by Eq. (3.4).

There, for a non-strange quark of type i $(i = u, d, \bar{u}, \bar{d})$ we have $s^i = s(x)$ as distribution function, while for the strange quarks $(i = s, \bar{s})$, $s^i = s'(x)$, which may very well be different from $s(x)$.

Whenever the purely valence diagram (Fig. 4a) cannot be drawn, the corresponding $\tilde{V}^h_{N-i_v}$ vanish:

$$\tilde{V}^{\pi^+}_{n-u_v} = \tilde{V}^{\pi^-}_{p-d_v} = 0 \quad . \tag{3.11}$$

Using Eqs. (3.3), (3.5) and (3.7)-(3.11) we obtain for the structure functions for pion production in the target fragmentation region the following expressions:

$$\begin{aligned} 2R^{ep}_{1,\pi^+} = {} & \frac{4}{9} v^u \tilde{V}^{\pi^+}_{p-u_v} + \frac{1}{9} v^d \tilde{V}^{\pi^+}_{p-d_v} + \left(\frac{4}{9} v^u + \frac{1}{9} v^d\right) \tilde{S}^{\pi}_{v} \\ & + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{V}^{\pi^+}_{p;s} + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{S}^{\pi}_{s} \quad , \end{aligned} \tag{3.12}$$

$$\begin{aligned} 2R^{ep}_{1,\pi^-} = {} & \frac{4}{9} v^u \tilde{V}^{\pi^-}_{p-u_v} + \left(\frac{4}{9} v^u + \frac{1}{9} v^d\right) \tilde{S}^{\pi}_{v} \\ & + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{V}^{\pi^-}_{p;s} + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{S}^{\pi}_{s} \quad , \end{aligned} \tag{3.13}$$

$$\begin{aligned} 2R^{en}_{1,\pi^+} = {} & \frac{1}{9} v^u \tilde{V}^{\pi^-}_{p-u_v} + \left(\frac{4}{9} v^d + \frac{1}{9} v^u\right) \tilde{S}^{\pi}_{v} \\ & + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{V}^{\pi^-}_{p;s} + \left(\frac{10}{9} s + \frac{2}{9} s'\right) \tilde{S}^{\pi}_{s} \quad , \end{aligned} \tag{3.14}$$

$$2R^{en}_{1,\pi^-} = \frac{1}{9}v^u\tilde{V}^{\pi^+}_{p-uv} + \frac{4}{9}v^d\tilde{V}^{\pi^+}_{p-dv} + \left(\frac{4}{9}v^d + \frac{1}{9}v^u\right)\tilde{S}^{\pi}_{v}$$
$$+ \left(\frac{10}{9}s + \frac{2}{9}s'\right)\tilde{V}^{\pi^+}_{p;s} + \left(\frac{10}{9}s + \frac{2}{9}s'\right)\tilde{S}^{\pi}_{s} \quad . \tag{3.15}$$

In the zero approximation for the Cabibbo angle one has:

$$R^{\nu p}_{1,\pi^+} = v^d\left(\tilde{V}^{\pi^+}_{p-dv} + \tilde{S}^{\pi}_{v}\right) + 2s\left(\tilde{V}^{\pi^+}_{p;s} + \tilde{S}^{\pi}_{s}\right) \quad , \tag{3.16}$$

$$R^{\nu p}_{1,\pi^-} = v^d\tilde{S}^{\pi}_{v} \qquad\qquad + 2s\left(\tilde{V}^{\pi^-}_{p;s} + \tilde{S}^{\pi}_{s}\right) \quad , \tag{3.17}$$

$$R^{\bar{\nu} p}_{1,\pi^+} = v^u\left(\tilde{V}^{\pi^+}_{p-uv} + \tilde{S}^{\pi}_{v}\right) + 2s\left(\tilde{V}^{\pi^+}_{p;s} + \tilde{S}^{\pi}_{s}\right) \quad , \tag{3.18}$$

$$R^{\bar{\nu} p}_{1,\pi^-} = v^u\left(\tilde{V}^{\pi^-}_{p-uv} + \tilde{S}^{\pi}_{v}\right) + 2s\left(\tilde{V}^{\pi^-}_{p;s} + \tilde{S}^{\pi}_{s}\right) \quad , \tag{3.19}$$

$$R^{\nu p}_{3,\pi^+} = -v^d\left(\tilde{V}^{\pi^+}_{p-dv} + \tilde{S}^{\pi}_{v}\right) \quad ,$$

$$R^{\nu p}_{3,\pi^-} = -v^d\tilde{S}^{\pi}_{v} \quad , \tag{3.20}$$

$$R^{\bar{\nu} p}_{3,\pi^+} = -v^u\left(\tilde{V}^{\pi^+}_{p-uv} + \tilde{S}^{\pi}_{v}\right) \quad ,$$

$$R^{\bar{\nu} p}_{3,\pi^-} = -v^u\left(\tilde{V}^{\pi^-}_{p-uv} + \tilde{S}^{\pi}_{v}\right) \quad .$$

The terms proportional to s and s' in Eq. (3.12) are equal to those in Eq. (3.15), and a similar equality holds between the corresponding terms in Eqs. (3.13) and (3.14). Similar equalities hold in the neutrino structure functions. These are a result of the fact that the sea in the target is an SU(2) singlet [the object "exchanged" in the current-current channel is an SU(2) singlet].

Due to the duality relations (3.7)-(3.11) the number of independent structure functions is reduced, and this implies new linear relations among experimental results. These will be discussed in Section 5.

4. EXOTICITY CRITERIA

We assume that the Regge limit of electroproduction can be achieved by taking the $\omega \to \infty$ limit of the scaling structure functions. Using the dual quark parton model, described in Sections 2 and 3, as a laboratory model, we study the problem of the approach to Feynman scaling [15] and the resulting exoticity criteria [13].

In hadronic reactions early scaling occurs when the $s^{-\frac{1}{2}}$ term is absent in the invariant differential cross-section.

In the total deep inelastic cross-section the valence quark contribution, corresponds to the resonance Regge term, and the sea quark contribution - to the background Pomeron term. Therefore, in the $\omega \to \infty$ limit one should have (see, for example, Kuti and Weisskopf [9]):

$$\begin{aligned} v(\omega) &\sim \omega^{\frac{1}{2}} \\ s(\omega),\ s'(\omega) &\sim \omega \end{aligned} \qquad (4.1)$$

Thus, early approach to Feynman [15] scaling occurs in the single particle inclusive cross-section when the $\omega^{\frac{1}{2}}$ term is absent from $L^{eN}_{1,h}$, $R^{eN}_{1,h}$. In our normalization the invariant cross-sections are proportional to $L^{eN}_{2,h} = (2/\omega)L^{eN}_{1,h}$ and $R^{eN}_{2,h} = (2/\omega)R^{eN}_{1,h}$. Therefore, in the cross-section this amounts to the absence of an $\omega^{-\frac{1}{2}}$ term. We study the two fragmentation regions separately and limit ourselves to the emission of mesons.

a. Current fragmentation

In this region the structure function $L^{eN}_{1,h}$, constrained by two-component duality counterterms of four types:

$$v(\omega)\,V(z,k_T^2)\ ,\ v(\omega)\,S(z,k_T^2)\ ,\ s(\omega)\,V(z,k_T^2)\ ,\ s(\omega)\,S(z,k_T^2) \tag{4.2}$$

Since the ω dependence for non-wee z (i.e., excluding the central region) is confined to v and s, the absence of an $\omega^{\frac{1}{2}}$ term implies the absence of the vV and vS terms. This can be achieved under either one of the following conditions (we denote the current by a, the target by b and the observed hadron by c):

$$\begin{array}{l} 1.\ ab \ \text{ exotic };\\ \quad \text{or} \\ 2.\ ab\bar{c}\ \text{ exotic and } S=0 \end{array} \tag{4.3}$$

The first case is clear. If ab is exotic, $v \equiv 0$. The second case is explained as follows: when $ab\bar{c}$ is exotic one cannot draw a pure valence diagram (Fig. 3a) irrespective of whether ab is exotic or not. Therefore, the vV term is absent. To eliminate the vS term in a non-trivial way one now needs $S = 0$.

The question that arises at this point is: under what conditions the decay of a single left moving quark into the neutral hadron component (S) is negligible in comparison to the decay into the quantum number carrying component (V). From the identification of V with the resonant term and S with the background component we know [see e.g., Feynman [2] and Ref. 12)]:

$$\frac{S^h(z,k_T^2)}{V_i^h(z,k_T^2)} \xrightarrow[z\to 1]{} 0 \tag{4.4}$$

However, one might expect S to be negligible in comparison to V over a wider range in z. The argument for this expectation is based on the observation that in the total deep inelastic cross-section the contribution of sea quarks (s,s') is limited to rather small values of x. In electroproduction [19] it seems to be limited to $x \lesssim 0.3$, and in neutrino production [20], even to $x \lesssim 0.1$. The physical principles behind the description of the nucleon in terms of its constituents and behind the description of the fragmentation of a quark into hadrons are similar (see Feynman [2]). One, therefore, expects that for a given statement about the quark distributions inside the nucleon, a similar one should hold for the distribution of the produced hadrons. Inside the nucleon the distribution of sea quarks (s, s') is limited to low momenta (that is, in the rapidity plot it falls off rapidly outside of the plateau). Thus, there exists some value x_0 $(\sim 0.2$, say) such that

$$\frac{s(x)}{v(x)} \ll 1 \quad , \qquad x > x_0 \quad . \tag{4.5}$$

Similarly, we expect that there exists some small value z_0 such that

$$\frac{S^h(z, k_T^2)}{V_i^h(z, k_T^2)} \ll 1 \quad , \qquad z > z_0 \quad . \tag{4.6}$$

Let us emphasize that the conjecture summarized by Eq. (4.6) is based solely on the experimental information concerning the distribution of partons inside the nucleon target.

Neglecting S in the deep fragmentation region (i.e., outside the plateau) here is similar to what is usually done in hadronic reactions [21]. There, for the fragmentation of a into c, $s_{a\bar{c}} = (p_a - p_c)^2$ is small (below threshold) and, therefore, the background (Pomeron) contribution is neglected in the $a\bar{c}$ vertex.

Comparing with hadronic reactions [13] we see that there the first criterion (ab exotic) has no counterpart in the study of single particle inclusive distributions. The second criterion ($ab\bar{c}$ exotic and $S = 0$), on the other hand, has a counterpart [21].

In electroproduction, where ab is not exotic, the second criterion may be useful from the phenomenological point of view when the large ω region is included (e.g., in K meson production).

b. Target fragmentation

With the same motivation we had in the current fragmentation region, we want to find criteria for the absence of $\omega^{-\frac{1}{2}}$ terms in the invariant differential cross-sections. A priori, we know that $\tilde{V}(x,z,h_T^2)$ and $\tilde{S}(x,z,h_T^2)$ [see Eq. (3.5)] have some ω dependence ($\omega = 1/x$). However, in the large ω limit, the leading ω dependence of the six-point function in the Mueller-Regge description is included in $v(\omega)$ and $s(\omega)$, $s'(\omega)$ [see Eq. (4.1)]. The reason is that here the various $\tilde{V}$ and $\tilde{S}$ become Regge residues, which are ω independent.

As a result, the two possible exoticity criteria (4.3) hold also here:

$$\begin{aligned} &1. \quad ab \quad \text{exotic}; \\ &\qquad \text{or} \\ &2. \quad a b \bar{c} \quad \text{exotic and} \quad \tilde{S}_v^h = 0 \quad . \end{aligned} \tag{4.7}$$

The arguments leading to Eqs. (4.4) and (4.6) can be also applied in the target fragmentation region. Namely, we expect

$$\frac{\tilde{S}^h(x,z,h_T^2)}{\tilde{V}^h(x,z,h_T^2)} \ll 1 \qquad \text{for } r \geq r_0 \quad . \tag{4.8}$$

where r_o is rather small (~ 0.2, say). This inequality is expected to hold both for $\tilde{S}_v^h$ and $\tilde{V}_{p-i_v}^h$ and for $\tilde{S}_s^h$ and $\tilde{V}_{p;s}^h$ although the criterion (4.7) only requires it for $\tilde{S}_v^h$.

5. SUMMARY OF RESULTS

a. Current fragmentation

$S^{\pi} = 0$ results

Results following from two-component duality constraints alone have been already discussed in Ref. 12). Here we present consequences of Eq. (4.6). Namely, of the existence of a non-trivial range of z for which S is negligible in comparison with V. Neglecting S for such a range ($z \geq z_o$) we obtain:

$$L^{ep}_{1,\pi^-} = \frac{1}{18}(v^d + 5s)\,V^{\pi} \quad ,$$

$$L^{ep}_{1,\pi^+} = \frac{1}{18}(4v^u + 5s)\,V^{\pi} \quad ,$$

$$L^{en}_{1,\pi^-} = \frac{1}{18}(v^u + 5s)\,V^{\pi} \quad , \tag{5.1}$$

$$L^{en}_{1,\pi^+} = \frac{1}{18}(4v^d + 5s)\,V^{\pi} \quad .$$

As a trivial consequence one has

$$L^{en}_{1,\pi^+} - L^{ep}_{1,\pi^+} = 4\left(L^{ep}_{1,\pi^-} - L^{en}_{1,\pi^-}\right) \quad , \quad z \geq z_0 \quad . \tag{5.2}$$

If for $x \to 1$ [e.g., $W = \sqrt{(p+q)^2}$ fixed and $|q^2|$ increasing] s/v^u, s'/v^u, v^d/v^u all go to zero [see Feynman [2] and Ref. 12)], we have the following predictions

$$\frac{L^{ep}_{1,\pi^-}}{L^{ep}_{1,\pi^+}} \longrightarrow 0 \; , \; \frac{L^{en}_{1,\pi^+}}{L^{ep}_{1,\pi^+}} \longrightarrow 0 \; , \; \frac{L^{en}_{1,\pi^-}}{L^{ep}_{1,\pi^+}} \longrightarrow \frac{1}{4} \; , \; \frac{L^{ed}_{1,\pi^-}}{L^{ed}_{1,\pi^+}} \longrightarrow \frac{1}{4} \tag{5.3}$$

$$x \to 1 \quad \text{and} \quad z \geq z_0 \quad .$$

Here d stands for the sum over proton and neutron structure functions. Notice, that for $z \geq z_0$ these ratios are independent of z and h_T. They depend only on x. Experimentally [22)], when ω changes from ~ 11 to ~ 6 (W fixed at 2.65 GeV and $|q^2|$ increases), the ratio $L^{ep}_{1,\pi^-}/L^{ep}_{1,\pi^+}$ already moves in the right direction, namely, decreases. The values of z covered in Ref. 22) are $z \geq \sim 0.4$.

The third relation in Eq. (5.3) is, actually, satisfied without the $z \geq z_0$ constraint. However, if we want to study the rate at which these limits are reached, we need all the information presented in Eqs. (4.5) and (4.6). Denote the ratio F_2^{en}/F_2^{ep} by r. Using Eq. (4.5), we have

$$r - \frac{1}{4} = \frac{F_2^{en}}{F_2^{ep}} - \frac{1}{4} = \frac{15 v^d}{16 v^u + 4 v^d} \quad , \quad x > x_o \tag{5.4}$$

Now, for $z \geq z_o$ we find

$$\frac{L^{ep}_{1,\pi^-}}{L^{ep}_{1,\pi^+}} = \frac{4}{15}\left(r - \frac{1}{4}\right) \quad ,$$

$$\frac{L^{en}_{1,\pi^+}}{L^{ep}_{1,\pi^+}} = \frac{16}{15}\left(r - \frac{1}{4}\right) \quad ,$$

$$\frac{L^{en}_{1,\pi^-}}{L^{ep}_{1,\pi^+}} = \frac{1}{4} \quad , \tag{5.5}$$

$$\frac{L^{ed}_{1,\pi^-}}{L^{ed}_{1,\pi^+}} = \frac{1}{4} \quad ,$$

$$z \geq z_o \quad , \quad x \to 1 \quad .$$

Therefore, under the assumptions (4.5) and (4.6), for $x \to 1$ the first two ratios in Eq. (5.3) approach their limits at a rate which is comparable to that of the approach of F_2^{en}/F_2^{ep} to its limit [the rate at which $(v^d/v^u) \to 0$]. On the other hand, the last two ratios should approach their limits much earlier [the rate at which $(s/v^u) \to 0$].

In neutrino reactions, even without neglecting the Cabibbo angle, both L_1 and L_3, and, therefore, the cross-sections satisfy the following relations

$$d\sigma^{\nu p}_{\pi^-} : d\sigma^{\nu p}_{\pi^0} : d\sigma^{\nu p}_{\pi^+} = 0 : 1 : 2 \quad ,$$

$$d\sigma^{\bar{\nu} p}_{\pi^-} : d\sigma^{\bar{\nu} p}_{\pi^0} : d\sigma^{\bar{\nu} p}_{\pi^+} = 2 : 1 : 0 \quad , \quad z \geq z_o \quad . \tag{5.6}$$

That is, in neutrino production, while for $z \to 0$ the distributions of π^-, π^0, π^+ should be equal, as z starts to increase the π^- distribution should fall off as function of z much faster than those of π^0, π^+. In anti-neutrino reactions the π^+ distribution plays this role.

When replacing the proton target in Eq. (5.6) by a neutron or a deutron ($\equiv$n+p) one obtains the same ratios.

The $\omega \to \infty$ limit for fast current fragments

Let us, first of all, remark that within the parton model the approach to Feynman scaling is always achieved from above. This is a consequence of the fact that all the contributions to the cross-section, namely, v, s, V, S, are positive quantities (being average numbers of partons or hadrons respectively).

Equation (5.1) together with Eq. (4.1) imply that the ratio of the non-leading $(\omega^{-\frac{1}{2}})$ terms is given by [we denote them by $(\gamma \xrightarrow{N} \pi)_R$]

$$\frac{(\gamma \xrightarrow{p} \pi^-)_R}{(\gamma \xrightarrow{n} \pi^+)_R} = \frac{1}{4} \quad ,$$

$$\frac{(\gamma \xrightarrow{n} \pi^-)_R}{(\gamma \xrightarrow{p} \pi^+)_R} = \frac{1}{4} \quad , \qquad (5.7)$$

$$\frac{(\gamma \xrightarrow{d} \pi^-)_R}{(\gamma \xrightarrow{d} \pi^+)_R} = \frac{1}{4} \quad ,$$

$$\frac{(\gamma \xrightarrow{p} \pi^-)_R}{(\gamma \xrightarrow{p} \pi^+)_R} = \frac{v^d}{4v^u} \quad , \qquad z \geq z_0 \quad .$$

We would like to obtain a numerical estimate to the last relation in Eq. (5.7). In the Regge limit v^u and v^d are related to the F and D couplings of Reggeons at the nucleon vertex [23]. One has:

$$v^u \propto 2F$$
$$v^d \propto F - D \qquad (5.8)$$

Therefore,

$$\frac{(\gamma \xrightarrow{p} \pi^-)_R}{(\gamma \xrightarrow{p} \pi^+)_R} = \frac{F-D}{8F} \quad , \quad z \geq z_0 \tag{5.9}$$

If we use the value $D/F \cong -0.2$ (obtained from hadronic reactions [24]) we have

$$\frac{(\gamma \xrightarrow{p} \pi^-)_R}{(\gamma \xrightarrow{p} \pi^+)_R} = \frac{1.2}{8} \quad , \quad z \geq z_0 \tag{5.10}$$

It is interesting to observe that in the symmetric quark parton model [9], where one has $v^u = 2v^d$ (corresponding to pure F coupling at the nucleon vertex), this ratio is 1:8.

In the total deep inelastic cross-section, the symmetric quark parton model [9] fails for $\omega \to 1$. However, it seems to be consistent with data on the average (see Kendall [19]). In our case (under the assumption that the Regge limit can be obtained by letting $\omega \to \infty$ in the inclusive deep inelastic structure functions, with the use of the symmetric quark parton model, the prediction (5.7) does not differ from that obtained by use of hadronic reaction data [Eq. (5.10)]. Clearly, a similar approximate equality between the symmetric quark parton prediction and the Regge prediction should hold already at the total cross-section level.

b. Target fragmentation

Isospin inequalities

The fragmentation functions E^h_{p-i} satisfy inequalities which follow from the positivity of the contribution of each isospin state of X in $p+\bar{i}+\bar{h}\to X$ *).

---

*) These inequalities depend only on the positivity constraint and isospin. Identical inequalities for inclusive cross-sections in hadronic reactions have been derived by Yao [25].

In general one has,

$$E^{h}_{p-i} \geq 0 \quad .$$

For the emission of pions, in addition,

$$\begin{aligned}
&E^{\pi^+}_{p-d} \geqslant \frac{1}{6} E^{\pi^-}_{p-d} \quad , \\
&E^{\pi^-}_{p-u} \geq \frac{1}{4} E^{\pi^-}_{p-d} \quad , \\
&E^{\pi^+}_{p-u} \geq \frac{1}{4} E^{\pi^-}_{p-d} \quad , \\
&\sqrt{2\left(E^{\pi^+}_{p-d} - \frac{1}{6} E^{\pi^-}_{p-d}\right)} \geq \left| \sqrt{E^{\pi^+}_{p-u} - \frac{1}{4} E^{\pi^+}_{p-d}} - \sqrt{E^{\pi^-}_{p-u} - \frac{1}{4} E^{\pi^-}_{p-d}} \right| \quad , \\
&E^{\pi^+}_{p-s} \geq \frac{1}{3} E^{\pi^-}_{p-s} \quad .
\end{aligned} \tag{5.11}$$

By substituting $d \to \bar{u}$, $u \to \bar{d}$ and $s \to \bar{s}$ in Eq. (5.11) one obtains the corresponding inequalities for the remaining structure functions.

Duality results

Due to the modification of the functions E^{h}_{p-i}, as suggested in this paper [see Eq. (3.4)], the inequalities (5.11) should be satisfied among $E^{h}_{p-i_V}$ and separately among $E^{h}_{p-i_S}$. For instance,

$$\begin{aligned}
&E^{\pi^+}_{p-d_V} \geq \frac{1}{6} E^{\pi^-}_{p-d_V} \\
&E^{\pi^+}_{p-d_S} \geq \frac{1}{6} E^{\pi^-}_{p-d_S} \quad , \quad \text{etc.}
\end{aligned} \tag{5.12}$$

All the duality constraints derived in this paper [see Eqs. (3.5) and (3.7)-(3.9)] are consistent with the isospin inequalities (5.11).

Using Eqs. (3.6)-(3.9) and (3.11)-(3.19) we obtain, in the zero approximation for the Cabibbo angle, the following duality sum rules:

$$R^{\bar{\nu}p}_{1,\pi^+} - R^{\nu p}_{1,\pi^+} = 6\left(R^{ep}_{1,\pi^+} - R^{en}_{1,\pi^-}\right) \quad ,$$

$$R^{\bar{\nu}p}_{1,\pi^-} - R^{\nu p}_{1,\pi^-} = 6\left(R^{ep}_{1,\pi^-} - R^{en}_{1,\pi^+}\right) \quad . \tag{5.13}$$

These, when combined, give

$$R^{\nu d}_{1,\pi^+} - R^{\nu d}_{1,\pi^-} = 6\left(R^{ed}_{1,\pi^+} - R^{ed}_{1,\pi^-}\right) \quad , \tag{5.14}$$

where d stands for the sum of proton and neutron structure functions. Relations (5.13) are the analogues of the duality relation

$$F_1^{\bar{\nu}p} - F_1^{\nu p} = 6\left(F_1^{ep} - F_1^{en}\right) \quad , \tag{5.15}$$

derived by Llewellyn Smith [9], Polkinghorne and Landshoff [7] and Parisi [26] for the total deep inelastic structure functions.

The left-hand side of Eq. (5.15), when integrated over x ($x = 1/\omega$) equals 1 (this is the Adler-Fubini-Dashen-Gell-Mann sum rule). This has been used [26] in order to obtain an integrated duality sum rule for the right-hand side of Eq. (5.15), namely

$$\int_0^1 \left(F_1^{ep} - F_1^{en}\right) dx = \frac{1}{6} \quad . \tag{5.16}$$

Since in the target fragmentation region the current-current light-cone dominates [10], one can, in principle, derive a similar integrated duality sum rule for Eq. (5.13). However, here we have a pion-nucleon system. Therefore, the right-hand side of the analogue of Eq. (5.16) becomes an unknown function of $h\cdot p$. In the parton model this is related to the x dependence of the E functions.

Let us stress here that the duality relations (5.13) do not have simple counterparts in the current fragmentation region [12]. In the parton model this results from the fact that the weak current changes the identity of the quark it hits before the latter fragments. In the light-cone approach this is related to the fact that in this region the current-current light-cone does not dominate [10].

The duality expressions for the various structure functions enable us to estimate the relative sizes of various cross-sections. From (3.12) and (3.13) we have

$$2R^{ep}_{1,\pi^+} - 2R^{ep}_{1,\pi^-} = \frac{4}{9} v^u \left(\tilde{V}^{\pi^+}_{p-u_v} - \tilde{V}^{\pi^-}_{p-u_v} \right) + \frac{1}{9} v^d \tilde{V}^{\pi^+}_{p-d_v}$$
$$+ \left(\frac{10}{9} s + \frac{2}{9} s' \right) \left(\tilde{V}^{\pi^+}_{p;s} - \tilde{V}^{\pi^-}_{p;s} \right) \quad . \tag{5.17}$$

We have, in general,

$$\tilde{V}^{\pi^+}_{p;s} \geq \tilde{V}^{\pi^-}_{p;s} \quad . \tag{5.18}$$

The reason being that in this case the quark ejected from the target is a sea quark, while the mesons are produced from the valence quarks of the target. From the experimental fact that $F_2^{en}/F_2^{ep} \leq 1$ we have $v^u \geq v^d$. The production of the valence component of the π^+ is through a u valence quark ($\pi^+ = u\bar{d} + \text{sea}$), and that of a π^- - through a d valence quark ($\pi^- = d\bar{u} + \text{sea}$). The latter should, therefore, be smaller than the former and hence Eq. (5.18).

For general values of x we cannot say much about the difference $\tilde{V}^{\pi^+}_{p-u_v} - \tilde{V}^{\pi^-}_{p-u_v}$. In these functions the valence component of pions is made from the valence quarks (u and d) remaining in the nucleon after a u valence quark has been hit. At $x \to 1$ one has good indications [see Feynman [2] and Ref. 23)] that the dominant configuration is the one in which a u valence quark carries all the momentum (it will be hit by the photon), while the remaining valence quarks (u+d) are in an antisymmetric state. Thus, they have equal distributions. Since in the pure valence term the π^+ comes from the remaining u quark, while the π^- - from the remaining d quark, we expect

$$\tilde{V}^{\pi^+}_{p-u_v} \cong \tilde{V}^{\pi^-}_{p-u_v} \quad , \quad x \to 1 \quad . \tag{5.19}$$

Therefore, for $x \to 1$ the expression in Eq. (5.17) is positive. Moreover, using Eq. (5.18) and the fact that for $x \to 0$ $(\omega = (1/x) \to \infty)$ the $s(x)$, $s'(x)$ terms dominate over the v^u, v^d terms, we expect the same to hold in the latter limit too. That is

$$2R^{ep}_{1,\pi^+} - 2R^{ep}_{1,\pi^-} \geq 0 \quad ; \quad x \sim 0 \text{ and } x \sim 1 \quad . \tag{5.20}$$

Or,

$$\frac{d\sigma(p \xrightarrow{\gamma} \pi^+)}{d\sigma(p \xrightarrow{\gamma} \pi^-)} \geq 1 \quad ; \quad x \sim 0 \text{ and } x \sim 1 \quad . \tag{5.21}$$

Similar arguments lead to

$$\frac{d\sigma(n \xrightarrow{\gamma} \pi^+)}{d\sigma(n \xrightarrow{\gamma} \pi^-)} \leq 1 \quad ; \quad x \sim 0 \text{ and } x \sim 1 \quad .$$

$$\frac{d\sigma(d \xrightarrow{\gamma} \pi^+)}{d\sigma(d \xrightarrow{\gamma} \pi^-)} \leq 1 \quad ; \quad x \sim 1 \quad (d = p + n) \quad . \tag{5.22}$$

It is interesting to compare these results with the predictions for the current fragmentation region [12], where the ratio of π^+/π^- production is larger than, or equal to 1 for the three targets (p,n,d).

In neutrino reactions the situation is somewhat more complicated due to the existence of R_3. The inequalities satisfied by the structure functions (3.16)-(3.20) due to Eqs. (5.18) and (5.19) are:

$$
\begin{aligned}
R^{\nu p}_{1,\pi^+} &\geq R^{\nu p}_{1,\pi^-} && \text{for all } x \quad , \\
R^{\nu n}_{1,\pi^+} &\leq R^{\nu n}_{1,\pi^-} && x \sim 0 \text{ and } x \sim 1 \quad , \qquad (5.23) \\
R^{\nu d}_{1,\pi^+} &\geq R^{\nu d}_{1,\pi^-} && x \sim 1 \quad .
\end{aligned}
$$

And

$$
\begin{aligned}
|R^{\nu p}_{3,\pi^+}| &\geq |R^{\nu p}_{3,\pi^-}| && \text{for all } x \quad , \qquad (5.24) \\
R^{\nu n}_{3,\pi^+} &\cong R^{\nu n}_{3,\pi^-} && x \sim 1 \quad .
\end{aligned}
$$

If θ, the scattering angle between the incoming neutrino and the outgoing muon, is very small, so that the $W_3^{\nu(\bar{\nu})N}$ term is negligible, Eq. (5.23) can be translated into relations among cross-sections:

$$
\begin{aligned}
&\frac{d\sigma(p \xrightarrow{\nu} \pi^+)}{d\sigma(p \xrightarrow{\nu} \pi^-)} \geq 1 && \text{for all } x \quad , \\
&\frac{d\sigma(n \xrightarrow{\nu} \pi^-)}{d\sigma(n \xrightarrow{\nu} \pi^+)} \geq 1 && x \sim 0 \text{ and } x \sim 1 \quad , \qquad (5.25) \\
&\frac{d\sigma(d \xrightarrow{\nu} \pi^+)}{d\sigma(d \xrightarrow{\nu} \pi^-)} \geq 1 && x \sim 1 \quad .
\end{aligned}
$$

Experimental results [27)] are consistent with Eq. (5.21) for small x.

Using [2),12)] v^d/v^u, s/v^u, $s'/v^u \to 0$ for $x \to 1$ and Eq. (5.19) we obtain from Eqs. (3.12)-(3.15):

$$\left.\begin{aligned}
&\frac{d\sigma(p \xrightarrow{\gamma} \pi^-)}{d\sigma(p \xrightarrow{\gamma} \pi^+)} \longrightarrow 1 \quad , \\[2ex]
&\frac{d\sigma(n \xrightarrow{\gamma} \pi^-)}{d\sigma(n \xrightarrow{\gamma} \pi^+)} \longrightarrow 1 \quad , \\[2ex]
&\frac{d\sigma(d \xrightarrow{\gamma} \pi^-)}{d\sigma(d \xrightarrow{\gamma} \pi^+)} \longrightarrow 1 \quad .
\end{aligned}\right\} x \to 1 \qquad (5.26)$$

The $\omega \to \infty$ limit for fast target fragments

Fast fragments are those with a finite (non-wee) fraction r of the momentum of the right moving quarks (see Fig. 2). As mentioned in Section 4, we expect that there exists some small value r_0 (~ 0.3, say) such that Eq. (4.8) is satisfied. That is, for $r \geq r_0$ the $\tilde{S}^\pi$ functions are negligible in comparison with the corresponding $\tilde{V}^\pi$. Equations (3.12)-(3.15) then become

$$\begin{aligned}
2R^{ep}_{1,\pi^+} &= \frac{4}{9} v^u \tilde{V}^{\pi^+}_{p-u_v} + \frac{1}{9} v^d \tilde{V}^{\pi^+}_{p-d_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right) \tilde{V}^{\pi^+}_{p;s} , \\
2R^{ep}_{1,\pi^-} &= \frac{4}{9} v^u \tilde{V}^{\pi^-}_{p-u_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right) \tilde{V}^{\pi^-}_{p;s} , \\
2R^{en}_{1,\pi^+} &= \frac{1}{9} v^u \tilde{V}^{\pi^-}_{p-u_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right) \tilde{V}^{\pi^-}_{p;s} , \\
2R^{en}_{1,\pi^-} &= \frac{1}{9} v^u \tilde{V}^{\pi^+}_{p-u_v} + \frac{4}{9} v^d \tilde{V}^{\pi^+}_{p-d_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right) \tilde{V}^{\pi^+}_{p;s} ,
\end{aligned} \qquad (5.27)$$

$$2R^{ed}_{1,\pi^+} = v^u\left(\frac{4}{9}\tilde{V}^{\pi^+}_{p-u_v} + \frac{1}{9}\tilde{V}^{\pi^-}_{p-u_v}\right) + \frac{1}{9}v^d\tilde{V}^{\pi^+}_{p-d_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right)\left(\tilde{V}^{\pi^+}_{p;s} + \tilde{V}^{\pi^-}_{p;s}\right),$$

$$2R^{ed}_{1,\pi^-} = v^u\left(\frac{1}{9}\tilde{V}^{\pi^+}_{p-u_v} + \frac{4}{9}\tilde{V}^{\pi^-}_{p-u_v}\right) + \frac{4}{9}v^d\tilde{V}^{\pi^+}_{p-d_v} + \left(\frac{10}{9}s + \frac{2}{9}s'\right)\left(\tilde{V}^{\pi^+}_{p;s} + \tilde{V}^{\pi^-}_{p;s}\right) \quad (5.27)$$

Since we assume that the Regge limit can be reached by letting $\omega \to \infty$ in the scaling functions, we have

$$R^{eN}_{1,h} \cong c_P\,\omega + c_R\,\omega^{1/2} \quad (5.28)$$

where c_P and c_R are ω independent. For large ω Eq. (4.1) holds. Thus, we have an ω^1 dependence in Eq. (5.27) coming from s and s' and an $\omega^{\frac{1}{2}}$ dependence coming from v^u and v^d. In principle, the approach to Feynman scaling (i.e., the coefficient c_R, which determines the approach to the ω independent term in the invariant cross-section) can have contributions from the ω dependence of the various $\tilde{V}^{\pi}$. However, as already mentioned in Section 4, we expect the latter to approach constant values for large ω. They play the role of Reggeon residues. As such, they cannot have essential singularities in the Reggeization variable, namely, in ω. Therefore, they cannot have, say $\omega^{-\frac{1}{2}}$ terms in their expansion. The term next to the constant in the expansion of the $\tilde{V}^{\pi}$ is $O(1/\omega)$.

In the parton language, the constancy of $\tilde{V}^{\pi}$ for large ω is a consequence of the fact that the ejected quark carries a vanishingly small fraction of the total momentum $(x = (1/\omega) \to 0)$. One, therefore, expects $\tilde{V}^{\pi}$, which describes the fragmentation of the remaining quark system, to depend only on the identity of the missing quark, but not on its momentum.

As a result, in the target fragmentation region, just as in current fragmentation, Feynman scaling is approached from above (all the additive terms are non-negative). The ratios of the $\omega^{-\frac{1}{2}}$ terms in the various processes are (we denote them by a subscript R):

$$\frac{(p \xrightarrow{\gamma} \pi^-)_R}{(n \xrightarrow{\gamma} \pi^+)_R} = 4 \quad , \tag{5.29}$$

$$\frac{(p \xrightarrow{\gamma} \pi^+)_R}{(n \xrightarrow{\gamma} \pi^-)_R} = \frac{4v^u \tilde{V}^{\pi^+}_{p-u_v} + v^d \tilde{V}^{\pi^+}_{p-d_v}}{v^u \tilde{V}^{\pi^+}_{p-u_v} + 4v^d \tilde{V}^{\pi^+}_{p-d_v}} \quad ,$$

$$\frac{(d \xrightarrow{\gamma} \pi^+)_R}{(d \xrightarrow{\gamma} \pi^-)_R} = \frac{v^u (4\tilde{V}^{\pi^+}_{p-u_v} + \tilde{V}^{\pi^-}_{p-u_v}) + v^d \tilde{V}^{\pi^+}_{p-d_v}}{v^u (\tilde{V}^{\pi^+}_{p-u_v} + 4\tilde{V}^{\pi^-}_{p-u_v}) + 4v^d \tilde{V}^{\pi^+}_{p-d_v}} \quad . \tag{5.30}$$

If we take the symmetric quark parton model [9] literally, then we have

$$v^u = 2v^d \tag{5.31a}$$

and

$$\tilde{V}^{\pi^+}_{p-u_v} = \tilde{V}^{\pi^-}_{p-u_v} = \frac{1}{2}\tilde{V}^{\pi^+}_{p-d_v} \quad . \tag{5.31b}$$

Which implies

$$(p \xrightarrow{\gamma} \pi^+)_R : (p \xrightarrow{\gamma} \pi^-)_R : (n \xrightarrow{\gamma} \pi^+)_R : (n \xrightarrow{\gamma} \pi^-)_R = 10 : 8 : 2 : 10 \quad ,$$

$$(d \xrightarrow{\gamma} \pi^+)_R : (d \xrightarrow{\gamma} \pi^-)_R = 12 : 18 \quad . \tag{5.32}$$

However, we do not expect (5.32) to be as close to experiment as the prediction of the symmetric quark parton model for current fragmentation $[(\gamma \xrightarrow{p} \pi^-)_R : (\gamma \xrightarrow{p} \pi^+)_R = 1:8]$ is close to experiment [Eq. (5.10)]. Here experiment means using Regge $D/F \cong -0.2$, obtained from hadronic reaction data [24]. The reason is that for $\omega \to \infty$, while $v^u \cong 2v^d$ is, probably, a good approximation, Eq. (5.31b) is, probably, incorrect. The emitted u valence quark has $x(=1/\omega) \to 0$. However, the remaining u and d valence quarks have all x values between zero and one, and we already know that the symmetric quark parton model fails for $x \to 1$. Therefore, for the production of fast pions we do not expect the $\tilde{V}^{\pi}$ functions to be related by Eq. (5.31b). Let us mention without giving the arguments that (5.31b) might be an acceptable approximation for the valence component in the production of slow pions $(r \to 0)$.

We would like to conclude this section by studying Eqs. (3.16)-(3.19) as an example of the way the $ab\bar{c}$ exoticity criterion works. For $r \geq r_0$ we neglect there the $\tilde{S}_s^{\pi}$ and $\tilde{S}_v^{\pi}$ terms. As a result, Eq. (3.17) now has only Pomeron terms in the ω variable:

$$R^{\nu p}_{1,\pi^-} = 2s\,\tilde{V}^{\pi^-}_{p;s} \qquad , \tag{5.33}$$

while in Eqs. (3.16), (3.18) and (3.19) some Regge (v^u, v^d) terms are left. In $\nu p \to \mu^- + \pi^-$+anything, $ab\bar{c}$ (a = current, b = target, c = observed hadron) has the quantum numbers of a $\pi^+ p\, \pi^+$ system, and is, therefore, exotic. Thus, this reaction is expected to have a flat cross-section for large ω, although ab is not exotic.

6. REMARKS AND CONCLUSIONS

In this paper we have extended the two-component duality idea to the target fragmentation region as described in the quark parton model. We have used the results obtained here and in Ref. 12) for the current fragmentation region in order to study questions arising in hadronic reaction physics such as the approach to Feynman scaling [15] and exoticity criteria [13]. We would like to conclude with a few remarks.

1) In the whole discussion we have treated the fragmentation regions only. The central region has not been studied in this paper.

2) In both the target and current fragmentation regions we find that the approach to Feynman scaling in the $\omega \to \infty$ limit is from above (the $\omega^{-\frac{1}{2}}$ contributions to the invariant cross-sections are non-negative). Whenever the incoming channel (a = current, b = target) satisfies ab = exotic, there are no valence quark terms (v^u, v^d) and hence no $\omega^{-\frac{1}{2}}$ terms in the large ω limit. That is, in our scheme, when the incoming channel is exotic, not only is the total cross-section flat as

a function of ω , but so are also the inclusive distributions for any produced hadron in the fragmentation regions.

This is different from the situation in hadronic reactions. There, some inclusive cross-sections approach the scaling limit from above, while others approach it from below, even when the incoming channel is exotic (pp).

The energy conservation sum rule seems to imply [17] (provided that the contribution of the central region to the integral can be neglected) that in the fragmentation region, if some inclusive cross-sections approach the scaling limit from above when the incoming channel is exotic, then others must reach it from below. The reason there [17] was that for an exotic hadronic reaction the total cross-section is flat. Therefore, in the inclusive sum rule, the $s^{-\frac{1}{2}}$ contributions, coming from the various inclusive cross-sections, have to add up to zero.

In our case, the solution to this constraint is the trivial one. For ab exotic - all $\omega^{-\frac{1}{2}}$ terms are separately zero for each inclusive cross-section.

3) We have found that for the fragmentation of a baryon target into an observed baryon there are, in some components, two contributing diagrams (see Fig. 5 for the pure valence component). Since these diagrams represent different <u>production</u> mechanisms, it was stated in Section 3 that one should add up their contributions. However, the diagram of Fig. 5b is expected to be smaller than that of Fig. 5a. The reason is that the probability of the two valence quarks ending up in <u>one</u> observed hadron is smaller than that for one quark only ending up in the same hadron, and the second quark, ending anywhere else.

4) Let us notice that in target fragmentation the current plays a secondary role. If we abstract the conclusion about the addition of diagrams from its specific context, we are tempted to conjecture that in the four-point function of baryon-antibaryon scattering one should, in principle, add the contributions of the two possible diagrams with a positive relative sign.

5) In the current fragmentation region, the pure valence term for the production of a baryon on a baryon target has only one contributing diagram just as in the production of a meson (Fig. 3a). This is a direct consequence of the production mechanism in the parton model [12].

6) In this paper we have treated single particle inclusive distributions. A natural question that now arises is whether the same physical principles, provided by the parton model, can be applied to the description of two particle inclusive correlations. We find that in the current fragmentation region and in e^+e^- annihilation into hadrons these correlations can be described by introducing new fragmentation functions. The new functions are analogous to the E^h_{p-i} describing target fragmentation. This problem will be treated elsewhere.

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REFERENCES

1) R.P. Feynman, Proceedings of the Third High Energy Conference held at Stony Brook, Gordon and Breach (1970);
J.D. Bjorken and E.A. Paschos, Phys.Rev. 185, 1975 (1969).

2) R.P. Feynman, "Photon Hadron Interactions", W.A. Benjamin, New York (1972), and Talk presented at the Neutrino 1972 Conference, Balatonfüred, Hungary (1972);
S.M. Berman, J.D. Bjorken and J.B. Kogut, Phys.Rev. D4, 3388 (1971).

3) S.D. Drell and T.M. Yan, Phys.Rev. Letters 24, 855 (1970);
J.D. Bjorken, Phys.Rev. D7, 282 (1973);
M. Cronau, F. Ravndal and Y. Zarmi, Nuclear Phys. B51, 611 (1973).

4) J. Ellis, Phys.Letters 35B, 537 (1971);
J.D. Stack, Phys.Rev. Letters 28, 57 (1972);
O. Nachtmann, IAS Princeton preprint (1972);
H. Fritzsch and P. Minkowski, CERN preprint TH.1575 (1972);
J. Ellis and Y. Frishman, CALTECH preprint 68-377 (1973);
P. Minkowski, CERN preprint TH.1628 (1973).

5) H.J. Lipkin and E.A. Paschos, Phys.Rev.Letters 29, 525 (1972).

6) H. Harari, Phys.Rev.Letters 20, 1395 (1968);
P.G.O. Freund, Phys.Rev.Letters 20, 235 (1968).

7) H. Harari, Phys.Rev.Letters 24, 286 (1970);
P.V. Landshoff and J.C. Polkinghorne, Nuclear Phys. B28, 225 (1971).

8) H. Leutwyler and J. Stern, Phys.Letters 31B, 458 (1970);
E.D. Bloom and F.J. Gilman, Phys.Rev.Letters 25, 1140 (1970);
V. Rittenberg and H.R. Rubinstein, Phys.Letters 35B, 50 (1971);
J. Iizuka, M. Kobayashi and H. Nitto, Progr.Theor.Phys. 45, 482 (1971);
S. Pallua and B. Renner, Nuclear Phys. B43, 331 (1972);
M. Chaichian, S. Kitakado, S. Pallua, B. Renner and J. De Azcárraga, Nuclear Phys. B51, 221 (1973);
E. Del Giudice, R. Musto, P. Di Vecchia and S. Fubini, CERN preprint TH.1553 (1972);
M. Chaichian, S. Kitakado and H. Satz, Bielefeld preprint Bi-73/01.

9) C.H. Llewellyn Smith, Nuclear Phys. B17, 277 (1970);
J. Kuti and V.F. Weisskopf, Phys.Rev. D4, 3418 (1971).

10) H. Harari, Phys.Rev. Letters 22, 562 (1969);
J.L. Rosner, Phys.Rev.Letters 22, 689 (1969).

11) P.V. Landshoff and J.C. Polkinghorne, Nuclear Phys. B33, 221 (1971);
C.E.A. Pantin, Nuclear Phys. B46, 205 (1972);
R.L. Kingsley, Cambridge preprint DAMTP 72/49 (1972);
S. Pallua and Y. Zarmi, CERN preprint TH.1587 (1972);
M. Chaichian and S. Kitakado, CERN preprint TH.1600 (1972).

12) M. Chaichian, S. Kitakado, S. Pallua and Y. Zarmi, CERN preprint TH.1626 (1973).

13) See, e.g.,
W.R. Frazer et al., Revs.Modern Phys. 44, 284 (1972) and references therein.

14) A.H. Müeller, Phys.Rev. D2, 2963 (1970).

15) R.P. Feynman, Phys.Rev.Letters 23, 1414 (1969).

16) G. Altarelli and L. Maiani, University of Rome preprint INFN-421 (1972).

17) S-H.H. Tye and G. Veneziano, CERN preprint TH.1552 (1972).

18) J.D. Bjorken in Ref. 3);
A. Schwimmer, CALTECH preprint 68-376 (1972).

19) H.W. Kendall, in Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies (N.B. Mistry, Editor) Cornell University (1972);
P.V. Landshoff and J.C. Polkinghorne, Phys.Letters 34B, 621 (1971).

20) D.H. Perkins, Review Talk given at the XVI International Conference on High Energy Physics, Chicago-Batavia (1972).

21) Chan Hong-Mo, C.S. Hsue, C. Quigg and J.M. Wang, Phys.Rev.Letters 26, 672 (1971);
Chan Hong-Mo and P. Hoyer, Phys.Letters 36B, 79 (1971).

22) I. Damman et al., DESY preprint 72/71 (1972).

23) M. Chaichian and S. Kitakado, CERN preprint TH.1640 (1973).

24) C. Michael, in Springer Tracts in Modern Phys. Vol. 55, Edited by G. Höhler (Springer Verlag 1970).

25) A. Yao, Harvard University, Ph.D. Thesis (1972).

26) G. Parisi, Nuovo Cimento Letters 3, 395 (1972).

27) J. Ballam et al., SLAC preprint, SLAC-PUB-1165 (1972).

FIGURE CAPTIONS

Figure 1 :

The process $\ell + N \rightarrow \ell' + h +$ anything.

Figure 2 :

The photon-parton Breit frame description of deep inelastic interactions.

a) Before the current interacts with a quark.

b) After the interaction has occurred. The ejected quark now moves to the left and fragments into left-moving hadrons, while the right-moving quarks fragment into right-moving hadrons.

Figure 3 :

The duality content of the current fragmentation region for a baryon target and an observed meson, in the quark parton model.

a) Resonance-resonance (vV) term.

b) Resonance-background (vS) term.

c) Background-resonance (sV) term.

d) Background-background (sS) term.

Figure 4 :

The duality content of the target fragmentation region for a baryon target and an observed meson, in the quark parton model.

a) $v\tilde{V}$ term.

b) $v\tilde{S}$ term.

c) $s\tilde{V}$ term.

d) $s\tilde{S}$ term.

Figure 5 :

The diagrams contributing to the pure valence term in the observation of a baryon (h) in the fragmentation region of a baryon target (p).

a) One valence quark common to target and observed hadron.

b) Two valence quarks common to target and observed hadron.

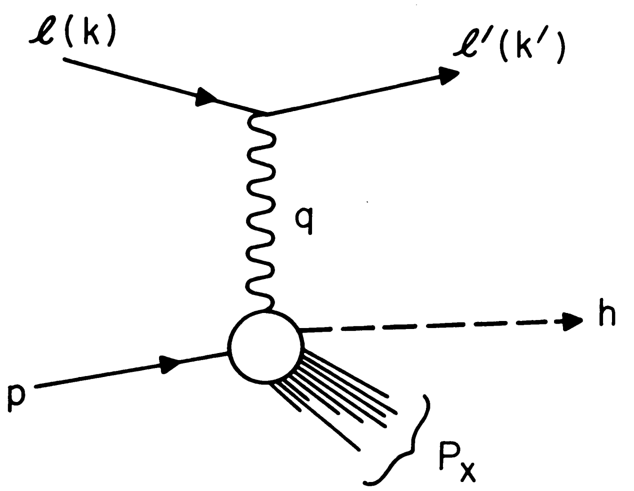

FIG.1

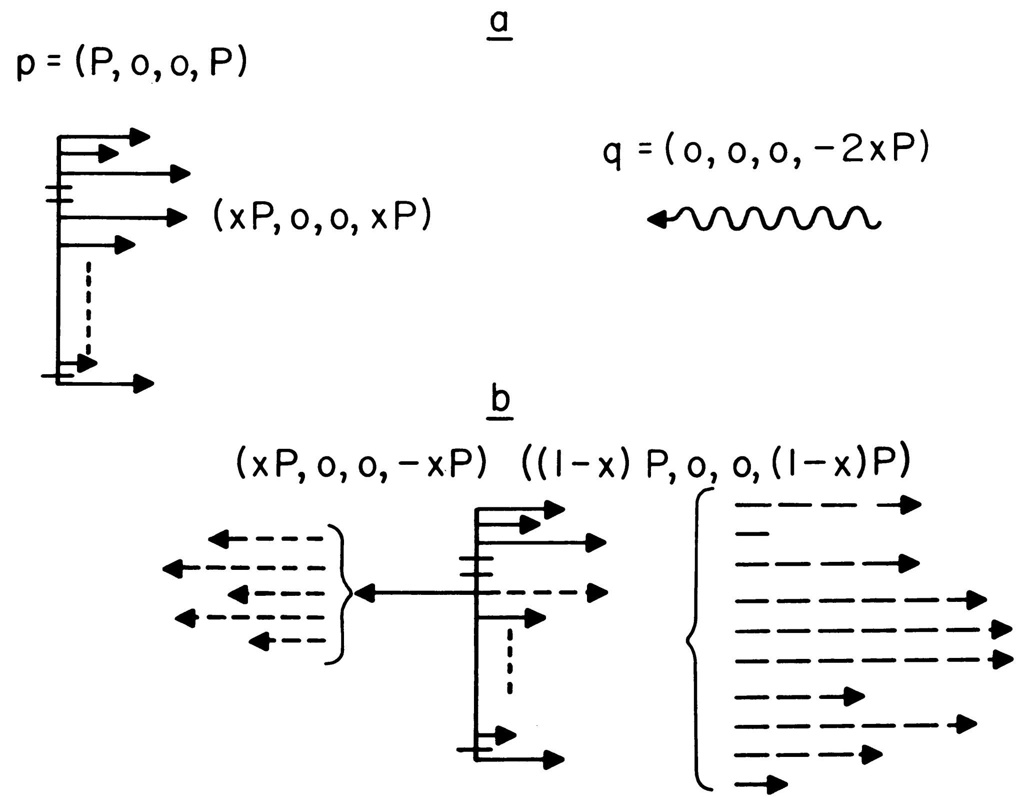

FIG.2

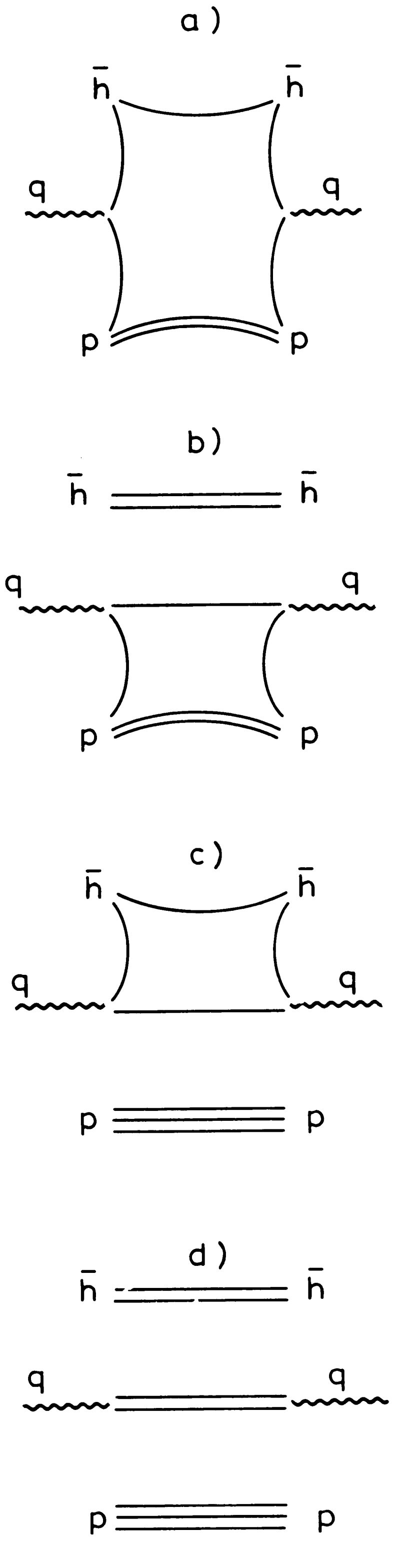
a)
$\bar{h}$ $\bar{h}$
q q
p p
b)
$\bar{h}$ $\bar{h}$
q q
p p
c)
$\bar{h}$ $\bar{h}$
q q
p p
d)
$\bar{h}$ $\bar{h}$
q q
p p

FIG. 3

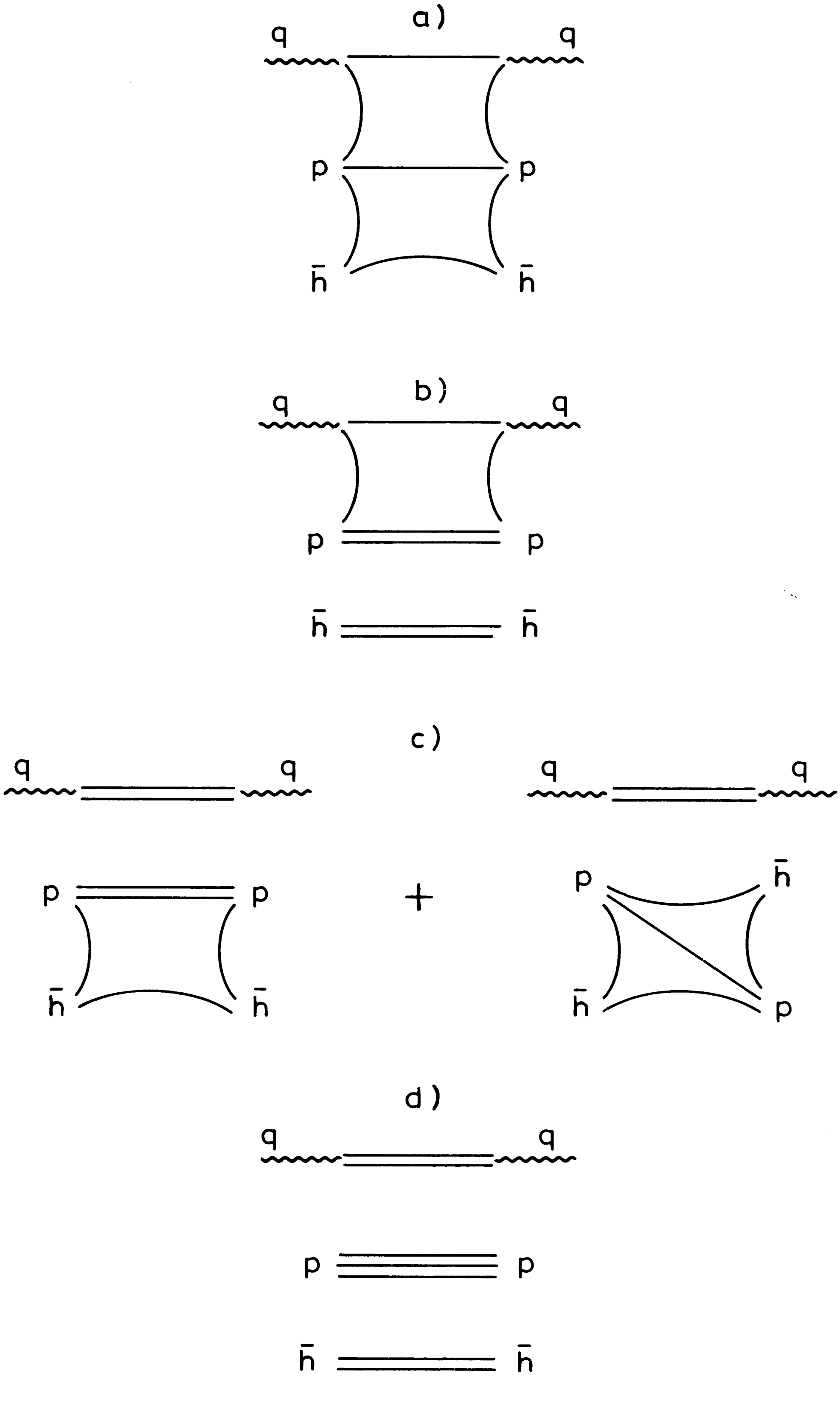

FIG.4

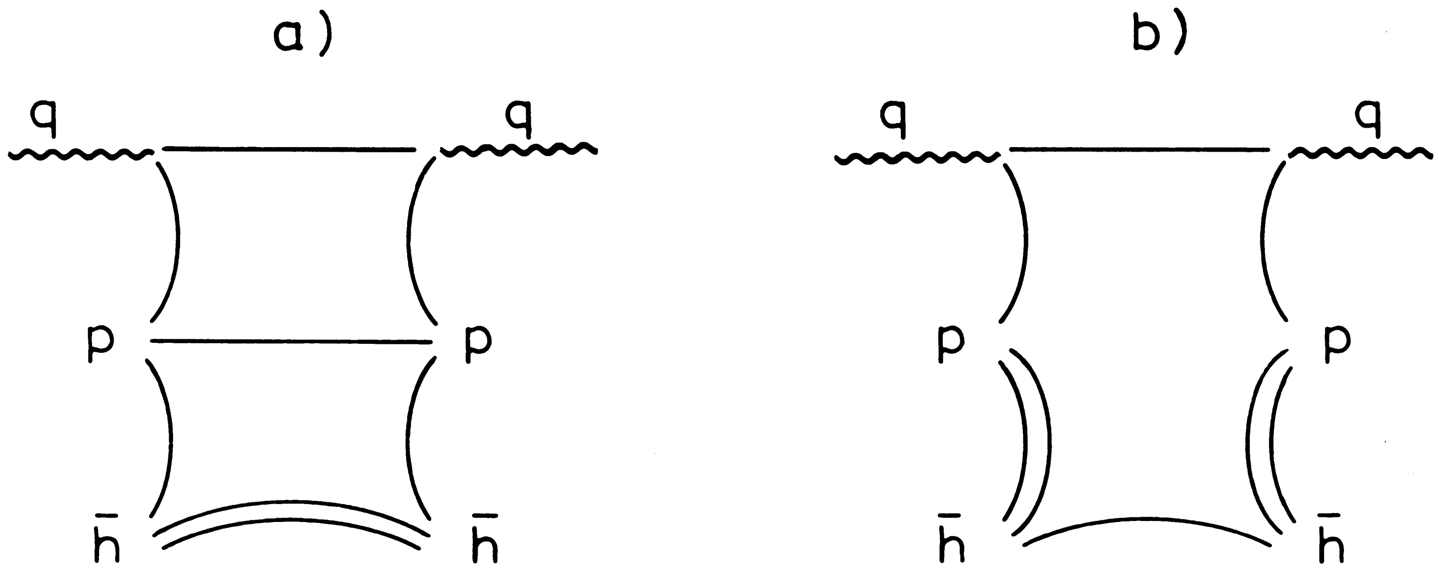

FIG.5