Archives

Ref.TH.1725-CERN

CANCELLATION OF THE LEADING DIVERGENCE IN DUAL LOOPS

M.B. Green

CERN -- Geneva
and
Cavendish Laboratory - Cambridge *)

A B S T R A C T

We show that the leading divergence of the single meson loop of the dual pion model is cancelled by a similar divergence of the fermion loop in the model with no ghosts. There is still one remaining divergence at the one-loop level.

---

*) Address after September 25, 1973.

Ref.TH.1725-CERN
8 August 1973

The elegant structure of dual models has been further enhanced [1] by the discovery of the consistency of the gauges operating in the Neveu-Schwarz model [2] (pion model) and the Ramond model [3] (fermion model). We now know that these models can only be consistent in 10 space-time dimensions with the leading meson Regge pole intercept $\alpha_0 = 1$ and the leading fermion intercept $\bar{\alpha}_0 = \frac{1}{2}$. It is also precisely in this critical space-time dimension that the renormalization procedure for dual loops [4],[5] runs into trouble.

It is the purpose of this letter to point out a beautiful and tantalizing similarity that exists between the fermion loop and meson loop contributions to meson scattering amplitudes in the critical dimension of space-time (i.e., 10) and when the trajectory intercepts take their critical values. We will show that the leading meson loop divergence [6],[7] is exactly cancelled by that of the fermion loop (without a counter term). However, we are still left with a lower lying divergence to deal with.

We will discuss the significance of these cancellations at the end of the paper.

The fermion model is constructed from the two sets of operators, a_r and b_r where :

$$[a_r^\mu, a_s^\nu] = \delta_{r,-s}\, g^{\mu\nu} \qquad \{b_r^\mu, b_s^\nu\} = \delta_{r,-s}\, g^{\mu\nu} \tag{1}$$

with $a_{-m} = a_n^+$ and $b_{-m} = b_m^+$.

The field $\Gamma_\mu(z)$ is defined by

$$\Gamma_\mu(z) = \gamma_\mu + i\sqrt{2}\,\gamma_5 \sum_{n=1}^{\infty} \left(b_n^\mu z^{-n} + b_{-n}^\mu z^n\right) \tag{2}$$

The Dirac matrices satisfy the algebra

$$\{\gamma^\mu, \gamma^\nu\} = -2 g^{\mu\nu} \tag{3}$$

These matrices have a dimension, E, that depends on the number of dimensions of space-time.

$$E = 2^{D/2} \tag{4}$$

is the minimum value of E consistent with the Clifford algebra and the natural value to consider in any model.

The vertex operator for a fermion emitting a ground state meson is :

$$V_2(z) = \Gamma_5 \; : e^{ik.Q(z)} : \; / i\sqrt{2} \tag{5}$$

where

$$\Gamma_5 = \gamma_5 \, (-)^{\sum_{n=1}^{\infty} b_{-n} b_n}$$

The position operator, $Q(z)$ is as defined, for example, in Ref. 3). An important point is that we have chosen the normalization in Eq. (5) to be consistent with the normalization of the three meson vertex also given in Ref. 3). This relative normalization is fixed at the tree diagram level by duality [see Corrigan and Olive, Ref. 8)]. The propagator takes the form $(F_o - m/\sqrt{2})^{-1}$ where m is the ground state fermion mass (which we will put equal to zero from hereon). We have :

$$F_o^{\,2} = L_o = \pi^2 + \sum_{r=-\infty}^{\infty} r\, a_{-r} a_r + \sum_{m=-\infty}^{\infty} m\, b_{-m} \cdot b_m \tag{6}$$

The expression for the fermion loop is thus (in the F_2 formalism which is known to have a satisfactory $m = 0$ limit [1]) :

$$M^F = \int d^4k \; \mathrm{Tr} \left\{ V_2(1) \frac{1}{F_o} V_2(1) \;\ldots\ldots\; \frac{1}{F_o} \right\} \tag{7}$$

which transforms simply [1] (by substituting $F_o/L_o = 1/F_o$) into :

$$M^F = \frac{1}{2} \int d^4k \; \mathrm{Tr} \left\{ V_1(1) \frac{1}{L_o} V_1(1) \;\ldots\ldots\; \frac{1}{L_o} \right\} \tag{8}$$

where

$$V_1(z) = k\cdot\Gamma(z)\, V_2(z) \tag{9}$$

We emphasize the factor of $\frac{1}{2}$ in Eq. (8) (which arises in the transformation essentially because of double counting of fermions flowing round the loop in opposite directions). By writing $(L_o)^{-1} = \int_0^1 dx\, x^{L_o - 1}$, Eq. (8) becomes :

$$M^F = \frac{1}{2}\int d^4k \int \prod_{i=1}^{N} x_i^{-\bar{\alpha}(\pi_i^2)-\frac{1}{2}} dx_i \; P(w)\; T_r\left\{ x_1^R V_1\, x_2^R \ldots\ldots V_1 \right\} \tag{10}$$

where

$$\bar{\alpha}(\pi^2) = \frac{1}{2} - \pi^2 \qquad \pi_i = k + p_1 + \ldots + p_{i-1} \qquad w = x_1 x_2 \ldots x_N$$

and $R = L_o - \pi^2$.

Equation (10) is analogous to Eq. (3.8) of Ref. 6). We have inserted a function $P(w)$ to account for projection onto the space of physical states. By analogy with the conventional model and Neveu-Schwarz model [9)] we shall take $P(w)$ to be an inverse power of the "partition function" :

$$P(w) = \prod_{n=1}^{\infty} (1-x^n)^2 (1+x^n)^{-2} \tag{11}$$

This should be checked by using an appropriate physical state projection operator as was done for the other models [10)].

In Eq. (10), the trace is to be taken over the spinor indices as well as the operator modes. The terms involving the b operators may be separated and the loop momentum performed in the standard way [11),12)] giving

$$M^F = 2\pi^2 \int \prod_{i=1}^{N} dx_i\; \omega^{-1} \left[\frac{f(w)}{\ln^2 w}\right]^{-D+2} \phi_0(w)^{-2} \;*$$

$$* \prod_{1\le i<j\le N} \left[\psi(x_{ij})\right]^{2p_i\cdot p_j}\; T_r\left\{ x_1^{R_b} V_{1b} \ldots\ldots x_N^{R_b} V_{1b} \right\} \tag{12}$$

where

$$\phi_0(w) = \prod_{n=1}^{\infty} (1 + w^n)^2 \tag{13}$$

$$f(w) = \prod_{n=1}^{\infty} (1 - w^n)^2 \tag{14}$$

$x_{ij} = x_i\, x_{i+1} \cdots x_{j-1}$. R_b and V_b are the terms in L_0 and V_2 which contain the b operators. The functions ψ are defined in Ref. 12). The trace occurring in Eq. (12) may be evaluated by tedious algebra or by the elegant method of Clavelli and Shapiro [7], noting that the vertices $V_{1b}(p_i, x_i)$ have similar commutation properties to those of the Neveu-Schwarz model. The trace becomes

$$\mathrm{Tr}\left\{ x_1^{R_b} V_{1b} \cdots\cdots x_N^{R_b} V_{1b} \right\} = \sum_{P \in P'} \left[\phi_0(w) \right]^D \times$$

$$\times (-)^P \prod_{j=1}^{N/2} p_{i_{2j-1}} \cdot p_{i_{2j}}\, \chi_0^+ \left(x_{i_{2j-1} i_{2j}}, w \right) \mathrm{Tr}\{1\} \tag{15}$$

where the sum is over all permutations P that pair the momenta, the ordering of the momenta in any pair being irrelevant [this is the same type of sum that occurs in Refs. 6),7)]. The function χ_0^+ is given by

$$\chi_0^+ = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{x^n + (w/x)^n}{1 + w^n} \tag{16}$$

It is very similar to χ^+ [defined in Ref. 6)] but the sum runs over integers. Note the zeroth modes contribute $\frac{1}{2}$ since they must only be counted once. The factor $\mathrm{Tr}\{1\}$ in Eq. (15) signifies that we are left with a trace of the unit matrix over the spinor indices, which gives [see Eq. (4)] $2^{D/2}$. Putting expression (15) in Eq. (12) gives a form for M^F that is directly comparable with the Neveu-Schwarz model [Eq. (3.9) of Ref. 6)]. However, it will be more instructive to compare the models after making a Jacobi transformation into the standard set of variables which correspond to ordering the particles in a disc with a hole in it.

Let

$$\rho_i = x_1 \ldots x_i \quad , \quad \nu_i = \frac{\ln \rho_i}{2\pi i} \qquad \tau = \frac{\ln W}{2\pi i} \tag{17}$$

We shall use :

$$\chi_o^+(\rho_i, W) = \frac{i}{2}\, \Theta_3(0|\tau)\, \Theta_4(0|\tau)\, \frac{\Theta_2(\nu_i|\tau)}{\Theta_1(\nu_i|\tau)} \tag{18}$$

(obtained by observing that χ_o^+ is related to the Jacobi dnu function [13]).

We now transform to disc variables :

$$\nu_i' = \frac{\ln \rho_i}{\ln W} \qquad \tau' = -1/\tau \qquad z_i = e^{2\pi i \nu_i'} \tag{19}$$

$$r = e^{i\pi\tau'} = e^{2\pi^2/\ln W}$$

This gives :

$$\begin{aligned} \chi_o^+(\rho_i, W) &= \frac{i}{2}\, \frac{1}{\tau}\, \Theta_3(0|\tau')\, \Theta_2(0|\tau')\, \frac{\Theta_4(\nu'|\tau')}{\Theta_1(\nu'|\tau')} \\ &= \frac{1}{\tau}\, \chi^+(z_i, r^2 e^{2i\pi}) \end{aligned} \tag{20}$$

[The notation indicates that χ^+ contains odd powers of $re^{i\pi}$]

$$\chi^+(\rho_i, W) = \sum_{n=1}^{\infty} \frac{(\rho_i)^{n-\frac{1}{2}} + (W/\rho_i)^{n-\frac{1}{2}}}{1 + (W)^{n-1/2}} = \frac{1}{\tau}\, \chi^+(z_i, r^2) \tag{21}$$

$$\left[\frac{\phi_o(W)}{f(W)}\right] = \frac{1}{\sqrt{2}}\, (r^2)^{-1/48}\, (W)^{-1/24} \left[\frac{\phi_+(r^2 e^{2i\pi})}{f(r^2 e^{2i\pi})}\right] \tag{22}$$

$$\left[\frac{\phi_+(w)}{f(w)}\right] \equiv \prod_{n=1}^{\infty} \frac{(1+w^{n+1/2})}{(1-w^n)}$$

$$= (r^2)^{-1/48}\,(w)^{1/48}\left[\frac{\phi_+(r^2)}{f(r^2)}\right] \tag{23}$$

We notice the startling similarity of the Jacobi transformed expressions relevant to the fermion loop [Eqs. (20) and (22)] to those relevant to the meson loop [Eqs. (21) and (23)]. In fact, substituting in Eqs. (15) and (12), we see that the ratio of the integrands in the fermion loop to that in the meson loop [which we obtain by suitable transformations of Eq. (3.9) of Ref. 6] is just :

$$\frac{\text{Integrand for fermion loop}}{\text{Integrand for meson loop}} = \frac{\frac{1}{2}\,\mathrm{Tr}\{1\}\,2^{(2-D)/2}\,w^{1/2}}{w^{(D-2)/16}}\,(1+O(r)) \tag{24}$$

where we have expanded the relevant χ functions about $r = 0$ where they are analytic. In ten dimensions $(D = 10)$ we see that the ratio of the integrands is one (to leading order). But to account for Fermi statistics, we should include a minus sign for the fermion loop (this can be checked by requiring consistency with the Feynman rules in the zero slope limit or from unitarity). This means that the leading power of $1/r^2$ in the integrand for the meson loop [extracted by Jacobi transforming the ψ functions in Eq. (12)] is exactly cancelled by that arising in the fermion loop.

We can rewrite the whole integral in terms of the "disc variables" by making a Jacobi transform on the ψ functions à la Clavelli and Shapiro [7] to obtain

$$M^F = \mathop{\mathcal{L}t}_{\epsilon\to 0} \frac{2i}{(4\pi\epsilon)^{D/2}}\, e^{-2q\cdot q'} \int_0^1 dr\, r^{-2}\left[\frac{f(r^2e^{2i\pi})}{\phi_+(r^2e^{2i\pi})}\right] \oint \prod_{i=2}^{N}\frac{dz_i}{z_i}\, \times$$

$$\times\, \mathrm{Tr}\left\{ (r^2e^{2i\pi})^{L_0} \prod_{i=1}^{N} V(\pi_i, z_i)\right\} \tag{25}$$

where $V(\pi, z)$ is the Neveu-Schwarz vertex defined in Ref. 9). We have used the notation and normalization of Ref. 7). The integration range is :

$$0 = \nu_1' < \nu_2' < \ldots < \nu_N'$$

(the ϵ limit takes care of the zero modes).

We can see that the integrand in Eq. (25) is identical to that for the meson loop with the variable r replaced by $re^{i\pi}$ (i.e., the radius of the disc is negative). This clearly suggests a cancellation of the leading divergence since all factors in the integrand have a power series expansion about $r = 0$. To be precise, since the integrals diverge we can first cut off both the fermion and meson loop integrals at $r = \delta$, subtract them, and then let $\delta \to 0$. Note that we actually have the freedom to choose the cut-offs in the meson and fermion integrals to be different (δ and δ') in which case we would be left with an extra term :

$$\mathcal{L}t_{\delta, \delta' \to 0} \left(\frac{1}{\delta} - \frac{1}{\delta'} \right) F(0)$$

where $F(0)$ is the coefficient of $1/r^2$ in the integrand of Eq. (25) evaluated at $r = 0$. But $F(0)$ is precisely the Born term of the N point function as was shown in the conventional model by Neveu and Scherk [5] and can be absorbed in a coupling constant renormalization.

To summarize, we find a cancellation of the leading fermion and meson loop divergences if :

1) we work in $D = 10$ space-time dimensions ;

2) the fermion intercept takes its critical value $\bar{\alpha}_0 = \frac{1}{2}$ and the meson intercept $\alpha_0 = 1$;

3) the coupling of two fermions to a meson is normalized as in Eq. (5) relative to the three meson coupling which we have assumed to be as defined in Ref. 8) ; this is the duality constraint for the tree diagrams ;

4) the fermion physical state projection operator gives a factor of an inverse partition function in a fermion loop as it does in a meson loop.

The following points are more speculative.

A) - The leading term that survives in the integrand of the meson loop minus fermion loop also diverges (like $1/r$). This divergence is responsible for the breakdown of the usual renormalization procedure (in the critical number of space-time dimensions). One may be tempted to find a consistent counter term but this appears to be difficult. The natural choice is, by complete analogy with Neveu and Scherk [4] :

$$\tilde{F}(p_1 \cdots p_N) = \int_0^1 \frac{dr}{r} \left(\frac{F_m - F_f}{r} \right)_{r=0} \tag{26}$$

where F_m and F_f denote the coefficients of $1/r^2$ in Eq. (25) in the meson and fermion cases respectively. The structure of Eq. (26) is exhibited by expanding the integrand of Eq. (25) to first order in r using :

$$\phi_+(r^2) - \phi_+(r^2 e^{2i\pi}) = 2r + O(r^2) \tag{27}$$

and

$$\chi^+(x_{ij}, r^2) - \chi^+(x_{ij}, r^2 e^{2i\pi}) = 8r \sin\pi(\nu_i' - \nu_j') + O(r^2) \tag{28}$$

The resulting integral does not look like a single Born term [because of the factors $\sin(\nu_i' - \nu_j')$ in Eq. (28)]. It does not therefore have a simple interpretation as a renormalization of the original Born coupling.

B) - An alternative interpretation [14] of the remaining divergence arises from the observation that the breakdown of the usual renormalization procedure is linked with the presence of a massless scalar daughter of the Pomeron, in the critical space-time dimension, which can couple to the vacuum. This may be symptomatic of perturbing about an incorrect vacuum in which case no simple subtraction procedure could be expected to be satisfactory.

C) - Since there is, as yet, no unambiguous way of constructing twisted fermion loops we have been unable to look at the Pomeron arising from such diagrams. However, in the light of the cancellations that are taking place between fermions and mesons it seems very possible that the unwanted negative G parity Pomeron that plagues the meson model will also be cancelled [15]. It is also natural to speculate that the divergence cancellations will generalize to an arbitrary number of loops.

D) - Dual models are known to have profound connections with gauge theories in the zero slope limit when isospin is added à la Chan and Paton [16] (although this is not yet checked for the fermion model). Gauge invariance then plays the rôle of duality in fixing the relative magnitudes of couplings [see point 4) above]. It might, therefore, be instructive to look further to gauge theories for guidance. It is known that gauge theories do exist in which divergence cancellations occur between fermion and meson loops [17]. In such theories there are very strong constraints on the spectrum of fermions for any given gauge group. [For example, a SU(2) gauge theory of massless fermions requires either 11 isospin $\frac{1}{2}$ fermions or one isospin $\frac{1}{2}$ and one isospin $\frac{3}{2}$ fermion for such cancellations [18].] In drawing conclusions about dual models we should bear in mind crucial features that are lost in the zero slope limit, namely : the only dimensional parameter in the theory (the Regge slope α'), the massless scalar Pomeron and the pion tachyon. However, the fermion-boson loop divergence cancellations do suggest a further analogy with gauge theories which may provide strong restrictions on the particle spectrum when quantum numbers are added.

I am very grateful to P. Ramond for continual advice and encouragement. I also thank D. Olive and D. Amati for helpful conversations. I am grateful to the CERN Theoretical Study Division for its hospitality.

REFERENCES

1) L. Brink, D. Olive, C. Rebbi and J. Scherk - CERN Preprint TH. 1695 (1973).

2) A. Neveu and J.H. Schwarz - Nuclear Phys. B31, 86 (1971) ;
A. Neveu, J.H. Schwarz and C.B. Thorn - Phys.Letters 35B, 529 (1971).

3) P. Ramond - Phys.Rev. D3, 2415 (1971) ;
A. Neveu and J.H. Schwarz - Phys.Rev. D4, 1109 (1971) ;
C.B. Thorn - Phys.Rev. D4, 1112 (1971).

4) A. Neveu and J. Scherk - Phys.Rev. D1, 2355 (1970).

5) A. Neveu and J. Scherk - Nuclear Phys. B36, 317 (1972) ;
P. Goddard - Nuovo Cimento 4A, 349 (1971).

6) P. Goddard and R.E. Waltz - Nuclear Phys. B34, 99 (1973).

7) L. Clavelli and J.A. Shapiro - Rutgers Preprint 02-73 (1973).
For our purposes, Appendix C of this paper is the most relevant section.

8) E. Corrigan and D. Olive - Nuovo Cimento 11A, 749 (1972).

9) L. Brink and D. Olive - Nuclear Phys. B58, 237 (1973).

10) L. Brink and D. Olive - Nuclear Phys. B56, 253 (1973).

11) D. Amati, C. Bouchiat and J.L. Gervais - Nuovo Cimento Letters 2, 399 (1969).

12) D.J. Gross, A. Neveu, J. Scherk and J.H. Schwarz - Phys.Rev. D2, 697 (1970).

13) A. Erdélyi, Ed. - Bateman Manuscript Project, Higher Transcendental Functions, Vol. II, McGraw-Hill, New York (1953), and Table I of Ref. 7).

14) This has been advanced by D. Amati. I am grateful to him for several discussions of dual loop renormalization.

15) See also : P. Ramond - CERN Preprint TH. 1716 (1973).

16) A. Neveu and J. Scherk - Nuclear Phys. B36, 155 (1972) ;
J. Scherk - Nuclear Phys. B31, 222 (1971).

17) G. 't Hooft - CERN Preprint TH. 1692 (1973).

18) I am grateful to G. 't Hooft for illuminating discussion of these gauge theories.