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FERMION MASSES AND WEAK ISOSPIN IN TECHNICOLOUR MODELS *)

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ABSTRACT

Isospin breaking is needed in models of dynamical symmetry breaking to generate up-down splittings in the quark and lepton mass matrices. It is shown how this isospin breaking can be introduced without upsetting the relation $M_W = M_Z \cos\theta_W$.

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In their seminal papers [1] on dynamical symmetry breaking, Susskind and Weinberg pointed out that the $M_W = M_Z \cos\theta_W$ relation (also called the "weak $\Delta I = \frac{1}{2}$ rule") would follow from isospin conservation by the "technicolour" (or "hypercolour") forces that break the electroweak gauge group. On the other hand, it is clear that isospin breaking must be present in the mechanism [2] through which quark and lepton masses are generated. The question thus arose whether these two requirements can be made compatible. The answer is simply yes. The general conditions under which one can have both $M_W = M_Z \cos\theta_W$ and $m_u \neq m_d$ will be found below. I follow here the discussion given in Ref. 3 although the question has also been discussed by several other authors [4].

The general requirement for an interaction to respect the weak $\Delta I = \frac{1}{2}$ rule is the following:

1. that it conserves electric charge
2. that it conserves a "custodial" SU(2), which is defined as a symmetry of both the Lagrangian and the vacuum, under which the generators (or gauge fields) of $SU_L(2)$ transform as a triplet.

It is easy to show [5] that the contributions to the $SU_L(2) \times U_Y(1)$ vector boson (mass)^2-matrix from any interaction that obeys these two requirements are such that: $\delta(M_Z)^2 = \delta(M_W)^2 \cos^2\theta_W$. The custodial symmetry must be SU(2) because SU(2) is the only group with a real representation of dimension three. The custodial SU(2) can but need not be isospin.

In the standard $SU_L(2) \times U_Y(1) \times SU^c(3)$ model with one elementary Higgs doublet, the strong colour interactions and the Higgs self-interactions do conserve "custodial" SU(2)'s, whereas the electroweak gauge interactions and the Yukawa interactions do not. Thus:

$$M_W = M_Z \cos\theta_W \left(1 + O(\alpha) + O\,\alpha\left(\frac{m_u^2 - m_d^2}{M_W^2}\right)\right) \qquad (1)$$

These corrections to the weak $\Delta I = \frac{1}{2}$ rule have been calculated [6] and should soon be tested experimentally. The custodial SU(2) for the strong interactions is of course isospin. The custodial SU(2) for the Higgs self-interactions is defined in Ref. 3. Note that if there are several Higgs doublets, the conservation of a custodial SU(2) is not automatic, and unless some special care is taken to impose one, Eq. (1) will be violated in higher orders of the Higgs self-interactions.

To assure the $\Delta I = \frac{1}{2}$ rule in technicolour models, we must require that the technicolour (TC) interactions conserve a custodial SU(2). We assume that the quarks and leptons acquire their masses from the TC condensates through broken "Extended Technicolour" (ETC) gauge interactions as described in Ref. 2. Clearly, if the custodial SU(2) is TC isospin, then ETC must violate isospin in order that we may have $m_u \neq$ $\neq m_d$, $m_c \neq m_s$, ... This strategy can in general be implemented since the ETC flavour symmetry is always a subgroup of (and therefore in general smaller than) the TC flavour symmetry (see Fig. 1a). As an example, consider the gauge group $SU_L(2) \times U_Y(1) \times SU^c(3) \times SU^{ETC}(3) \times Sp^{T'C}(6)$ and the (left-handed) fermion representation content:

$$\begin{aligned} \begin{pmatrix} U \\ D \end{pmatrix} &= (2, \tfrac{1}{6}, 3, 3, 1) \\ \bar{U} &= (1, +\tfrac{2}{3}, \bar{3}, \bar{3}, 1) \\ \bar{D} &= (1, +\tfrac{1}{3}. \bar{3}, 3, 1) \\ \mathcal{F} &= (1, 0, 1, \bar{3}, 6) \end{aligned} \qquad (2)$$

T'C and the fermions $\mathcal{F}$ have been introduced to break $SU^{ETC}(3) \rightarrow SU^{TC}(2)$ at a scale $\Lambda^{T'C} \simeq 30$ TeV through the condensate:

$$<\mathcal{F}^a_i(-i\sigma_2)\ \mathcal{F}^b_j\ \eta^{ij}\ \varepsilon_{abc}>_0\ \sim\ \delta_{c3} \qquad (3)$$

which is a Lorentz scalar, T'C singlet and ETC triplet. The quark multiplets break up into techniquarks (TC doublets) and ordinary quarks (TC singlets):

$$\begin{aligned} \begin{pmatrix} U \\ D \end{pmatrix} &= \left(\begin{array}{cc|c} U_1 & U_2 & u \\ D_2 & D_2 & d \end{array}\right) = (2, \tfrac{1}{6}, 3, 2+1, 1) \\ \bar{U} &= (\bar{U}_1\ \ \bar{U}_2 \ |\ \bar{u}) = (1, -\tfrac{2}{3}, \bar{3}, 2+1, 1) \\ \bar{D} &= (\bar{D}_1\ \ \bar{D}_2 \ |\ \bar{d}) = (1, +\tfrac{1}{3}, \bar{3}, 2+1, 1) \end{aligned} \qquad (4)$$

of $SU_L(2) \times U_Y(1) \times SU^c(3) \times SU^{TC}(2) \times SU^{T'C}(6)$. The TC flavour symmetry includes the chiral isospin group $SU_L(2) \times SU_R(2)$ which gets broken spontaneously down to TC isospin $SU_{L+R}(2)$ when the techniquarks condense:

$$<\bar{U}U + \bar{D}D>_0\ \simeq\ (\Lambda^{TC})^3\ \simeq\ (\tfrac{1}{2}\ \text{TeV})^3 \qquad (5)$$

TC isospin is a custodial symmetry and therefore the weak $\Delta I = \frac{1}{2}$ rule will be assured to all orders of the strong TC interactions. On the other hand, we will have $m_u \neq m_d$ since ETC violates isospin ($U_R \sim 3^{ETC}$ whereas $D_R \sim \bar{3}^{ETC}$). Indeed we find in lowest order of ETC/TC vector boson exchange:

$$m_u \simeq \frac{g^2_{ETC}}{m^2_{ETC}} \langle \bar{U}U \rangle_0 \ , \quad m_d = 0 \qquad (6)$$

The corrections to the weak $\Delta I = \frac{1}{2}$ rule due to the isospin violation in the ETC interactions are of the same order of magnitude as those due to the isospin violation in the Yukawa couplings of an elementary Higgs doublet. They are acceptably small provided $m_u^2 - m_d^2 \lesssim M_W^2$ (see Eq. (1)).

In general a gauge interaction is isospin symmetric if its gauge group has real fermion representation content [3]. It is then interesting to note that if ETC "tumbles" [7] down to TC, ETC will automatically violate isospin whereas TC will conserve isospin. See Ref. 8 for an attempt to combine the ideas of "custodial SU(2)" and "tumbling" into a realistic model.

If ETC conserves isospin, TC will conserve isospin as well and the only way in which one can have $m_u \neq m_d$ is by having the TC condensates break isospin spontaneously. However, this will upset $M_W = M_Z \cos\theta_W$ unless we can impose a custodial SU(2) other than isospin. This can be done provided the flavour symmetry of the TC interactions is sufficiently large (see Fig. 1b). An example of such a model was constructed in Ref. 3.

Finally we note that $\nu_\ell - \ell$ splittings can be obtained in the lepton mass matrix by giving large Majorana masses to the right-handed neutrinos [9].

In summary, Fig. 1 represents the two different ways in which $M_W = M_Z \cos\theta_W$ can be made compatible with $m_u \neq m_d$. In the first mechanism (Fig. 1a), isospin is conserved by the TC interactions and the TC condensates but is not a symmetry of ETC. In the second mechanism, isospin is a symmetry of both the ETC and TC interactions but is violated spontaneously by the TC condensates. The custodial symmetry is a SU(2) other than isospin.

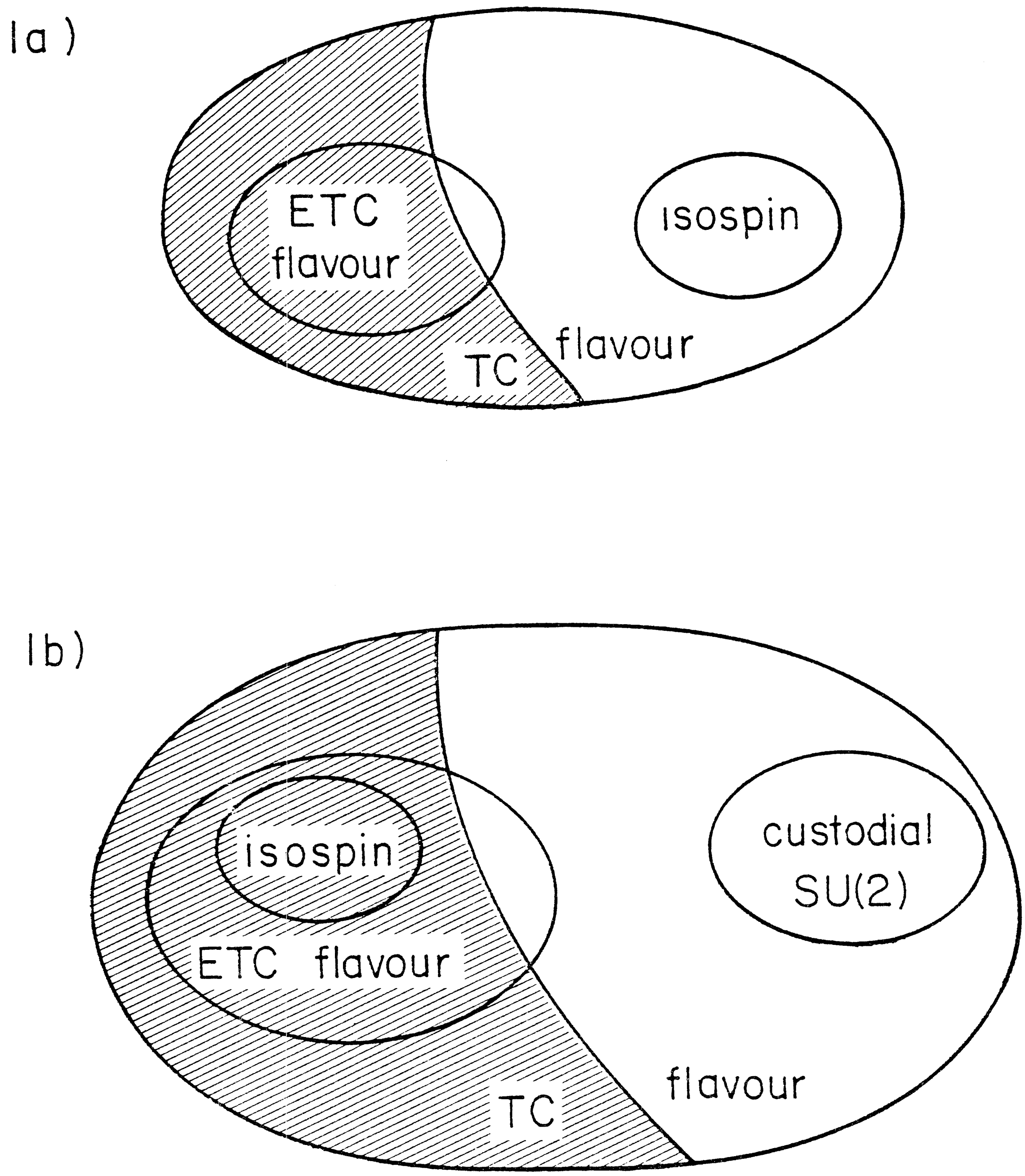

Fig. 1 : The two flavour symmetry geographics that assure simultaneously $M_W = M_Z \cos\theta_W$ and $m_u \neq m_d$. The shaded areas represent the flavour symmetries which are spontaneously broken by the TC condensates.

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