EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

THE ANOMALOUS MAGNETIC MOMENT OF POSITIVE AND NEGATIVE MUONS

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ABSTRACT

The anomalous g-factor $a \equiv (g-2)/2$ has been measured for muons of both charges in the Muon Storage Ring at CERN. The two results, $a_{\mu^+} = 1165910(12) \times 10^{-9}$ and $a_{\mu^-} = 1165936(12) \times 10^{-9}$, are in good agreement with each other, and combine to give a mean $a_\mu = 1165922(9) \times$ $\times\ 10^{-9}$, which is very close to the most recent theoretical prediction $1165921(10) \times 10^{-9}$. For the experimental results, the total statistical and systematic error is given. The measurements thus confirm the remarkable QED calculation plus hadronic contribution, and serve as a precise verification of the CPT theorem for muons.

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This letter presents the latest results on the muon g-factor anomaly, $a \equiv (g-2)/2$, from the CERN Muon Storage Ring. Measurements of the muon lifetime and electric dipole moment will be reported separately.

Preliminary results for the positive muon were presented early in 1975 [1] together with a description of the measurement technique. The experiment has already been discussed in several review articles [2], but nevertheless before detailing the new results for muons of both signs we would like to recall the principles of the method.

Owing to radiative corrections, the g-factor of the charged leptons is slightly larger than the value 2 predicted by Dirac theory, and an observable consequence of this is that, in a magnetic field, the particle spin rotates faster than its momentum vector. The angular frequency of this relative precession is

$$\vec{\omega}_a = \frac{-e}{mc}\, aB, \qquad 2\pi f_a = |\vec{\omega}_a| \,, \tag{1}$$

and its measurement is used as a means of obtaining the value of the anomaly. The precision with which this can be done depends largely on how well the mean magnetic field, seen by the muon population, is known. This requirement is more easily satisfied if the field is uniform, but in such a case the particle motion is not constrained in the direction of the field. Using an electric field to achieve the necessary trapping, shifts the angular frequency of the relative precession to the value

$$\vec{\omega}_a' = \vec{\omega}_a - \frac{e}{mc}\left\{\frac{1}{\gamma^2 - 1} - a\right\} \vec{\beta} \times \vec{E} \,, \tag{2}$$

where we have assumed throughout that both $\vec{B}$ and $\vec{E}$ are transverse to $\vec{\beta}$. As can be seen, the magnitude of this frequency shift depends upon the particle energy, and is zero if $\gamma = [1 + (1/a)]^{1/2} = 29.3$. At this energy and for this combination of fields the spin motion closely approximates to that of the ideal situation in which the particles circulate in a uniform magnetic field. For muons the corresponding momentum is 3.094 GeV/c.

effects by dividing them into various subgroups according to different characteristics such as beam intensity, counter position, decay electron energy, and starting time of the fit. No effects were found beyond those consistent with statistical fluctuations.

The experiment consists essentially of a measurement of the ratio R of two frequencies: the relative spin precession frequency ω_a; and the mean magnetic field seen by the muon sample, expressed in terms of the effective mean proton resonance frequency in vacuum $\bar{\omega}_p$. This latter is related to the mean field $\bar{B}$ and the mean muon Larmor frequency in vacuum $\bar{\omega}_\mu$, by

$$\frac{e\bar{B}}{mc} = \frac{2}{g}\,\bar{\omega}_\mu = \frac{2}{g}\,\lambda\bar{\omega}_p \qquad (\lambda = \omega_\mu/\omega_p) \;. \tag{5}$$

In the following we summarize the findings of an examination of the possible sources of error, dividing them into two groups in so far as they affect these two frequency measurements.

Possible sources of systematic effects on the observed muon spin precession frequency derive from the conditions under which the data are collected. In addition to the decay electrons, the 22 shower detectors are exposed to an initial flash (bunch length $\sim$ 10 nsec) of about 10^7 particles, which the pion beam transports to the storage ring every PS burst. The photomultipliers are blanked off during this time, thereby largely protecting the fast electronics, but the tubes themselves exhibit subsequent gain changes during the muon storage time. The over-all gain changes of the photomultiplier and fast electronics system, measured by a method using light-emitting diodes, were shown to have no effect on f_a. The uniformity of the timing of the arrival of pulses was established with an accurate determination of the time slewing, which was combined with this gain measurement. The effect of time slewing on the frequency f_a was shown to be less than 1 ppm.

The correct functioning of the time-measuring devices or digitrons [6] was established by the internal consistency of the data and by periodical checks with a special test program. The linearity of the digitrons, for time intervals of the order of the stored muon lifetime, was shown to be good to a small fraction

The crystal clock used for measuring the NMR frequency both when mapping and monitoring the field, was the same as that used for measuring ω_a; thus the effect of drifts in its frequency largely cancelled out. The absolute values of the magnetic field given by the monitoring probes were consistent with those obtained using the mapping device to better than 0.5 ppm. Thus the slow changes in the magnetic field level between full maps could be reliably followed and corrected for. This correction is combined with that due to the small drift in master clock frequency on the third line of Table 2.

The next correction arises because the field maps are made in the absence of vacuum tank, electrodes, and inflector. Each of these effects has been measured separately.

The actual distribution of the muon orbits can be obtained by observing how the initial bunch of muons circulating round the ring spreads out in time [4]. This information is used in a Monte Carlo orbit tracking program to obtain the correction for the previously assumed ideal phase-space distribution as given in the fifth line of Table 1.

The spread in the orbits also means that not all the muons have the momentum (3.094 GeV/c) necessary to nullify the effect of the electric field on the muon spin. This can be treated as a small shift in the magnetic field, and the relevant correction is shown in the sixth line, while immediately below, the pitch correction [11] due to the betatron motion of the particles is given.

The result of all these terms is seen in the statement of $\bar{f}_p$, for a particular run, given at the bottom of Table 2. In Table 1 we give the individual frequencies f_a and $\bar{f}_p$ together with their ratio R for each of the nine experimental runs, and also the weighted average values of R for μ^+, μ^-, and the combined data. The errors quoted are statistical on f_a and systematic on $\bar{f}_p$. All twelve R values are plotted in Fig. 2.

The over-all average value of R is $3.707213(27) \times 10^{-3}$. The error contributions are 7.0 ppm (statistical on f_a) and 1.5 ppm (systematic on the magnetic field), over-all 7.2 ppm.

We can examine this agreement in terms of the muon being a composite structure. Then as Brodsky [17] has pointed out, the coupling to an internal charge current would lead to a modification $\Delta a_\mu \simeq 0(m_\mu/m^*)$ where m^* is a characteristic internal mass, the mass of the first excited state, or continuum threshold. Then the lower limit for m^* is of the order of 10^5 GeV/c^2.

Finally, any inequality between the g-factors for positive and negative muons would violate CPT. These measurements yield $(g_{\mu^-} - g_{\mu^+})/g = 0.026 \pm 0.017$ ppm and represent the most accurate test of the CPT theorem applied to muons.

It is a pleasure to express our thanks to those who, over the two-year period of data-taking, have both helped in the running of the experiment and made specific contributions. We specially thank R.W. Williams and S. Wojcicki for their active participation in many different phases of the experiment. We warmly appreciate the work of E.M. McMillan on the stability of the muon orbits and selection of the field index value. We thank M. Rousseau for his excellent work on the fast electronics, W. Lysenko for his calculation of the effect of the electric field on machine stability, and G. Frémont and K. Mühlemann for their unfailing technical support. We thank M. Comyn who also cheerfully took part in the runs. Warm thanks are due to G. Lebée and O. Runolfsson for their crucial contributions regarding the inflector and the field mapping, respectively.

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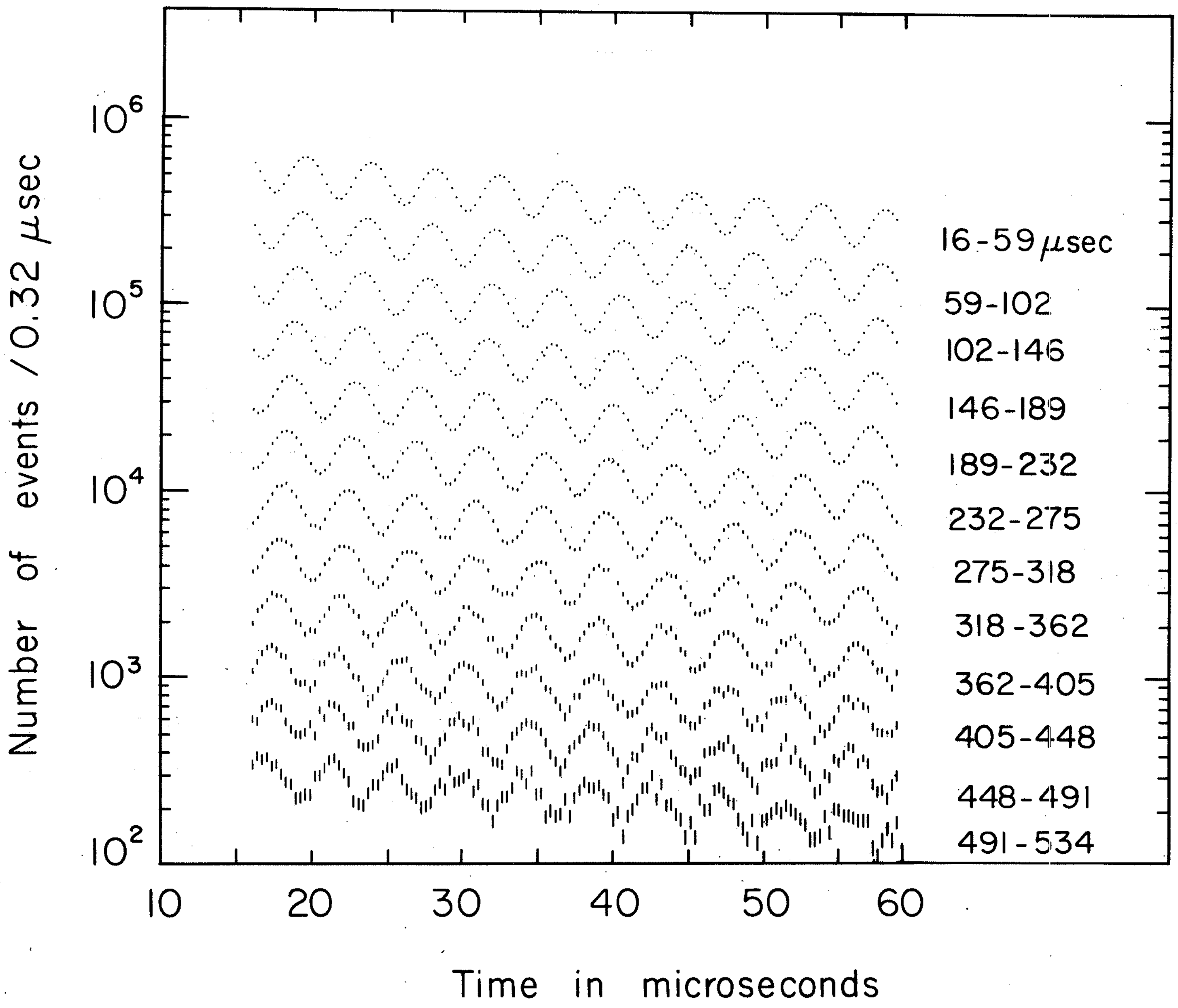
Table 3

Contributions to the theoretical value of a_μ

a_μ (QED)	$1165851.8(2.4) \times 10^{-9}$
a_μ (hadronic)	$66.7(9.4) \times 10^{-9}$
a_μ (weak)	$2(2) \times 10^{-9}$
a_μ (theory)	$1165921(10) \times 10^{-9}$



Number of events / 0.32 μsec
10^6
10^5
10^4
10^3
10^2
10
20
30
40
50
60
Time in microseconds
16-59 μsec
59-102
102-146
146-189
189-232
232-275
275-318
318-362
362-405
405-448
448-491
491-534

Fig. 1