SINGLE-PARTICLE STATES AROUND DOUBLE-MAGIC ^{132}Sn

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Abstract

Double-magic ^{132}Sn exhibits the strongest shell closure of any observed nucleus. The experimental data on single-particle states in the surrounding nuclei ^{131}In, ^{131}Sn, ^{133}Sn and ^{133}Sb are reviewed. Single-particle energies are calculated with a standard Woods-Saxon potential, and corrections resulting from a comparison with the ^{208}Pb region are applied.

1. The shell closure in ^{132}Sn

The ^{208}Pb region has an established position of simple structure, where the properties of elementary modes of excitation can be studied under well defined conditions. There is a long history $^{1)}$ of shell-model calculations for nuclei with a few valence nucleons outside of ^{208}Pb, moving in a restricted number of shells above and below the Fermi surface. Collective particle-hole $^{2)}$ and pair $^{3)}$ excitations have been identified and described with the aid of tractable calculations. Polarization phenomena corresponding to the coupling of simple-particle and collective degrees of freedom can also be investigated under sufficiently simple conditions.

More recently it has been found $^{4)}$ that ^{132}Sn shows a similar strong shell closure as ^{208}Pb. This has opened the possibility to explore the properties of simple states in the ^{132}Sn region under conditions which are comparable to but not identical with those in the ^{208}Pb region. The experimental difficulties around ^{132}Sn relate to the circumstance that these nuclei are displaced ~ 10 nucleons away from the line of β-stability to the neutron-rich side (Fig. 1). This implies that nuclear reaction studies are essentially excluded. The nearest approach $^{5)}$ seems to be the ^{136}Xe(d,^{3}He) reaction which leads to ^{135}I, 3 protons away from ^{132}Sn. The production method for ^{132}Sn and its closest neighbours is in practice limited to fission of actinides, in which case the heavy fragment has a reasonable chance to end up near ^{132}Sn. Spectroscopic studies of specific fission fragments have so far required the use of mass separation to clear up the spectra.

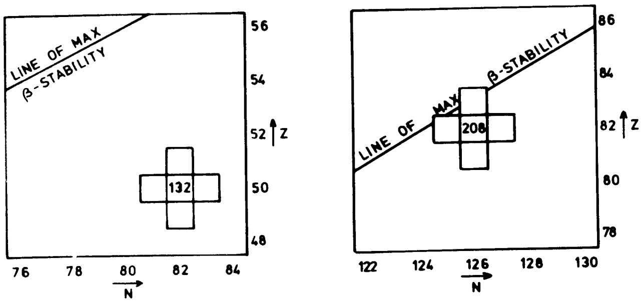

Fig. 1 The ^{132}Sn and ^{208}Pb regions relative to the line of maximum β-stability.

The presently known levels in ^{132}Sn are shown in Fig.2.

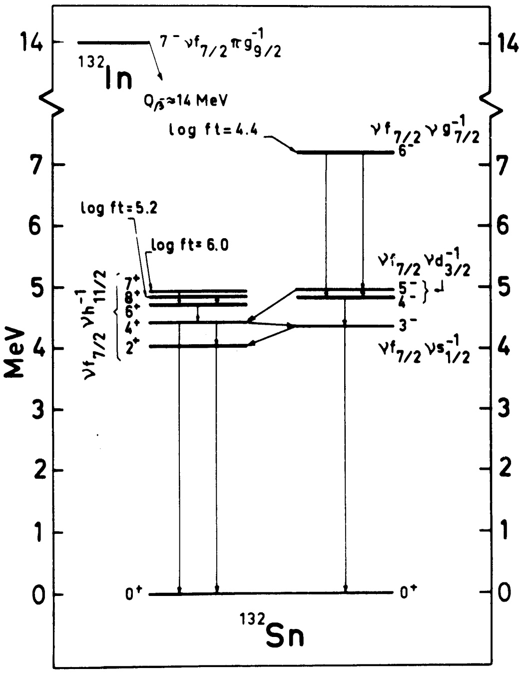

Fig. 2 Levels in ^{132}Sn

The information comes partly from the β-decay $^{6)}$ of ^{132}In, partly from the direct production in fission $^{7)}$ of the isomeric 8^+ state. The β-decay proceeds mainly by a fast Gamow-Teller transition to a particle-hole state at 7.2 MeV. The very large Q_β value and the predicted existence of other configurations which can be reached by strong 1st forbidden transitions implies that it should be possible to observe more levels in ^{132}Sn with reasonable intensities.

The energies of the lowest levels in ^{132}Sn and ^{208}Pb are compared in Fig.3.

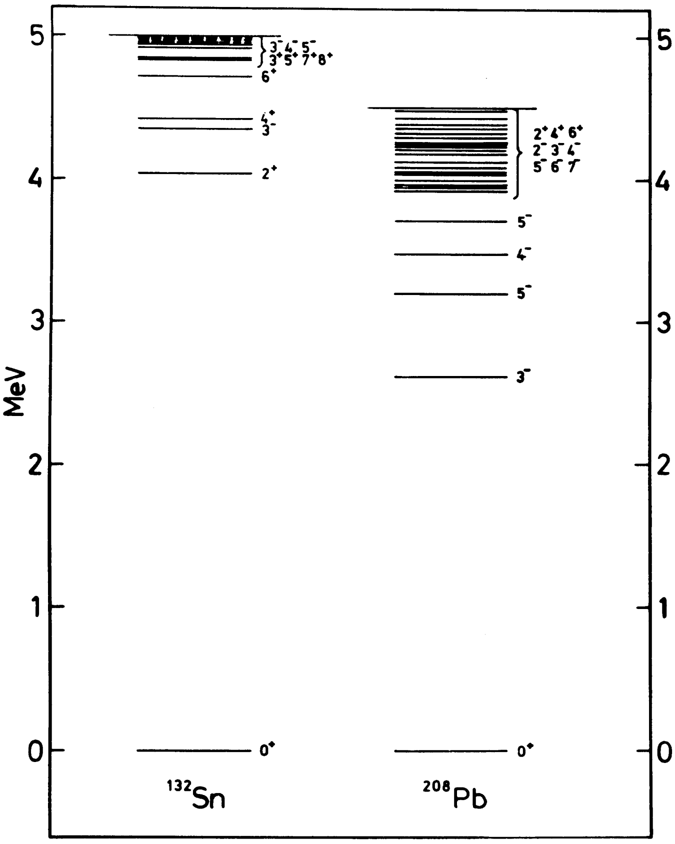

Fig. 3 All levels in ^{132}S below 5 MeV, and in ^{208}Pb below 4.5 MeV. The three dashed levels in ^{132}Sn are predicted by a shell-model calculation.

The 24 levels shown in ^{208}Pb have all been observed, and no further ones are expected to occur below 4.5 MeV. In ^{132}Sn three predicted levels have been included in order to complete the set of levels up to 5 MeV.

The most obvious feature in Fig.3 is the larger gap between the ground state and the first excited state in ^{132}Sn compared to ^{208}Pb. In fact, no other nucleus above ^{16}O has a gap as large as 4 MeV. Even allowing for an $A^{-1/3}$ dependence reflecting the variation with the size of the nucleus the gap is considerably larger in ^{132}Sn than in ^{208}Pb. From this point of view ^{132}Sn exhibits the strongest shell closure of any nucleus.

Looking in more detail at the excited levels one notices that the negative-parity states are very high in ^{132}Sn, while the positive-parity states occur at about the same excitation energies as in ^{208}Pb. The rapid lowering from ^{132}Sn to ^{208}Pb of the collective negative-parity states, in particular the 3$^-$ state, is not expected in the liquid drop model. It is probably connected with the smaller octupole strength of the lowest stretched non-spin-flip particle-hole excitations in ^{132}Sn,

$(\pi g_{7/2} p_{1/2}^{-1},\ \pi d_{5/2} p_{1/2}^{-1},\ \nu f_{7/2} s_{1/2}^{-1},$

$\nu h_{9/2} d_{3/2}^{-1})$

compared with the analogous excitations in

^{208}Pb $(\pi h_{9/2} d_{3/2}^{-1},\ \pi f_{7/2} s_{1/2}^{-1},\ \nu g_{9/2} p_{3/2}^{-1},$

$\nu i_{11/2} f_{5/2}^{-1})$.

The lowest positive-parity states are dominated by the $\Delta N = 0$ neutron excitation

$\nu f_{7/2} h_{11/2}^{-1}$ in ^{132}Sn and $\nu g_{9/2} i_{13/2}^{-1}$ in

^{208}Pb, with nearly equal single-particle energy differences.

2. Single-particle states in ^{131}In, ^{131}Sn, ^{133}Sn and ^{133}Sb.

Of the four nuclei differing by one nucleon from ^{132}Sn, the two farthest from stability (^{131}In and ^{133}Sn) have not yet been studied in any detail. Their ground state assignments are known, and a $p_{1/2}$ isomer has been found in ^{131}In at an unknown excitation energy, but not other excited states have been observed. The experimental difficulty is due to the rapid decrease of the fission yield with increasing N - Z. The other two single-particle nuclei have been studied in some more detail. The level schemes are shown in Fig.4 (^{131}Sn) and Fig.5 (^{131}Sb).

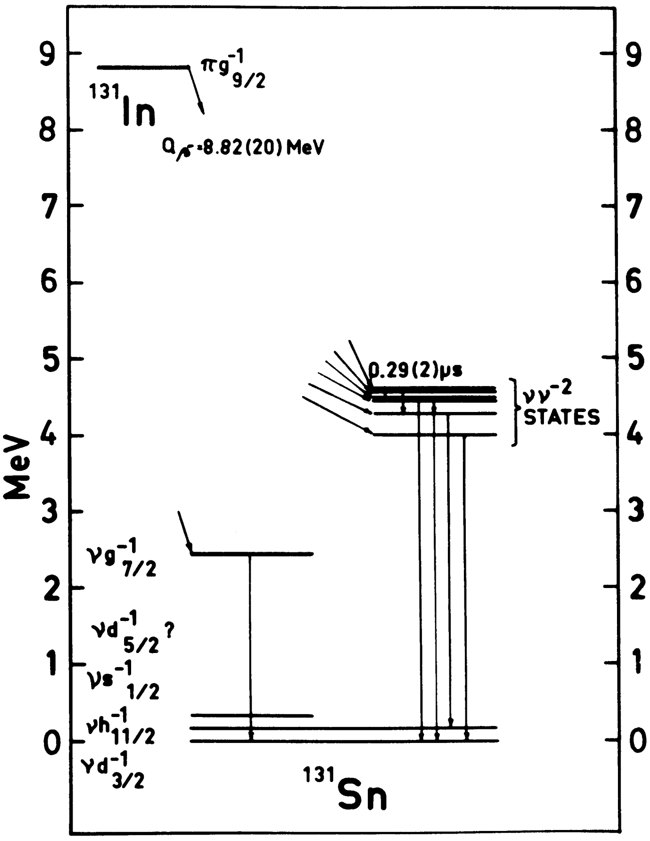

Fig. 4 Levels in ^{131}Sn.

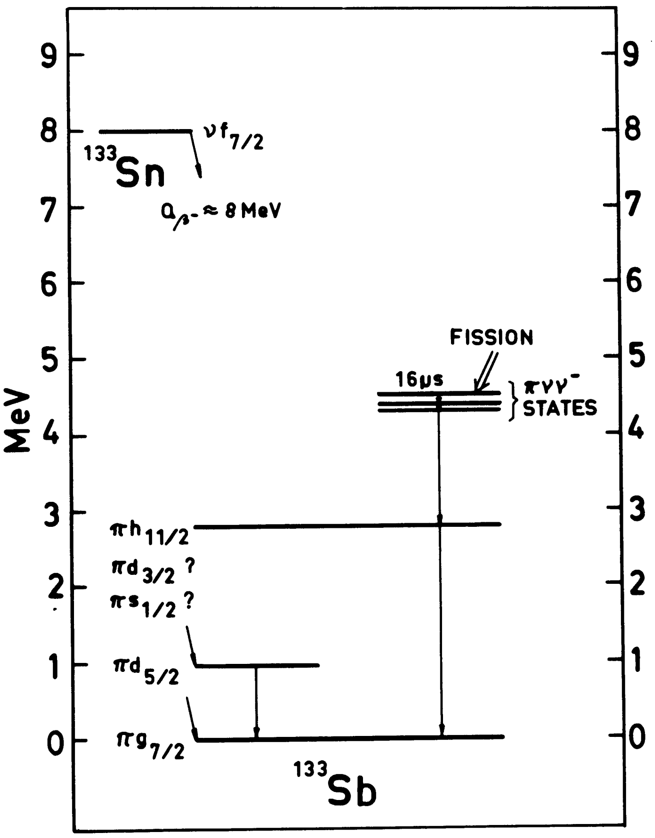

Fig. 5 Levels in ^{133}Sb.

The β-decay [8] of ^{131}In to ^{131}Sn proceeds mainly by a Gamow-Teller transition

$\pi g^{-1}_{9/2} \rightarrow \nu g^{-1}_{7/2}$.

An isomeric state in ^{131}In, which can be assigned

$\pi p^{-1}_{1/2}$

based on systematics from lighter In isotopes, decays to the

$\nu s^{-1}_{1/2}$ and $\nu d^{-1}_{3/2}$ states.

Recently a number of weak decay branches have been observed [9], leading to higher core-excited states in ^{131}Sn. One of these levels was found to be isomeric with

$T_{1/2} = 0.29(2)$ μs.

The detailed nature of the core-excited states is not known. The only neutron-hole state in the major shell between 50 and 82 which has not been identified in ^{131}Sn is $d_{5/2}$.

Levels in ^{133}Sb have been observed both in the β-decay [10] from ^{133}Sn and in direct fission [11] to a 16μs isomer in ^{133}Sb. The observed core-excited states between 4 MeV and 5 MeV are not understood in detail. Three of the five proton states in the major shell between 50 and 80 are firmly established in ^{133}Sb, but

$\pi s_{1/2}$ and $\pi d_{3/2}$

have not yet been identified.

In summary, 9 single-particle states have been observed out of a total of 20 in the four major shells above and below Z = 50, N = 82. In order to put these states on an absolute energy scale the nucleon separation energies $S_p(^{132}\mathrm{Sn})$, $S_n(^{132}\mathrm{Sn})$, $S_n(^{133}\mathrm{Sn})$ and $S_p(^{133}\mathrm{Sb})$ are needed.

3. Ground state masses of ^{131}In, ^{131}Sn, ^{132}Sn, ^{133}Sn and ^{133}Sb.

The masses of these nuclei are not all known. The experimental values that exist are based on measurements of total β-decay energies. The following mass excess values are adopted.

$ME(^{131}\mathrm{In}) = -68.50(25)$ MeV	ref. 8
$ME(^{132}\mathrm{Sn}) = -76.59(\ 8)$ MeV	ref.12
$ME(^{133}\mathrm{Sb}) = -78.98(21)$ MeV	ref.13

In order to estimate the masses of ^{131}Sn and ^{133}Sn we can apply mass relations derived from a shell model analysis of a simple $\pi\nu^{-1}$ state in ^{132}Sb and another $\nu\nu^{-1}$ state in ^{132}Sn.

The ground state of ^{132}Sb has $I^{\pi} = 4^-$ and the dominant configuration

$\pi g_{7/2}\ \nu d^{-1}_{3/2}$.

The particle-hole interaction energy is given by the difference between the proton separation energies

$$INT = S_p(^{133}\mathrm{Sb}) - S_p(^{132}\mathrm{Sb}).$$

This interaction energy is expected to be small due to the large angle between the

$g_{7/2}$ and $d_{3/2}$

angular momentum vectors in the I = 4 coupling. It may be taken equal to the interaction energy in the analogous

$\pi h_{9/2}\ \nu f^{-1}_{5/2}$, $I^{\pi} = 6^-$ state in ^{208}Bi

$$INT = S_p(^{209}\mathrm{Bi}) - S_p(^{208}\mathrm{Bi}) + E(6^-,^{208}\mathrm{Bi})$$
$$-E(^5/_2{}^-.^{207}\mathrm{Pb})$$
$$= +\ 0.03\ \mathrm{MeV}.$$

This value together with the measured mass excess 12) $ME(^{132}Sb) = -79.67(7)$ MeV gives

$$ME(^{131}Sn) = ME(^{132}Sn) + ME(^{132}Sb) - ME(^{133}Sb) - INT = -77.31(24) \text{ MeV}.$$

It is assumed that the error in the estimated interaction energy is smaller than the error from the measured mass excesses.

A similar analysis can be made for the 4^- state in ^{132}Sn at 4.83 MeV, which is described as a

$$\nu f_{7/2}\, d_{3/2}^{-1}$$

excitation. The particle-hole interaction energy is estimated by a comparison with the

$$\nu g_{9/2} f_{5/2}^{-1},\ I^\pi = 6^-$$

state in ^{208}Pb at 3.92 MeV to be

$$INT = -0.08 \text{ MeV}.$$

From this follows the mass excess

$$ME(^{133}Sn) = 2\, ME(^{132}Sn) - ME(^{131}Sn) + E(4^-, ^{132}Sn) - INT = -70.96(22) \text{ MeV}.$$

The single-nucleon separation energies derived from the mass excesses are

$S_p(^{133}Sb) = 9.68$ MeV $S_n(^{133}Sn) = 2.44$ MeV

$S_p(^{132}Sn) = 15.38$ MeV $S_n(^{132}S_n) = 7.35$ MeV

all with uncertainties of about 0.2 MeV.

4. Calculation of single-particle energies.

For somparison with the empirical energies and in order to fill the holes in the systematics of single-particle states around ^{132}Sn a calculation has been performed with a standard Woods-Saxon potential. This phenomenological approach is known to reproduce the experimental single-particle energies around ^{208}Pb with similar or better accuracy than the more fundamental approach based on Hartree-Fock theory. The results of a Woods-Saxon calculation for the ^{208}Pb region is shown in Fig.6.

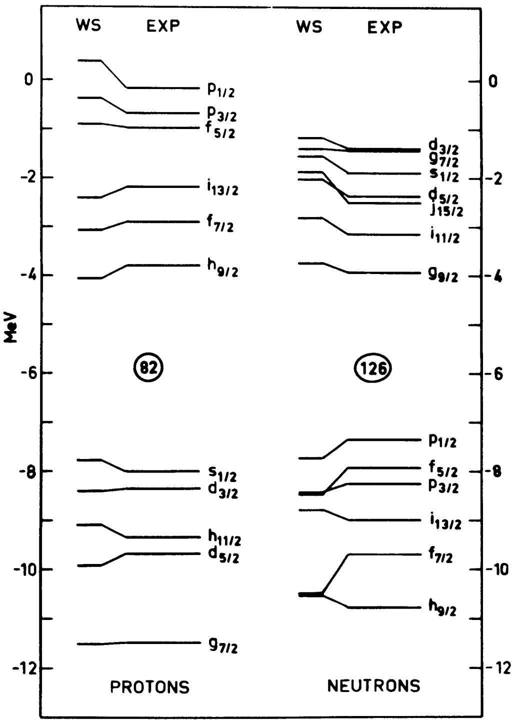

Fig. 6 Single-particle energies in the major shells around ^{208}Pb. The parameters of the Woods-Saxon potential are

$V_{op} = 59.0$ MeV, $V_{on} = 44.5$ MeV,

$V_{lsp} = 20$ MeV, $V_{lsn} = 19$ MeV,

$r_o = 1.27$ fm, $r_{ls} = 1.15$ fm,

$a = 0.70$ fm.

The Woods-Saxon potential summarizes different physical effects in a simplified manner. It is known from Hartree-Fock calculations that the bare single-particle energies around the Fermi surface are spread out, corresponding to an effective mass considerably smaller than the nucleon mass. This is compensated by a coupling of the single-particle states to collective vibrations, which compresses the spectrum so much that the resulting effective mass comes close to the nucleon mass. The extra large shifts of the neutron $j^{15}/_2$ and $f^7/_2$ states seen in Fig.6 are probably connected with strong near-resonance couplings to the low 2.6 MeV 3^- excitation.

The Woods-Saxon calculation for the ^{132}Sn region is illustrated in Fig.7.

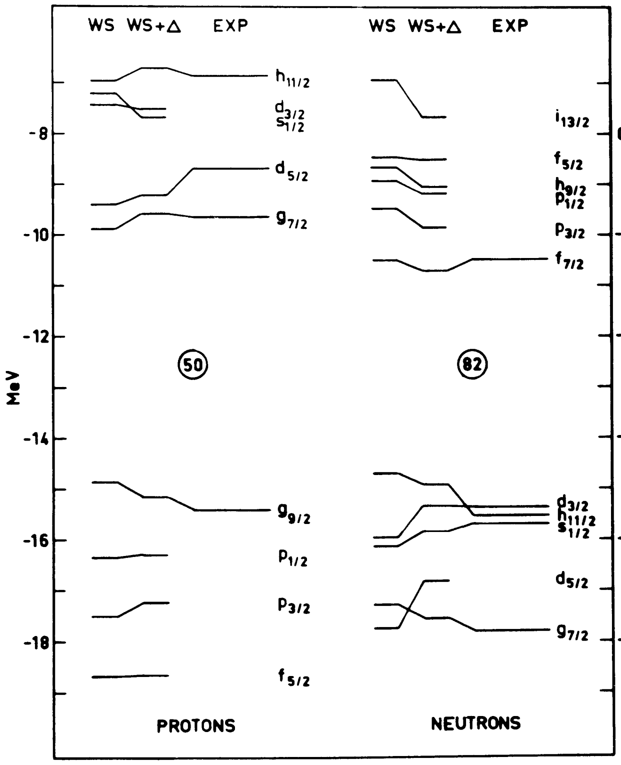

Fig. 7 Single-particle energies in the major shells around ^{132}Sn. The parameters of the Woods-Saxon potential are

V_{op} = 60.6 MeV, V_{on} = 43.5 MeV,

V_{lsp} = 20 MeV, V_{lsn} = 19 MeV,

r_o = 1.27 fm r_{ls} = 1.15 fm,

a = 0.70 fm.

The errors in the energies are expected to be of the same magnitudes in the ^{132}Sn and ^{208}Pb regions. Lacking a quantitative understanding of the finer details of the single-particle energies it may still be possible to correlate the differences between the Woods-Saxon and the experimental energies in the two regions. By comparing Figs. 6 and 7 one will notice a close similarity between the shell structures in the two regions. Every state in the ^{132}Sn region corresponds to one particular state in the ^{208}Pb region with the same radial quantum number n but one unit larger angular momenta l and j. With a few exceptions the correspondence also applies to the ordering and energy spacings of the single-particle states. This makes it natural to assume that the discrepancies between calculated and experimental energies have similar origin and should be roughly equal for the related levels in the two regions. For definiteness we may assume

$$\Delta_{nlj}(^{132}\mathrm{Sn}) = \left(\frac{208}{132}\right)^{1/3} \Delta_{nl+1\ j+1}(^{208}\mathrm{Pb})$$

where Δ is the difference

$$\Delta = \varepsilon_{WS} - \varepsilon_{EXP}.$$

These shifts are added to the Woods-Saxon energies in the ^{132}Sn region. The resulting energies and a comparison with the known experimental energies are shown in Fig.7 and Table 1.

Table 1. Calculated and experimental single-particle energies in the ^{132}Sn region (MeV).

Protons			Neutrons		
lj	WS+Δ	Exp	lj	WS+Δ	Exp
$h_{11/2}$	- 6.70	- 6.89	$i_{13/2}$	+0.33	
$d_{3/2}$	- 7.52		$f_{5/2}$	-0.51	
$s_{1/2}$	- 7.67		$h_{9/2}$	-1.05	
$d_{5/2}$	- 9.21	- 8.72	$p_{1/2}$	-1.18	
$g_{7/2}$	- 9.57	- 9.68	$p_{3/2}$	-1.86	
			$f_{7/2}$	-2.71	-2.44
$g_{9/2}$	-15.16	-15.38			
$p_{1/2}$	-16.30		$d_{3/2}$	-7.35	-7.35
$p_{3/2}$	-17.23		$h_{11/2}$	-6.92	-7.52
$f_{5/2}$	-18.65		$s_{1/2}$	-7.85	-7.68
			$d_{5/2}$	-8.83	
			$g_{7/2}$	-9.56	-9.78

The rms deviation of the calculated and measured values is 0.3 MeV. A similar accuracy is expected for those states which have not yet been observed.

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DISCUSSION

K. Aleklett: Which error bars on the masses in the ^{132}Sn region will improve your calculations?

J. Blomqvist: An improvement of the accuracy of some of the Q_β values to 50-100 keV would be very welcome.

K. Bleuler: Had you to change the parameters of the Wood-Saxon potential when going so far off from the stability line?

J. Blomqvist: Yes, the Woods-Saxon potential was allowed to vary from Pb to Sn. However, the changes were small, of the order of 1 MeV for the depth of the potentials.

F. Tondeur: When fitting your Woods-Saxon potential, did you allow a full free variation of all parameters, e.g. different radii for protons and neutrons, or different diffuseness?

J. Blomqvist: No. Of the 7 parameters only the two central potential depths were allowed to vary. The radii and diffuseness parameters were kept constant.