STUDY OF HEAVY Sn,Sb AND Te NUCLEI:A SHELL-MODEL DESCRIPTION OF THE INTERPLAY OF SINGLE-PARTICLE AND COLLECTIVE EXCITATIONS.

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Abstract.
A shell-model approach in order to study the interplay of collective and non-collective excitations, which is feasible for numerical calculations,is suggested. Study of some specific applications to odd-even and even-even nuclei near the doubly-closed shell nucleus $^{132}$Sn has been carried out.Finally, for the even-even nucleus $^{132}$Te,the fermion residual proton-neutron interaction is mapped onto a boson-boson interaction which enables us to describe the collective quadrupole excitations near $^{132}$Sn.

## 1. Introduction

In describing nuclei where the number of valence nucleons is not far from a closed shell configuration(± 1,±2,±3,±4),specific nucleon configurations remain such as to make a description in terms of collective excitations only,difficult or impossible. This happens to be the case for nuclei near the doubly-closed Z=50, N=82 $^{132}$Sn nucleus[1-3]. In trying to couple the extra proton degree of freedom,in describing heavy Sb,Te,I and Xe nuclei,with the underlying Sn core nuclear excitations,large deviations from the conventional macroscopic particle-core coupling mechanism are observed[4-8] (see fig.1). In order to attempt a full description

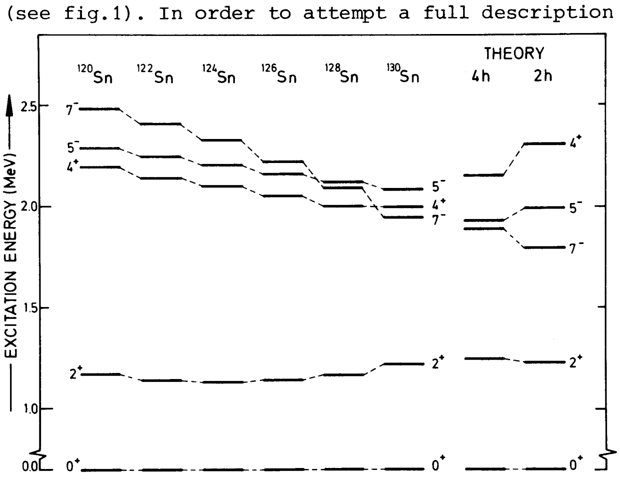


*Figure 1: Systematics for the $J_i^\pi = 2_1^+,4_1^+,5_1^-,7_1^-$ levels from $^{120}$Sn towards $^{130}$Sn. Comparison with the theoretical 2-hole($^{130}$Sn) and 4-hole($^{128}$Sn)spectra is made.*

of either the core system as well as of the particle core coupled configurations, one has to start from a shell-model approach.

Therefore, we first derive(sect.2)the basic equation for a shell-model approach to particle-core coupling,which is able to describe as well odd-even,odd-odd as even-even nuclei. In sect.3, we discuss some specific applications to nuclei near $^{132}$Sn i.e. $^{129}$Sb and $^{132}$Te.

## 2. Shell-model approach

The aim is to derive a general expression for matrix elements of the two-body interaction $V_{i,k}$, between wave functions involving $n_i$ particles of type i and $n_k$ particles of type k(with type we mean:protons and neutrons,particle-and hole excitations,particle(-hole)and boson excitations). We call furthermore $J_i$ $(J_k)$ the resulting angular momentum of the $n_i$ $(n_k)$particles and J the total angular momentum from coupling $J_i$ with $J_k$ in that order.Usually,the interaction $V_{ik}$ is expanded as+

$$V_{i,k} = \Sigma < p'n';J_1M_1|V|pn;J_1M_1> a^+_{\rho'_i} a^+_{\eta'_k} a_{\eta_k} a_{\rho_i} \times$$

$$\times <j'_p m'_p, j'_n m'_n|J_1M_1><j_p m_p, j_n m_n|J_1M_1> . \quad (1)$$

In order to obtain more efficiently written expressions,we shall use the Wigner covariant notation of the 3j-symbols[9].Moreover,we make the tacit assumption throughout this paper that in the two-body matrix elements $<p'n';J_1M_1|V|pn;J_1M_1>$,we denote with p,p',n,n' not only the quantum numbers but also the exact nature of the type of excitations i.e. for fermion hole excitations p,p' means $p^{-1}$, $p'^{-1}$.

We can now write an alternative expression for (1) as+

$$V_{i,k} = \Sigma\, U((p'p)(n'n);J') a^+_{\rho'_i} a^+_{\eta'_k} a_{\eta_k} a_{\rho_i} \begin{pmatrix} m'_p & M' & j_p \\ j'_p & J' & m_p \end{pmatrix} \times$$

$$\times \begin{pmatrix} m'_n & J' & j_n \\ j'_n & M' & m_n \end{pmatrix} \quad (2)$$

The quantities U(...),appearing in eq(2)can be calculated once for all and be stored. Obviously,the U-matrix elements of(2)are related to the normal two-body matrix elements of(1),by equating(1)and(2), with as a result

$$U((p'p)(n'n);J') = \hat{J}'^2 \sum_{J_1} \hat{J}_1^2 (-1)^{j_p+j'_n+J'-J_1} \begin{Bmatrix} j'_p & J_1 & j'_n \\ j_n & J' & j_p \end{Bmatrix}$$

$$\times <p'n';J_1M_1|V|pn;J_1M_1>. \quad (3)$$

If we now call $|c_iJ_iM_i>$ $(|c_kJ_kM_k>)$the resulting wave functions of the $n_i$ $(n_k)$particles of type i(k), $c_i$ $(c_k)$denoting all other quantum numbers necessary to label the wave functions uniquely,then the matrix elements for the resulting coupled wave function$|c_iJ_i,c_kJ_k;JM>$ become

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+ In the summations of (1) and (2),all quantum numbers associated with the greec letters of the four operators are implied. A greec letter ρ denotes $\rho \equiv (p,m_p)$ with $p \equiv \{n_p,\ell_p,j_p\}$. Moreover, the coupled angular momenta $J_1M_1$ or J'M' are summed.

$$<c_i'J_i',c_k'J_k';JM|V|c_iJ_i,c_kJ_k;JM>= \sum_{\{p',p,n',n,J'\}} U((p'p)(n'n);J')\times$$

$$\times(-1)^{J_i+J_k'+J-J'} \begin{Bmatrix} J_k & J' & J_k' \\ J_i' & J & J_i \end{Bmatrix} <c_i'J_i'\|(p'p)J'\|c_iJ_i> \times$$

$$<c_k'J_k'\| (n'n)J' \| c_kJ_k>, \tag{4}$$

using as a definition for the reduced single-particle matrix elements

$$<c_i'J_i'\|(p'p)J'\|c_iJ_i>\equiv \frac{1}{\hat{J'}} <c_i'J_i'\| [a_{p'}^{+},\tilde{a}_p]_{J'} \| c_iJ_i>, \tag{5}$$

where $[a_\alpha^{+}\tilde{a}_\beta]_\lambda$ denotes the usual vector coupling and where the annihilation operator is defined as

$$\tilde{a}_\beta \equiv (-1)^{j_b+m_b} a_{-\beta} .$$

Here, one also remarks that the reduced matrix elements in eq.(4) can be calculated independently and only depend on the particular states $|c_iJ_i>$ for the first one and on $|c_kJ_k>$ for the second one. In this sense, the eq.(4) lends itself to possible approximations when taking specific values for these reduced matrix elements (using a macroscopic collective wave function to describe core excitations, the reduced matrix elements reduce to the boson c.f.p.).

We now show the equivalence of eq(4), in the particular case of one particle-core coupling, with the well-known macroscopic particle-core coupling matrix elements. Using for $V_{i,k}$ a sum of multiple-multipole forces

$$V_{i,k} = \Sigma\, \chi_\lambda\, \vec{Q}_\lambda(\vec{r}_i).\vec{Q}_\lambda(\vec{r}_k) , \tag{6}$$

$$\vec{Q}_\lambda(\vec{r}) \equiv (\sqrt{\tfrac{m\omega}{h}}r)^\lambda\, Y_\lambda(\hat{r}) , \tag{7}$$

one can reduce eq(4), in a straightforward way into

$$<p',c_k'J_k';JM|V|p,c_kJ_k;JM>= \sum_\lambda \chi_\lambda (-1)^{j_p+J_k'+J} \begin{Bmatrix} J_k & \lambda & J_k' \\ j_p' & J & j_p \end{Bmatrix}$$

$$\times<p'\| \vec{Q}_\lambda \| p><c_k'J_k'\| \vec{Q}_\lambda\| c_kJ_k>. \tag{8}$$

Here, we used the definition

$$<c_k'J_k'\| \vec{Q}_\lambda\| c_kJ_k>= \sum_{n,n} <n'\| \vec{Q}_\lambda\| n><c_k'J_k'\| (n'n)\lambda\| c_kJ_k>. \tag{9}$$

Expression(8) is completely equivalent with the macroscopic particle-core coupling matrix element. The core matrix elements(9) are however defined in terms of their shell-model description. Thus, the expression(4) is a most general expression, giving possibilities to treat odd-even, even-even as well as odd-odd nuclei on the same footing. The advances of this expression is that

- the residual interaction $V_{ik}$ is treated in a correlated basis,
- the form of eq(4) allows for approximations in an easy way,
- the residual interaction matrix elements of $V_{ik}$ are easily programmable on a computer,
- weak-coupling can easily be obtained from the present formulation.

## 3. Application near Z=50, N=82

### 3.1. Parameters and two-body interactions

In order to carry out calculations near the doubly-closed shell nucleus $^{132}$Sn, for which the first excited state occurs at $E_x$=4.041 MeV, single-particle(-hole) energies and effective two-body matrix elements are needed. In table 1, we give the

Table 1

| PROTON | | NEUTRON | |
|---|---|---|---|
| $\varepsilon_{1g_{7/2}}$ | 0.0 | $\tilde{\varepsilon}_{2d_{3/2}}$ | 0.0 |
| $\varepsilon_{2d_{5/2}}$ | 1.0 | $\tilde{\varepsilon}_{3s_{1/2}}$ | 0.3 |
| $\varepsilon_{1h_{11/2}}$ | 2.0 | $\tilde{\varepsilon}_{1h_{11/2}}$ | 0.4 |
| $\varepsilon_{2d_{3/2}}$ | 2.4 | $\tilde{\varepsilon}_{1g_{7/2}}$ | 2.4 |
| $\varepsilon_{3s_{1/2}}$ | 2.0 | $\tilde{\varepsilon}_{2d_{5/2}}$ | 2.8 |
| $V_o$ | -39.0 | $V_o$ | -39.0 |
| t | +0.2 | t | +0.2 |

*The parameters for the proton-proton and neutron-neutron interactions as well as the proton single-particle and neutron single-hole energies.*

proton single-particle and neutron single-hole energies as obtained from the experimental level schemes of $^{133}$Sb(ref.10) and $^{131}$Sn(ref.2), respectively. Neutron single-particle and proton single-hole energies could be obtained from the experimental level schemes of $^{133}$Sn and $^{131}$In respectively. However, lack of experimental data does not allow a good determination of these energies.

Two-body p-p and n-n interactions matrix elements have been obtained from a gaussian interaction $V=V_o\, e^{-\beta r^2}(P_S + tP_T)$, with parameters as given in table 1. This particular interaction has proven its ability to describe many even-even nuclei in this particular mass region[11-15]. For the proton-neutron interaction, a sum of quadrupole and octupole forces has been used with parameters $\chi_2$ and $\chi_3$ (see eq.6), determined via a best fit to low-lying experimental levels in $^{129,130,132}$Sb. Thus, for the particular $^{132}$Sn mass region, the values $\chi_2$ = - 0.15 MeV and $\chi_3$= -0.06 MeV have been used[9,16]. In some cases, calculations have also been carried out using a $\delta$-interaction with spin exchange as proton-neutron interaction[9].

Starting from these basic nuclear parameters and using eq.4 after diagonalising in the identical nucleon systems in order to obtain the wave functions $|c_iJ_iM_i>$ and $|c_kJ_kM_k>$), the actual energy matrices can be constructed and diagonalised.

### 3.2. Application to $^{129}$Sb

Results for the core nuclei $^{128,129,130}$Sn have already been discussed at some length in refs.9 and 17. These nuclei can serve as a core in order to describe more complex nuclei. The nucleus $^{131}$Sb (1 particle-2 hole system) is discussed in much detail by F.Schussler et al.in the present proceedings[18].

In this section, we point out that using eq.(4) and having a good description of the 4 hole nucleus $^{128}$Sn as well as of the odd-proton moving in all the available proton single-particle orbits($1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$), the $^{129}$Sb level scheme and its decay properties can be studied in a shell-model approach to the particle-core coupling description. (see fig.2).

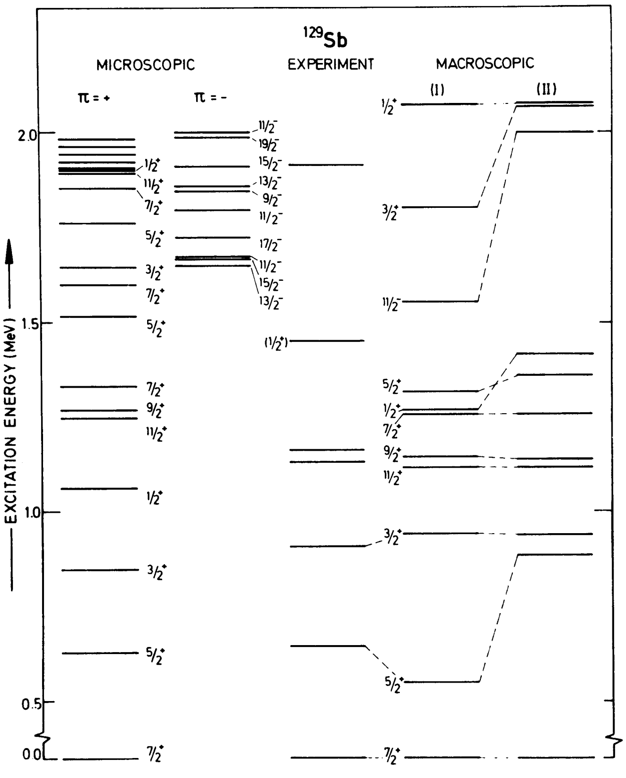


*Figure 2: Comparison of the experimental data, the results of a shell-model approach to particle-core coupling (eq. 8) using a quadrupole+octupole proton-neutron interaction as discussed in the text and the macroscopic particle-core coupling calculations for two different $\xi_3$ values (see text) in $^{129}Sb$.*

For $^{129}Sb$, macroscopic particle-core coupling calculations have been carried out[19,20]. Here, we perform the macroscopic particle-core coupling calculations with the proton single-particle energies as discussed here (table1) and collective parameters $\hbar\omega_2$, $\hbar\omega_3$ (the $2_1^+$ and $3_1^-$ excitation energy in $^{128}Sn$) $\xi_2$ =1.35 and $\xi_3$ = 1.50 (fig.2, calculation (I)). Here, also the calculation with $\xi_3$ = 0 was carried out (fig.2, calculation (II)) to show the strong octupole dependence for the macroscopic coupling calculations [5,21,22]. The results are shown in fig.2, where also the experimental data are given.

We now discuss, in some detail, the similarities and differences between both the macroscopic and shell-model calculations

i) the $J_i^\pi = 7/2_1^+$ state has about equal single-particle amplitudes in both models. The strongest coupling occurs with the $|1g_{7/2},2_1^+;\ 7/2^+>$ configuration in both the shell-model and macroscopic calculations,

ii) the $J_i^\pi = 5/2_1^+$ level has again comparable single-particle amplitudes in both models. For the $|1h_{11/2},3^-;5/2^+>$ configuration, about equal admixtures occur although in the shell-model approach, the coupling occurs preferentially with two high-lying $J^\pi = 3^-$ levels ($|1h_{11/2},3_4^-;5/2^+>$ at $|1h_{11/2},3_5^-;5/2^+>$ at 3.97 MeV). These $3^-$ levels in $^{128}Sn$ show very strong B(E3) transition probabilities to the ground state and therefore serve as a description of the collective $3^-$ level.

iii) the $J_i^\pi = 11/2_1^-$ state also has comparable single-particle character in both calculations. For the $|2d_{5/2},3^-;11/2^->$ configurations, the same comments as for the $J_i^\pi = 5/2_1^+$ level apply

iv) for the $J_i^\pi = 3/2_{1,2}^+$ levels, some differences occur when comparing both calculations. In the macroscopic calculation, the $J_i^\pi = 3/2_1^+$ state is mainly the $|1g_{7/2},2_1^+;3/2^+>$ configuration as is also the case in the shell-model approach. In the latter model, a $J_i^\pi = 3/2_2^+$ level occurs, containing as the most important configuration $|1g_{7/2},2_2^+;3/2^+>$, a configuration with no obvious counterpart in the macroscopic model

v) for the $J_i^\pi = 1/2_1^+$ state, about equal fractions of the $3s_{1/2}^+$ and the $|2d_{5/2},\ 2_1^+;1/2^+>$ configurations result in both descriptions. A second $1/2^+$ level ($J_i^\pi = 1/2_2^+$) is obtained in the macroscopic model as mainly the $|1g_{7/2},4_{2ph}^+;1/2^+>$ configuration ($4_{2ph}^+$ means the quadrupole two-phonon $4^+$ state) and in the shell-model calculation, the corresponding $|1g_{7/2},4_1^+;1/2^+>$ configuration dominates.

As a conclusion we find very strong similarities between both models. For the quadrupole degree of freedom, in both the macroscopic and the shell-model approach, coupling of the single-particle configuation goes preferentially via the $|n\ell j,2_1^+>$ configurations. For the octupole degree of freedom, high-lying $|n\ell j,3_i^->$ configurations are strongly admixed via the $3_4^-$ and $3_5^-$ levels ($E_x \simeq$ 4.0 MeV). Also, the strong octupole force dependence of some particular levels is completely analoguous in both models, due to the large non-spin flip reduced $Y_3$ matrix elements resulting in both models.

Similar comparisons can be made for the levels which have the quadrupole one-phonon $|1g_{7/2},2_1^+;J^\pi>$ configuration as their main configuration. For the $|1g_{7/2},7_1^-\ ;\ J^\pi>$ and $|1g_{7/2},5_1^-;\ J^\pi>$ configurations, negative parity multiplets result (see also fig.2) that are not easily obtained within the macroscopic particle-core coupling model.

In this subsect. 3.2, we have discussed the application of a shell-model approach to particle-core coupling (see eqs. 4 and 8) near closed shells. We have carried out an extensive comparison with the macroscopic particle-core coupling model calculations in $^{129}Sb$ and obtain very similar results concerning energy spectra and nuclear wave functions. Nuclear levels, in which the neutron 2p, 4p, ... excitations show up explicitely can only be obtained in a consistent way from the shell-model approach to particle-core coupling.

### 3.3. Study of $^{132}Te$.

Recently, we have reported on detailed shell-model studies, along the lines discussed in eq(4), for the even-even $^{132}Te$ nucleus [9,16,23,24]. In this particular nucleus, the wave functions $|c_iJ_iM_i>$ and $|c_kJ_kM_k>$ describe the single-closed shell $^{134}Te$ and $^{130}Sn$ nuclei. Thus, the proton-neutron interaction will be decisive in describing the kind of coupling between both systems (weak- or strong mixed final wave functions).

The basic experimental features are shown in fig.3, where the idea of weak coupling particular proton excitations ($4_1^+,6_1^+$) to the neutron $0^+$ ground state or of specific neutron excitations ($7^-,5^-,4^+$) to the proton $0^+$ ground state, becomes clear. In the particular case of $^{132}Te$, the proton-neutron interaction used was the $\delta$-interaction with spin exchange, $V = V_{eff}.\delta(\vec{r}_p-\vec{r}_n)(P_S+tP_T)$, with $V_{eff}$=84 MeV.fm$^3$ and t=5.

In this nucleus, a representation of the wave functions obtained by coupling the proton 2p and neutron 2h wave functions for $^{134}Te$ and $^{130}Sn$ respectively,

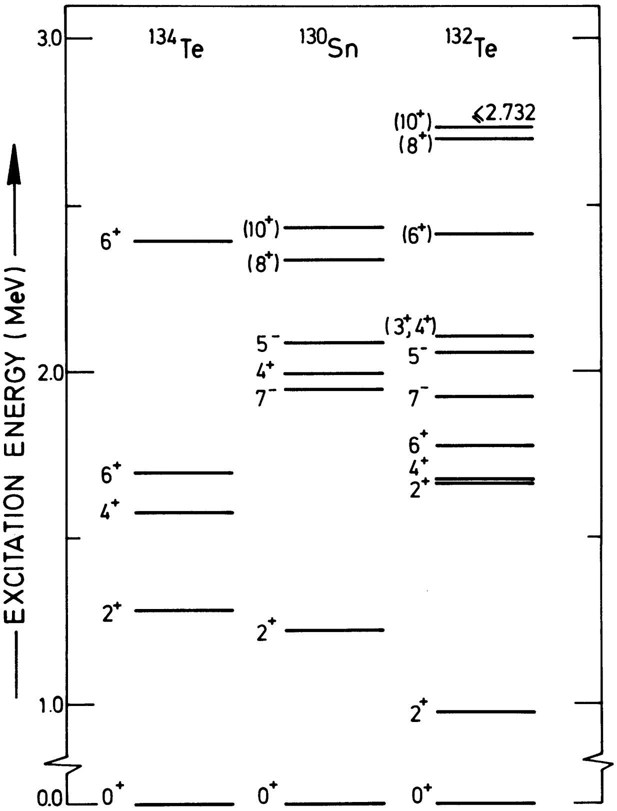


*Figure 3: The most important experimental low-lying levels in $^{134}$Te, $^{130}$Sn and $^{132}$Te, indicating the weak-coupling pattern.*

is particularly transparent and moreover gives a possibility of comparing with a boson model interpretation of the purely collective quadrupole excitations.

Diagonalising within the separate proton and neutron spaces as well as by diagonalising within the full proton 2p-neutron 2h configuration space (dimensions of 1000x1000 occur), the wave functions

$$|k,J_n(\nu)\rangle \equiv \sum_{h_1,h_2} A^k(h_1h_2,J_n)\ |(h_1h_2),J_n\rangle , \quad (10)$$

$$|\ell,J_p(\pi)\rangle \equiv \sum_{p_1,p_2} B^{\ell}(p_1p_2,J_p)\ |(p_1p_2),J_p\rangle , \quad (11)$$

and

$$|i,J\rangle \equiv \sum_{\substack{h_1,h_2,J_n \\ p_1,p_2,J_p}} C^i(h_1h_2J_n,p_1p_2J_p;J)\,|(h_1h_2)J_n(p_1p_2)J_p;J\rangle , \quad (12)$$

result for $^{130}$Sn, $^{134}$Te and $^{132}$Te, respectively. Inverting the relations (10) and (11), one can express the states (12) as

$$|i,J\rangle = \sum_{\ell,J_p,k,J_n} D^i(kJ_n,\ell J_p;J)\,|kJ_n,\ell J_p;J\rangle . \quad (13)$$

Wave functions (13) for some particular important states are shown in table 2.

The nucleus under study, $^{132}$Te, now presents an interesting case in order to study relations to the IBA model of Arima and Iachello[25]. In this approach, particular collective states in nuclei with 2p active protons and 2n active neutrons (outside closed shells) are supposed to result from the interaction of p proton and n neutron bosons, with only spin o (s boson) an 2 (d boson). In the IBA approximation, a

Table 2

| $E_x$ (MeV) | Wave function |
|---|---|
| 0.000 | $\|0_1^+\rangle=-0.98\|0_1^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 1.467 | $\|0_2^+\rangle= 0.86\|0_3^+(\nu)\otimes 0_1^+(\pi)\rangle+0.49\|0_2^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 1.566 | $\|0_3^+\rangle= 0.83\|0_2^+(\nu)\otimes 0_1^+(\pi)\rangle-0.47\|0_3^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 2.384 | $\|0_4^+\rangle=-0.91\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle+0.30\|2_2^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 1.977 | $\|1_1^+\rangle= 0.96\|1_1^+(\nu)\otimes 0_1^+(\pi)\rangle-0.24\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 2.425 | $\|1_2^+\rangle= 0.94\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle+0.24\|1_1^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 1.109 | $\|2_1^+\rangle=-0.78\|2_1^+(\nu)\otimes 0_1^+(\pi)\rangle+0.61\|0_1^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 1.306 | $\|2_2^+\rangle= 0.75\|0_1^+(\nu)\otimes 2_1^+(\pi)\rangle+0.60\|2_1^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 2.440 | $\|2_5^+\rangle= 0.93\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle-0.20\|2_3^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 2.408 | $\|3_1^+\rangle= 0.97\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 1.582 | $\|4_1^+\rangle= 0.97\|0_1^+(\nu)\otimes 4_1^+(\pi)\rangle$ |
| 2.310 | $\|4_2^+\rangle= 0.84\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle+0.49\|4_1^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 2.480 | $\|4_3^+\rangle= 0.81\|4_1^+(\nu)\otimes 0_1^+(\pi)\rangle-0.46\|2_1^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 1.705 | $\|6_1^+\rangle= 0.97\|0_1^+(\nu)\otimes 6_1^+(\pi)\rangle$ |
| 2.697 | $\|8_1^+\rangle= 0.96\|8_1^+(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 2.764 | $\|8_2^+\rangle= 0.98\|2_1^+(\nu)\otimes 6_1^+(\pi)\rangle$ |
| 2.704 | $\|10_1^+\rangle= 0.95\|10_1^+(\nu)\otimes 0_1^+(\pi)\rangle-0.24\|10_1^+(\nu)\otimes 2_1^+(\pi)\rangle$ |
| 2.035 | $\|5_1^-\rangle= 0.97\|5_1^-(\nu)\otimes 0_1^+(\pi)\rangle$ |
| 1.823 | $\|7_1^-\rangle= 0.96\|7_1^-(\nu)\otimes 0_1^+(\pi)\rangle-0.22\|7_1^-(\nu)\otimes 2_1^+(\pi)\rangle$ |

*Wave functions for some important states in $^{132}$Te, expressed in the coupled basis of $^{134}$Te and $^{130}$Sn wave functions (see also eq. 13). The excitation energy (in MeV), using a proton-neutron $\delta$-interaction, is also given.*

boson is related to a pair of fermion particles (-holes). In the ideal situation, the boson-boson interaction should be calculated from the residual fermion interaction[26]. Now in the particular nucleus $^{132}$Te, one is left with one boson of each kind and one can make the following correspondences

-associate one $\underline{s}$ boson $s_\pi(s_\nu)$ with the $0^+$ ground state in $^{134}$Te ($^{130}$Sn)

-associate one $\underline{d}$ boson ($d_\nu$) with the $2_1^+$ state in $^{134}$Te ($^{130}$Sn).

The necessary boson matrix elements for all possible angular momenta ($J^\pi = 0^+, 1^+, 2^+, 3^+, 4^+$) can easily be calculated. Indeed, we have at our disposal the shell-model matrix elements $\langle (h_1'h_2')J_n'(p_1'p_2')J_p';J|V|(h_1h_2)J_n(p_1p_2)J_p;J\rangle$. Then a unitary transformation gives us the matrix elements of V in the coupled basis of the states of $^{130}$Sn and $^{134}$Te. These matrix elements can than be interpreted as matrix elements of the boson-boson interaction. They are given in Table 3. In diagonalising the latter interaction within the restricted space (one $\pi$-one $\nu$ boson) and comparing with the full shell-model calculations (fig. 4), a very good agreement results as well in energy as for the nuclear wave functions (Table 4). By inspecting the wave functions corresponding with the $2^+$ states, one observes that the wave function antisymmetric for exchange of proton and neutron coordinates corresponds to the lowest $2^+$ state.

This feature can be easily pointed out to correspond to the lowest $2^+$ state for a proton 2-particle-neutron 2-hole system, when simplifying towards a single j-shell, and a pairing (for identical particles) +quadrupole (for non identical particles) interaction within this scheme and taking into account the two unperturbed $J^\pi = 2^+$ states $|J_p = 2^+, J_n = 0;\ J = 2^+\rangle$ and

$|J_p = 0^+, J_n = 2^+ ; J = 2^+\rangle$ (neglecting other configurations).

Table 3

| | | |
|---|---|---|
| $\langle s_\nu s_\pi;0\|V\|s_\nu s_\pi;0\rangle$ | = | 0.94 |
| $\langle s_\nu s_\pi;0\|V\|d_\nu d_\pi;0\rangle$ | = | 0.32 |
| $\langle d_\nu d_\pi;0\|V\|d_\nu d_\pi;0\rangle$ | = | 0.90 |
| $\langle d_\nu d_\pi;1\|V\|d_\nu d_\pi;1\rangle$ | = | 0.82 |
| $\langle s_\nu d_\pi;2\|V\|s_\nu d_\pi;2\rangle$ | = | 0.92 |
| $\langle s_\nu d_\pi;2\|V\|d_\nu s_\pi;2\rangle$ | = | 0.14 |
| $\langle s_\nu d_\pi;2\|V\|d_\nu d_\pi;2\rangle$ | = | -0.01 |
| $\langle d_\nu s_\pi;2\|V\|d_\nu s_\pi;2\rangle$ | = | 0.86 |
| $\langle d_\nu s_\pi;2\|V\|d_\nu d_\pi;2\rangle$ | = | 0.04 |
| $\langle d_\nu d_\pi;2\|V\|d_\nu d_\pi;2\rangle$ | = | 0.86 |
| $\langle d_\nu d_\pi;3\|V\|d_\nu d_\pi;3\rangle$ | = | 0.82 |
| $\langle d_\nu d_\pi;4\|V\|d_\nu d_\pi;4\rangle$ | = | 0.76 |

*Proton-neutron boson residual interaction matrix elements for $^{132}$Te as obtained from the fermion two-body δ interaction.*

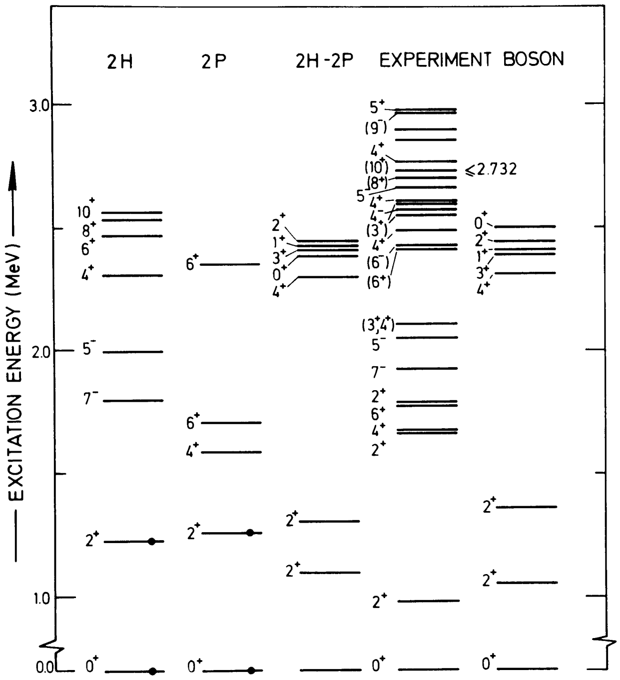


*Figure 4: The separate 2-hole($^{130}$Sn), 2-particle($^{134}$Te) and 2-particle 2-hole($^{132}$Te) shell-model calculations (only the low-lying levels that can be interpreted in terms of collective excitations, are given) are compared with the experimental data. In the last column, the results from the boson-boson interaction in a one proton-one neutron basis, are given.*

The only non-vanishing matrix element is the off-diagonal matrix element :

$$\langle J_p=2^+,J_n=0^+;J=2^+|-\chi_\lambda \vec{Q}^p_\lambda\cdot\vec{Q}^n_\lambda|J_p=0^+,J_n=2^+;J=2^+\rangle$$
$$=\chi_\lambda\langle j\|Q_\lambda\|j\rangle^2\cdot\frac{4}{5(2J+1)} \quad . \qquad (14)$$

The energy matrix for $J^\pi = 2^+$ then simplifies into the 2 x 2 matrix (calling $E_{pair} = -G(j+1/2)$ and taking $\varepsilon_j = 0$)

$$\begin{bmatrix} E_{pair} & b \\ b & E_{pair} \end{bmatrix} , \qquad (15)$$

with eigenvalues $E_{pair} - b$ and $E_{pair} + b$ corresponding respectively with the antisymmetric and symmetric wave functions :

$$\frac{1}{\sqrt{2}}|J_p = 2^+, J_n = 0^+;J=2^+\rangle - \frac{1}{\sqrt{2}}|J_p=0^+,J_n=2^+;J=2^+\rangle$$
$$(16)$$
$$\frac{1}{\sqrt{2}}|J_p = 2^+,J_n=0^+;J=2^+\rangle + \frac{1}{\sqrt{2}}|J_p=0^+,J_n=2^+;J=2^+\rangle.$$

For a proton 2-particle-neutron 2-particle system the off-diagonal matrix element changes sign, thus the same eigenvalues are obtained, but the wave functions (16) interchange.

Table 4

| |
|---|
| $\|0_1^+\rangle = 0.99\|0_1^+(\nu) \otimes 0_1^+(\pi)\rangle$ |
| $\|0_4^+\rangle = 0.99\|2_1^+(\nu) \otimes 2_1^+(\pi)\rangle$ |
| $\|2_1^+\rangle = -0.60\|0_1^+(\nu) \otimes 2_1^+(\pi)\rangle + 0.80\|2_1^+(\nu) \otimes 0_1^+(\pi)\rangle$ |
| $\|2_2^+\rangle = -0.60\|2_1^+(\nu) \otimes 0_1^+(\pi)\rangle - 0.80\|0_1^+(\nu) \otimes 2_1^+(\pi)\rangle$ |
| $\|2_5^+\rangle = 1.00\|2_1^+(\nu) \otimes 2_1^+(\pi)\rangle$ |

*Wave functions for $^{132}$Te, resulting from diagonalising the proton-neutron boson interaction in the restricted basis of collective excitations only.*

Starting now from this π-ν boson interaction, it should become possible to proceed towards more complex nuclei with p bosons of the proton type and n bosons of the neutron type. The π-ν Hamiltonian is completely equivalent with the one discussed in ref.[26] although an other microscopic proton-neutron interaction is used by Otsuka, i.e. a quadrupole-quadrupole interaction whereas the delta interaction with spin exchange is used here. One must have in mind however, that the interaction derived above is obtained near closed shells whereas, strictly speaking, the IBM is valid far from closed shells. Restricting to strongly collective states only, the approach discussed above may well give good results.

In conclusion, we can say that, by means of the shell-model calculations carried out for $^{134}$Te, $^{130}$Sn and $^{132}$Te, it became possible to derive a macroscopic π-ν boson interaction from a residual proton-neutron (delta) interaction, when identifying the respective $0_1^+$ and $2_1^+$ states in $^{130}$Sn and $^{134}$Te, as collective s and d boson excitations.

## 4. Conclusion

In this study, we have proposed a shell-model approach for studying, in a unified way, the interplay of collective and non-collective excitations. The formalism extends the simplified macroscopic particle-core coupling models in such a way as to allow for single-particle coupling to non-collective excitations. Applications for odd-mass and even-even nuclei near the doubly-closed shell nucleus $^{132}$Sn have been discussed in some detail. In the even-even nucleus $^{132}$Te, we have made an attempt to derive a collective proton-neutron interaction in terms of $s(J^{\pi}=0^{+})$ and $d(J^{\pi}=2^{+})$ boson-like excitations only.

The general method, as discussed in sect.2 has also been applied to other mass region i.e. the $^{96-100}Pd$ nuclei[27].

Going beyond 4p(4h) configurations away from closed shells will definitly need certain approximations such as taking only the lowest $J^{\pi}=0^{+}$ and $2^{+}$ two particle(-hole) states acting on the 4 particle (-hole) space, in order to produce 6 particle(-hole), 8 particle(-hole),...configurations. In a certain sense, the approximations of the IBA have to be used.

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