DECAY EXPERIMENTS TO TEST β- AND γ-STRENGTH FUNCTIONS

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Abstract

The decays of six delayed-proton precursors have now been studied with the particle-X-ray coincidence technique, yielding a more or less detailed view of level widths in the proton emitting nuclei. The results lead to a reevaluation of our techniques for calculating level densities and average gamma-decay widths in exotic nuclei. With these aspects of β-delayed proton decay specified, the proton energy spectra of a number of precursors ($65 \leq A \leq 121$) are examined for evidence of structure in the β-strength functions. Overall, the "gross theory" β-strength functions are remarkably reliable, although some systematic deviations are observed in the xenon and tellurium isotopes.

I. Introduction

Beta-delayed proton decay provides a sensitive window on the properties of exotic nuclei, opening up a view of excited states that can hardly be matched by any other process, even in nuclei near β-stability. Its drawback is that in heavier nuclei ($A \gtrsim 65$) it seems to reveal too much at once. As a two-step process - β-decay followed by proton emission - it reflects not only the β-transition rates to a large body of states, but also the interplay of proton and γ-ray widths in the decay of these states; furthermore, because the level density is usually high, the effects of individual states cannot be resolved from one another and so must be dealt with in toto. Thus, if only the proton energy spectrum is measured, its analysis can hardly be expected to yield very specific information about any one of the physical properties involved.

Yet, experimentally, it is often not easy to obtain more-complete data since proton emission is rarely a strong decay mode. The emitters nearest stability, though produced most prolifically, exhibit proton branches of less than $\sim 10^{-4}$ per disintegration; while the more remote examples make up for a larger branching ratio with significantly reduced production cross sections. This handicap has not prevented new techniques from being developed and applied to enlarge our knowledge of the decay process, but it has meant that even now there are very few precursors to whose decay all available techniques have been applied.

In what follows, I shall consider in detail these few well-measured cases in order to establish what physical properties are uniquely specified by experiment. Since much of the crucial data is new, as yet unpublished, this will be a more demanding test than has been possible previously. Then, armed with what constraints these data provide, I shall examine the more-fragmentary data from other precursors for any incontrovertible trends or deviations. The results, while surprising in some aspects, indicate that the basic features of the decay process are quite well understood.

II. β-Delayed Proton Decay

A schematic description of β-delayed proton decay appears in fig. 1. If the density of proton-emitting states is

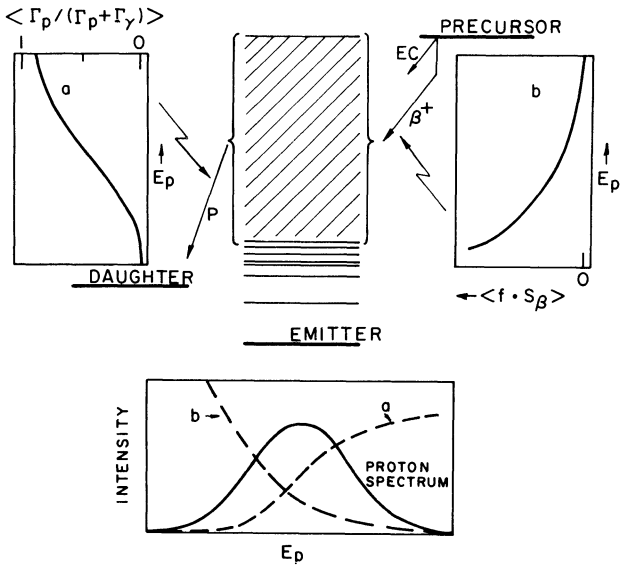


Fig. 1: The upper central portion of the figure illustrates the general decay scheme of a delayed-proton precursor. The graphs labelled a and b at the upper left and right show the behaviour of $<\Gamma_p/(\Gamma_p + \Gamma_\gamma)>$ and $<f \cdot S_\beta>$, respectively, as a function of proton energy. The graph at the bottom demonstrates how these two functions combine in determining the delayed-proton spectrum.

sufficiently high that individual transitions are not resolved in the proton spectrum, then the average intensity of protons with energy E_p is given by[1]

$$I_p(E_p) = \sum_{if} \langle I_\beta^i \rangle_{E_p} \left\langle \frac{\Gamma_p^{if}}{\Gamma_p^i + \Gamma_\gamma^i} \right\rangle_{E_p} \qquad (1)$$

where I_β^i is the intensity of β-decay from the precursor to excited state i in the emitter, Γ_p^{if} is the partial width for proton emission between state i and final state f in the daughter, Γ_p^i is the total proton decay width of state i, and Γ_γ^i is its γ-decay width. Here <> denotes the (Porter-Thomas) statistical mean, and the summation is extended over all pairs of states i and f betwen which protons of energy E_p can be emitted.

If, for illustration, we consider a case where only one final state f plays a

role, then $\Gamma_p^{if} = \Gamma_p^i$ and the summation in equation (1) disappears. (This situation actually occurs in the decays of ^{65}Ge and ^{69}Se.) The overall shape of the proton spectrum is simply determined by the product of two factors, one describing the β-decay of the precursor, the other being the probability of subsequent proton decay from the emitter. The approximate energy dependence of these factors and their effect in determining the bell-shaped proton spectrum are shown in fig. 1. Obviously, the population of more than one final state in the daughter will complicate this simple picture, but not to the extent of distorting its qualitative outline.

The energy dependence of the two factors to some extent separates their influence on the proton spectrum. For example, in lighter precursors, such as ^{69}Se, the high-energy end of the proton spectrum almost entirely reflects the behaviour of the β-decay, since $\Gamma_p/(\Gamma_p + \Gamma_\gamma)$ approaches unity in that region. However, in general the two components are sufficiently intertwined that it is difficult to draw firm conclusions about either the β-decay or the level widths from the proton spectrum alone. This masking of the essential physics can be illustrated by referring to fig. 2. In the case of high

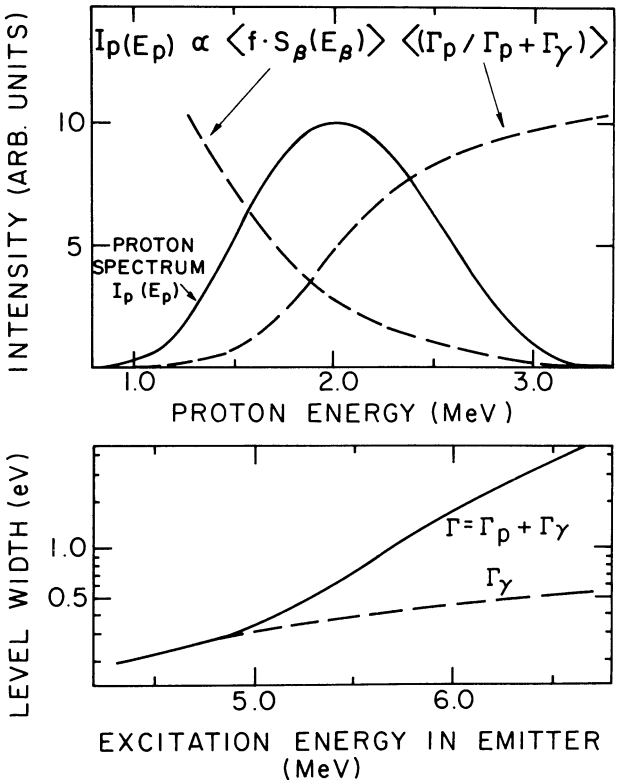


Fig. 2: The top portion of the figure again shows the effect of the two factors that comprise the total proton spectrum. The bottom portion shows for comparison the corresponding level widths; the origins of the energy scales have been adjusted to differ by the proton binding energy. The features are appropriate to an odd-A emitter with A ∿ 70.

level-density it is convenient to write the β-decay intensity as

$$\langle I_\beta^i \rangle_{E_p} = \langle f \cdot S_\beta^i(E_\beta) \rangle \qquad (2)$$

where the kinematic and phase-space effects are accounted for in f, the statistical rate function, while the nuclear structure information is contained in the β-strength function, S_β. The latter, which is proportional to the average Gamow-Teller matrix element squared per energy interval, is expected to exhibit relatively modest energy dependence compared to f, but quite comparable to that of the level widths. Thus, quite reasonable variations in any one of S_β, Γ_p or Γ_γ could lead to the same observed variation in the proton spectrum, especially at lower energies.

These ambiguities can be reduced somewhat by complementary data, for example measurements of the proton branching ratio or the level density; but they can only be effectively removed by a direct measurement of the level widths themselves. This is now possible in selected cases with the particle X-ray coincidence technique (PXCT), and it is these cases that provide the only unequivocal picture of the energy dependence of S_β, Γ_p and Γ_γ. Here is the key to extracting nuclear structure information from delayed-proton decay.

III. PXCT Results

The PXCT was developed at Chalk River[2] and the first measurement using it was described at the last conference in this series[3]. In the intervening 5 years these difficult measurements have been extended to five more cases and have involved teams not only at Chalk River[4] but at CERN[5-7] and GSI, Darmstadt[8].

The principle of the technique is illustrated in fig. 3. It depends upon comparing the decay time of a nuclear state

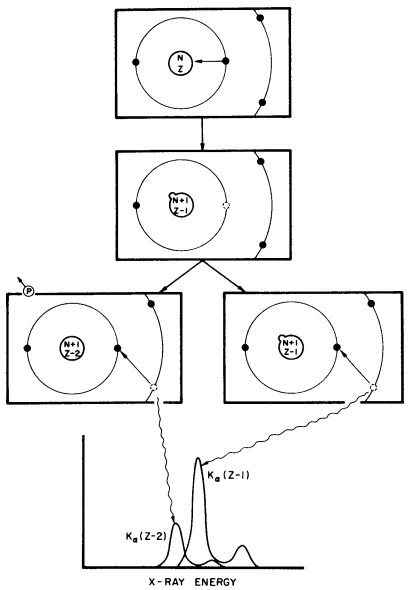


Fig. 3: Pictorial representation of the p-X coincidence technique for lifetime measurements on proton-unstable states produced by electron capture.

with the filling time of a vacancy in the atomic K shell. When the precursor (with atomic number Z) decays by electron capture to excited nuclear states in the emitter (Z-1) it produces simultaneously a vacancy

in the atomic K shell. If the excited states are proton unstable, then the energy of the X-ray emitted with the filling of the atomic vacancy will depend upon whether the proton has already been emitted (in which case the X-ray would be characteristic of a Z-2 element) or has not yet been emitted (characteristic of a Z-1 element). If the nuclear and atomic lifetimes are comparable, then the K_α X-rays observed in coincidence with protons will lie in two peaks whose relative intensities uniquely relate one lifetime with the other. The relative peak intensities measured as a function of proton energy relate directly to the excited state lifetimes (or widths) as a function of excitation.

As with any coincidence measurement of a relatively rare decay mode, PXCT data suffer from statistical limitations. These are compounded for precursors with A $\gtrsim$ 100 by a ratio of X-ray peaks that is of order 0.01. Nevertheless, two decays - those of ^{69}Se(refs. 2,3) and ^{73}Kr(refs.5,6) - have yielded a detailed view of the X-ray ratio as a function of energy; results from the latter are shown in fig. 4. In the remaining four studied cases - the decays of

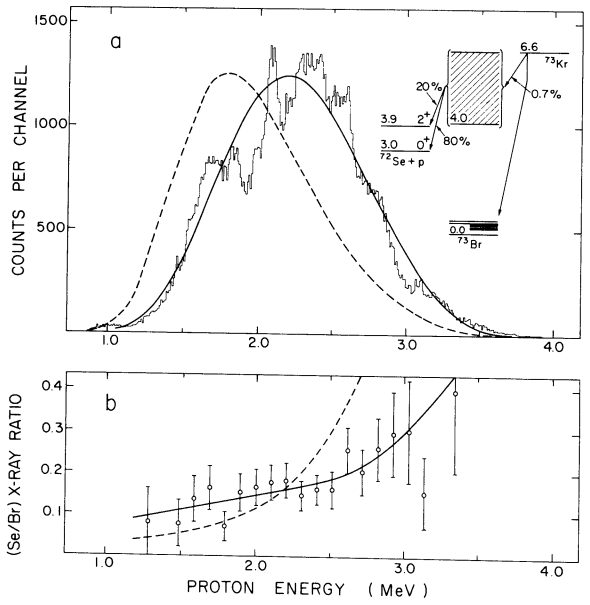


Fig. 4: a) Spectrum of protons observed following the decay of ^{73}Kr. In the simplified decay scheme, which is inset, all energies are given in MeV relative to the ^{73}Br ground state.
b) Ratio of X-rays from Se relative to those from Br, plotted as a function of coincident proton energy. The smooth curves in (a) and (b) are the results of calculations described in the text. Data are from ref.[6].

^{65}Ge(ref. 4), ^{99}Cd(ref. 7), ^{113}Xe(ref. 8) and ^{115}Xe(ref. 7) - the average ratio over the whole spectrum was obtained. Fig. 5 shows a spectrum of X-rays, from which such a ratio is extracted; the spectrum was measured in coincidence with delayed protons of all energies in the decay of ^{115}Xe.

The measured X-ray ratio is determined by the total width, Γ, of excited states where

$$\Gamma = \Gamma_p + \Gamma_\gamma \qquad (3)$$

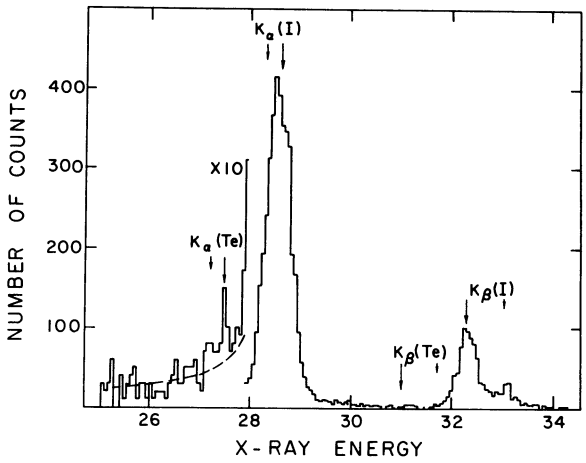
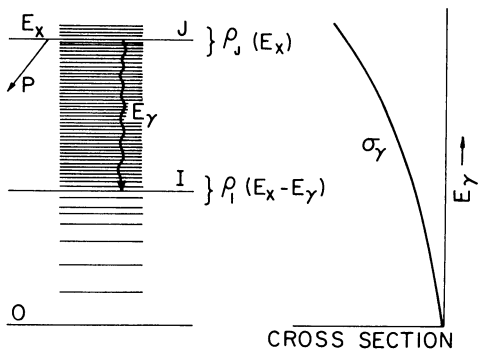


Fig. 5: Spectrum of X-rays measured in coincidence with delayed protons in the decay of ^{115}Xe. Data are from ref.[7].

At low proton energies $\Gamma_\gamma >> \Gamma_p$, so although one is only observing proton decay, the coincident X-ray ratio is actually sensitive to Γ_γ. This is a useful observation since it allows the behaviour of Γ_γ to be established in this region, and extrapolated to higher energies where $\Gamma_p \gtrsim \Gamma_\gamma$, thus permitting a somewhat decoupled test of Γ_p. Obviously, the results are everywhere independent of S_β.

In analyzing PXCT results it is useful to calculate the level widths first, then test the results against the measured X-ray ratios. The method is outlined in fig. 6.



$$\Gamma = \Gamma_p + \Gamma_\gamma$$

$$\Gamma_p(E_x) = \frac{T}{2\pi\rho(E_x)}$$

$$\Gamma_\gamma(E_x) = \int_0^{E^{max}} E_\gamma^3 f_{E1}(E_\gamma) \sum_{I=J-1}^{J+1} \frac{\rho_I(E_x - E_\gamma)}{\rho_J(E_x)}\, dE_\gamma$$

$$f_{E1}(E_\gamma) = 8.7 \times 10^{-8}\, \frac{\sigma_\gamma(E_\gamma)}{E_\gamma}$$

Fig. 6: The formulas used in the calculation of level widths are illustrated by a sample decay scheme, in which some useful symbols are defined, and a typical plot of the photonuclear cross section as a function of γ-ray energy.

The proton decay width for the channel $i \to f$

$$\langle \Gamma_p^{if}(E_x) \rangle = \left[2\pi \, \rho_{J_i}(E_x) \right]^{-1} \sum_{\ell} T_{\ell}(E_p) \qquad (4)$$

where $\rho_{J_i}(E_x)$ is the density of states at excitation energy E_x with the same spin and parity as state i, $T_{\ell}(E_p)$ is the transmission coefficient for protons of energy E_p and angular momentum ℓ, and the sum extends over all values of ℓ allowed by the spins and parities of states i and f. Without a detailed spectroscopic model, it is convenient to use the optical model for calculating T_{ℓ}, with parameters[9] taken from low-energy proton scattering on medium-weight nuclei. It should be noted that this latter procedure may be better suited to some decays than others. In all cases where PXCT measurements are available, the daughter nuclei are even-even with states f following the usual 0^+, 2^+, 4^+ pattern. Thus, the spectroscopic factors for proton emission to these states are presumably all similar, in which case the optical model calculations should be valid. If the daughter nucleus were odd, the spectroscopic factors to low-lying states f could vary considerably from state to state, necessitating more detailed calculations for a nucleus about which little spectroscopic information is known. Since this would introduce fresh uncertainties, I have chosen in what follows to consider only decays that lead to even-even daughter nuclei.

The γ-ray decay width is dominated by E1 radiation[9], the average radiation width of states with spin J_i at excitation E_x being given by

$$\langle \Gamma_{\gamma}(E_x) \rangle = \int_0^{E_{max}} E_{\gamma}^3 f_{E1}(E_{\gamma}) \sum_{I=J_i-1}^{J_i+1} \frac{\rho_I(E_x - E_{\gamma})}{\rho_{J_i}(E_x)} dE_{\gamma} \qquad (5)$$

where f_{E1}, the strength function for E1 γ-decay, is related to the photoabsorption cross section (in mb), σ_{γ}, through the relation

$$f_{E1} = 8.7 \times 10^{-8} \sigma_{\gamma}(E_{\gamma})/E_{\gamma} \quad (MeV)^{-3} \qquad (6)$$

It is well known that the cross section for photonuclear reactions (see fig. 6) on spherical nuclei can be described by a single Lorentzian whose parameters have been determined experimentally for a variety of nuclei[10]. The results lend themselves to expressing these parameters in terms of global prescriptions[10-12], a necessity for an unstable nucleus, which cannot itself serve as a target. With these experimentally-based global prescriptions the strength function, f_{E1}, is specified with no remaining free parameters.

Inspection of equations (4) to (6) reveals that the only remaining unspecified quantity is the level density, ρ, which plays a crucial role in determining $\langle \Gamma_p \rangle$ and a less important one (because it appears both in the numerator and denominator) in $\langle \Gamma_{\gamma} \rangle$. I shall return to the question of level density later, but for the moment let us consider the usual - for exotic nuclei, at least - Gilbert and Cameron prescription[13] with parameters specified by Truran et al[14]; this combines a Fermi-gas expression at high excitations with a constant-temperature representation at low. In the analysis of PXCT data, the level-density parameter, a, is treated as "free" and adjusted to optimize agreement with the data.

Unfortunately, while these methods achieve qualitative similarity between experiment and theory, there remains a quantitative discrepancy at low energies where $\langle \Gamma_{\gamma} \rangle$ predominates. The least disruptive approach to correcting this difficulty is to introduce a multiplicative constant K on the right-hand side of equation (5), thereby maintaining the calculated energy-dependence of $\langle \Gamma_{\gamma} \rangle$ while leaving its magnitude (through K) to be freely adjusted to match experiment. The effects of changes in the two parameters, a and K, are illustrated in figs. 7 and 8, respectively, for the ^{73}Kr data. Since they have quite different, and rather large, effects on the calculated X-ray ratios, there is no difficulty in determining optimum a and K values with $\sim$ 10% precision.

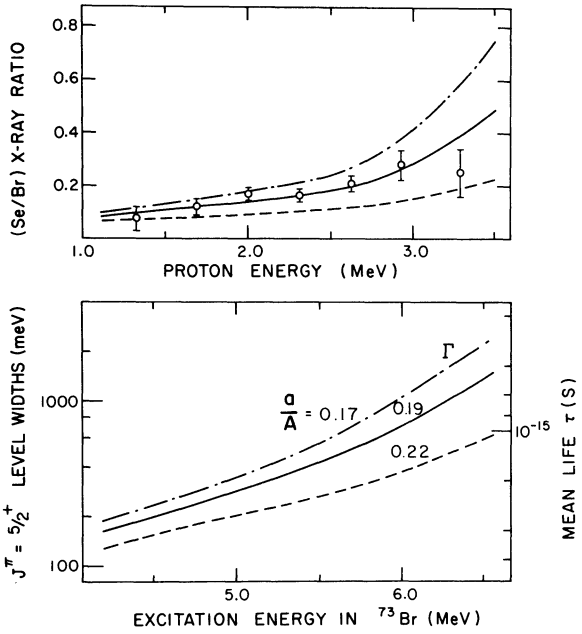


Fig. 7: a) Measured X-ray ratios for ^{73}Kr plotted in 300 keV intervals as a function of coincident proton energy. The curves are calculated ratios corresponding to level widths shown in (b).
b) Total level widths, Γ, calculated with Gilbert-Cameron level-densities and equations (3) to (6). In all cases K=8.5; for the dashed curve, the level density parameter $a = 0.220\ A$, for the solid curve $a = 0.187\ A$, and for the dot-dashed curve $a = 0.172\ A$.

There is much to recommend this solution. Not only is the agreement with the X-ray data good, but the same parameters that lead to this agreement also produce a calculated proton spectrum that reproduces experiment with remarkable fidelity (the solid curves in fig. 4). On the other hand, the required value of a is 13.65 MeV^{-1}, which is 40% higher than the value

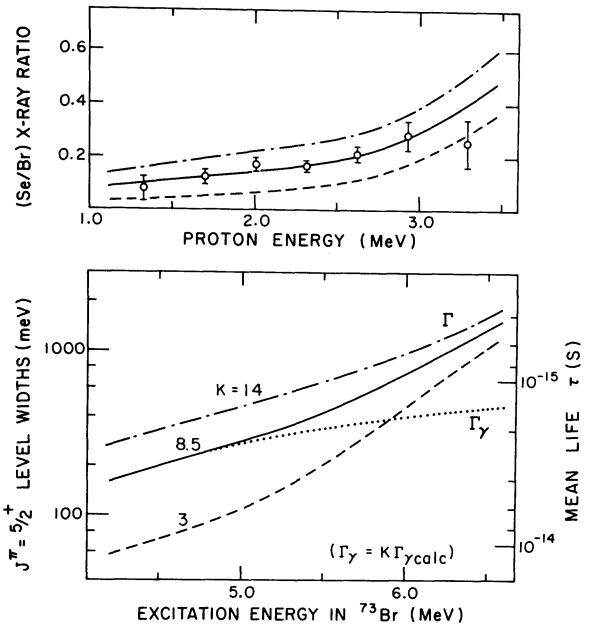


Fig. 8: a)Measured X-ray ratios for ^{73}Kr plotted in 300 keV intervals as a function of coincident proton energy. The curves are calculated ratios corresponding to level widths shown in (b).
b)Total level widths, Γ, calculated with Gilbert-Cameron level-densities and equations (3)to(6). In all cases a=0.187 A; for the dashed curve K=3.0, for the solid curve K=8.5, and for the dot-dashed curve K=14.0. The dotted curve shows the partial γ-decay width, Γ_γ, for the case with K=8.5.

prescribed by Truran et al[14], and K is a staggering 8.5, indicating that equation (5) underestimates $\langle\Gamma_\gamma\rangle$ by that factor. Before accepting these unexpected conclusions, it would be circumspect if we ask how well the Gilbert and Cameron formulas, together with equation (5), do in reproducing other $\langle\Gamma_\gamma\rangle$ measurements, particularly in nuclei near stability where a large body of established data exists.

IV. $\langle\Gamma_\gamma\rangle$ Survey

Experimental values of $\langle\Gamma_\gamma\rangle$ and the level density have been determined at the neutron threshold for many nuclei from studies of (n,γ) reactions on stable targets[15]. The mass range to be considered here, $60 \leq A \leq 147$, was chosen to include only nuclei with high enough level densities that a statistical-model analysis is not inappropriate, while excluding those that are obviously deformed. The range also brackets the region of delayed proton precursors (see fig. 9) to which we ultimately wish to apply the results.

The method used is analogous to that already described for ^{73}Kr. The Gilbert-Cameron density parameter a was determined in each case by requiring agreement with the measured density at the neutron threshold. Then $\langle\Gamma_\gamma\rangle$ was calculated according to equation (5), and the ratio of experiment to calculation, $\langle\Gamma_\gamma\rangle_{exp}/\langle\Gamma_\gamma\rangle_{calc}$ determined[12]. This ratio, which is equal to the "parameter" K already defined, is plotted for 63 cases (the solid dots) in fig. 10a. All are considerably lower than the value obtained for levels in ^{73}Br (the β-decay daughter of ^{73}Kr): the average value is 1.9, with two-thirds of the points lying in the range ±40% about this value.

Where do the other exotic nuclei measured by the PXCT fit into this scheme? While ^{69}Se decay can be analyzed similarly to ^{73}Kr, the other cases - where only an overall X-ray ratio has been measured - could obviously not yield independent measurements of two quantities, a and $\langle\Gamma_\gamma\rangle$. Fortunately, however, the level density in each of these cases (^{65}Ga,^{99}Ag,^{113}I,^{115}I) can be determined independently, either by direct counting (^{65}Ga) or by fluctuation analysis of a high-resolution delayed proton spectrum[16,17], although in fact the data for ^{113}I have not yet been analyzed. For the three of these cases where the measurements are complete, the Gilbert-Cameron density parameter was determined by requiring agreement with the experimental density, and this value was used both in the analysis of the PXCT data and in the calculation of $\langle\Gamma_\gamma\rangle$. The five, in all, PXCT results are plotted as open circles with error bars or as limits in fig. 10a; not only are there large discrepancies between experiment and calculation but these discrepancies do not follow the systematics of the (n,γ) data and they scatter so widely that any hope of predictive capacity must be abandoned.

Some hint of the source of the problem, however, has already appeared. It was noted in the discussion of ^{73}Kr decay that the a value required by the data was significantly higher than systematics[14] based largely on (n,γ) results. Similar discrepancies occur for the other cases among exotic nuclei. Since the density is only measured in one region of excitation, the large single-parameter adjustment required in the formula to achieve agreement at this one energy may well introduce considerable distortions at other energies. The $\langle\Gamma_\gamma\rangle$ calculation (equation (5)), which depends on the rate of change of density with excitation energy rather than its absolute value, would suffer accordingly.

To correct the difficulty, another level-density prescription was used: the back-shifted Fermi gas model[18]. With two operable parameters, the "fictive ground state" (Δ) as well as a, this model is known to reproduce experimental level densities over a wide region of excitation energy[18], and to account for the shell-correction shifts that appear in the energy scale when detailed spectroscopic calculations are made. With parameter values taken directly from Dilg et al[18], $\langle\Gamma_\gamma\rangle$ was calculated[12] for the same 63 (n,γ) cases and then compared with experiment as black dots in fig. 10b. Evidently the agreement is much better than with the Gilbert and Cameron formula: the average ratio, $\langle\Gamma_\gamma\rangle_{exp}/\langle\Gamma_\gamma\rangle_{calc}$, is 1.08 and two-thirds of the values lie within ±33% of that value.

The improvement is even more impressive when the PXCT data are added (open circles and limits). Not only is the agreement excellent between calculated and experimental $\langle\Gamma_\gamma\rangle$ values, but the a and Δ parameters required to satisfy the measured level densities fall naturally into the

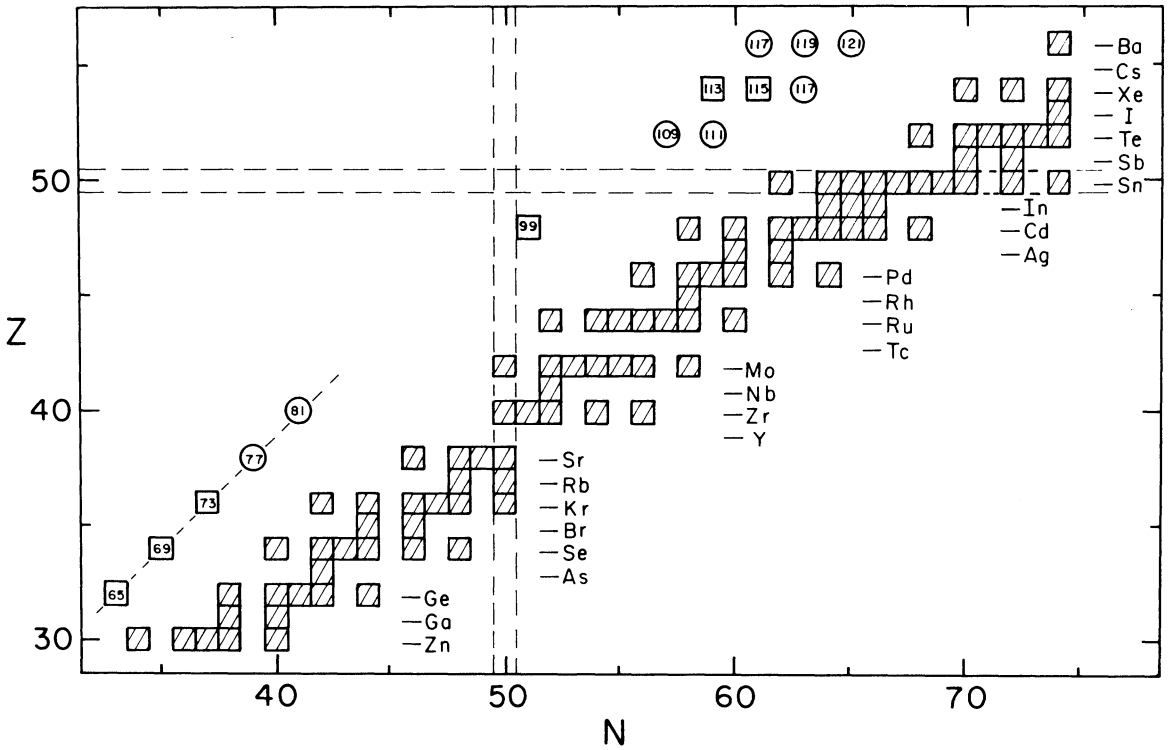


Fig. 9: A portion of the chart of nuclides showing the stable nuclei as cross-hatched squares. The numbered (with A) squares correspond to precursors for whose decay PXCT measurements have been made. Numbered circles are all other precursors whose spectra are known in this region and that have even-even daughter nuclei. These are the cases considered in this work.

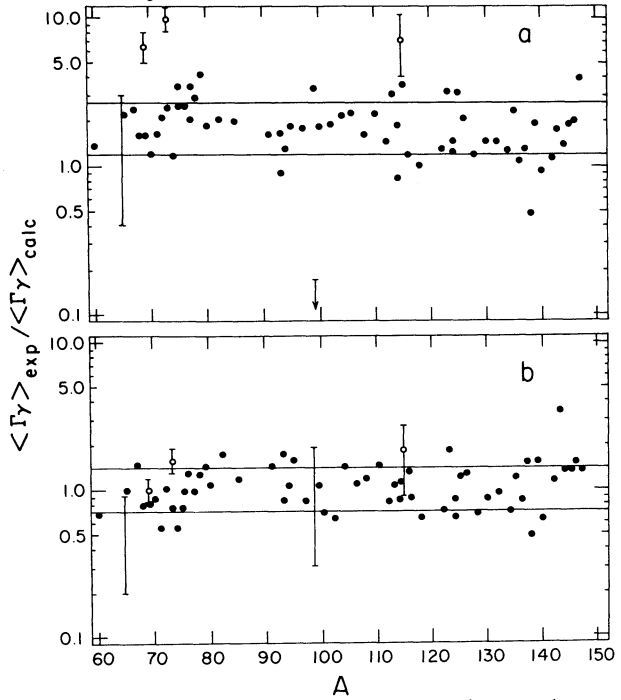


Fig. 10: a) The ratios of experimental-to-calculated $\langle\Gamma_\gamma\rangle$-values for a number of nuclei, plotted as a function of A. Where experimental data are from (n,γ) results[15] near β-stability the ratios appear as solid dots; the PXCT data on exotic nuclei yield ratios shown as open circles and/or limits. Since experimental uncertainties are frequently not assigned in ref.[15], error bars have been omitted entirely for the (n,γ) ratios; this does not imply that they are negligible. Gilbert-Cameron level densities were used in the calculation. b) Same as fig.10a except that the back-shifted-Fermi-gas level densities were used in the calculation.

systematics[18] established for the near-stable nuclei.

In terms of predictive capacity, we may certainly conclude that if the level density of an exotic nucleus is known at some energy, then the back-shifted Fermi-gas formula (incorporating parameters that yield agreement with experiment) will, when used in equation (5), determine $\langle\Gamma_\gamma\rangle$ to reasonable accuracy - at least within a factor of 2 in the mass-range considered here. It also appears that for a completely unknown nucleus, $\langle\Gamma_\gamma\rangle$ can best be calculated with the same formula but with level density parameters (a and Δ) interpolated from the systematics of Dilg et al[18].

V. β-Strength Functions

So far, I have shown that, for the cases where level widths and densities have been measured, it is possible to reproduce the data quite reliably with soundly-based calculations. With the average decay-widths thus specified, the delayed-proton energy spectrum can be analyzed for its β-decay information more confidently than has been possible in the past. In effect, $S_\beta(E_\beta)$ is the only unknown in a calculation of $I_p(E_p)$ from equations (1) and (2).

As one might suspect, reality is not quite as simple as that. Although we have a reasonable idea of $\langle\Gamma_\gamma\rangle$ and $\langle\Gamma_p\rangle$ we are less secure in our knowledge of the individual Γ_p^{if} values, relying entirely upon the optical model and the belief that an even-even daughter nucleus will exhibit approximately equal spectroscopic factors to its various states f. Also $S_\beta(E_\beta)$ is, in principle, a complicated function of energy and

even possibly of spin. Nevertheless, we have specified the parameters of the problem with more precision than before, and it is now possible to test simple models for S_β against experimental data, reasonably assured that it is S_β that one is testing and not some other property that has been arbitrarily defined.

In choosing a general model for testing S_β in exotic nuclei, it is necessary to avoid spectroscopic detail since that is rarely known for nuclei very far from β-stability. Besides, by accounting only for general trends we allow the proton data to speak for themselves in determining the specific features of local spectroscopy. Accordingly, we adopt the "gross theory" of β-decay[19,20] by using a Gaussian strength function and a modified Fermi gas model to calculate the Gamow-Teller matrix elements - and hence S_β - as a function of energy. That this approach is reasonable is borne out by the excellent agreement between the total β-decay half-lives predicted[20] and those measured for all fourteen precursors of interest here (the numbered boxes and circles in fig. 9).

The results for ^{73}Kr are illustrated in fig. 4. The dashed lines in both parts of the figure are the outcome when level densities are calculated from the Gilbert-Cameron prescription[13] with unmodified parameters from Truran et al[14], and $\langle\Gamma_\gamma\rangle$ is taken unscaled (i.e. $\overline{K=1}$) from equation (5). They do not agree particularly well with experiment. The solid lines in the same figure show the excellent agreement obtained when the parameters are optimized to achieve agreement with the X-ray data only; evidently the calculated proton spectrum shape also improves significantly. The optimization can be achieved in two ways, both leading to essentially the same calculated curves:

1) With the Gilbert-Cameron prescription, a and K are adjusted (to 13.65 MeV^{-1} and 8.5) to produce the desired result; or

2) The back-shifted Fermi-gas model is used, a and Δ being adjusted (to 9.2 MeV^{-1} and -1.8 MeV) for agreement, and a value of $\langle\Gamma_\gamma\rangle_{exp}/\langle\Gamma_\gamma\rangle_{calc} = 1.6 \pm 0.3$ extracted.

For the reasons already discussed the second choice is seen as the most desirable, the parameters being consistent with systematic trends[18].

We see here the importance of the X-ray data, for without it we might well have attributed the discrepancy between the first calculation (dashed line) and experiment to remarkable structure in S_β rather than the systematic error in level-widths that was actually responsible. It is a warning we should heed, even with improved calculational techniques, for any case where X-ray data are unavailable.

The "gross-theory" β-strength function used in generating the calculated spectra in fig. 4 is shown in fig. 11, which also shows the corresponding strength functions for other known precursors in the T_z=1/2 series. All are illustrated only for the

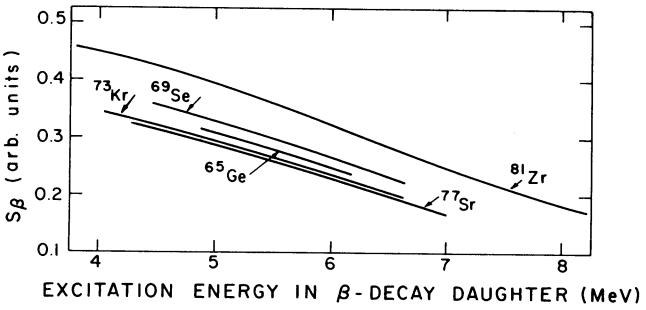


Fig. 11: Strength functions calculated according to the "gross theory" for the β-decay of the T_z=1/2 precursors. The results are plotted as a function of excitation energy in the emitters (i.e. the β-decay daughters) and show correct relative intensities.

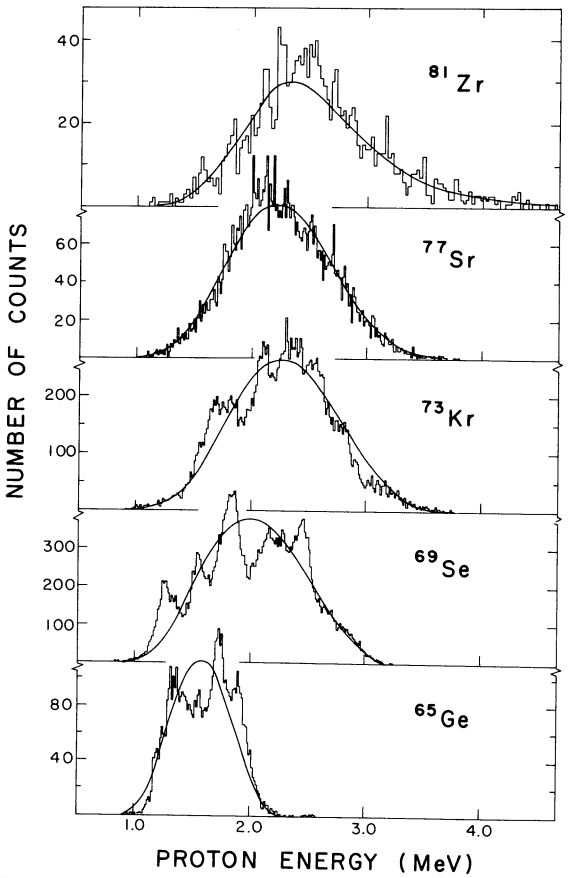


Fig. 12: Delayed proton spectra observed for precursors in the T_z=1/2 series. Curves are the results of calculations described in the text; J^π values for the precursors, from lightest to heaviest, were taken to be $3/2^-$, $3/2^-$, $5/2^+$, $3/2^+$ and $3/2^-$. Data are from references[4] (^{65}Ge, ^{77}Sr, ^{81}Zr), [1] (^{69}Se) and [5,6] (^{73}Kr).

region of excitation (in the emitter) accessed by the delayed-proton decay channel. All decrease smoothly with increasing excitation at a similar rate.

The corresponding proton-spectrum calculations for this series are compared with experiment in fig. 12. The lightest three precursors, ^{65}Ge, ^{69}Se, and ^{73}Kr, all have measured X-ray ratios, which are also well reproduced, and so constitute tests of S_β

that are as rigorous as any so far possible with delayed proton data. Indeed ^{65}Ge and ^{69}Se essentially populate only the ground state in their daughter nuclei (^{64}Zn and ^{68}Ge), thus removing even the possible ambiguity of the Γ_p^1 decomposition. The decays of ^{77}Sr and ^{81}Zr have not yet yielded adequate X-ray data for analysis; accordingly their spectrum calculations in fig. 12 used the back-shifted Fermi-gas level densities (as did all other spectrum calculations) with parameters interpolated from the systematic data of Dilg et al[18]. Agreement with the overall behaviour of the spectra is excellent. Similarly good agreement is obtained with other measured decay properties, such as the total β-branching ratio to proton emitting states (see table 1) and the subsequent proton branching to excited states in the daughter nucleus[1,6,21], with one exception: the β-branching in ^{73}Kr. This has already been noted[6] as a feature worthy of remeasurement.

Table 1: Comparison between experiment and theory for β-branching to proton emitting states

Precursor	β-branching	
	Experiment	Calculation[a]
^{65}Ge	$(1.3 \pm 0.5) \times 10^{-4}$ [4]	0.4×10^{-4}
^{69}Se	$(6.0 \pm 1.0) \times 10^{-4}$ [1]	3.7×10^{-4}
^{73}Kr	$(6.8 \pm 1.2) \times 10^{-3}$ [29]	2.2×10^{-4}
^{77}Sr	$< 2.5 \times 10^{-3}$ [21]	2.4×10^{-4}
^{81}Zr	–	2.1×10^{-2}
^{99}Cd	$(1.7 \,^{+1.1}_{-0.5}) \times 10^{-3}$ [16]	1.3×10^{-3}
^{109}Te	$(9.2 \pm 3.1) \times 10^{-2}$ [28]	2.0×10^{-3}
^{111}Te	–	8.2×10^{-3}
^{113}Xe	–	2.1×10^{-3}
^{115}Xe	$(3.4 \pm 0.6) \times 10^{-3}$ [23]	1.0×10^{-3}
^{117}Xe	$(2.9 \pm 0.6) \times 10^{-5}$ [23]	2.0×10^{-5}
^{117}Ba	–	1.3×10^{-2}
^{119}Ba	–	1.8×10^{-3}
^{121}Ba	$(2 \pm 1) \times 10^{-4}$ [25]	1.2×10^{-4}

a) with "gross theory" strength function

The excellent overall agreement is strong testimony against any large deviations from the smooth behaviour of the "gross theory" strength functions, a result made all the more striking by the observation that the five T_z=1/2 precursors traverse a region of dramatically changing deformation, as illustrated in fig. 13. If local nuclear structure is to play a dominant role in controlling S_β, one might have expected to see its effects here.

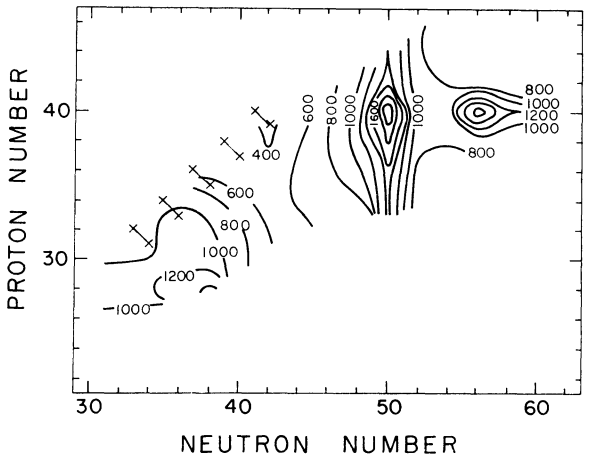


Fig. 13: Contour diagram of excitation energies of first excited 2^+ states in even-even nuclei; contours are labelled in keV. The T_z=1/2 precursors and their β-decay daughters (the emitters) are marked by X. (They are odd nuclei, however, and consequently are otherwise unrelated to the contours).

There are some "features", of course. These arise from a well-known[22] statistical property associated with determining any strength function from a limited sequence of levels. This point has been illustrated earlier[21] with the particularly simple decay of ^{69}Se. Fig. 14 is taken from ref. [21] but applies as well to the calculations reported here. It shows the β-strength function that would be required in order to

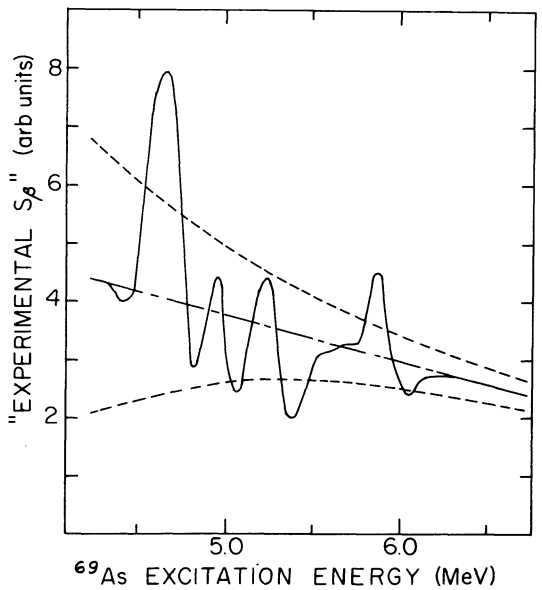


Fig. 14: The solid line gives the "β-decay strength function" extracted from the experimental delayed proton spectrum of ^{69}Se. It is only for illustration and depends upon a number of simplying assumptions. The dot-dashed line is the "gross theory" strength function, and the dashed lines are ±σ uncertainties associated with determining a strength function from a limited number of levels. From ref. [21].

reproduce in detail the peak structure observed in the proton spectrum of ^{69}Se. Although at first glance this curve might appear to be an "experimental" strength function, correctly speaking it is not, since it is derived assuming that there are no fluctuations in the level widths. However, it does represent an overestimate of structure in the strength function. The same figure shows the "gross theory" strength function as well as the fundamental uncertainty associated with a limited sample[22] in terms of plus and minus one standard deviation. Such features that exist do not show any deviation from "gross theory" significantly outside expected nuclear statistics.

A similarly coordinated set of calculations is shown in fig. 15 for the three xenon isotopes ^{113}Xe, ^{115}Xe and ^{117}Xe. Although for the lighter two isotopes corresponding X-ray data exist[7,8], it should be noted that only ^{115}Xe has had its level density so far determined; thus it is the only one of the three that is fully specified by data. The most notable feature of these results is that the "gross theory" allowed strength functions (dashed lines) do not properly account for the proton spectra, particularly at high energies. A more rapidly decreasing function, such as the one shown in solid lines at the bottom of the figure, is required to achieve reasonable agreement with the proton spectrum shape. (The unaltered "gross theory" S_β already does quite well in reproducing the measured branching ratios and X-ray ratios.) The S_β function actually used was the "gross theory" S_β scaled by $k(E_\beta + 2.5\ E_\beta^2)$ where E_β is the β-decay (to be exact, electron-capture) energy and k is an arbitrary constant; however, this form was used simply for computational convenience, and should not be taken seriously in detail. Beyond the change in overall energy-dependence, no deviation from smoothness in S_β appears outside the limits set by nuclear statistics, limits that are tighter here than those illustrated for ^{69}Se decay in fig. 14 because the level density in the iodines is higher than in ^{69}As.

The only other precursor for which PXCT data exist is ^{99}Cd. The spectrum of ^{99}Cd protons, together with the corresponding calculation, is given in fig. 16. Here, as with the T_z=+1/2 series, the "gross theory" works well in reproducing the spectrum shape and the measured branching ratio, although fluctuations in the former are more obvious since the level density is low in a nucleus so near a doubly-closed shell.

The last figures, 17 and 18, show spectra and calculations for odd-A precursors in the tellurium and barium isotopes (two atomic numbers below and two above xenon). Curiously enough, ^{109}Te and ^{111}Te show the same behaviour as the xenon precursors while the three barium precursors do not indicate any major deviation from the "gross theory". It should be emphasized, though, that no PXCT data are available for any of these five decays and, while the level-density (and Γ_γ) systematics are "anchored" at ^{115}Xe, some flexibility in the choice of parameters remains. Thus, the conclusions drawn from these cases must be considered more tentative than those claimed from figs. 12, 15 and 16.

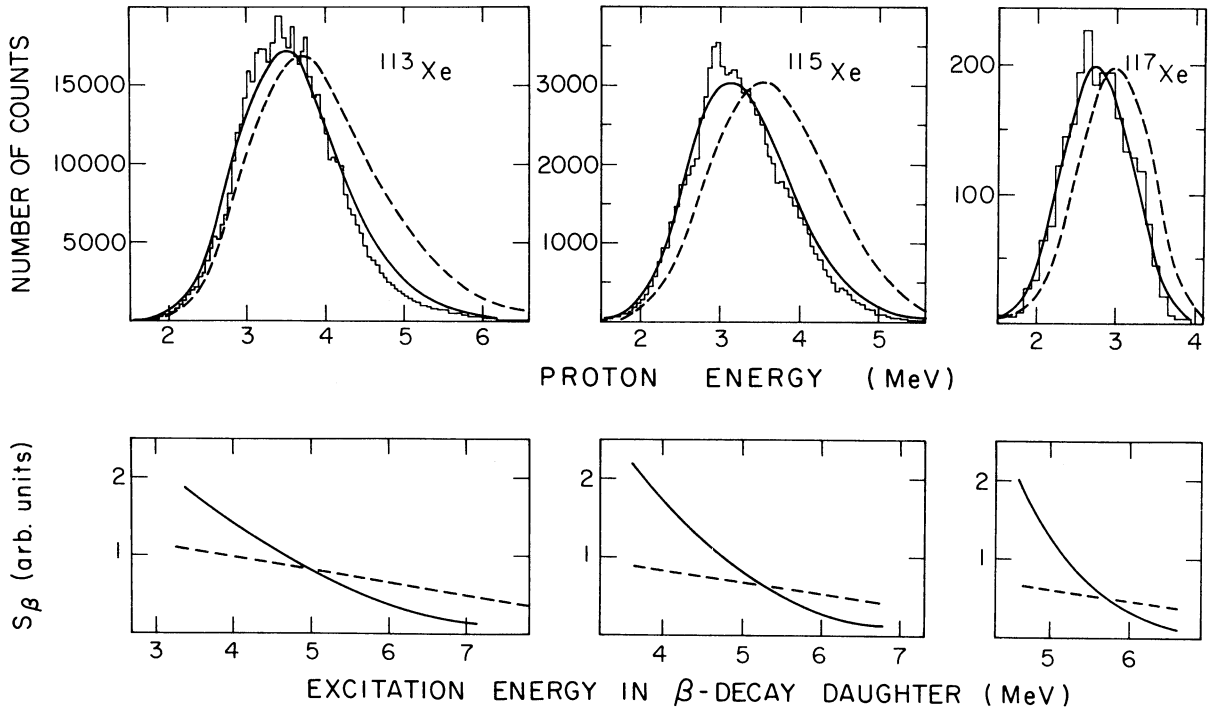


Fig. 15: (top) Delayed proton spectra observed for three Xenon precursors. Curves are the results of calculations described in the text and correspond to strength functions shown below; the precursor J^π values were all taken to be $7/2^+$. Data are from references[8](113Xe), [7](115Xe) and [23](117Xe).

(bottom) β-strength functions, S_β, used in calculating the delayed proton spectra shown above; they are plotted as a function of excitation energy in (left to right) ^{113}I, ^{115}I and ^{117}I. The dashed curves were obtained from the "gross theory" of β-decay[19,20]; the solid curves were derived empirically to produce improved agreement with the delayed proton data.

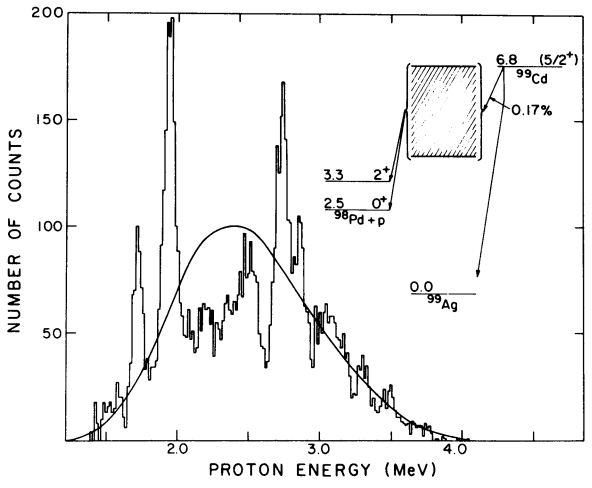


Fig. 16: Spectrum of delayed protons observed following the decay of ^{99}Cd (from ref.[7]). The smooth curve is the result of calculations, described in text, which employed the "gross theory" strength function; precursor J^{π} value was taken to be $5/2^{+}$. A decay scheme is inset with energies indicated in MeV.

VI. Conclusions

In the foregoing investigation, pains have been taken to follow the most conservative course. Only those precursors whose decays are fully specified by direct measurements of level-widths and -densities were used initially to investigate decay properties. This accomplished, other precursors were examined for apparent patterns - but the cases chosen were restricted to those which, like the fully specified ones, led to even-even daughter nuclei. This was intended insofar as possible to remove ambiguities of interpretation.

Cautious as this approach already is, a few extra words of warning should be appended. The five fully-specified precursors (^{65}Ge, ^{69}Se, ^{73}Kr, ^{99}Cd and ^{115}Xe) indeed admit of very little flexibility in analysis. It would be extraordinarily difficult to alter any aspect of the calculated decay process - the β-strength function, for example - and maintain agreement with all experimental data. The other precursor for which PXCT data are available, ^{113}Xe, is also reasonably tightly constrained, but it will not be completely secure until the ^{113}I level density is known as well. However, the remaining eight precursors allow room to maneuver, even though they were chosen for minimum ambiguity. Although the properties of levels in these emitters were calculated using the established principles, the level density parameters in particular can still be varied somewhat while remaining consistent with A-dependent systematics[18]. I believe that the calculations shown in the figures best represent a self-consistent set, but at the same time it must be admitted that even a radical change in one of the input parameters cannot be ruled out where only the proton spectrum is known experimentally.

Even with these reservations several significant conclusions can be drawn. First, the back-shifted Fermi-gas level density prescription is more successful than that of Gilbert and Cameron in representing such neutron-deficient nuclei. Second, it is now possible to calculate average γ-decay widths reliably in these nuclei. Third, the "gross theory" β-strength functions are remarkably successful in describing the observed β-decay behaviour, although a different rate of change with energy has definitely been observed in the xenon isotopes and with less assurance in the telluriums. Finally, no strong "features" or bumps have been identified in any of the β-strength functions beyond the expectations of Porter-Thomas statistics; certainly no evidence has been seen of the maxima predicted by Bykov and Naumov[30].

The number of cases studied is still too small to allow any more than speculation on the detailed behaviour of the β-strength function. In general, it seems clear that the basic precepts of the "gross theory" are justified, that indeed the centre of gravity of the Gamow-Teller strength lies far from the region of observation, presumably near the analogue (actually the supermultiplet) state, with its width determined by the extent of spin-

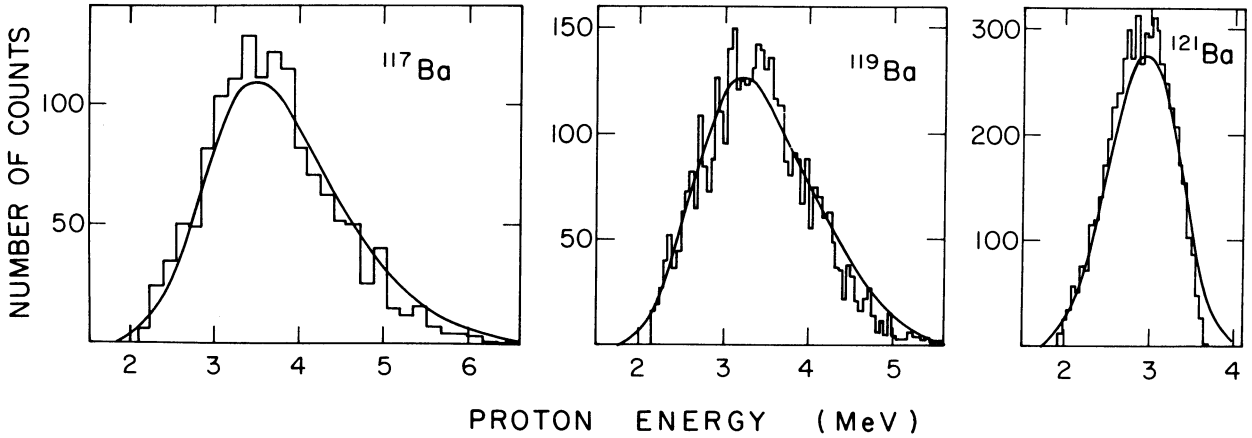


Fig. 17: Delayed proton spectra observed for three barium precursors. Curves are the results of calculations described in the text and correspond to "gross theory" strength functions; precursor J^{π} values were all taken to be $7/2^{+}$. Data are from references[24] (^{117}Ba) and [25] (^{119}Ba & ^{121}Ba).

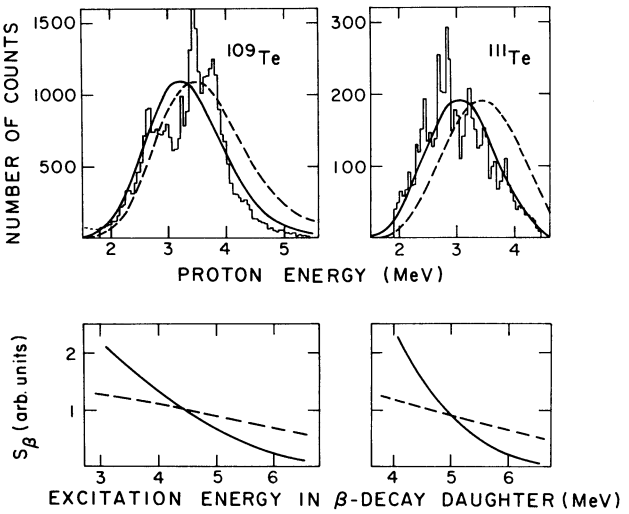


Fig. 18: (top) Delayed proton spectra observed for two tellurium precursors. Curves are the results of calculations described in the text and correspond to strength functions shown below; precursor J^{π} values were all taken to be $5/2^{+}$ (although $7/2^{+}$ gives nearly the same results). Data are from references[26] (^{109}Te) and [27] (^{111}Te). (bottom) β-strength functions, S_{β}, used in calculating the delayed proton spectra shown above; they are plotted as a function of excitation energy in ^{109}Sb and ^{111}Sb. The dashed curves were obtained from the "gross theory" while the solid curves were derived empirically to produce improved agreement with the delayed-proton data.

dependent forces and the availability of states with which to mix. The observation that the β-strength function usually behaves as though the Gamow-Teller resonance is Gaussian in its tail seems fortuitous enough that it is difficult to know whether to take seriously deviations from this behaviour. If we do take them seriously we must remain puzzled as to whether they result from a local configuration effect, possibly with a resonance just outside the energy window, or from a more general change in the form of the resonance caused perhaps by deformation. The first possibility is contradicted by the fact that the branching ratios (see table 1) are given quite accurately by the "gross theory" for ^{115}Xe and ^{117}Xe (though not for ^{109}Te) which should not be true if the predominant effect is a local resonance. But the second possibility is also contradicted, in this case by the fact that the $T_z=1/2$ precursors, though evidently deformed, are all in agreement with the "gross theory".

The evidence is tantalizing but inconclusive, and we must surely await the availability of more data. In the meantime, the "gross theory" provides a reasonably accurate first approximation to the β-decay strength function where no other information exists.

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DISCUSSION

P.G. Hansen: The importance of having cross checks in this kind of analysis emerges very clearly from your talk. I was wondering whether you have compared the level densities measured by fluctuation analysis (^{115}Xe, ^{99}Cd) with those used in your analysis?

J.C. Hardy: I have actually used those results. The full analysis (for Γ_γ and a)of PXCT data is only possible if the X-ray ratio has been measured as a function of proton energy. In the cases you mention, only a single ratio (i.e. coincidences with all proton energies) was possible from experiment. Consequently, I used the CERN fluctuation analysis to extract the level-density parameters and then extracted Γ_γ from the X-ray ratio.

M. Finger: Can you comment on the structure you see on some of the proton spectra?

J.C. Hardy: In fact I have done so in the proceedings but left it short in my talk as this has been dealt with before. Briefly, the structure in all cases is within the expectation of Porter-Thomas fluctuations. It always seems necessary to emphasize that this statement does not mean that they are somehow any less real. Since in most cases, however, the structure cannot be uniquely attributed to Sβ (and not to Γ_p for example), it only seems fruitful to discuss the average behaviour.

E. Roeckl: You introduced an "ad-hoc" beta strength function for reproducing the proton spectra of the precursors of tellurium (Z=52) and xenon (Z=54, whereas the beta strength function from the gross theory appear to do well for barium (Z=56) precursors as well as below Z=50. How do you interpret this observation?

J.C. Hardy: For the moment I cannot, particularly in the light of "gross theory" success for the $T_z = \frac{1}{2}$ series as well. As usual, one needs data on new cases - and possibly more complete (i.e. PXCT) data on the less well known old ones.

H. Morinaga: Do you use the word fluctuation in the different meaning from structures?

J.C. Hardy: Within the statistical-model approach, each class of transition amplitudes is seen as exhibiting a Porter-Thomas statistical distribution about some average behaviour. Although it is the average behaviour (or strength function) that is supposed to contain the essential physics, the "fluctuations" about this average are in every way real, and reproducible from one measurement to another. Indeed, with the perfect untruncated shell-model code you could presumably calculate these small features in detail. Whether you would improve your understanding of the important physics by doing so is another question.