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POLARIZATION EFFECTS IN DEEP ELECTROPRODUCTION AND THE QUARK PARTON MODEL

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A B S T R A C T

Deep inelastic scattering of longitudinally polarized leptons from polarized nucleons is studied phenomenologically and the parallel antiparallel asymmetry is computed using the quark parton model.

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1. - INTRODUCTION

In Bjorken's terminology this paper is a kindergarten exercise on the parton model for deep inelastic electroproduction on a polarized target.

The analysis is made to lowest order in the electromagnetic interaction assuming the one-photon exchange approximation to be valid. The cross-section is then a linear combination of structure functions which are scalar under Lorentz transformations. Two of these functions describe unpolarized scattering and three are needed for the polarization effects.

In Section 2, we introduce the spin dependent part of the hadronic tensor to fix the notations and we recall what type of scaling is expected for the five structure functions. The inequalities due to the Hermitian and semi-definite character of the hadronic tensor are written in the scaling limit.

The various asymmetries one can experimentally measure are defined in Section 3 with their limits at high energy in the scaling region.

The quark parton model is used in Section 4 to compute the non-trivial function for polarization in the scaling limit. This formalism gives naturally a sum rule already derived by Bjorken from current algebra and we discuss numerical estimates for this sum rule. The three-quark model of the nucleon is also considered as an illustration.

A generalization of the Bjorken sum rule is given in the Appendix.

2. - STRUCTURE FUNCTIONS

1) - The hadronic tensor is the Fourier transform of the average value between one-nucleon states of the product of two components of the electromagnetic current

$$T_{\mu\nu} + S_{\mu\nu} = \frac{M}{2\pi} \int d_4x \, e^{iq\cdot x} \langle p | J_\nu(0) J_\mu(x) | p \rangle$$

where M is the nucleon mass, p the nucleon energy momentum and q the virtual photon energy momentum.

The spin independent part, $T_{\mu\nu}$ was already studied in inelastic lepton scattering on an unpolarized target. The spin dependent part $S_{\mu\nu}$ is linear in the target polarization N where N is a unit space-like pseudovector orthogonal to p.

From Lorentz covariance, space reflection invariance and the conservation of the electromagnetic current we define three covariants for $S_{\mu\nu}$ and therefore three structure functions depending on two scalar variables q^2 and W^2 where W is the effective mass of the unobserved hadronic system $W^2 = -(p+q)^2$ [1)-6)]

$$S_{\mu\nu} = \frac{1}{2Mi}\, \varepsilon_{\mu\nu\rho\sigma}\, q^{\rho} N^{\sigma} X_1(q^2, W^2) + \frac{1}{2M^2 i}\left(\eta_\mu P_\nu - P_\mu \eta_\nu\right) X_2(q^2, W^2) + \frac{1}{2M^2}\left(\eta_\mu P_\nu + P_\mu \eta_\nu\right) Y(q^2, W^2)$$

where the two vectors n and P, orthogonal to p are given by

$$\eta_\mu = \frac{1}{M}\, \varepsilon_{\mu\nu\rho\sigma}\, p^{\nu} q^{\rho} N^{\sigma} \qquad\qquad P_\mu = p_\mu - \frac{p.q}{q^2}\, q_\mu$$

From the Hermitians of the electromagnetic current, the three-structure functions X_1, X_2 and Y are real. If, moreover, time reversal invariance holds, X_1 and X_2 are real but Y must be purely imaginary so that it vanishes. The structure function Y measures a violation of the time reversal invariance in electromagnetic interactions [2)].

Inequalities between structure functions can be derived from the positivity of the hadronic tensor [2),5),6)]. They will be written later in the specific form needed in this paper.

2) - The two structure functions for unpolarized inelastic lepton scattering are associated to the total photoabsorption cross-section with polarized space-like photons σ_T and σ_L where T means transverse and L longitudinal [7)].

It has been proposed by Bjorken [8)] that these structure functions for large q^2 and W^2 scale in the sense

$$\lim_{\substack{q^2 \Rightarrow \infty \\ W^2 \Rightarrow \infty \\ \xi = \frac{q^2}{2M\nu} \text{ fixed}}} \frac{1}{\pi e^2}\, \nu M\, \sigma_{T,L}(q^2, W^2) = F_{T,L}(\xi) \tag{1}$$

where the photon laboratory energy ν is defined in an invariant way by $\nu M = -p.q$ or $W^2 = M^2 - q^2 + 2M\nu$.

Experiments performed at SLAC and DESY strongly support this conjecture [9)].

3) - If a scaling a la Bjorken holds for the polarization structure functions X_1, X_2 and Y, it is expected to be of the form [5),10)]

$$\begin{cases} \mathrm{LIM} \ \dfrac{\nu}{M} X_1(q^2, W^2) = 2 F_{\parallel}(\xi) \\ \mathrm{LIM} \ \dfrac{\nu^2}{M\sqrt{q^2}} X_2(q^2, W^2) = 2 F_{\perp}(\xi) \\ \mathrm{LIM} \ \dfrac{\nu^2}{M\sqrt{q^2}} Y(q^2, W^2) = 2 F_N(\xi) \end{cases}$$

where LIM is the Bjorken limit defined in Eq. (1).

The positivity constraints take particularly simple forms in the scaling region

$$F_{\parallel}^2(\xi) \leq F_T^2(\xi) \tag{2}$$

$$F_N^2(\xi) \leq F_T(\xi) F_L(\xi) \tag{3}$$

$$F_{\perp}^2(\xi) + F_N^2(\xi) \leq \frac{1}{2}\left[F_T(\xi) + F_{\parallel}(\xi)\right] F_L(\xi) \tag{4}$$

It is obvious on these expressions that if the longitudinal scaling function $F_L(\xi)$ is zero as suggested by the experiments [9)], so are both $F_{\perp}(\xi)$ and $F_N(\xi)$.

4) - In what follows we shall study the high-energy limit in the scaling region and we introduce a second scaling variable $\rho = E/E'$ where $E(E')$ is the ingoing (outgoing) lepton energy in the laboratory frame. The relation between E, ρ, ξ and the laboratory scattering angle θ is simply

$$\rho^{-1} = 1 + \frac{2E \sin^2 \frac{\theta}{2}}{M\xi}$$

3..- ASYMMETRIES

1) - For a nucleon at rest the four-vector N has only space components and we introduce a three-dimensional frame of reference where the lepton momenta $\vec{k}$ and $\vec{k}'$ have the following components

$$\vec{k} = k\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, 0\right)$$

$$\vec{k}' = k'\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, 0\right)$$

The polarized differential cross-section is related to the unpolarized one by

$$d^2\sigma = \varepsilon\, d^2\sigma_{NP}\ (1 + \Delta)$$

where $\varepsilon = 1$ if the initial lepton is polarized and $\varepsilon = \frac{1}{2}$ when the final lepton polarization is measured. The general asymmetry Δ is linear in the nucleon spin orientation $\vec{N}$ $(N^2 = 1)$

$$\Delta = \vec{\Delta} \cdot \vec{N}$$

and we have to study the three independent asymmetries Δ_1, Δ_2 and Δ_3.

2) - When the nucleon has a polarization orthogonal to the scattering plane $(N_1 = N_2 = 0)$ there is no correlation between the lepton and nucleon spin orientations. The asymmetry Δ_3 measures a violation of time reversal invariance.

In the high energy limit, Δ_3 scales

$$\lim_{\substack{s,q^2,\nu \Rightarrow \infty \\ \rho,\,\xi \text{ fixed}}} \Delta_3(s, q^2, \xi) = \Delta_N(\rho, \xi)$$

with

$$\Delta_N(\rho,\xi) = -\frac{2(1+\rho)\sqrt{\rho}}{1+\rho^2+2\rho R(\xi)}\ \frac{F_N(\xi)}{F_T(\xi)}$$

From the positivity condition (3), we deduce an upper limit for the modulus of this asymmetry [5]

$$|\Delta_N(\rho,\xi)| \leq \frac{2(1+\rho)\sqrt{\rho R(\xi)}}{1+\rho^2+2\rho R(\xi)}$$

where, as usual, R is the ratio of the two scaling functions

$$R(\xi) = \mathrm{LIM}\ \frac{\sigma_L(q^2, W^2)}{\sigma_T(q^2, W^2)}$$

3) - We now study the case where the nucleon polarization lies in the scattering plane $(N_3 = 0)$. The asymmetries measure a correlation between the longitudinal polarization of the incoming or the outgoing lepton and the polarization of the nucleon target. Again in the high energy limit Δ_1 and Δ_2 scale

$$\lim_{\substack{s,q^2,\nu\Rightarrow\infty \\ \rho,\xi \text{ fixed}}} \Delta_1(s,q^2,\nu) = \Delta_{\parallel}(\rho,\xi)$$

$$\lim_{\substack{s,q^2,\nu\Rightarrow\infty \\ \rho,\xi \text{ fixed}}} \Delta_2(s,q^2,\nu) = \Delta_{\perp}(\rho,\xi)$$

with

$$\Delta_{\parallel}(\rho,\xi) = -\frac{1-\rho^2}{1+\rho^2+2\rho R(\xi)} \frac{F_{\parallel}(\xi)}{F_T(\xi)}$$

$$\Delta_{\perp}(\rho,\xi) = \frac{2(1-\rho)\sqrt{\rho}}{1+\rho^2+2\rho R(\xi)} \frac{F_{\perp}(\xi)}{F_T(\xi)}$$

From the positivity conditions (2) and (4) we deduce upper limits for the modulae of these asymmetries

$$|\Delta_{\parallel}(\rho,\xi)| \leq \frac{1-\rho^2}{1+\rho^2+2\rho R(\xi)}$$

$$|\Delta_{\perp}(\rho,\xi)| \leq \frac{2(1-\rho)\sqrt{\rho R(\xi)}}{1+\rho^2+2\rho R(\xi)}$$

4. - QUARK PARTON MODEL

1) - In the parton model, the electromagnetic current scatters incoherently on the point-like partons so that the structure functions have the scaling properties already given. When the partons are quarks and non-interacting gluons [11)-13)], only the transverse scaling function $F_T(\xi)$ for unpolarized scattering is non-trivial. We introduce the standard notations :

- P_N probability for a configuration with N partons in the hadron ;
- N_j number of quarks of type j in this configuration ;
- $f_j^N(\xi)$ density of probability of the parton j longitudinal momentum viewed in the $P \to \infty$ frame of the hadron ;
- Q_j electric charge of the quark of type j,

with the normalization constraints

$$\sum_N P_N = 1 \qquad \int_0^1 f_j^N(\xi)\,d\xi = 1 \qquad (5)$$

In the quark parton model $F_T(\xi)$ has the nice representation

$$2F_T(\xi) = \sum_j Q_j^2 \, D_j(\xi) \tag{6}$$

where the quark j distribution in the hadron is given by

$$D_j(\xi) = \sum_N{}' P_N \, N_j \, f_j^N(\xi)$$

The ratio $R = (\sigma_L(q^2,W^2))/(\sigma_T(q^2,W^2))$ scales trivially and is simply given by

$$R = \frac{q^2}{\nu^2}$$

Therefore from the positivity constraints (3) and (4) the polarization structure functions X_2 and Y have a zero scaling limit in the sense previously defined

$$F_\perp(\xi) = 0 \qquad\qquad F_N(\xi) = 0 \tag{7}$$

and the asymmetries $\Delta_\perp(\rho,\xi)$ and $\Delta_N(\rho,\xi)$ vanish in this limit. The only measurable quantity is then $\Delta_{||}(\rho,\xi)$ and we now study the scaling function $F_{||}(\xi)$:

2) - In the N parton configuration the partons are labelled by an index α. We are interested by the value of the spin third component measured along the momentum direction. We call it s_α for the parton α and s_o for the composite hadron and we introduce the reduced variable $\sigma_\alpha = s_\alpha / s_o$.

Let us define the N dimensional correlation function $S^N(\sigma_1, \sigma_2, \ldots, \sigma_N)$ with the normalization condition

$$\sum_{\sigma_1}{}' \sum_{\sigma_2} \cdots \sum_{\sigma_N} S^N(\sigma_1, \sigma_2, \ldots, \sigma_N)\, \delta\Big[\sum_\alpha \sigma_\alpha - 1\Big] = 1 \tag{8}$$

where $\delta[\;]$ is a Kronecker symbol.

The probability for the parton α to have the polarization σs_o in the N parton configuration is simply obtained summing S^N over all variables but σ_α

$$S_\alpha^N(\sigma) = \sum_{\sigma_1} \sum_{\sigma_2} \cdots \sum_{\sigma_N}{}' S^N(\sigma_1, \sigma_2, \ldots, \sigma_N)\, \delta\Big[\sum_\beta{}' \sigma_\beta - 1\Big]\, \delta[\sigma_\alpha - \sigma] \tag{9}$$

and from (8) we deduce the obvious condition :

$$\sum_{\sigma} S_{\alpha}^{N}(\sigma) = 1 \tag{10}$$

Another interesting property concerns the mean value of σ given by

$$\bar{\sigma}_{\alpha} = \sum_{\sigma} \sigma S_{\alpha}^{N}(\sigma) \tag{11}$$

Using the definition (9) of $S_{\alpha}^{N}(\sigma)$ and the normalization condition (8), it is straightforward to deduce the equality

$$\sum_{\alpha}{}' \bar{\sigma}_{\alpha} = 1 \tag{12}$$

3) - The structure functions X_1, X_2 and Y are easily computed for elastic scattering on a polarized spin $\frac{1}{2}$ particle and the result is expressed in terms of the familiar Dirac form factors *)

$$\left| \begin{array}{l} X_1^{el}(q^2,W^2) \Rightarrow 2M^2 \delta(W^2-M^2)\left[\bar{F}_1(q^2)+\bar{F}_2(q^2)\right]^2 \\ X_2^{el}(q^2,W^2) \Rightarrow 2M^2 \delta(W^2-M^2)\left[\bar{F}_1(q^2)+\bar{F}_2(q^2)\right]\bar{F}_2(q^2) \\ Y^{el}(q^2,W^2) \Rightarrow 0 \end{array} \right.$$

Therefore in the quark parton model we immediately recover the relations (7) and only the scaling function $F_{||}(\xi)$ can be non-trivial.

Its computation is straightforward using the standard technics and the result is

$$2F_{||}(\xi) = \sum_{j,\sigma} Q_j^2 \sigma D_{j\sigma}(\xi) \tag{13}$$

where the distributions $D_{j\sigma}(\xi)$ are now defined by

$$D_{j\sigma}(\xi) = \sum_{N} P_N N_j f_j^N(\xi) S_j^N(\sigma) \tag{14}$$

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*) The vanishing of Y for elastic scattering is equivalent to the well-known fact that time reversal violation cannot be measured in lepton hadron elastic scattering, when the hadron has spin 0 or $\frac{1}{2}$.

Comparing now the expressions (6) and (13) we can check the positivity constraint

$$|F_{\parallel}(\xi)| \leq F_T(\xi)$$

taking into account the normalization relation (10) $S_j^N(1) + S_j^N(-1) = 1$.

4) - We integrate the scaling function $F_{\parallel}(\xi)$ over ξ

$$Z = \int_0^1 2 F_{\parallel}(\xi)\, d\xi$$

Using the Eqs. (13), (14) and the normalization integral (5) we can write Z in the convenient form

$$Z = \sum_{j,\sigma}' D_{j\sigma} \langle j,\sigma | \sigma_3 Q^2 | j,\sigma \rangle \tag{15}$$

where the $D_{j\sigma}$'s can be interpreted as the mean values, in the hadron, of the number of quarks of type j having a spin parallel ($\sigma = +1$) or antiparallel ($\sigma = -1$) to the hadron spin. It is clear now, on Eq. (15), that Z is the average value of the operator $\sigma_3 Q^2$.

By definition of the symmetric coefficients of the Gell-Mann algebra Z can be decomposed into

$$Z = \frac{1}{2} d^{QQ}{}_{c}\; g_A^c$$

where g_A^C, given by

$$g_A^c = \sum_{j,\sigma}' D_{j\sigma} \langle j,\sigma | \sigma_3 F^c | j,\sigma \rangle$$

is the average value of the U(6) algebra operator $\sigma_3 F^C$. It follows (see the Appendix) that g_A^C is the coupling constant of the hadronic axial vector current of U(3) index C.

This result is identical to the Bjorken sum rule [3),14)] derived from the algebra of space components of the electromagnetic current

$$[J_j^Q(x), J_k^Q(0)]\,\delta(x_0) = \frac{1}{i}\, \varepsilon_{jkl}\, d^{QQ}{}_{c}\, A_l^c(0)\, \delta_4(x)$$

where $A_\ell^C(0)$ is an axial vector current by parity conservation.

5) - For a given hadronic multiplet the integrals Z are linear combination of three reduced constants *)

$$Z = \frac{2}{3} f_1 + \frac{1}{3}\left[f_a Q + f_s \Omega_Q^{(8s)} \right] \tag{16}$$

where $\Omega_Q^{(8s)}$ is the symmetric isometry of weight Q.

The singlet coupling constant f_1 is related to the axial vector baryonic current and it cannot be directly measured. The octuplet coupling constants f_a and f_s are obtained, in the Cabibbo theory, from the leptonic decay of hadrons.

Let us now restrict ourselves to the baryon octuplet $J^P = \frac{1}{2}^+$ to which the nucleon belongs. The vector part for neutron β decay being normalized to unity, $f_a + f_s$ is the axial vector part for the same transition and as an example $\sqrt{\frac{2}{3}}\, f_s$ is the axial vector part for $\Sigma^- \to \Lambda + e^- + \bar{\nu}_e$. From an over-all fit of hyperon β decay we get [15]

$$f_a + f_s = -\frac{G_A}{G_V} = 1{,}23$$

$$\frac{f_s}{f_a + f_s} = 0{,}6 \qquad **)$$

With Clebsch-Gordan coefficients, formula (16) gives

$$Z^p = \frac{2}{3} f_1 + \frac{1}{3} f_a + \frac{1}{9} f_s$$

$$Z^n = \frac{2}{3} f_1 \qquad\quad - \frac{2}{9} f_s$$

A prediction independent of f_1 already obtained by Bjorken is

$$Z^p - Z^n = \frac{1}{3}\left(-\frac{G_A}{G_V}\right) \simeq 0{,}41$$

The constant f_1 being associated to the axial vector baryonic current has a simple representation in the quark parton model

$$f_1 = \frac{1}{3} \sum_{j,\sigma} \sigma D_{j\sigma}$$

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*) The third term is obviously absent for SU(3) multiplet $D(\lambda_1, \lambda_2)$ with $\lambda_1 \lambda_2 = 0$.

**) This value of the D/F ratio is associated to a Cabibbo angle θ_C with $\sin\theta_C \simeq 0.25$. Experiment can accommodate small changes of both parameters but this is of minor importance for the estimates of Z^p and Z^n.

We assume the spin of the gluons to be uncorraleted with the nucleon spin so that in the first moment relation (12) the sum over gluons vanishes and we obtain $f_1 = \frac{1}{3}$ *). We are now in position to make numerical predictions on Z^p and Z^n

$$Z^p \simeq 0.47 \qquad\qquad Z^n \simeq 0,06 \tag{17}$$

so that Z^p and Z^n differ essentially by one order of magnitude.

6) - It can be interesting to look at the simplest model of a nucleon consisting of three quarks : $p = q_1 q_1 q_2$, $n = q_2 q_2 q_1$. The proton and neutron wave functions have been computed from U(6) symmetry by Itzykson and Jacob [16] and we easily obtain the probabilities $S_j(\sigma)$

$$S_1(1) = \frac{5}{6} \qquad\qquad S_1(-1) = \frac{1}{6}$$
$$S_2(1) = \frac{1}{3} \qquad\qquad S_2(-1) = \frac{2}{3}$$

The first momentum relation (10) is trivially satisfied and the scaling functions F_T^p, $F_{||}^p$, F_T^n, $F_{||}^n$ are given in terms of two longitudinal momentum distributions $f_1(\xi)$ and $f_2(\xi)$

$$2F_T^p(\xi) = \frac{8}{9} f_1(\xi) + \frac{1}{9} f_2(\xi) \qquad\qquad 2F_{||}^p(\xi) = \frac{16}{27} f_1(\xi) - \frac{1}{27} f_2(\xi)$$

$$2F_T^n(\xi) = \frac{2}{9} f_1(\xi) + \frac{4}{9} f_2(\xi) \qquad\qquad 2F_{||}^n(\xi) = \frac{4}{27} f_1(\xi) - \frac{4}{27} f_2(\xi)$$

From these expressions we obtain upper and lower bounds for the asymmetries $\Delta_{||}^p$ and $\Delta_{||}^n$

$$-\frac{1}{3} \leq \frac{F_{||}^p(\xi)}{F_T^p(\xi)} \leq \frac{2}{3}$$

$$-\frac{2}{3} \leq \frac{F_{||}^n(\xi)}{F_T^n(\xi)} \leq \frac{1}{3}$$

If the momentum distribution is symmetrical, we have only one independent function $f(\xi)$. The scaling function $F_{||}^n(\xi)$ vanishes and we have predictions for the asymmetries

$$\Delta_{||}^p = -\frac{5}{9}\,\frac{1-\rho^2}{1+\rho^2} \qquad\qquad \Delta_{||}^n = 0$$

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*) Let us remark that this value of f_1 is independent of the SU(3) multiplet considered and is only due to the quark baryonic number.

In this model one can exactly compute the integrals Z^p and Z^n

$$Z^p = \frac{5}{9} \qquad\qquad Z^n = 0 \tag{18}$$

and we recall the theoretical prediction

$$-\frac{G_A}{G_V} = \frac{5}{3}$$

Let us remark that these values (18) are qualitatively the same as the phenomenological one (17).

5. - CONCLUSION

We have studied the inelastic scattering of longitudinally polarized charged leptons from a polarized nucleon with a particular attention to the deep inelastic region. In a model with spin zero and spin $\frac{1}{2}$ partons the structure functions describing a nucleon polarization orthogonal to the incident lepton momentum scale in a trivial way and it will not be possible to observe any asymmetry at high energy. The situation is more favourable when the proton polarization is parallel or antiparallel to the lepton ones. The associated structure function scales in the ordinary way and it has been computed assuming the spin $\frac{1}{2}$ partons to be quarks. The Bjorken sum rule for polarization has been derived as a consequence of this quark parton model.

Numerical estimates based on experimental data for hyperon β decay suggest that the asymmetry on a neutron target is, in average, smaller by one order of magnitude than the asymmetry on a proton target. This qualitative feature is confirmed by the naive three-quark model of the nucleon which can be considered as a crude but realistic approximation of the true nucleon. A more detailed information on the nucleon structure is needed to obtain quantitative predictions for the asymmetry $\Delta_{\parallel}$ as a function of ξ . It is precisely the aim of these experiments to bring such an information.

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APPENDIX

We consider a forward scattering amplitude

$$\gamma^{a} + \alpha \Rightarrow \gamma^{\bar{b}} + \beta$$

with different U(3) indices for the currents and the hadrons ($\bar{b}$ means the Hermitian conjugate of b). We can define an integral associated to polarization and which generalizes the equation (15)

$$Z^{\beta\alpha;ba} = \sum_{m,n}' D^{\beta\alpha}_{mn} \langle m | G_3 F^{b} F^{a} | n \rangle$$

where m, n are quark and antiquark indices including the polarization ($m = j\sigma$). Using the quark model algebra to reduce the product of two infinitesimal generators we obtain

$$Z^{\beta\alpha;ba} = \frac{1}{2}\left(d^{ba}{}_{c} + f^{ba}{}_{c}\right) g_{A}^{c;\beta\alpha}$$

where d and f are the symmetric and skew symmetric couplings. The quantity $g_A^{c;\beta\alpha}$ is given by

$$g_{A}^{c;\beta\alpha} = \sum_{m,n}' D^{\beta\alpha}_{mn} \langle m | G_3 F^{c} | n \rangle$$

Now it is well known that in the quark model a Lie algebra of the U(6) type can be generated by the space integrals

$$\left| \begin{array}{l} V^{c} = \int q^{*}(\vec{x})\, F^{c}\, q(\vec{x})\, d^3x \\ A^{c}_{k} = \int q^{*}(\vec{x})\, G_{k}\, F^{c} q(\vec{x})\, d^3x \end{array} \right.$$

where $q(\vec{x})$ is the Dirac quark field. As usual, V means vector and A axial vector. It follows immediately a generalized Bjorken sum rule

$$g_{A}^{c;\beta\alpha} = \langle \beta | A^{c}_{3} | \alpha \rangle$$

From SU(3) covariance we can introduce three reduced matrix elements

$$g_{A}^{c;\beta\alpha} = \delta_{c0}\,\delta_{\beta\alpha}\, f_1 + (1-\delta_{c0})\left[f_a\, F^{c}_{\beta\alpha} + f_s\, \Omega^{8s}_{c;\beta\alpha}\right]$$

where $\Omega_{c}^{(8s)}$ is the symmetric isometry associated to the weight C.

REFERENCES

1) C.K. Iddings - Phys.Rev. 138, B446 (1965).

2) N. Christ and T.D. Lee - Phys.Rev. 143, 1310 (1960).

3) J.D. Bjorken - Phys.Rev. 148, 1467 (1966).

4) N. Dombey - Revs.Modern Phys. 41, 236 (1969).

5) M.G. Doncel and E. de Rafael - I.H.E.S. Preprint (1971)

6) M. Gourdin - Contribution to the TEPP Project (1971).

7) M. Gourdin - Nuovo Cimento 21, 1094 (1961).

8) J.D. Bjorken - Phys.Rev. 179, 1547 (1966).

9) See, for instance : R. Taylor - Report at the Kiev Conference (1970).

10) Y. Georgelin, J. Stern and J. Jersak - Nuclear Phys. B27, 493 (1971).

11) J.D. Bjorken and E. Paschos - Phys.Rev. 185, 1975 (1969)

12) Ch. Llewellyn Smith - Talk given at the Naples Meeting (1970), CERN Preprint TH. 1188 (1970).

13) M. Gourdin - Nuclear Phys. B29, 601 (1971).

14) J.D. Bjorken - Phys.Rev. D1, 1376 (1970).

15) See, for instance : M. Roos - CERN Preprint TH. 1349 (1971).

16) C. Itzykson and M. Jacob - Nuovo Cimento 48, 909 (1967).