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CP NON-INVARIANCE AND THE $K_S \to \mu^+\mu^-$ DECAY RATE

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A B S T R A C T

Christ and Lee suggested CP non-invariance as an explanation of the low experimental $K_L \to \mu^+\mu^-$ decay rate. We discuss Hamiltonian realizations of this mechanism, and the lower bounds on the $K_S \to \mu^+\mu^-$ decay rate implied by them. The lower bound on the $K_S \to \mu^+\mu^-$ branching ratio varies in these models from $10 * 10^{-7}$ (for the most economical model) to $2 * 10^{-7}$.

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Christ and Lee [1] have suggested that the disagreement between the upper bound [2] on the $K_L \to \mu^+\mu^-$ decay rate and the corresponding theoretical lower bound [3],[4] can be explained by invoking CP non-invariance. The idea is that the $K_1 \to \mu^+\mu^-$ amplitude is so large that the small admixture of the CP even component K_1 in the state K_L becomes important. The large $K_1 \to \mu^+\mu^-$ decay amplitude implies a large $K_S \to \mu^+\mu^-$ decay rate; they obtain [1]

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 5 * 10^{-7} \tag{1}$$

where Γ_S is the total K_S decay rate. Gaillard [5] has shown that the Christ-Lee mechanism requires only that

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 1.6 * 10^{-7} . \tag{2}$$

The purpose of this note is to discuss Hamiltonian realizations of the Christ-Lee mechanism, and the lower limits on the decay rate $\Gamma(K_S \to \mu^+\mu^-)$ implied by them.

The basic requirements of the Christ-Lee mechanism are new interactions allowing : 1) the large amplitude for $K_1 \to \mu^+\mu^-$ and 2) CP non-invariance for the K_1 and/or K_2 decay amplitude into the $\mu^+\mu^-$ channel. The simplest type of Hamiltonian, as discussed in class A models below, involves only a single new interaction which allows a direct CP non-invariant $K_1 \to \mu^+\mu^-$ transition. For class A models, we show *)

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 10 * 10^{-7} \tag{3}$$

This result is especially interesting because there are preliminary reports [7] that the experimental upper bound has been reduced to a value approaching the limit (3) and may be improved soon beyond this. Our other theoretical limits (class B and class C models) serve to point out the "interpretation" of possible experimental $K_S \to \mu^+\mu^-$ decay rates between the limits (2) and (3).

Our notation and assumed experimental data are the same as those of Ref. 5). We assume CPT invariance and express the physical states K_S and K_L in terms of the usual CP eigenstates K_1 and K_2 as

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*) This point has been made by us previously in unpublished manuscripts [6].

$$|K_S\rangle = |K_1\rangle + \epsilon\,|K_2\rangle\,, \tag{4.a}$$

$$|K_L\rangle = |K_2\rangle + \epsilon\,|K_1\rangle \tag{4.b}$$

and

$$\epsilon \simeq 2 * 10^{-3}\, e^{i\pi/4} \tag{4.c}$$

where terms of relative higher order in ϵ have been dropped (as also done hereafter, throughout). The decay amplitudes T_{an} are normalized so that the squares of their magnitudes give the relevant partial decay rates; here, a represents the K meson state (K_1, K_2, K_S or K_L), and n stands for the decay channel. The two $\mu^+\mu^-$ ($\gamma\gamma$) decay channels are denoted as $\mu_\pm$ ($\gamma_\pm$), where the subscript is the CP value. We assume that there are no anomalously large effective couplings *) of the states 3π, $\pi\pi\gamma$, or other unknown intermediate states to $\mu^+\mu^-$, so that the unitarity relations may be written as [5)]

$$\mathrm{Im}\, T_{2\mu_-} = T_{2\gamma_-} \cdot \sqrt{\varphi_-} \tag{5.a}$$

$$\mathrm{Re}\, T_{1\mu_-} = i\, T_{1\gamma_-} \cdot \sqrt{\varphi_-} \tag{5.b}$$

$$\mathrm{Re}\, T_{2\mu_+} = i\, T_{2\gamma_+} \cdot \sqrt{\varphi_+} \tag{5.c}$$

$$\mathrm{Im}\, T_{1\mu_+} = \tfrac{1}{2}\, \mathrm{Re} \sum_m \left(T_{1m}\, T^*_{m\mu_+} \right) \tag{5.d}$$

where the sum may include the $\pi\pi$ state (including associated photons) as well as $\gamma\gamma$; here,

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*) The normal (two-photon exchange) contribution from 3π states has recently been shown [8)] to be very small. If the $K_L \rightarrow \pi\pi\gamma$ decay rate equals its present upper limit, there may be small corrections [9)] needed to Eqs. (5.a) and (5.c). The possibility of other unknown intermediate states has been considered [10)] as a possible solution of the $K_L \rightarrow \mu^+\mu^-$ puzzle without CP non-invariance.

$$\varphi_- \simeq 1.2 * 10^{-5} \tag{6.a}$$

and

$$\varphi_+ \simeq 0.8\ \varphi_- \tag{6.b}$$

are the $\gamma\gamma \to \mu^+\mu^-$ amplitudes for the values of CP indicated by the subscripts. Im ϵ in Eq. (4.c) depends on the Wu-Yang phase convention; Eq. (5.c) depends, in addition, on the apparent experimental equality $\eta_{+-} = \eta_{oo}$ in $K_L \to 2\pi$ decays.

<u>Class A Models</u>

The essential feature of these models is one new effective term H' in the Hamiltonian *)

$$H' = G' \cdot \sin\theta \cdot A^7_\lambda \cdot \bar{\psi}_\mu \gamma_\lambda \gamma_5 \psi_\mu \tag{7}$$

where A^7_λ is the strangeness changing component of the axial current, that is odd with respect to C. The CP violating term (7) contributes to the dispersive amplitude Im $T_{1\mu_-}$, but since this provides a direct transition from K_1 to $\mu^+\mu^-$, one has Re $T_{1\mu_-} = 0$, and

$$T_{L\mu_-} = i\ \mathrm{Im}\ T_{2\mu_-} + \mathrm{Re}\ T_{2\mu_-} + i\epsilon\ \mathrm{Im}\ T_{1\mu_-} \tag{8.a}$$

and

$$T_{S\mu_-} = i\ \mathrm{Im}\ T_{1\mu_-}\ , \tag{8.b}$$

where the term $\epsilon\ T_{2\mu_-}$ has been dropped in comparison with the much larger $K_1 \to \mu_-$ amplitude in Eq. (8.b).

For the class A, the $K \to \gamma\gamma$ amplitudes are CP invariant **), so that

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*) There exist many models which contain a term like (7). A general discussion of such models is given in Ref. 11). The application to the present problem has been discussed in Ref. 12) where it is shown that G' must be about 0.01 times the usual weak coupling G. The numerical value of G' given in Ref. 12) should be increased by a factor $\sqrt{2}$. Such a term occurs naturally in the effective Hamiltonian in the Okubo theory 13) of CP violation.

**) Another definition of the class A models could be that only $K \to \mu^+\mu^-$ decay amplitudes are allowed to be CP non-invariant.

$$T_{L\gamma_+} = \epsilon\, T_{1\gamma_+} \;;\quad T_{S\gamma_+} = T_{1\gamma_+} \,, \tag{9.a}$$

$$T_{S\gamma_-} = \epsilon\, T_{2\gamma_-} \;;\quad T_{L\gamma_-} = T_{2\gamma_-} \,, \tag{9.b}$$

$$T_{1\gamma_-} = T_{2\gamma_+} = 0 \,. \tag{9.c}$$

Using the experimental information

$$\Gamma(K_S \to \gamma\gamma) \leq 1.6 * 10^{-3}\; \Gamma_S \,, \tag{10.a}$$

$$\Gamma(K_L \to \gamma\gamma) = 5 * 10^{-4}\; \Gamma_L \tag{10.b}$$

where Γ_L is the total K_L decay rate, one finds

$$\Gamma(K_L \to \gamma\gamma) \geq |T_{L\gamma_-}|^2 \geq 0.99\, \Gamma(K_L \to \gamma\gamma)$$

so that, to a very good approximation,

$$| T_{2\gamma_-} | = \sqrt{\Gamma(K_L \to \gamma\gamma)} \,. \tag{11}$$

From the imaginary part of Eq. (8.a), and the unitarity relation (5.a), one gets

$$(\mathrm{Re}\,\epsilon) \cdot \sqrt{\Gamma(K_S \to \mu^+\mu^-)} \geq \sqrt{\varphi_- \cdot \Gamma(K_L \to \gamma\gamma)} - \sqrt{\Gamma(K_L \to \mu^+\mu^-)} \,, \tag{12}$$

which gives the bound [Eq. (3)]

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 10 * 10^{-7}$$

where we have used Eqs. (4.c), (6.a), (10.b) and (11), and the experimental limit

$$\Gamma(K_L \to \mu^+\mu^-)/\Gamma_L \leq 1.8 * 10^{-9} \,. \tag{13}$$

The model requires a non-vanishing value of $\mathrm{Re}\, T_{2\mu_-}$ for the limit (13) to be satisfied. The required value of r ($\equiv \mathrm{Re}\, T_{2\mu_-}/\mathrm{Im}\, T_{2\mu_-}$) is about -0.45 for the minimum value (3) of $\Gamma(K_S \to \mu^+\mu^-)$. Such a magnitude is not unreasonable for the dispersive contribution from the intermediate $\gamma\gamma$ state *).

Other new terms beside that in Eq. (7) may be present in the Hamiltonian with couplings of the order of G' without changing the bound (3). In fact, this bound is independent of any assumption about the decay amplitudes to the CP even state μ_+. This bound could be lowered by a factor of two if the absorptive amplitude $\mathrm{Re}\,T_{1\mu_-}$ were not restricted to be zero; however, there seem to be no reasonable models for achieving this.

Class B Models

In contrast to the class A models where the CP = −1 states were involved, the class B ones involve the CP = +1 states, μ_+ and γ_+. The essential new terms in the Hamiltonian are : 1) a CP invariant neutral current interaction allowing, in lowest order, the needed large $K_1 \to \mu^+\mu^-$ amplitude, and 2) a CP violating interaction allowing the decay $K_2 \to \gamma_+$; this may be either an electromagnetic [15] or a weak-electromagnetic interaction [16] **).

The neutral current yields the dispersive amplitude $\mathrm{Re}\,T_{1\mu_+}$, but the absorptive part ***) $\mathrm{Im}\,T_{1\mu_+} = 0$, except possibly for a small contribution from the $\gamma\gamma$ state discussed below. Of course, the absorptive part $\mathrm{Re}\,T_{2\mu_+}$ is associated with the $\gamma\gamma$ intermediate state and is non-zero, Eq. (5.c). On the basis of known interactions, one expects some non-zero CP conserving amplitudes $T_{2\mu_-}$ and $T_{2\gamma_-}$; however, it can be shown [1),5)] that in order to get the lowest bound on $\Gamma(K_S \to \mu^+\mu^-)$, one can neglect all amplitudes

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*) The apparent disagreement between the required sign of r and that in an explicit model calculation [3] need not be considered serious because of the uncertainty in the form factor for the $K_2 \to \gamma\gamma$ vertex. On the other hand, there exist class A models [for example, that of Okubo [13]] which allow the CP violating decay amplitude $K_2 \to 2\pi$ in an I = 0 state. For such models, the quantity $\mathrm{Re}\,T_{2\mu_-}$ would have, in addition to the contribution from the $\gamma\gamma$ state, a term $= \alpha \cdot (\mathrm{Im}\,T_{1\mu_-})$ where the factor

$$\alpha = \left[\langle \pi^+\pi^-, I=0 | H' | K_2 \rangle / \langle \pi^+\pi^-, I=0 | H | K_1 \rangle\right]$$

takes into account [14] the transformation to the Wu-Yang phase convention.

**) The model discussed by Barshay [17] is, effectively, a class B model.

***) For the possibility $\mathrm{Im}\,T_{1\mu_+} \neq 0$, see the class C models below.

to CP odd final states. In that case, if the branching ratio for $K_S \to \gamma\gamma$ decay is sufficiently small ($<10^{-4}$) so that the contribution of the amplitude $T_{1\gamma_+}$ to $T_{L\gamma_+}$ can be ignored, the analysis is similar to that for class A models. The relation

$$\mathrm{Re}\, T_{L\mu_+} = (\mathrm{Re}\,\epsilon)\cdot \mathrm{Re}\, T_{1\mu_+} + \mathrm{Re}\, T_{2\mu_+} \tag{14}$$

and Eq. (5.c) leads to the bound (12) with φ_- replaced by φ_+, yielding

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 6 * 10^{-7} . \tag{15}$$

This result is the Christ-Lee lower bound with our input data.

The bound (15) holds also if the $K_S \to \gamma\gamma$ branching ratio is not negligibly small. Since the $\gamma\gamma$ intermediate state determines the absorptive amplitudes $\mathrm{Re}\, T_{2\mu_+}$ and $\mathrm{Im}\, T_{1\mu_+}$, this bound does not [1),5)] depend on the $K_S \to \gamma\gamma$ decay rate.

Class C Models

To obtain a still lower bound, we continue to consider the $CP = +1$ final states, but now allow the amplitude $T_{1\mu_+}$ to have a large absorptive part. Since the main product of K_1 decay is the $\pi\pi$ state, the reasonable way to achieve this is to replace assumption 1) of the class B models by the introduction of a direct CP conserving $\pi\pi \to \mu^+\mu^-$ interaction with a coupling of the order of electromagnetism - much stronger than that provided by a reasonable calculation of the 2γ exchange contribution. The sequence $K_1 \to \pi^+\pi^- \to \mu^+\mu^-$, then, provides the amplitudes $\mathrm{Re}\, T_{1\mu_+}$ and $\mathrm{Im}\, T_{1\mu_+}$. Barshay [18)] has given an explicit discussion of such an interaction.

If the $K_S \to \gamma\gamma$ branching ratio is very small ($<10^{-4}$) and we assume $\eta_{+-} = \eta_{oo}$ for $K_L \to \pi\pi$ decay [as done in Eq. (5.c)], the introduction of a non-vanishing $\mathrm{Im}\, T_{1\mu_+}$ gives

$$|\epsilon| \cdot \sqrt{\Gamma(K_S \to \mu^+\mu^-)} \geq \sqrt{\varphi_+ \cdot \Gamma(K_L \to \gamma\gamma)} - \sqrt{\Gamma(K_L \to \mu^+\mu^-)} \tag{16}$$

which lowers the previous limit by a factor of two :

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 3 * 10^{-7} . \tag{17}$$

It seems strange that the result (16) involves $|\epsilon|$, a quantity having a value depending on the $K/\bar{K}$ relative phase, rather than $\mathrm{Re}\,\epsilon$, as in (12). However, in the present case, because of the essential role of the $\pi\pi$ intermediate state, $|\epsilon|$ is correctly evaluated as $|\eta_{+-}| = |\eta_{oo}|$. The bound (17) can be lowered to some extent if one allows *) a small non-zero $(\eta_{+-}-\eta_{oo})$.

If the $K_S \to \gamma\gamma$ branching ratio is higher, the bound (17) gets lowered. If the present limit (10.a) is saturated, one gets, assuming $\mathrm{Im}\, T_{1\gamma_+} = 0$,

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma_S \geq 2 * 10^{-7} . \qquad (18)$$

Gaillard [5] finds a slightly lower value (2) by allowing a large absorptive part in $T_{1\gamma_+}$ which seems difficult to obtain dynamically. The points is that this absorptive part should arise predominantly from the $\pi\pi$ intermediate state, and any reasonable estimate [4] of this gives a contribution to the rate much smaller than the limit (10.a).

In conclusion, the Table shows a summary of our results. In using our numerical results, or similar ones of others [1),5)], it should be remembered that there may be some **) correction due to the $\pi\pi\gamma$ intermediate state, and that no attempt has been made to include the experimental errors of the input data. We consider model A as the only "economical" realization of the Christ-Lee mechanism. It should be noted that model C (needed to go from the Christ-Lee number to that of Gaillard) requires an anomalous muon-hadron interaction. As emphasized by Barshay [18], such anomalous interactions can be constructed so as to solve the $K_L \to \mu^+\mu^-$ puzzle without CP violation.

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*) Present data [19] allow such a small number : $|\eta_{oo}/\eta_{+-}| = 1.00 \pm 0.06$, $\phi_{oo} \simeq \phi_{+-} \pm 20^{\circ}$. There is, of course, no reason to rule out $\eta_{+-} \neq \eta_{oo}$ in these models since there is an electromagnetic origin of CP violation.

**) The estimate of a 11% correction [9] is somewhat too high [20].

<table>
<tr><th rowspan="2">Class of Model</th><th rowspan="2">Essential $\mu^+\mu^-$ Decay Channel</th><th colspan="2">New Interaction</th><th colspan="2">New Amplitude</th><th rowspan="2">Assumed $K_S \to \gamma\gamma$ Branching Ratio</th><th rowspan="2">Lower Bound on $K_S \to \mu^+\mu^-$ Branching Ratio</th></tr>
<tr><th>CP Invariant</th><th>CP Violating</th><th>CP Invariant</th><th>CP Violating</th></tr>
<tr><td>A</td><td>$^1S_0$</td><td>none</td><td>centiweak neutral current</td><td>none</td><td>$\mathrm{Im}\, T_{1\mu_-}$</td><td>$\leq 1.6*10^{-3}$</td><td>$10 * 10^{-7}$</td></tr>
<tr><td>B</td><td>$^3P_0$</td><td>centiweak neutral current</td><td rowspan="3">electro-magnetic
or
weak electro-magnetic</td><td>$\mathrm{Re}\, T_{1\mu_+}$</td><td rowspan="3">$T_{2\gamma_+}$
and
$T_{2\mu_+}$</td><td>$\leq 1.6*10^{-3}$</td><td>$6 * 10^{-7}$</td></tr>
<tr><td rowspan="2">C</td><td rowspan="2">$^3P_0$</td><td rowspan="2">$\pi\pi \to \mu\bar{\mu}$ of electro-magnetic strength</td><td rowspan="2">$\mathrm{Re}\, T_{1\mu_+}$
and
$\mathrm{Im}\, T_{1\mu_+}$</td><td>negligible ($<10^{-4}$)</td><td>$3 * 10^{-7}$</td></tr>
<tr><td>$\leq 1.6*10^{-3}$</td><td>$2 * 10^{-7}$</td></tr>
</table>

<u>TABLE</u> : Summary of various classes of models and the lower bounds on the $K_S \to \mu^+\mu^-$ branching ratio in those models.

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