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LOW ENERGY PHENOMENOLOGY *)

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PART I : Low Energy Interactions : $\pi\pi$, $K\pi$, πN.

PART II : Duality and Symmetry.

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PART I : LOW ENERGY INTERACTIONS : $\pi\pi$, $K\pi$, πN

1. - THEORETICAL TOOLS

In $\pi\pi$ interactions the theoretical tools have become directly useful for phenomenology. In πN scattering, dispersion relations have been an obvious tool for a long time. In $\pi\pi$, dispersion relations have been used by Morgan and Shaw [1] ; however, dispersion relations alone are less useful, since the total cross-sections are difficult to measure over a large energy range. On the other hand, crossing is much more restrictive for $\pi\pi$.

In order to implement unitarity simply, one uses partial wave amplitudes. But then crossing becomes complicated. From crossing alone follow the relations of Roskies [2]. They involve a finite number of partial waves, e.g. :

$$\int_0^4 ds\,(s-4)\left[2f_0^0(s) - 5f_0^2(s)\right] = 0$$

where $m_\pi = 1$. Note that the integral goes over the unphysical region below threshold.

Using in addition fixed t dispersion relations and positivity (of the imaginary parts of elastic partial wave amplitudes) one obtains Martin's inequalities [3], e.g., for $\pi^0\pi^0$ scattering :

$$f_0(4) \geq f_0(0)$$

In the last year these tools have been refined [4] and derived also for $K\pi$ and πN scattering [5]. Moment inequalities, i.e., inequalities involving integrals from $s = 0$ to $s = 4$, have been derived in the same spirit [6].

All these theoretical tools apply to the unphysical region below threshold, $0 \leq s \leq 4$. In order to make them useful for the physical region, one needs an explicit parametrization or a model. Then these tools become powerful, they have killed many models in the recent past [7]. But even with a parametrization or a model, the usefulness of these constraints will be limited to $m_{\pi\pi} < m_\rho$.

Therefore, one needs relations involving directly the physical region. Such relations have been recently derived by Roy [8] and Basdevant, Le Guillou and Navelet [9]. They are of the form

$$f_\ell^I(s) = \int_4^\infty ds' \sum_{\ell' I'} K_{\ell\ell'}^{II'}(s,s')\, \mathrm{Im}\, f_{\ell'}^{I'}(s') \; +$$

+ subtractions for $\ell = 0, 1$.

They are useful since the kernel is highly convergent, $(s')^{-3}$, therefore the Regge contribution is weak. They are axiomatically valid up to $\sqrt{s} \sim 1100$ MeV, but practically they are particularly useful below ~600 MeV, because in that case one needs only S and P waves on the right-hand side. These relations are a direct test for experimental data or theoretical models. For example, Basdevant, Le Guillou and Navelet [9] show how these sum rules discriminate between two possible forms of the $I = 0$ S wave once the scattering lengths, the $I = 2$ S wave and the P wave are given.

2. - MODELS

Let us now consider models constructed with the tools discussed above. One needs some physical input. The two questions asked are :

(a) can one predict the S waves from the knowledge of the P wave ?

(b) can one predict the energy dependence of S and P waves from the low energy input of current algebra ?

A. - Phenomenological P wave input

As an example for a model with a phenomenological input we discuss the one of Le Guillou, Morel and Navelet (GMN) [10]. Related work has been done by Piguet and Wanders [11] and by Bonnier and Gauron [12]. GMN use as their input the P wave from 500 to 1100 MeV, the mass and width of the f_0, and the asymptotic ρ exchange contribution. They satisfy unitarity by a K matrix parametrization involving seven parameters. For S and P waves there exist five Roskies' relations. Martin's inequalities further restrict the remaining two parameters to a small domain. As output they obtain the S waves. For $I = 0$ they get a broad ε, the phase shift below 750 MeV is in a band containing solutions of the "between" and "up" type. The $I = 2$ phase shift is small and negative in agreement with the experimental one. The scattering lengths are close to Weinberg's. Thus one predicts the S waves semi-quantitatively just from a P wave input.

B. - Current algebra input

As an example we take the recent work of Kang and Lee [13], where current algebra is used to fix subtraction constants. Unitarity is achieved with the N/D method, where the left-hand cuts are parametrized by eight poles. The five Roskies' relations are used to enforce crossing for S and P waves, and the inequalities of Martin are checked. The resulting scattering lengths show only a small deviation from

Weinberg's, therefore one has a unitarized, crossing symmetric solution close to Weinberg's linear solution. The I = 0 S wave has a broad ε , the I = 2 S wave agrees with the experimental one, and the P wave below 750 MeV looks qualitatively like the ρ .

It is satisfying that the mathematical tools combined with widely different physics input (experimental P wave or current algebra) leads to similar low energy $\pi\pi$ phase shifts.

3. - PADE APPROXIMANTS

A. - The σ model

The Padé approach is more ambitious, one attempts to calculate everything from a Lagrangian with one or two free parameters. The σ model is renormalizable, soft π limits are valid in each order. Basdevant and Lee [14] calculated the $[1,1]$ Padé approximants for S, P, D waves and obtained unitarized current algebra amplitudes close to Weinberg's. The σ becomes unstable and surprisingly very broad. The ρ and f_0 are predicted at about the correct masses. Tests with Roskies' relations and Martin's inequalities show that crossing is approximately fulfilled.

B. - Massive Yang-Mills model

Basdevant and Zinn-Justin [15] used a ρ dominance input in the form of a Yang-Mills model. As output they obtained the S waves (with a broad σ) and the f_0. One has here a reciprocal situation where a ρ input generates a σ output, and a σ input (in the σ model) generates the ρ as output.

C. - Contact term model for π N

Filkov and Palyushev [16] have submitted the first successful πN Padé calculation. In lowest order they take the nucleon Born terms and a $N\bar{N}\pi\pi$ contact term, whose strength is the only free parameter. The corresponding Lagrangian is strictly speaking not renormalizable, but at the one-loop level only one subtraction is needed, which is absorbed in the coupling constant of the contact term. The $[1,1]$ Padé approximant gives the results shown in Fig. 1 by the solid line labelled $\beta = 0$. The qualitatively correct predictions of the $P_{33}(1238)$, $D_{13}(1520)$, $P_{11}(1480)$ resonances and of the S_{11} wave are striking even more so, when compared to the σ model Padé calculation of Mignaco and Remiddi [17]. The other six partial waves (S, P, D) show an equally good qualitative agreement with experiment. It will be interesting to understand the physics of this success.

4. - SUMMARY OF THEORETICAL PREDICTIONS OF $\pi\pi$ S WAVES

A summary of various theoretical predictions for the $I = 0$ and $I = 2$ S waves is shown in Fig. 2. All models agree that the $I = 2$ S wave is small and negative, $\delta_0^2(750\ \mathrm{MeV}) = -(15 \pm 5)^\circ$. The $I = 0$ S wave below 750 MeV is predicted in a 30° wide band from a "between" type to an "up" type. At and above the ρ mass, the various models predict different behaviour. The mathematical tools referring to $0 \leq s \leq 4m_\pi^2$ are not very restrictive for this region, and the effect of the $K\bar{K}$ channel becomes important. It is satisfying that different tools and distinctly different physical input give about the same answer for $m_{\pi\pi} < m_\rho$.

5. - $\pi\pi$ PHASE SHIFTS FROM K_{e4}

A K_{e4} experiment with ca. 1600 events has been reported by a Saclay-Geneva collaboration [18]. In K_{e4} one studies the $\pi\pi$ final state interaction undisturbed by other hadrons. The strong interaction effects can be separated from the weak interactions (determination of weak form factors) by the method of Pais and Treiman [19], which has been applied here for the first time. The intensity depends on five variables : dipion mass s, dilepton mass s_ℓ, dipion decay angle ϑ_π, dilepton decay angle ϑ_ℓ, and the angle ϕ between the two decay planes. The method makes use of the (ϑ_ℓ, ϕ) correlation. The dependence on these two variables is written explicitly :

$$I = \sum_i I_i(s_\pi, s_\ell, \vartheta_\pi)\, Y_i(\vartheta_\ell, \phi)$$

where the Y_i are simple trigonometric functions. The ratio of two correlation coefficients, I_4 and I_7, averaged over s_ℓ and ϑ_π gives directly the phase shift difference :

$$\frac{1}{2}\frac{\langle I_7\rangle}{\langle I_4\rangle} = \tan(\delta_S - \delta_P)$$

In this correlation method the sign of $(\delta_S^0 - \delta_P^1)$ is determined. This experiment gives a positive sign and resolves the sign ambiguity of earlier experiments. The magnitude and energy dependence of $(\delta_S^0 - \delta_P^1)$ are shown in Fig. 3 together with Weinberg's prediction and with the upper and lower limits of the theoretical models discussed above. The best fit gives $(\delta_S^0 - \delta_P^1)$ about twice as large as Weinberg, but its error bars make it consistent with Weinberg. One does not measure the scattering length in this experiment. Theoretically one expects the scattering lengths to be small and the effective range to be large, therefore the measured phase shifts are mainly due to the effective range.

6. - CHEW-LOW EXTRAPOLATION IN $\pi^- p \to \pi^- \pi^+ n$

A SLAC wire chamber spectrometer group [20] has collected good statistics at $p_L = 15$ GeV/c. This makes very small t values accessible and the Chew-Low extrapolation more reliable. In the $\chi^2(\delta_0^0)$ plot of Fig. 4, one sees that they obtain a well-defined unique $I = 0$ S wave for $M_{\pi\pi} = 550, 600, 650$ MeV. It is of the "between" type suggested by Morgan and Shaw [1]. Uniqueness comes about, because they trust the absolute magnitude of their $\pi\pi$ differential cross-section. The same applies to Baton et al. [21]. Earlier analyses fitted only the shape of the angular distributions and therefore were left with an up-down ambiguity. Near and above the ρ the ambiguity in δ_0^0 still persists, anything from "down" to "up" is acceptable ; except that at $M_{\pi\pi} = 950$ MeV a unique $\delta_0^0 \approx 150^\circ$ seems to be required.

Figure 5 shows the S wave phase shifts of Baton et al. [21] and Baillon et al. [20]. We see again the uniqueness of δ_0^0 below the ρ, and we note that the two analyses agree within 5°. Near and above the ρ the solutions are very badly determined ; note that the error bars of Baton et al. do not reflect the true errors, but rather they are the result of a smoothing procedure.

Similarly the theoretical predictions for δ_0^0 (see Fig. 2) are all within a 30° band (from "between" to "up") below the ρ, but they differ strongly above the ρ.

The $I = 2$ S wave phase shifts up to 800 or 900 MeV are well-determined both experimentally and theoretically.

7. - THE $K\bar{K}$ CUSP IN $\pi\pi - \pi\pi$

A striking effect helps us to resolve the δ_0^0 ambiguity in the region 750 - 1000 MeV. Figure 6 [22] shows the $< Y_1^0 >$ moment, which is, for these $\pi\pi$ masses, primarily due to S - P interference. We see a striking cusp at the $K\bar{K}$ threshold. Such an effect is expected for the S wave, since K^+K^- and $K_1^0K_1^0$ production shoot up to almost the unitarity limit within about 30 MeV above threshold [23]. Since the 4π inelasticity is very small below 1000 MeV [23], the $I = J = 0$ wave must move along the periphery of the Argand diagram and accelerate counter-clockwise to infinite speed as we reach the $K\bar{K}$ threshold. After threshold it must shoot towards the centre of the Argand plot. At $K\bar{K}$ threshold $\delta_0^0(\pi\pi)$ must be $\lesssim 180^\circ$ otherwise $< Y_1^0 >$ would go through zero and become negative below the $K\bar{K}$ threshold. Since δ_0^0 must make a large counter-clockwise excursion before $K\bar{K}$ threshold, δ_0^0 must start from a "down" solution in the region 750 - 900 MeV and then shoot up towards an "up" solution, as one approaches the KK threshold. [Note that the down solution of Baton et al. and Baillon et al. gives $\delta_0^0(m\quad) = 65^\circ - 75^\circ$.] The value of δ_0^0 at $K\bar{K}$ threshold should be somewhere between 150° and 180°

We see that the $K\bar{K}$ threshold effect is crucial in resolving the δ_0^0 ambiguities in the region 750 - 1000 MeV. In any future analysis coupled-channel unitarity and analiticity near the $K\bar{K}$ threshold must be taken into account explicitly.

8. - $K\pi$ PHASE SHIFTS

As in the $\pi\pi$ interaction the interest centres on the non-exotic ($I = \frac{1}{2}$) S wave, particularly on the question of a daughter resonance of the vector meson.

In Figure 7 we show the $I = \frac{1}{2}$ S wave phase shift obtained by the CERN-Brussels-UCLA collaboration [24]) from data of the international K^+ collaboration. They extrapolated the shape of the $K^+\pi^- - K^+\pi^-$ angular distribution in $K^+p - K^+\pi^-\Delta^{++}$ to the π pole and performed various checks (correct prediction of π^+p on-shell moments, no dependence on p_{lab}). In this particular phase shift solution they assumed the $I = \frac{1}{2}$ P wave to be given by a Breit-Wigner resonance, and they took the $I = \frac{3}{2}$ phase shifts from previous studies of $K^-p - K^-\pi^-\Delta^{++}$, $K^-n - K^-\pi^-p$: $\delta_1^3 = 0$ and δ_0^3 corresponding to $\sigma_{\frac{3}{2}} = 1.8$ mb. We see that the $I = \frac{1}{2}$ S wave is unique below and above the $K^*890(1^-)$. But in a narrow region near 890 there is an up-down ambiguity : either δ_0^1 increases slowly (but stays near 45^o), or it shoots up by 180^o and represents a very narrow resonance ($M = 865$ MeV, $\Gamma = 35$ MeV). If one now uses the magnitude of the extrapolated cross-section, this second solution ("up") is disfavoured but not excluded. Similar studies have been done by a group from Johns Hopkins University [25]), which excludes the "up" solution.

From the point of view of the Veneziano model, both solutions differ strongly from the simple B_4 prediction. The down one, because the κ is shifted to well beyond $M = 1100$ MeV, since $\delta_0^1(1100 \text{ MeV}) \approx 60^o$; the up solution because $\Gamma = 35$ MeV is much too narrow.

It will be interesting to see whether one can make strong theoretical arguments against the up solution on the basis of the theoretical tools discussed in Sections 1 and 2.

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FIGURE CAPTIONS

Figure 1 The predictions of the πN phase shifts in the contact term model Padé calculation of Filkov and Palyushev (solid line with $\beta = 0$). Also shown are the predictions of the Padé calculation of Mignaco, Pusterla and Remiddi [17] (dashed line).

Figure 2 Theoretical predictions of $\pi\pi$ S waves : Morgan and Shaw [1] ; Le Guillou, Morel and Navelet [10] ; Kang and Lee [13] ; σ model (Padé) [14] ; Yang-Mills (Padé) [15].

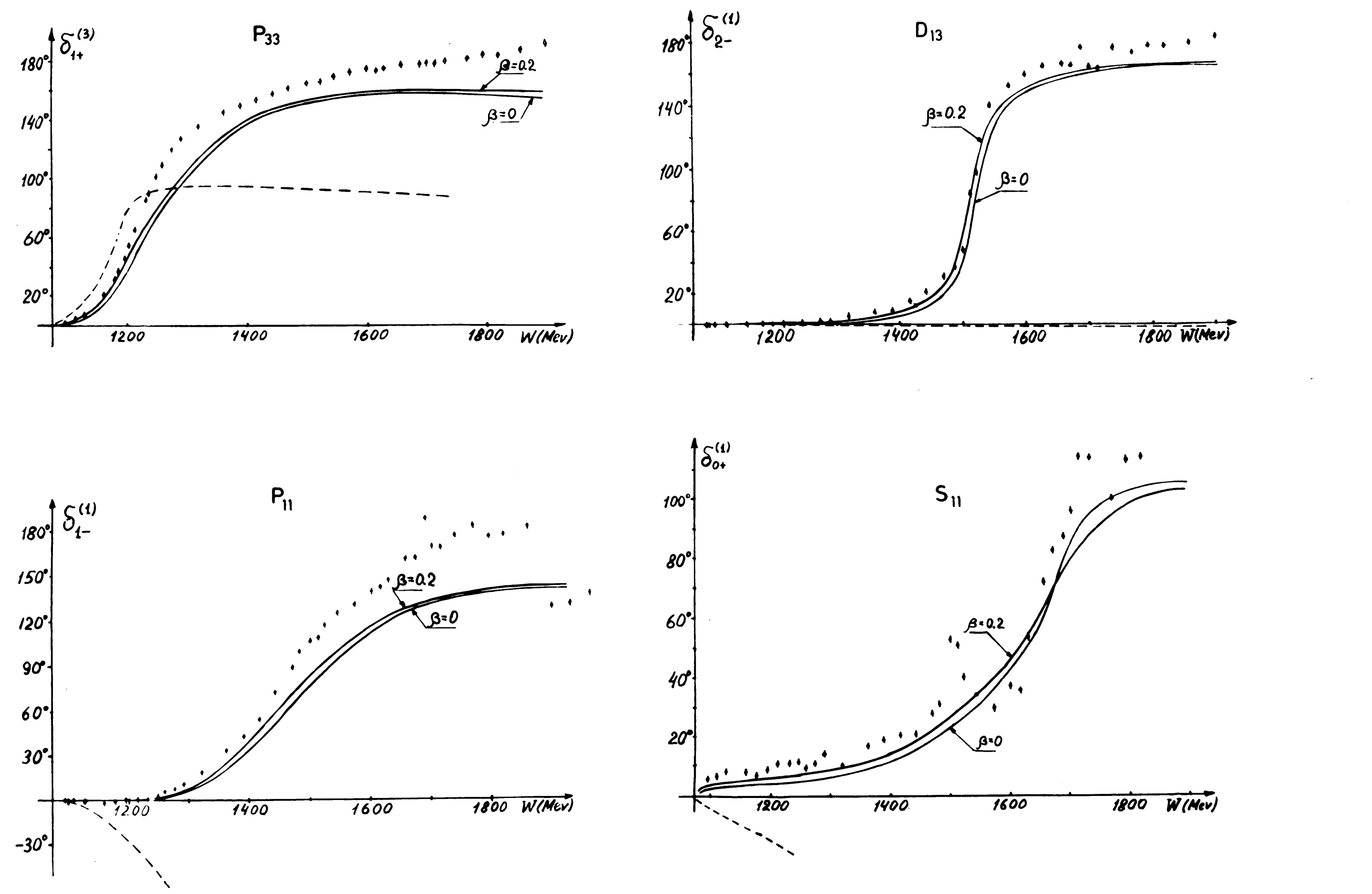
Figure 3 The $\pi\pi$ phase shift difference $(\delta_S^0 - \delta_P^1)$ determined in the K_{e4} experiment [18]. Also shown are Weinberg's prediction and the highest and lowest predictions of the models discussed in the text ; Bonnier and Gauron [12] (top curve) ; Padé σ model [14] (bottom curve).

Figure 4 $\chi^2(\delta_0^0)$ from the extrapolated $\pi^+\pi^-$ differential cross-section [20].

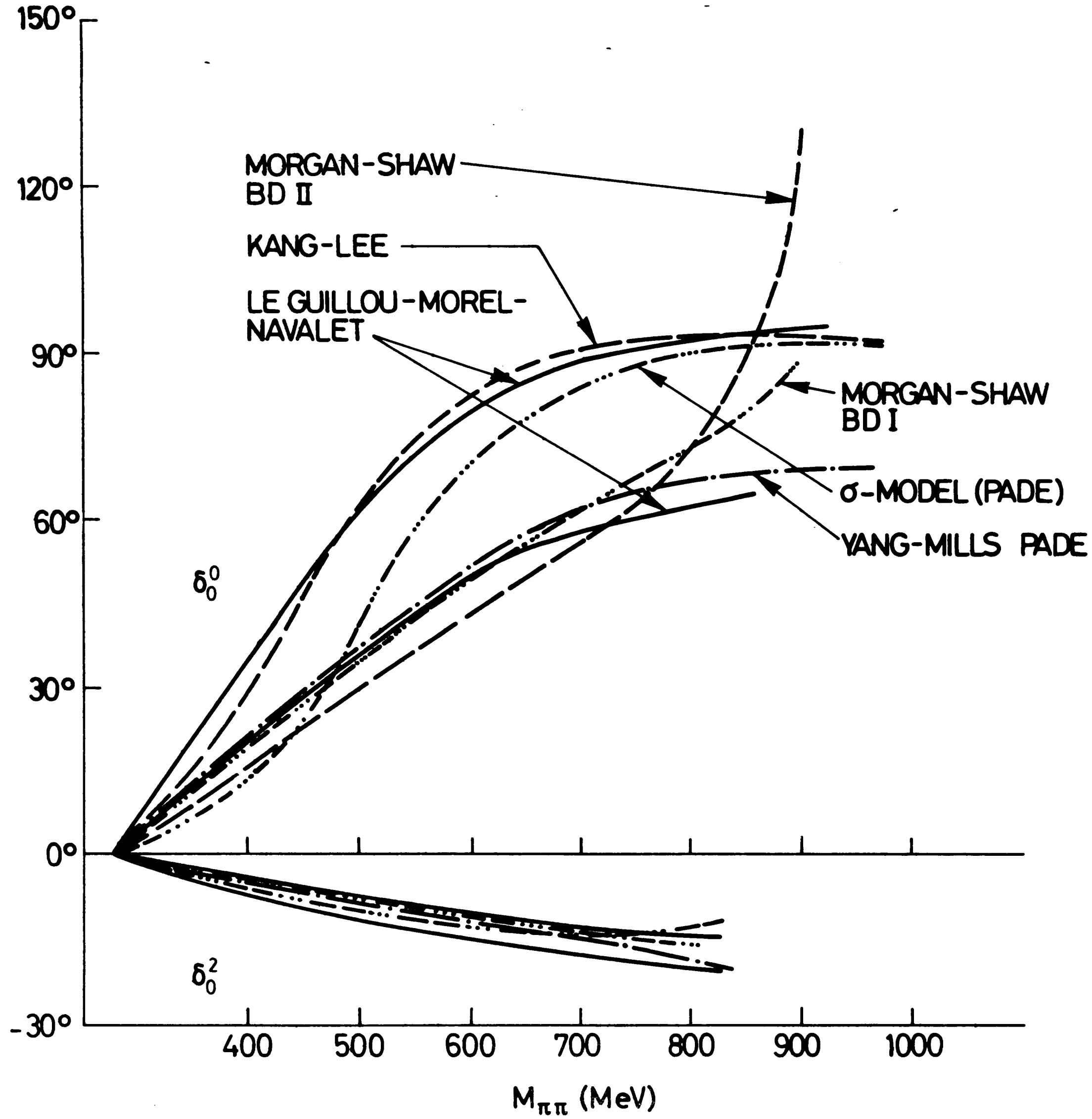
Figure 5 S wave phase shifts from Chew-Low extrapolations of $\pi\pi$ scattering ; Baillon et al. [20] ; Baton et al. [21].

Figure 6 The $< Y_1^0 >$ moment as a function of the $\pi^+\pi^-$ mass in $\pi^+ p \to \Delta^{++}\pi^+\pi^-$ at $p_L = 7.1$ GeV/c, with $t' < 0.1$ GeV^2, and $M_{p\pi^+} < 1.4$ GeV from Ref. 22).

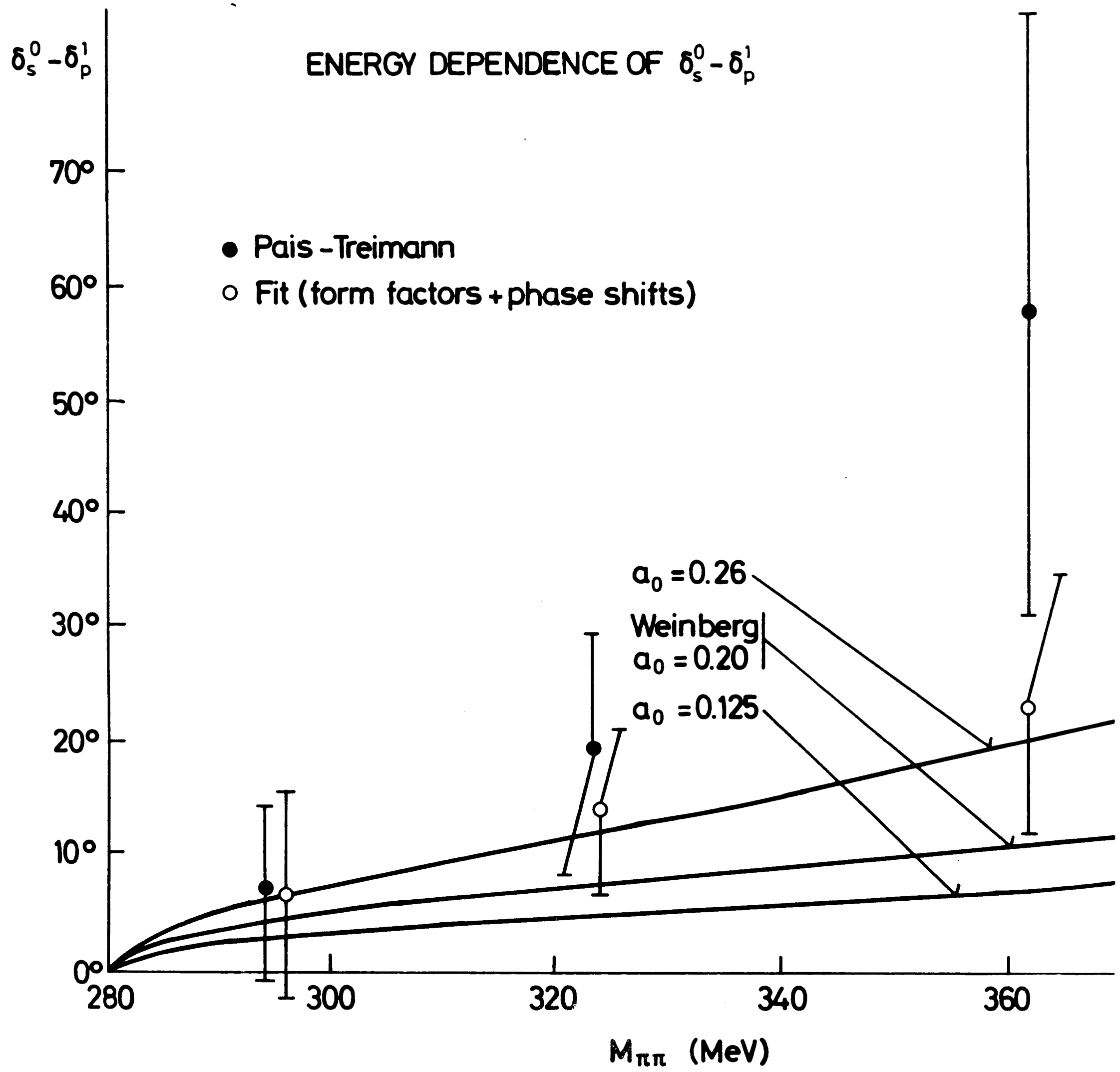
Figure 7 The $I = \frac{1}{2}$ S wave $K\pi$ phase shift from the extrapolated $K^+\pi^- \to K^+\pi^-$ moments in $K^+p \to K^+\pi^-\Delta^{++}$ and using the contraints δ_1^1 = Breit-Wigner with $M = 891$ MeV and $\Gamma = 50$ MeV, $\delta_1^3 = 0$, δ_0^3 corresponding to $\sigma_{\frac{3}{2}} = 1.8$ mb. [Ref. 24)].


P33
$\delta_{1+}^{(3)}$
180°
140°
100°
60°
20°
1200
1400
1600
1800
W(Mev)
β=0.2
β=0
D13
$\delta_{2-}^{(1)}$
180°
140°
100°
60°
20°
1200
1400
1600
1800
W(Mev)
β=0.2
β=0
P11
$\delta_{1-}^{(1)}$
180°
150°
120°
90°
60°
30°
-30°
1200
1400
1600
1800
W(Mev)
β=0.2
β=0
S11
$\delta_{0+}^{(1)}$
100°
80°
60°
40°
20°
1200
1400
1600
1800
W(Mev)
β=0.2
β=0

- Figure 1 -

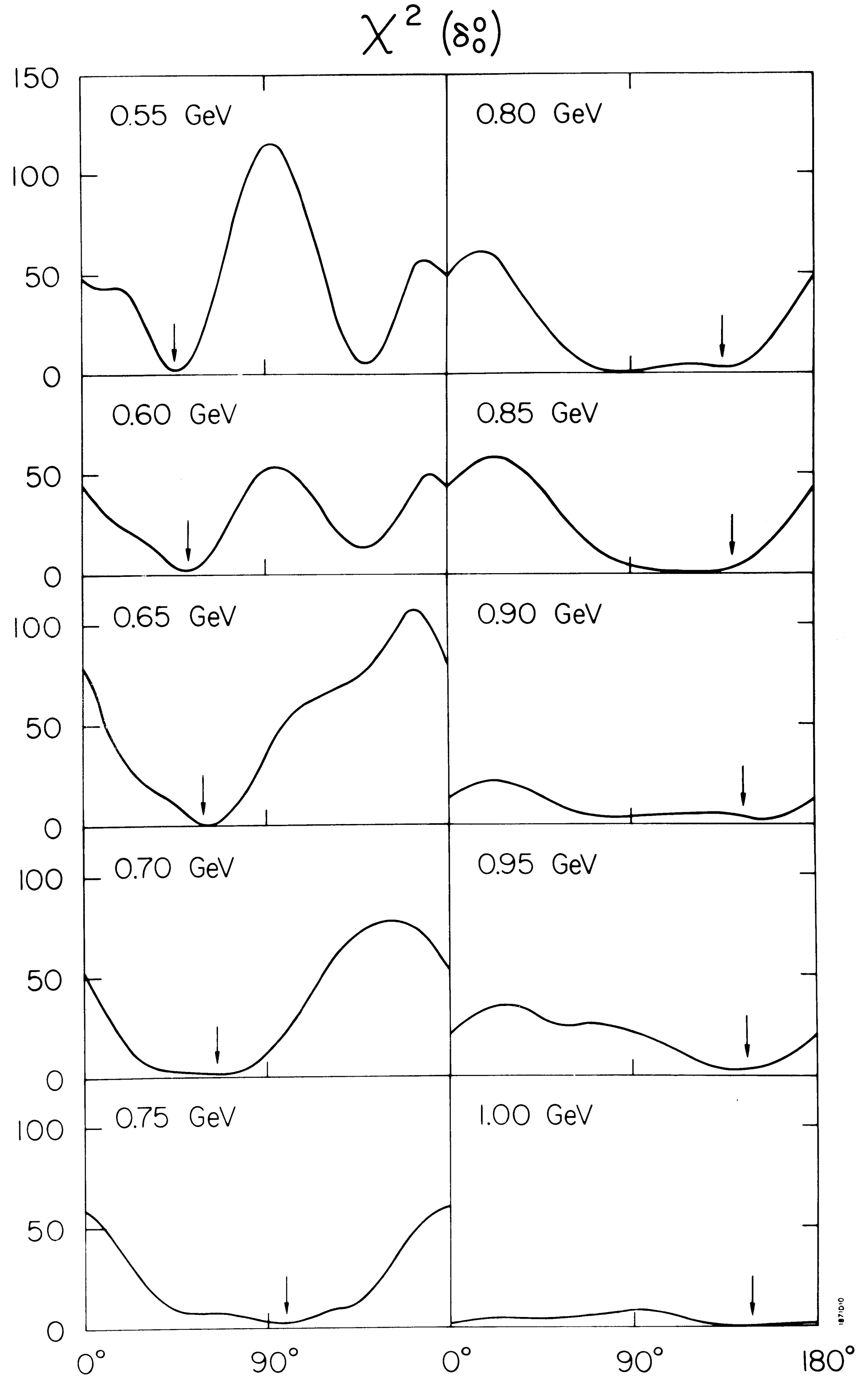

150°
120°
90°
60°
30°
0°
-30°
MORGAN-SHAW
BD II
KANG-LEE
LE GUILLOU-MOREL-
NAVALET
MORGAN-SHAW
BD I
σ-MODEL (PADE)
YANG-MILLS PADE
δ_0^0
δ_0^2
400
500
600
700
800
900
1000
$M_{\pi\pi}$ (MeV)

- Figure 2 -



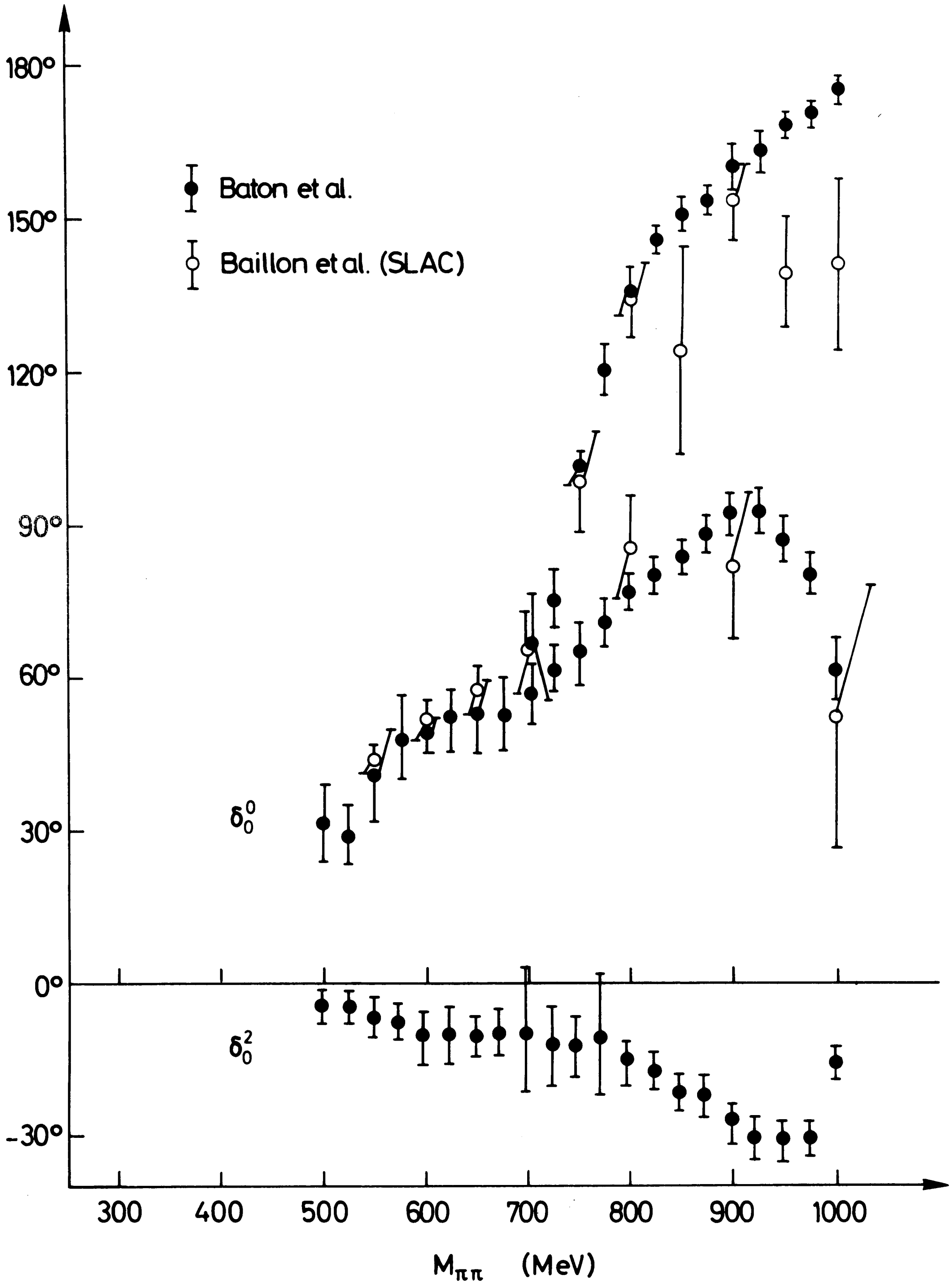
$\delta_s^0 - \delta_p^1$
ENERGY DEPENDENCE OF $\delta_s^0 - \delta_p^1$
70°
60°
50°
40°
30°
20°
10°
0°
● Pais - Treimann
○ Fit (form factors + phase shifts)
a_0 = 0.26
Weinberg
a_0 = 0.20
a_0 = 0.125
280
300
320
340
360
$M_{\pi\pi}$ (MeV)

- Figure 3 -

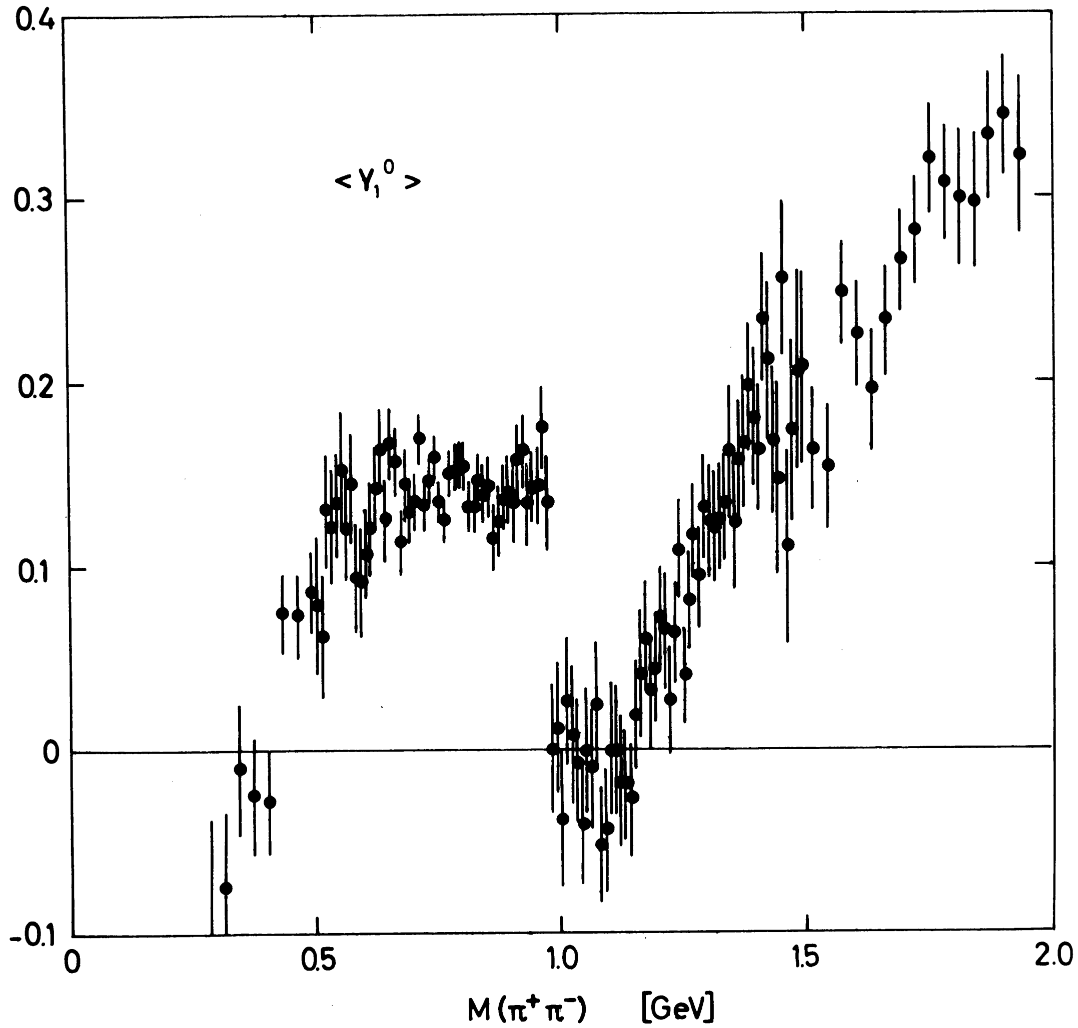


χ^2 (δ^0_0)
150
100
50
0
0.55 GeV
0.80 GeV
0.60 GeV
0.85 GeV
0.65 GeV
0.90 GeV
0.70 GeV
0.95 GeV
0.75 GeV
1.00 GeV
0°
90°
180°

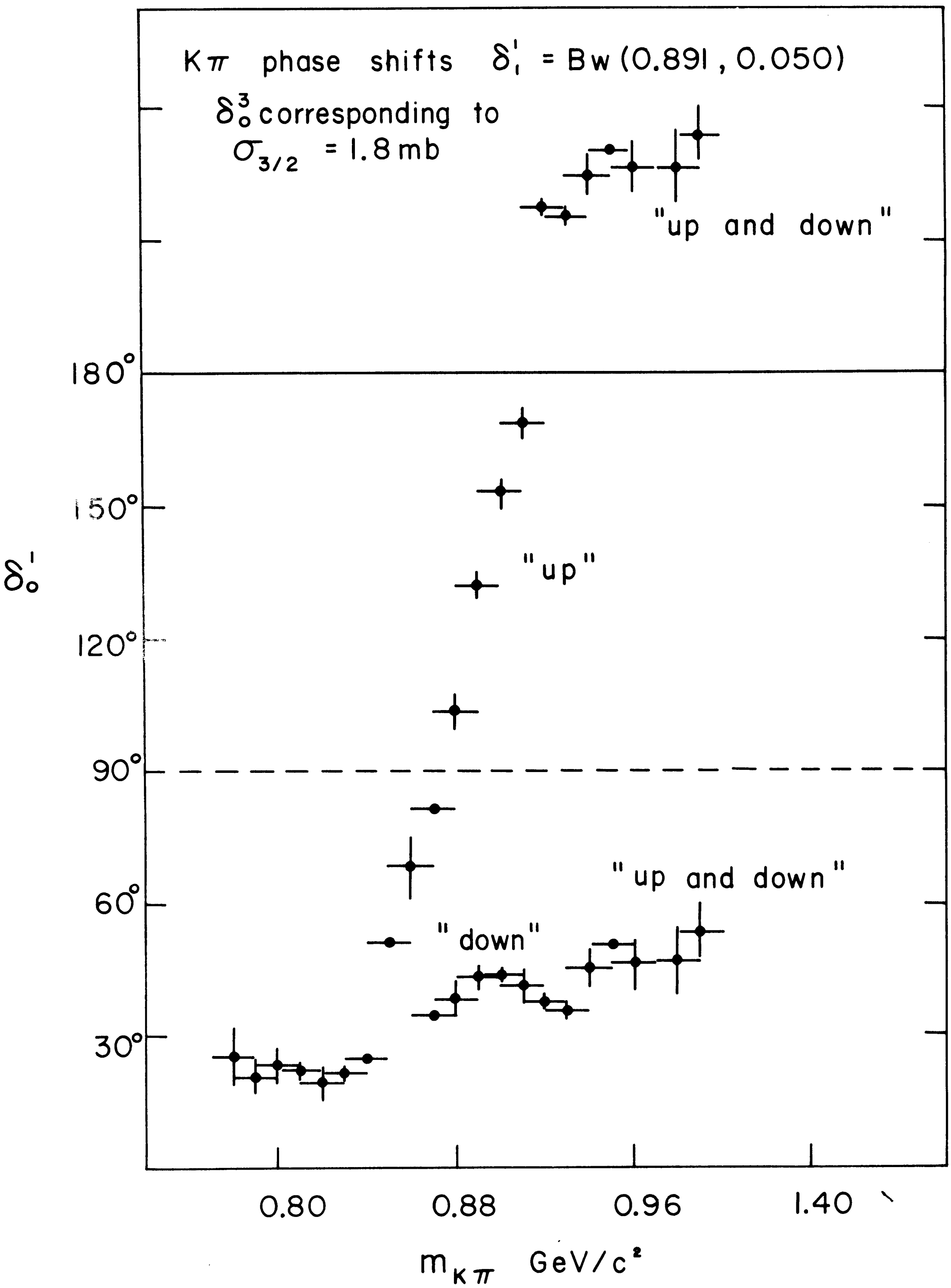
- Figure 4 -


Baton et al.
Baillon et al. (SLAC)
180°
150°
120°
90°
60°
30°
0°
-30°
δ_0^0
δ_0^2
300
400
500
600
700
800
900
1000
$M_{\pi\pi}$ (MeV)

- Figure 5 -



- Figure 6 -



Kπ phase shifts δ¹₁ = Bw (0.891, 0.050)
δ³₀ corresponding to
σ3/2 = 1.8 mb
"up and down"
180°
150°
"up"
δ¹₀
120°
90°
60°
"up and down"
"down"
30°
0.80
0.88
0.96
1.40
mKπ GeV/c²

- Figure 7 -

PART II : DUALITY AND SYMMETRY

1. - INTRODUCTION

Work on duality follows several lines :

- FESR duality ;
- B_4 and B_5 phenomenology ;
- self-consistent pole models (trees) ;
- dual loops.

While the consistency aimed at or achieved increases from top to bottom, the similarity to the real world and the predictive power decrease.

In formal duality there have been interesting developments since the Kiev conference ; since they are highly technical, and since it is not clear, whether these efforts will be rewarded by the discovery of a realistic and consistent model, I shall not attempt to review the progress along these lines [1]. On the other hand, in phenomenological duality (FESR, B_4, B_5) we do have a picture, which is close to nature, which has a lot of power in the predicting and correlating experimental facts, but which is only approximately consistent. Here not much has changed in the last year (except for inclusive reactions, see the Rapporteur's Talk of Satz). I shall try to give you the picture we now have about a few selected topics.

Phenomenological schemes with predictive power (for two-body scattering) are :

(i) FESR with simplicity ansatz ("FESR duality") ;

(ii) B_4 with simplicity ansatz (only a minimum number of B_4 terms).

For phenomenological work involving baryons, it is more direct to work with FESR rather than B_4, because of the problem of parity doubling in B_4, and because of the strong absorption required in the low partial waves of B_4.

What is FESR duality ? Operationally it is defined as what follows from :

(1) analyticity and crossing ;

(2) resonance dominance at low energy, $\nu \leq N$. Experimentally this is well established e.g., in $\mathrm{Im}\, f_\ell(\pi^- p - \pi^0 n)$ for $p_L \leq 1.5$ GeV/c [2] ;

(3) a simple Regge expression at high energy, $\nu \geq N$. For $t \leq 0$ one needs one pole and a cut per quantum number, for $t \geq m^2(1^-)$ no cuts seem to be needed.

The Pomeron corresponds to non-resonating background at low energy [3].

This scheme is approximate, but it has predictive power : it relates two different things, resonances and Regge expressions, which are both known (or directly observable), in contrast to the high mass, low spin resonances occuring in B_4.

The relative signs of parent resonances in inelastic channels provide important tests of duality on one hand and SU_3 on the other hand. Experimentally, they are determined by interference effects between overlapping resonance tails in phase shift analyses. In the Table, we give these signs ; in the columns $\frac{1}{2}^+$ and $\frac{3}{3}^+$ the SU_3 signs [with $F/D(\frac{1}{2}^+) \approx \frac{2}{3}$], in the other columns the experimental signs. The signs are in complete agreement *) with SU_3 (read vertically) [7] and with the anti-exchange-degeneracy implied by duality (read horizontally).

The alternating sign in the $\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \ldots$ sequence is achieved for the Y_1^*'s by alternating between $F/D < 1$ and $F/D > 1$, for the Y_0^*'s by alternating between an octet (with $F/D \approx \frac{2}{3}$) and a state which is dominantly singlet. See Fig. 2 [9].

In the $\frac{3}{2}^+ - \frac{5}{2}^-$ sequence there is no $Y_0^*(\frac{3}{2}^+)$, therefore the $Y_0^*(\frac{5}{2}^-)$ must decouple from channels with exchange degeneracy, i.e., from $N\bar{K} - N\bar{K}$ and $N\bar{K} - \Xi K$. Decoupling from $N\bar{K}$ implies $F/D(\frac{5}{2}^-) = -\frac{1}{3}$, see Fig. 2.

Exchange degeneracy gives us a qualitative understanding of the SU_3 coupling patterns of the observed baryon parent resonances. But as soon as one looks for an exact algebraic solution to the exchange degeneracy equations for the couplings, one is in trouble, because there are too many equations. In the $\frac{3}{2}^+ - \frac{5}{2}^-$ sequence, one must either introduce an unobserved $\frac{3}{2}^+$ octet [10], or one must break duality, e.g., by neglecting the constraints imposed by exoticity in the t channel [11] or by fulfilling all equations only approximately. (Note that the unwanted $\frac{3}{2}^+$ octet of the exact solution couples only weakly.) In the $\frac{1}{2}^+ - \frac{3}{2}^-$ sequence with a $8(\frac{1}{2}^+)$ and a 1-8 mixture for the $\frac{3}{2}^-$ (on the leading trajectory) unbroken duality implies $F/D(\frac{1}{2}^+) = F/D(\frac{3}{2}^-) = 1$, which clearly disagrees with the experimental values although it is in the right ball park compared to the F/D value for the $\frac{5}{2}^-$.

Another trouble of the exact exchange degeneracy scheme is that it predicts a magic mixing angle not only for the 1^- and 2^+ mesons, but also for all other meson multiplets, whereas the 0^- looks much more like an octet. Finally in baryon-antibaryon channels one must either break duality strongly or introduce exotics (which couple mainly to $B\bar{B}$).

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*) Note that the experimental signs [8),9)] of the $Y_1^*(1910, \frac{5}{2}^+)$ couplings have always been in agreement with SU_3 [assuming $F/D(\frac{5}{2}^+) \approx F/D(\frac{1}{2}^+) < 1$] and with anti-exchange degeneracy (in $N\bar{K} - \Lambda\pi, \Sigma\pi$). On the other hand the magnitude of the experimental $Y_1^*(1910)$ coupling is about a factor two too large in $(N\bar{K} - N\bar{K})$ and about a factor two too small in $N\bar{K} - \Sigma\pi$, if we assume $F/D = \frac{2}{3}$. However the phase shift errors on these two amplitudes are large.

At $t = m^2(1^-)$ one can make the direct connection to the vector meson dominance model (VMD) of the electromagnetic form factors, in particular to the isoscalar : isovector ratio. Note that the upper vertices in Fig. 3b,c are well known, while the lower vertices are common to both graphs and involve the F/D ratios for flip and non-flip. Here we are only interested in the symmetry ($\rho : \omega$ ratio) implied by VMD, but not in the dynamics, since ρ', ρ'', etc, must play an important role in a pole dominance model.

$(A + \nu B, A)$ correspond to (F_1, F_2), while $(A', \nu B)$ correspond to (G_E, G_M). At $t = 0$, $J = 1$, universality gives $(D/F)_{F_1} = 0$, and the near-vanishing of the isoscalar anomalous magnetic moment gives $(F/D)_{F_2} \approx \frac{1}{3}$. We conclude that the F/D ratios are independent of t, at least between $t = 0$ and $t = m^2(1^-)$, for (F_1, F_2) but not at all for (G_E, G_M). This predicts the F_1 charge radius of the neutron to be zero, in agreement with experiment.

Tensor meson dominance (TMD) for the gravitational coupling is a generalization [17] of VMD. It implies universality of the f_1 (2^+, SU_3 singlet) coupling [18]. But the universality implied by the vector meson dominance of electromagnetic couplings combined with exchange degeneracy predicts [19] for the non-flip couplings

$$(f_1 - MM) : (f_1 - B\bar{B}) = 2 : 3$$

where $M = 0^-$ mesons, $B = \frac{1}{2}^+$ baryons. This is in contradiction to the universality implied by TMD for the gravitational coupling. On the other hand, TMD has also been used for Pomeron couplings [20]. In this case the ratio 2:3 is a welcome prediction, it agrees with the quark model prediction $\sigma^{\infty}_{tot}(MB) = \frac{2}{3}\sigma^{\infty}_{tot}(BB)$.

4. - CLASH OF t CHANNEL STRUCTURE AND s CHANNEL STRUCTURE IN THE POLE MODEL. DAUGHTERS AND ABSORPTION

Resonance saturation requires that exotic u channels, e.g., K^+p, have Im F(non-Pomeron) = 0 at all t. In t channel language this implies exchange degeneracy (ρ-A_2, ω-f), and this forces the imaginary part of each Regge pole to have a simple zero at $\alpha(t) = 0, -1, -2, \ldots$ (NWSZ, nonsense wrong signature zero). The position of these zeros depends on $\alpha(t)$ and not on the spin structure. We call this the t channel or exchange degeneracy structure.

In contrast in the s channel, K^-p, FESR with resonance saturation require Im F = 0 at $t = -0.15$ GeV^2 for non-flip and at $t = -0.5$ GeV^2 for flip. This reflects the spin structure of the parent resonances, which dominate empirically. We call this the s channel or spin structure.

When discussing daughters, we must distinguish three points :

(a) daughter waves are certainly needed in a peripheral band with an impact parameter $b \approx 1$ fermi ($\ell = kb_0$) and with Δb logarithmically expanding ; at high energies they dominate over the parents, which become ultraperipheral ($\ell \sim s \gg kb_0$) ; at low energies the peripheral band and the ultraperipheral parents are one and the same ; the average strength of the peripheral daughters at higher energies follows directly from the Regge factor $s^{\alpha(0)}$;

(b) the strength of the low daughter waves, $b \lesssim \frac{1}{2}$ fermi, is quite wrong in the B_4 model ; in Section 4, we saw that B_4 must be strongly absorbed in the low partial waves ; see also Ref. 23) ;

(c) are the daughter waves dominated by genuine resonances which are mass degenerate with the parents ? we saw in Section 8 of Part I that the 0^+ daughter of the $K^*_{890}(1^-)$ is shifted well beyond 1100 MeV, if it exists ; the $\Delta(1238)$ has no $\frac{1}{2}^-$ daughter, and the $\Lambda(1520)$ has no $\frac{1}{2}^+$ daughter nearby.

At high energies the predictions of strengths and shapes of forward and backward peaks from crossed channel resonances via B_4 present an impressive step forward compared to the complete lack of predictive power without some form of duality. However, the B_4 model (without absorption) predicts the high energy amplitudes too large by about a factor two (if on uses the strength of the low energy parent resonances as input). This points again to the need for strong absorption in low partial waves. Unfortunately, we lack a satisfactory prescription how to perform the required absorption (see the Rapporteur's Talk by R.J.N. Phillips).

6. - ODORICO ZEROS

In view of the practical complications with explicit B_4 fits, it is useful to focus on qualitative features of the Veneziano formula, e.g., on its zero [24]. The simple Veneziano term

$$\frac{\Gamma(\frac{1}{2}-\alpha_s)\,\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_s-\alpha_t)}$$

has the structure (s poles) * (t poles) * (zeros at fixed u). The u values of the zeros correspond to the crossing points of the s channel and t channel poles.

For each pair of complex conjugate zeros in $f(z)\cdot f^*(z^*)$ there is the two-fold ambiguity : which of the two zeros is the zero of $f(z)$, $z_k = z_i$ or $z_k = z_i^*$? The position and the half-width of the dip give directly $\mathrm{Re}\, z_k$ and $|\mathrm{Im}\, z_k|$. Each phase shift solution gives an answer to the sign ambiguity in $\mathrm{Im}\, z_k$. As an example, we show in Fig. 6 the imaginary part of one of the zeros in the t plane, $t_k = -2q^2\,(1-z_k)$, as a function of s (= real) for π^+p scattering according to the solution "Berkeley, path 1". One sees that at two energies the shortest path method (on the Argand plots) has selected the wrong sign of $\mathrm{Im}\, t_k(s)$. The sign of $\mathrm{Im}\, z_k$ is a simple tool for helping us to connect phase shift solutions at different energies. It is unlikely to introduce a theoretical prejudice in contrast to the shortest path method.

These zeros are useful because they are close to the physical region. Figure 6 shows that $\mathrm{Im}\, t_k \sim 0.02\ \mathrm{GeV}^2$ while a resonance pole has typically $(-\mathrm{Im}\, s) = m\Gamma \sim 0.2\ \mathrm{GeV}^2$.

Figure 7 shows the trajectories of the real part of the zeros (of z_k or t_k) versus s in the Mandelstam plane. The regularity of the trajectories, as we move across resonances, is striking.

The practically important case is $0^-\ \frac{1}{2}^+$ scattering with $d\sigma/d\Omega$ and P measured. The useful pair of amplitudes is $F^\pm$, because their magnitudes are known experimentally :

$$|F^\pm|^2 = (1 \pm P)\, d\sigma/d\Omega$$

while the phases are completely unrestricted by the data. (The connection to the conventional pair is : $F^\pm = f \pm ig$.) Barrelet zeros are useful, if the data show dips, or if $(1 \pm P)$ has sharp minima, i.e., $P \approx \mp 1$. This is the case for non-exotic channels, see, e.g., the π^+p polarization [28] at $p_L = 1.4$ GeV/c in Fig. 8, where three zeros of $|F^+|$ and $|F^-|$ are directly visible. On the other hand K^+p shows no such structure, and we expect that there will be no helpful nearby zeros in exotic channels.

$F^\pm$ have a square root singularity in $z = \cos\vartheta$ at $z = \pm 1$, therefore these amplitudes are unsuited for dispersion relations in t and for duality discussions.

We discuss the question of ambiguities of phase shift analyses for the spinless case in the inelastic region. One attempts to determine the phase of f, if $|f|$ is measured. One chooses ℓ_{max} as large as required by the statistical quality of the data. The phase shift solutions have ℓ_{max} zeros ; the solutions are completely determined by these zeros and by the known forward amplitude :

$$f(z) = f(1) \prod_{k=1}^{\ell_{max}} \left(\frac{z - z_k}{1 - z_k} \right)$$

- TABLE -

		$\frac{1}{2}^+$ 8	$\frac{3}{2}^-$ 1–8	$\frac{5}{2}^+$ 8	$\frac{7}{2}^-$ 1–8	$\frac{3}{2}^+$ 10	$\frac{5}{2}^-$ 8	$\frac{7}{2}^+$ 10
$N\bar{K} - \Sigma\pi$	Y_1^*	–	+	–	?	–	+	–
	Y_0^*	–	+	–	+	0	≈0	0
$N\bar{K} - \Lambda\pi$	Y_1^*	–	+	–	?	+	–	+

Signs of the couplings of parent resonances in inelastic channels.
For $\frac{1}{2}^+$, $\frac{3}{2}^+$: SU_3 signs.
For the other resonances : relative signs determined experimentally in phase shift analyses [8].
Vertical : SU_3 tests [7),9)].
Horizontal : duality tests.

DISCUSSION

H. Burkhardt

I should like to comment on the zero-tracking technique discussed by Barrelet. I agree that it is a useful way of energy smoothing and may have some advantages over the shortest path method, particularly when there is a dip which is stable in energy. It will certainly eliminate some phase shift solutions but I do not think that it will give uniqueness. In the realistic situation where the high partial waves are not set identically zero, we have shown that a continuum phase ambiguity exists which does not alter the zeros of the amplitude ; I believe that in order to remove it explicit theoretical input is needed - fixing the discontinuity of the nearby cut in $\cos\theta$ is sufficient in principle.

Ch. Schmid

Tracking nearby zeros will always be helpful, but in many cases it will not lead to uniqueness. However, I do believe that <u>some</u> cases, e.g., $K^-p - K^-p$ near 1 GeV correspond essentially to the extreme case No 1, which leads to uniqueness.

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FIGURE CAPTIONS

Figure 1 Anti-exchange-degeneracy of the Y_1^* resonances in $N\bar{K} + \Lambda\pi$. The vertical axis gives the strength $C(\mathrm{Im}\,B) \equiv \int \mathrm{Im}\,B\,d\nu$ at $t = m_{K^*}^2$ [6].

Figure 2 Experimental F/D ratios for the decay of the parent baryon octets into $0^- \frac{1}{2}^+$ [9].

Figure 3 The connection between :
(a) the Y^* resonances in $\bar{K}N - \bar{K}N$,
(b) the vector meson trajectories in $N\bar{N} - K\bar{K}$, and,
(c) the electromagnetic form factors of the nucleon.

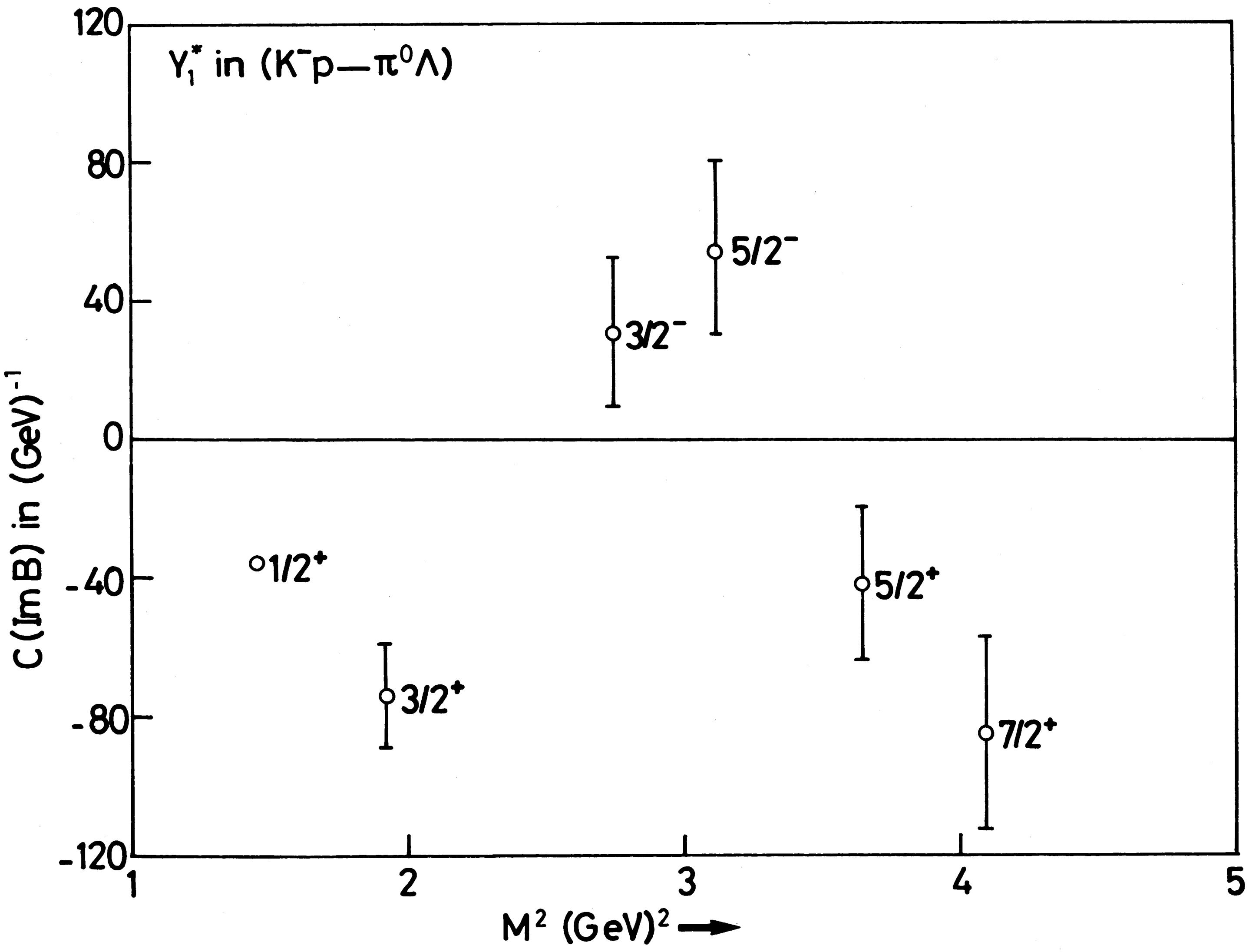
Figure 4 The Mandelstam plot (light lines) for $K^-p - \bar{K}^0 n$ in the s channel. The heavy lines are resonances (s channel : $\Lambda_\alpha - \Lambda_\gamma$, t channel : ρ). The dotted lines give the zeros predicted by a simple Veneziano term for B involving only $\Lambda_\alpha - \Lambda_\gamma$ in the s channel.

Figure 5 $K^-p - \bar{K}^0 n$ angular distributions between $p_L = 0.45$ and 1.8 GeV/c. The solid lines are hand drawn to guide the eye. The figure shows three dips which are remarkably constant in u (top scale), $u \simeq -0.1, -0.7, -1.7\ \mathrm{GeV}^2$, but which move in t (bottom scale) [24].

Figure 6 The imaginary part of one of the zeros of the π^+p scattering amplitudes $F^\pm$ in the t plane ($\sim\cos\theta$ plane) plotted versus s (= real) [26]. Based on the phase shift analysis Berkeley path 1.

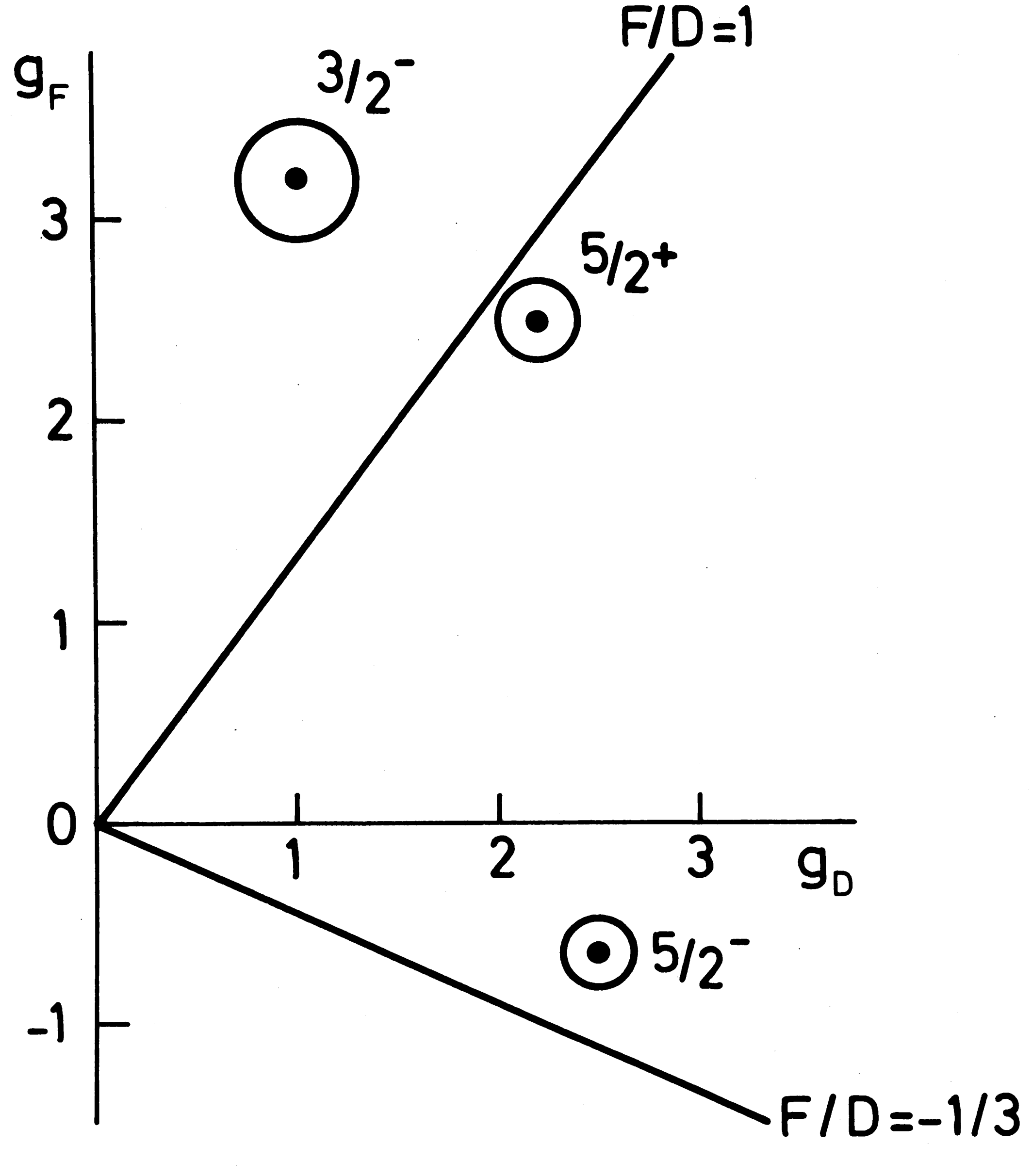
Figure 7 The real parts of the zero trajectories of the π^+p scattering amplitudes $F^\pm$ (crosses and dots respectively) [26]. Based on the CERN experimental phase shift analysis.

Figure 8 The π^+p polarization at $p_L = 1.40$ GeV/c [28].



Y_1^* in ($K^-p \rightarrow \pi^0\Lambda$)
120
80
40
0
-40
-80
-120
1
2
3
4
5
3/2⁻
5/2⁻
1/2⁺
3/2⁺
5/2⁺
7/2⁺
C (Im B) in (GeV)⁻¹
M² (GeV)²

- Figure 1 -



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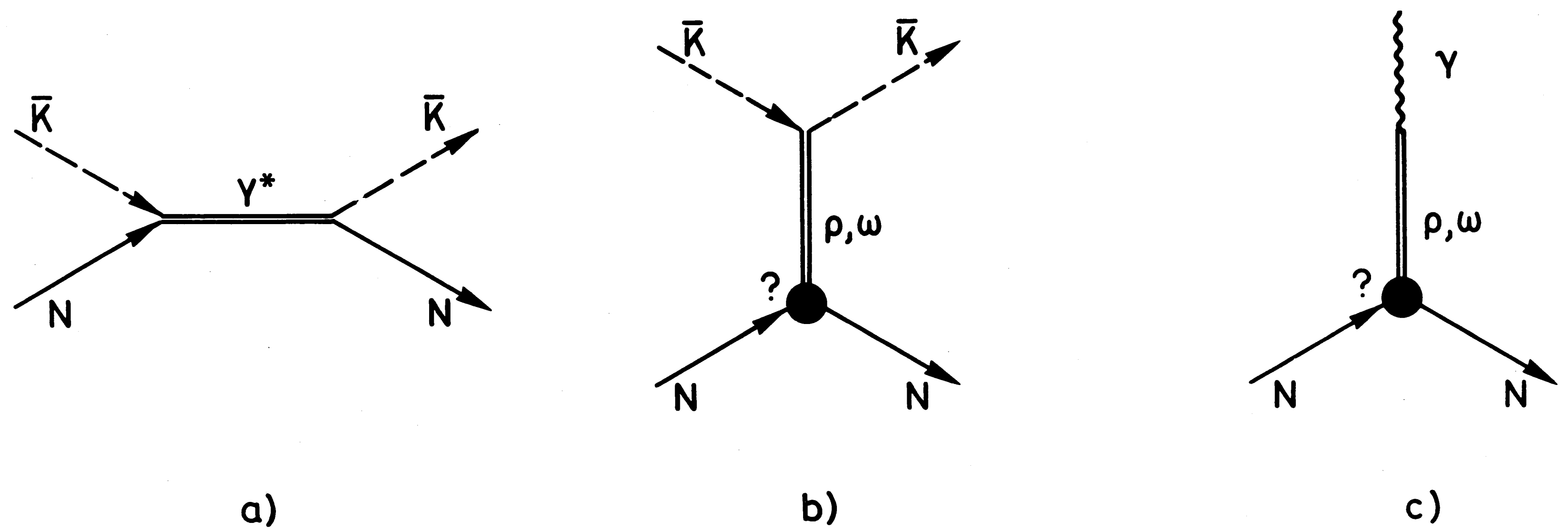

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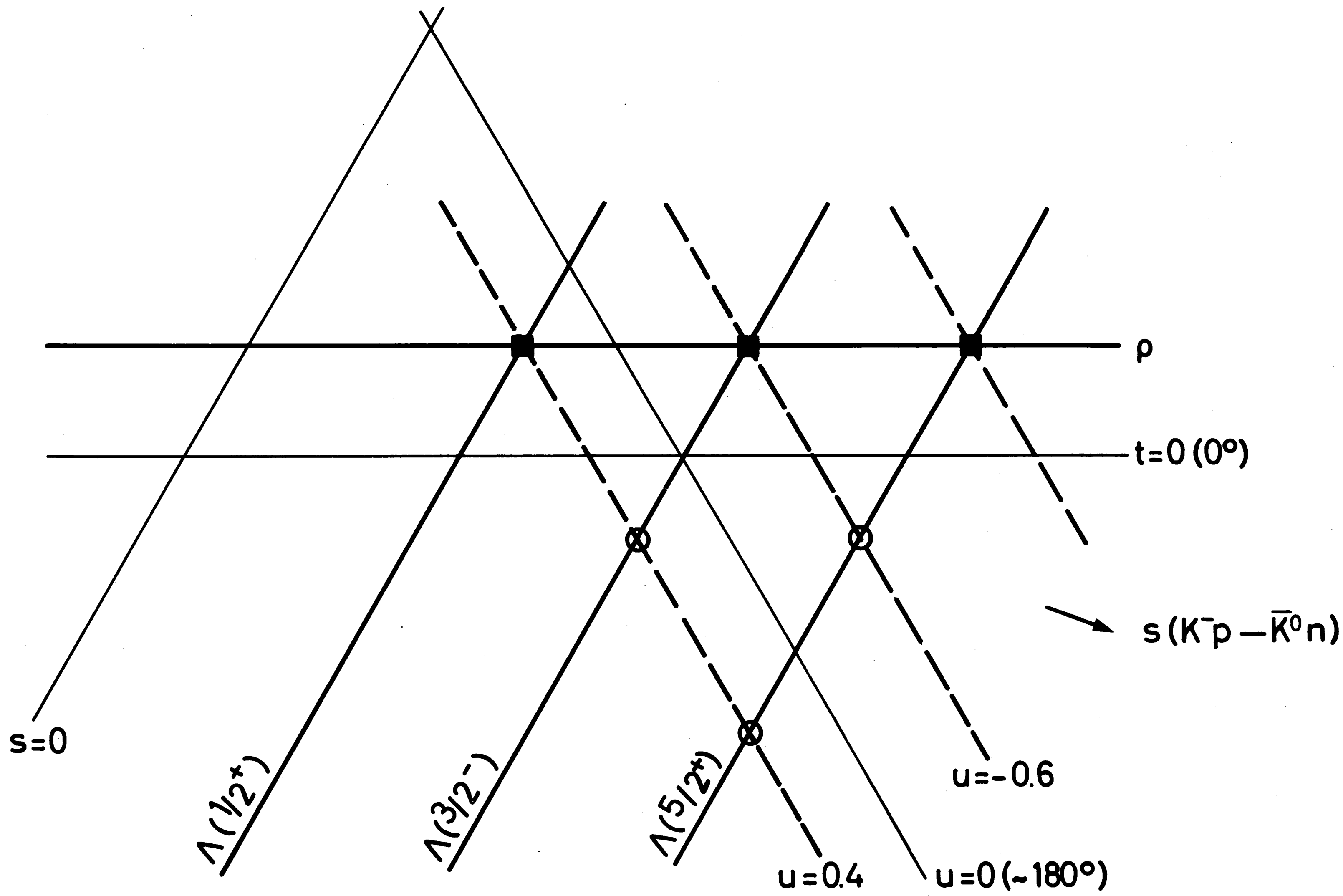
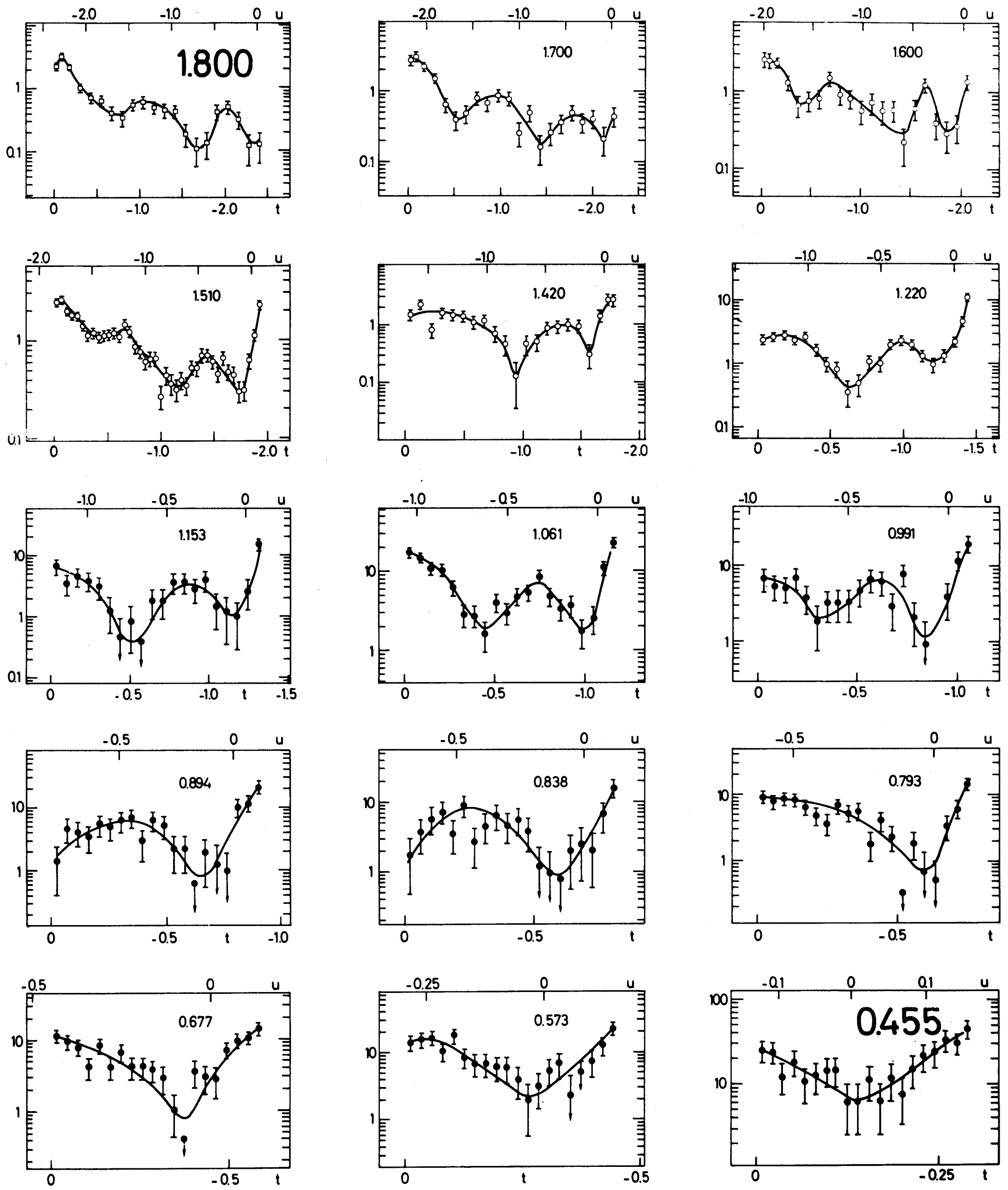


- Figure 4 -

$K^- p \longrightarrow \bar{K}^0 n$

$$\frac{d\sigma}{dt}\left[\frac{mb}{(GeV/c)^2}\right]$$



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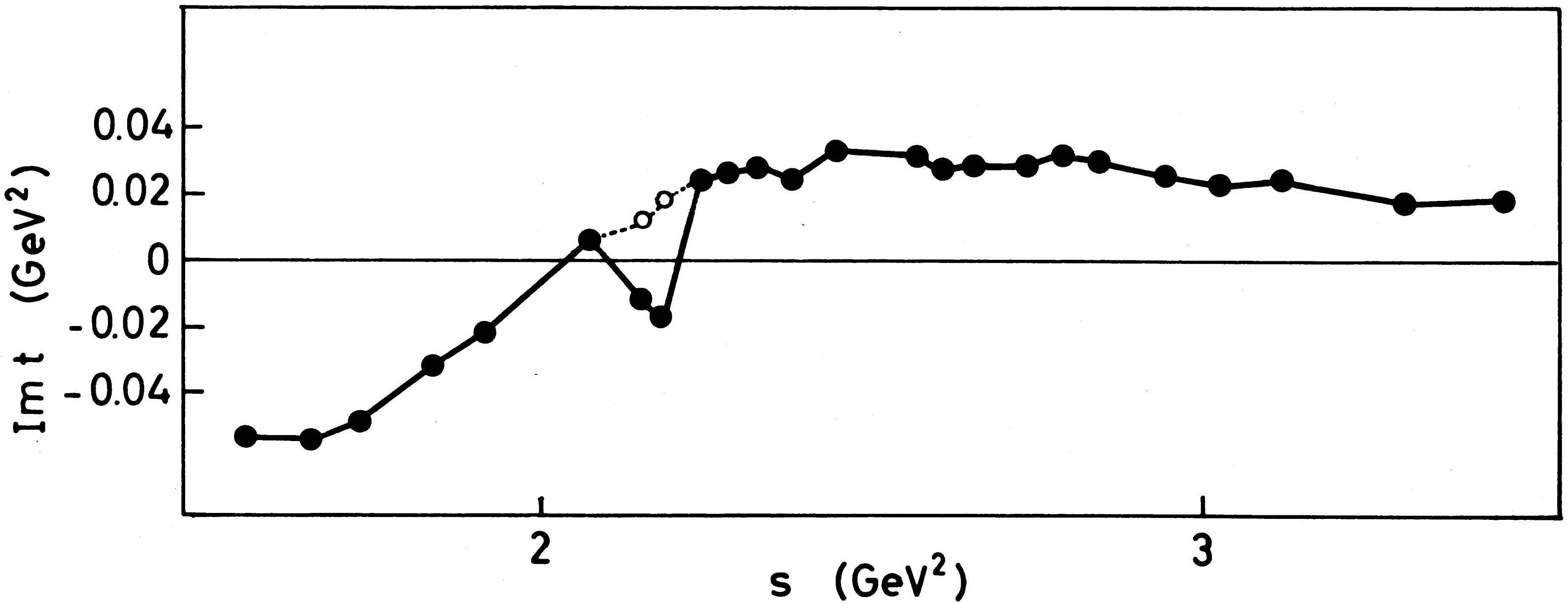
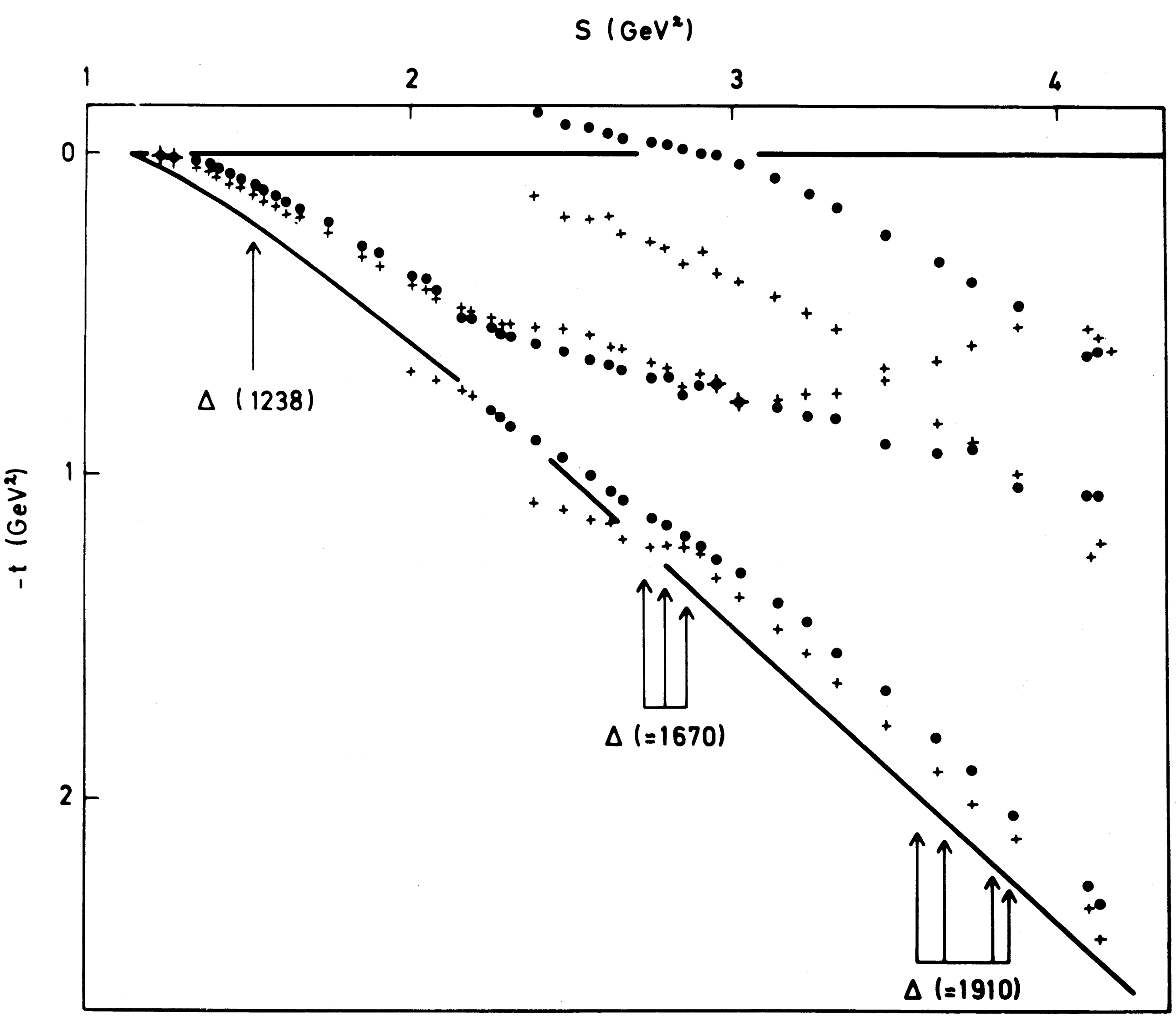
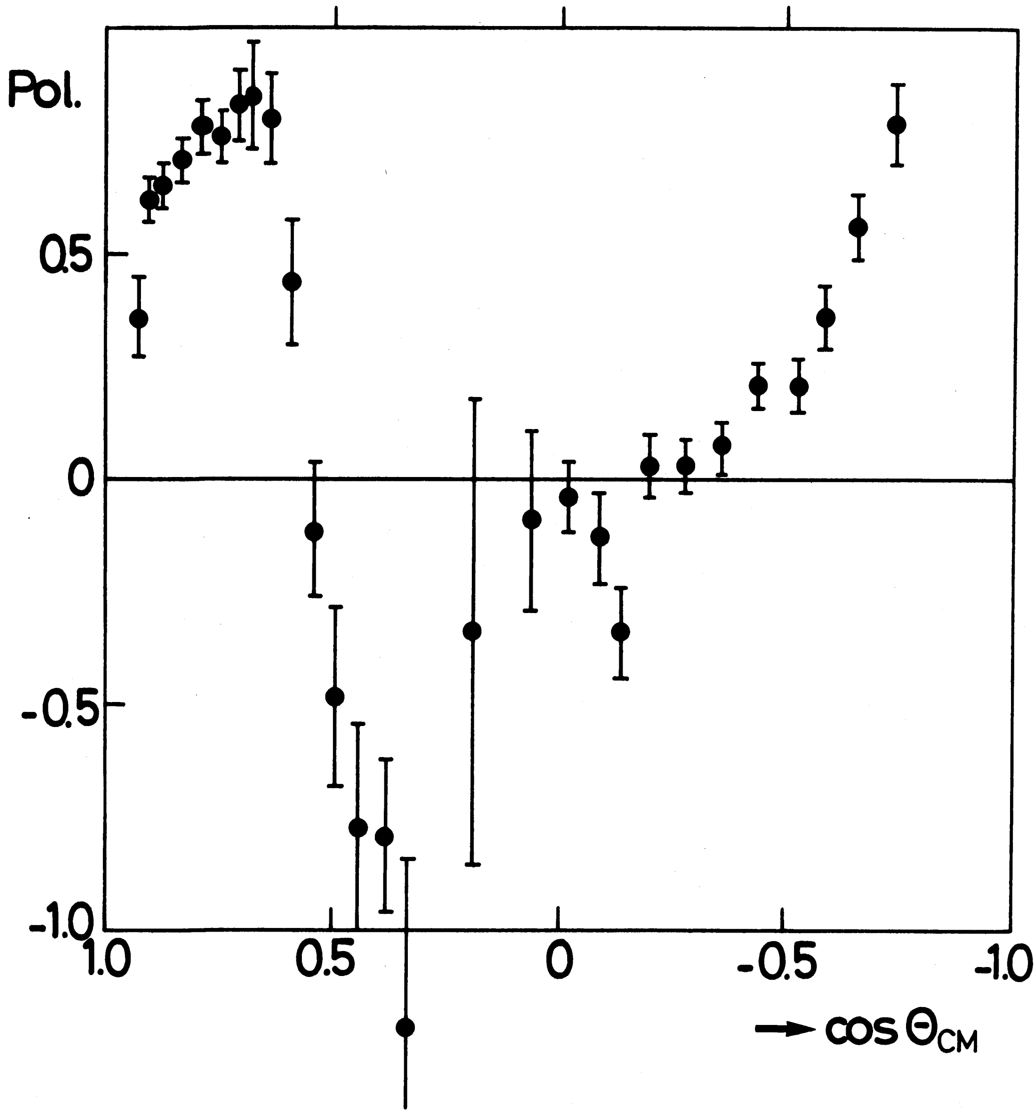


- Figure 6 -



- Figure 7 -



- Figure 8 -