MODELS FOR NUCLEON-ANTINUCLEON ANNIHILATIONS

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1. INTRODUCTION

This is a partial review. It concentrates on a few general features of phenomenological character and describes models of statistical type and related, which attempt at systematizing the properties like the structure of the multiplicity distribution, the magnitude and energy variation of the multipion cross-sections.

Nucleon-antinucleon annihilations provide multiple production (mainly of pions) already at low kinetic energies. Since the reaction implies the conversion of the nucleon masses, a rather large total energy is available for the products, but it is remarkable that a substantial (∿ 40%) fraction is used up for the pion (and kaon) masses. Table 1 shows the difference in the average multiplicity of produced pions between production in pp collisions and $p\bar{p}$ annihilations (Q is the available energy, equal to $\sqrt{s}$ for annihilation, to $\sqrt{s} - 2\,M_p$ for pp collisions).

Table 1

Average multiplicity of produced pions

	lab. momentum (GeV/c)	Q(GeV)	$\langle n_\pi \rangle$
pp	19	4.25	4.0 ± 0.1
	25	5.10	5.3 ± 0.04
	0	1.88	5.0 ± 0.15
$p\bar{p}$	7	3.85	6.7 ± 0.3

Nucleon-antinucleon annihilation thus appears as a special type of multiple production at low and intermediate energies. The connections with other aspects of hadron collisions have not yet been much explored. For example, there might exist a relation between the annihilation and the process of pionization in high energy collisions, if such a mechanism distinct from the excitation of the incident particles exists.

The annihilation cross-section is large at low kinetic energies and is still of the same magnitude as the non-annihilation inelastic channels near 6 GeV/c. If the difference $\sigma_{tot}(\bar{p}p) - \sigma_{tot}(pp)$ at high energy is attributed to annihilation, σ_{ann} is still equal to 5mb at 50 GeV (lab. energy). The empirical formula

$$\sigma_{ann} = \frac{45}{\sqrt{s} - 1.4} \text{ (millibarn)} \qquad (1)$$

reasonably fits the annihilation cross-section (including channels with kaon production) from 1.2 GeV/c ($\sqrt{s}$ = 2.15 GeV^2) to 50 GeV ($\sqrt{s}$ = 10 GeV^2). Formula (1) should not be extrapolated down towards threshold. The annihilation cross-section in the very low energy region grows very large; it is not known precisely close to threshold. The onset of partial waves is rapid; the application of the unitarity limits shows that the minimum number of partial waves is already 2 at 50 MeV lab. kinetic energy and 3 beyond 100 MeV; the analysis of the angular distribution of elastic scattering requires up to at least D-waves between 60 and 100 MeV and at least F-waves from 100 MeV cm[1].

The relative magnitude of the absorption in the various partial waves is not known. Evidence that S-capture for annihilation strongly dominates in annihilation at rest, in similarity to other hadronic atoms, has recently been challenged by an evidence for a substantial $2\pi^0/\pi^+\pi^-$ ratio[2].

The comparatively large value of the average pion multiplicity - and the narrowness of the multiplicity distribution - in annihilation reflects the fact that the final particles use up much more of the available phase space than the particles produced in high energy collisions do. In that sense, annihilations could be regarded as "central" collisions as opposed to peripheral ones. Let

$$\langle p_t^{(0)} \rangle \gtrsim 0.35 \text{ GeV/c} \tag{2}$$

be the "universal" average transverse momentum. In an isotropic distribution (as results from uniform phase space population)

$$\langle p_t \rangle = \frac{\pi}{4} \langle p \rangle . \tag{3}$$

Therefore, a necessary condition for the realization of a peripheral event (i.e. one where extra energy is drained into the longitudinal motion) is :

$$\langle e \rangle \gtrsim \langle p \rangle > \frac{4}{\pi} \langle p_t^{(0)} \rangle \gtrsim 0.45 \text{ GeV} = e^0 \tag{4}$$

A critical multiplicity of produced pions :

$$N^{(0)} = \frac{\sqrt{s}}{e^0} \gtrsim 2.2 \sqrt{s} \tag{5}$$

defines a kind of threshold for peripherality. Comparing the values of $N^{(0)}$ with those of $\langle N \rangle$ in Table 1, it is seen that even in the higher region of the studied energy domain the kinematical limitations are important for an appreciable fraction of the multiplicity distribution.

2. THE MULTIPLICITY DISTRIBUTION OF PIONS

2.1 The multiplicity of charged pions is determined more easily than the multiplicity including the neutral ones. The dependence of $<n_c>$ as a function of $\sqrt{s}$ can be fitted by a formula

$$<n_c> \;=\; a + b\,\sqrt{s} \qquad (6)$$

in reasonable approximation from 0 to 7 GeV/c (s = 4 to 15) but the data suggest a flattening at higher energy.

The shape of the multiplicity distribution (including neutral and charged pions) obtained for annihilation at rest[3] is shown in Fig. 1, where the predictions of the statistical model of uncorrelated pion production are shown for comparison. A Poisson shape with $<N> = 5$ is also shown. The qualitative agreement (the fit is very poor, see Appendix 1) between experiment and the statistical model suggests that the narrowness of the multiplicity distribution derives from the kinematical restrictions. This feature still holds at higher energies, but we believe that comparisons with Poisson shapes or variants should then be applied to the whole inelastic channels, including pion production without annihilation.

2.2 Branching ratios between charge configurations

The cross-sections for annihilation into pions are given in terms of "topologies" :

$$p + \bar{p} \rightarrow k(\pi^+ + \pi^-) \qquad (7a)$$

$$\rightarrow k(\pi^+ + \pi^-) + \pi^0 \qquad (7b)$$

$$\rightarrow k(\pi^+ + \pi^-) + MM \qquad (7c)$$

where k = 1, 2, etc., and where MM means a system of at least two neutral pions. The bubble chamber technique presently does not allow the specification of the MM content in individual events. The contribution of channels of type c) is large and increases with energy : ∿ 60% of all annihilations at rest, ∿ 85% at 5.7 GeV/c. We call "leading" channel the channel of type a) for multiplicity 2k, of type b) for multiplicity (2k + 1).

The structure of the pion collection in charge and number is governed by : a) the multiplicity distribution : what percentage of the annihilations gives 2π, 3π, 4π, etc.; b) the branching ratios between the various charge configurations for each multiplicity, for example : in the 5π states, how much do $2\pi^+2\pi^-\pi^0$, $\pi^+\pi^-3\pi^0$, and $5\pi^0$ contribute. Topologies of the type c) of course contain the contribution of several multiplicities; as long as the neutral pions will not be identified nor their momenta measured, the study of the properties of annihilation will be hindered.

Already welcomed is the determination of the average number of neutral pions associated to a given number 2k of charged pions. The failure of isospin statistical branching ratios in this matter is to be recalled; more details are found in Appendix 2.

2.3 How to estimate the average number of neutral pions associated to a given number of charged pions ?

The average number $<\ell_k>$ of missing π^o's in (MM) for a given k depends of course on k and $\sqrt{s}$. One tends to believe that $<\ell_k>$ should decrease when k increases, as it is the case for example in statistical models. This is not necessarily the rule and it is amusing to mention a simple model case (not realized in annihilation !) : if pions were produced through I = 0 pairs exclusively, with a Poisson law for the pair multiplicity M, then

$$<\ell_k> = \sum_{i=k}^{\infty} 2(i-k)e^{-<M>} \frac{<M>^i}{i!} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{i-k} \binom{i}{k} /\ldots$$
$$/ \sum_{i=k}^{\infty} e^{-<M>} \frac{<M>^i}{i!} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{i-k} \binom{i}{k} = \frac{2}{3}<M> = \frac{1}{3}<N> \qquad (8)$$

would be independent of k.

The simplest method to find out the average number of missing π^o's assumes that - in all pion multiplicities - the total energy is shared equally between the pions irrespectively of charge, an assumption which can be checked in the leading channels and seems to be a very good approximation, at least for the higher multiplicities. Some (weak) deviation should come from the association of charge with larger longitudinal momentum.

A slightly different method which is in principle a finer analysis consists in fitting the missing mass distribution (mass spectrum of the MM object) as a combination of contributions with ℓ = 2, 3, etc. of variable weights. A model has thus to be chosen for the properties in multiplicities (2k + 2), (2k + 3), etc. The simplest model is "phase space", namely the assumption that the invariant mass of the group of π^o's corresponds to a constant matrix element in every multiplicity. The approach is especially worthwhile when it extrapolates the information gained in the detailed study of "leading" channels, thus replacing the constant matrix element by a definite structure with resonant factors, etc. This is what has been done for annihilation at rest.

The information obtained by these methods must in principle be used when comparison is made between experiment and the models which predict globally the properties of the multiplicity distribution. However, the models, some of which will be reviewed now, have not yet reached the stage of such moderately ambitious tests.

3. FERMI STATISTICAL MODELS

3.1 The relative probability for observing $N(=n_+ + n_o + n_-)$ particles of mass m, members of an isospin multiplet, is written :

$$P(\sqrt{s};I;n_+,n_o,n_-) = \frac{1}{n_+!n_o!n_-!} W(I,n_+,n_o,n_-)\ R_N(\sqrt{s};N.m)\ \frac{K^{4-2N}}{(2\pi)^{3N}} \qquad (9)$$

where W is a spin-isospin weight, R_N is the invariant momentum space integral and K is a parameter with the dimensions of a mass. The correspondence :

$$m_\pi^{-1} K^{-2} = \Omega = \lambda\Omega_o \tag{10}$$

where $\Omega_o = \frac{4}{3}\pi m_\pi^{-3}$ is the "normal" interaction volume is often made. It is known that the traditional model of uncorrelated pion production requires $\lambda \gtrsim 5$-10 to fit the average pion multiplicity[4,5]. The large value of λ is often supposed to indicate that resonances should be incorporated into the model.

3.2 A treatment[6] by Lamb has included resonance production by extending the usual statistical treatment of isospin invariance to SU(3) invariance and by introducing several multiplets. The following set was retained : 0^-, 1^- and 2^+ mesons; any selection is of course arbitrary.

There are successes in this approach : a) fits of topologies are fairly good; b) symmetry violation is interpreted by a ratio Ω_K/Ω_π independent of energy. But the energy dependence of the interaction volume is strong and even leads to a volume equal to zero at finite energy. Even then, the fast decrease of the 2π channel when energy increases is not accounted for (see Fig. 2); the disagreement between the 2π contribution and the average multiplicity is a recurrent shortcoming in variants of the statistical hypothesis.

3.3 Note that Lamb has formulated the model to compute not simply rates but also the magnitude and energy variation of the cross-sections. Quite generally we can write the cross-section for a particular channel in the form :

$$\sigma_N\,(\sqrt{s};n_+,\, n_o,n_-) = \frac{1}{p_c\sqrt{s}}\;\frac{1}{n_+!n_o!n_-!}\;R_N\,(\sqrt{s};N.m)\,\frac{<|T_N|^2>}{(2\pi)^{3N}} \tag{11}$$

where it is again assumed for simplicity that one definite isospin multiplet is treated. $<|T_N|^2>$ is the average matrix element squared, including the spin-isospin dependence. Muirhead and Poppleton[7] have used the experimental values of the multipion cross-sections to study the behaviour of the dynamics of pion production. We shall come to this later.

4. RESTRAINED MULTIPERIPHERALISM

The model initiated by Chan et al.[8] is devised to interpolate between high energy peripheral scattering, described by the multi-Regge behaviour, and low energy scattering dominated by phase space behaviour. The application to $p\bar{p}$ annihilations has been made by Chen[9]. The cross-section for the production of $N(=n_+ + n_- + n_o)$ pions in a definite ordering (multiperipheral graph) is written:

$$\sigma_N(\sqrt{s}) = \frac{1}{p_c\sqrt{s}}\;\frac{1}{n_+!n_o!n!}\int |A|^2 dR_N \tag{12}$$

where

$$|A| \propto g^{N-1} \prod_{i=1}^{N-1} \left(1 + \frac{h}{1+\frac{si}{a}}\right)\left(1 + \frac{si}{a}\right)^{\alpha}\left(1 + \frac{si}{b}\right)^{\beta t_i} ; \qquad (13)$$

the s_i and t_i are the sub-energies and momentum transfer squared; α and β are the parameters of the linearized nucleon trajectory, taken as -0.38 and 0.88 from high energy fits. h is taken equal to 0.077, the value found by Chan et al.[8] for other reactions and the two energy scale factors a and b are chosen 0.1 and 3.0 respectively to bring the best agreement to experimental data.

A rather good fit is obtained for the pion transverse momentum, longitudinal momentum, and c.m. angular distributions. The results are not very sensitive to the choice of parameters, but it is essential to set b much larger than a, to make the angular distributions sufficently isotropic. This underlines the qualitative difference between annihilation and high energy production mentionned in the opening.

The cross-sections are normalized individually. A fair fit to the energy variation of the cross-sections in the lab. momentum range 1.6 - 7 GeV/c is then obtained (an extrapolation for 4π and 5π at 12 GeV/c also fits but a result[10] for 9π at 7 GeV/c : (0.65 ± 0.05)mb is in disagreement with the prediction : 0.37 mb). It must be noticed that for the 2π channel the sharp decrease of the cross-section is not understood and that the observed angular distribution is much more isotropic than predicted.

When the relative magnitudes of the cross-sections for the various multiplicities are considered, the model is in difficulty. In conclusion, the model offers a parametrization of the amplitude which rather well reproduces properties like the variation of the average transverse momentum with multiplicity and the magnitude of the forward-backward asymmetry; this is obtained by deviating from a parameter value obtained in high energy fits. The relevance of the model for the calculation of the magnitude of the cross-sections is not obvious.

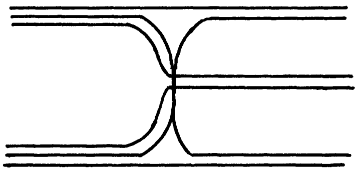
5. QUARK MODELS

5.1 The quark rearrangement model[11] goes beyond SU(3) invariance by the dynamical assumption that the number of quarks and antiquarks is conserved (at least in first approximation). Therefore, nucleon-antinucleon annihilation would yield three and only three mesons. The particular scheme of rearrangement (no spin exchange, which implies uncoupling of spin and orbital momenta, and no isospin and strangeness exchange) implies the production of pseudoscalar and vector non strange mesons, with definite rates. A detailed comparison with experiment has shown that there are severe discrepancies between the model and the experiment[12]. The predominance of three-meson production, even in a more general framework than rearrangement does not correspond to the data. If the 2π mode is indeed low, other two-body modes ($\rho\pi$,$\rho\rho$,$\rho\omega$,$f\pi$, etc.) exist and they make up globally a non negligible fraction of the total cross-section, at least at low energy. Less conventional two-body modes become perhaps operative at higher energy. The average multiplicity predicted by the quark rearrangement model, correct at threshold, increases too slowly with energy; two possible corrections exist :

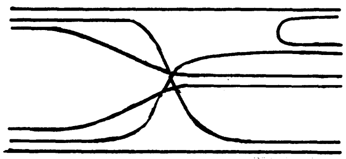
a) the production of heavier mesons;

b) the emission of "bremsstrahlung" pions before the actual reorganization of the quark matter.

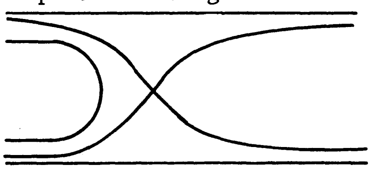
5.2 A possible recasting of the quark rearrangement or recombination idea was conceivable by referring to diagrams with quark lines[13]. A diagram for three-meson annihilation would then be :



The creation of an extra meson would be represented by the intrusion of a new quark line :

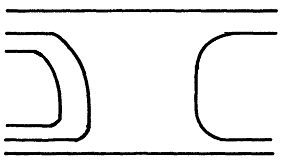


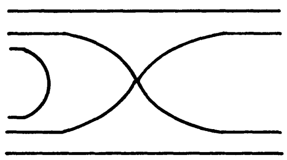
and these contributions would be treated in a kind in a kind of perturbation approach. Two-meson annihilation would correspond to a diagram :



and both the smallness and fast decrease of these processes would be associated to the large change in momentum for the returning quark line.

The approach fails for several reasons, the main one being the following : the comparison of the 2K and 2π modes





requires $\sigma(K\bar{K}) << \sigma(\pi\pi)$ and a faster decrease of the former when s increases. But over the whole energy range where measurement have been made the two cross-sections are in a practically constant ratio $\sim 1/3$. The consideration of the quark content does not seem a fruitfull way to examine annihilation processes.

6. A TWO-BODY MODEL

The three-body filter does not work. We switch to another filter and present a model[14)] based on the assumption that quasi two-body states dominate the annihilation. As an essential ingredient, the two-body amplitude implements the principel of dominance of nearly threshold in the s-channel.

We have pointed out that the 2π and 2K cross-sections show a very rapid decrease when s increases; this behaviour seems to be shared by the other identified two-body modes ($\rho\pi, \rho f, \omega\rho \ldots$). It is then tempting to assume that it is a fundamental feature and to investigate whether a dominance of two-body processes with parallel energy dependence would account for the properties of the multipion cross-sections.

The cross-section of a particular channel

$$p + \bar{p} \to A + B$$

where A and B are any meson (resonance) is written :

$$\sigma(p\bar{p} \to AB) = \frac{1}{p_c^2 \sqrt{s}} \frac{\alpha(AB)}{J+2} \sum_{\Delta\lambda=0}^{J+1} \int_{t_o}^{t_1} |R_{AB}\,(s,t,\Delta\lambda)|^2 dt \tag{14}$$

where $J = J_A + J_B$ (spins) and $\alpha(AB)$ is the spin-isospin weight; $\Delta\lambda$ is the helicity-flip; t_o and t_1 of course depend on s and on the masses.

What is the structure of the amplitude ? R_{AB} is written as the product of two factors :

$$R_{AB}\,(s,t,\Delta\lambda) = R_{AB}^{(s)}\,(s)\;.\;R_{AB}^{(t)}\,(s,t,\Delta\lambda) \tag{15}$$

where $R^{(t)}$ is the conventional dual amplitude and $R^{(s)}$, assumed to depend only on the direct-channel parameters is a correction which is expected to be important because of the failure of the dual approach to describe annihilation processes. The specific forms retained for $R^{(s)}$ and $R^{(t)}$ are :

$$R^{(t)}\,(s,t\Delta\lambda) = \left(\frac{t_1-t}{t_o}\right)^{\Delta\lambda} \exp\left[A(t_1-t)\right] \qquad t_o = 1\ (\mathrm{GeV/c})^2 \tag{16}$$

$$R^{(s)}\,(s) = C\left[\frac{s-s_{AB}}{s}\right]^{1/2} \exp\ -a\left[(s-s_{AB})\right]^{1/2} \qquad s_{AB} = (m_A+m_B)^2 \tag{17}$$

The assumption that the normalization constant C is the same for all channels is of course a crude approach. An elaborate fit procedure has not been attempted; to fix the parameters, a and C/A have been determined by fitting the magnitude and energy variation of the 2π cross-section (Fig. 2), and the parameter A by fitting the ratio between the 2π and the total annihilation cross-section at a fixed s ($s = 4.5\ (\mathrm{GeV})^2$); the results do not critically depend on A. The retained values are $a = 1$, $A = 2$.

The essential content of the model is a rapid increase of each two-body cross-section at the channel threshold followed by an exponential decrease (remember that a number of channels have their threshold below $2M_p$).

The correct order of magnitude is obtained for the 3π, 4π, and 5π cross-sections (see Fig. 3). For higher multiplicities, a severe underestimation occurs but this is to be expected as a consequence of the incomplete knowledge of the meson spectrum, especially beyond 1700 MeV. (Note that the branching ratio for $g \to 4\pi/2\pi$, here taken to be 3/1, has a serious impact on the results; cf. the results of Matthews et al. (Nucl. Phys. B33 (1971) 1) on the g meson).

In conclusion, quasi-particles seem to play a very important role and it seems plausible that most final states are made of them. The principle of nearby threshold dominance seems to be required by the data and its implementation leads, for the magnitude and energy variation, to a qualitative agreement between the 2π channel and the higher multiplicities, a result which was not reached in other models.

7. THE EXTRACTION OF THE AVERAGE MATRIX ELEMENT SQUARED

7.1 The cross-section for the production of N pions is written:

$$\sigma_N(\sqrt{s};n_+,n_o,n_-) = \frac{1}{n_+!n_o!n_-!}\ \frac{1}{p_c\sqrt{s}}\ \frac{1}{(2\pi)^{3N}} \int |T_N|^2\ dR_N$$

$$= \frac{1}{n_+!n_o!n_-!}\ \frac{1}{p_c\sqrt{s}}\ \frac{1}{(2\pi)^{3N}} <|T_N|^2>R_N. \qquad (18)$$

Muirhead and Poppleton[7)] have investigated the properties of $<|T_N|^2>$ by calculating

$$\sigma_N \cdot \frac{p_c\sqrt{s}}{R_n} \qquad (19)$$

[The $(n_+!n_o!n_-!)$ factor does not appear in their formulation] . The gross behaviour can be expressed by the equality of (19) with

$$A_N\ s^{-5}\ . \qquad (20)$$

The s-dependence is thus the same for all N; besides, A_N is of the form a^N, where a is a constant; more precisily, the value of a is different for N odd and for N even. The existence of (20) may suggest the relevance of uncorrelated pion production, possible final state interactions being then dependent on the basic processes of pion creation, but in our opinion, such a conclusion is not at all compulsory.

The agreement between the form (20) and the data is crude. Fluctuations can certainly be tolerated in such an approach, but there furthermore appears a systematic trend depending on multiplicity, as we shall soon show.

Hansen et al.[15] have used a different version to analyse many-body cross-sections from various hadron collisions by considering the following quantities :

$$\sigma_A = \sigma_N \frac{p_{1ab}^{N-2}}{R_N} \quad . \tag{21}$$

Interesting systematics arises; it is found that

$$\sigma_A \propto p_{1ab}^{-n_A} \tag{22}$$

where n_A (not the multiplicity !) depends on the type of reaction, more precisely on the dominant exchange in a multi-Regge graph but not on the multiplicity. However, the reactions of annihilation are found anomalous in that framework : when $n_A \approx 4$ is obtained for nucleon exchange in high energy reactions, it is equal to ≈ 3 for small annihilation multiplicities and ≈ 2 for higher multiplicities (the anomaly persists with s instead of p_{1ab} as variable).

7.2 In view of these contradictions, we have made again the calculations along the same lines[7], and computed

$$<|T_N|^2> = n_+!n_o!n_-! \frac{p_c\sqrt{s}}{R_N} \sigma_N \quad . \tag{23}$$

The presence of the factor $(n_+!n_o!n_-!)$ brings about a more uniform behaviour for even and odd multiplicities (although we do not claim that it is the adequate factor for the branching ratios within multiplicities). Fig. 4-6 show the comparison of the data with two laws

$$<|T_N|^2> \propto s^{-4} \; 8^N \tag{24}$$

and

$$<|T_N|^2> \propto s^{-5} \; 8^N \quad . \tag{25}$$

It is seen that a rather regular variation of the exponent in the s-dependence variation from ≈ 4 to ≈ 6 is observed when the multiplicity increases, an effect already partially noticed[10].

It is obvious that a s^{-c} dependence with fixed c cannot be tolerated at high energy, since this would entail[15) :

$$\sigma_N \rightarrow \frac{1}{s} \; s^{-c} \; s^{N-2} \; = \; s^{N-c-3} \tag{26}$$

as R_N is proportional to s^{N-2} asymptotically. The cross-sections for $N > c+3$ would grow without limit!

In terms of a statistical model, the observed behaviour implies a very weak dependence on energy of the parameter governing the ratio between the pion multiplicities but a drastic dependence of an extra factor governing the reaction probability. The heuristical importance of the approach seems limited. Quite a range of models are certainly apt to reproduce the observed gross features.

In connection with phase space effects, the consideration of the $K\bar{K}\,n\pi$ channels might be instructive and their comparison with the purely pionic channels would shed light on the way the kinematical limitations operate. For example, the fact that the ratio of $\sigma(K\bar{K})$ to $\sigma(2\pi)$ seems practically independent if s rises the question whether this becomes true at sufficiently high energy for $\sigma(K\bar{K}n\pi)/\sigma((n+2)\pi)$.

7.3 Many aspects have not been touched in this review, in the first place the Veneziano approach. Let us just mention very briefly two points. It is possible that s-channel resonances below threshold play a role. The low multiplicity cross-sections could therefore be governed by the tails of such resonances whereas higher multiplicities would originate in other mechanisms. Another way to look at the problem is to ask whether to different channels could correspond different interaction ranges : would this explain the $2\pi^o$ - $K^o\bar{K}^o$ contradiction in annihilation at rest ?

A more differential approach than those reported here is probably necessary to gain insight into the annihilation mechanism. A feature such as the rapid change with energy of the angular distribution of $\bar{p}p \rightarrow \pi^+\pi^-$ is certainly a very significant information, whose understanding will reach beyond the particular considered channel.

The interrelation between the hadrons is dramatically illustrated by the fact at the annihilation of the nucleon with its twin particle, the antinucleon, -a reaction which reorganizes at least two GeV of matter- energy- is a marvellous factory of mesons. We can pick up unknown mesons there, find out their mass, spin, etc., but the elucidation of the reaction mechanism remains a huge task.

Appendix 1

STATISTICAL MODEL AND ANNIHILATION AT REST

Fig. 1 presents the percentage distribution of the pion multiplicity in annihilation at rest[3] compared with :

A) the Poisson shape (normalized to 1 from N = 2 to ∞);

B) the predictions of the statistical model of pion production, without resonances, adjusted to <N> = 5 (Exp. 5.0 ± 0.15);

C) the same adjusted to $<N_c>$ = 3.05 (Exp. : 3.05 ± 0.10).

The fit of the statistical model is very poor, but the trend suggests that the narrowness of the multiplicity distribution is related to phase space restrictions; annihilations, at least at low energy, should therefore not be tackled with asymptotic models. When the table of topologies is examined, it is seen that a systematic discrepancy exists between the statistical model and the data at the level of branching ratios. Especially, the properties of the 5π-states, as they are presented[3] ($2\pi^+2\pi^-\pi^0/5\pi$ = 0.43 ± 0.03), deviate very much from the statistical predictions. The introduction of selection rules in the statistical framework[4] is no cure (see Table A1.I). Neither is there a straightforward explanation in terms of resonances in the statistical framework; the occurence of complicated interferences is probably the source of the difficulty. Pais' scheme[18] defines classes with definite permutation symmetry (in general several classes correspond to one value of the isospin), the branching ratios varying greatly from class to class). It might perhaps be feasible to find out the dominant classes and then check the compatibility between $\bar{p}p$ and $\bar{p}n$ results.

TABLE A1.I

Frequencies (%) of the topologies in annihilation at rest. Several sets of weights for the four S-sectors, compatible with R_p/R_n = 1.33 in deuterium, have been tried for the calculation "with selection rules"; they always give comparable results.

Topology	Experiment	Without selection rules	With selection rules
zero-prong	∿ 3.	2.2	1.1
$\pi^+\pi^-$	0.375 ± 0.03	0.2	0.1
$\pi^+\pi^-\pi^0$	6.9 ± 0.35	4.1	4.8
$\pi^+\pi^-$MM	35.8 ± 0.8	32.9	32.4
$2\pi^+2\pi^-$	6.9 ± 0.6	10.0	8.5
$2\pi^+2\pi^-\pi^0$	19.6 ± 0.7	26.0	30.4
$2\pi^+2\pi^-$MM	20.8 ± 0.7	17.9	16.4
$3\pi^+3\pi^-$	2.1 ± 0.25	4.3	3.8
$3\pi^+3\pi^-\pi^0$	1.85 ± 0.15	2.1	2.4
$3\pi^+3\pi^-$MM	0.3 ± 0.1	0.2	0.2

APPENDIX 2

BRANCHING RATIOS

The use of statistical branching ratios, which assumes that all allowed isospin states for the final particles have the same weight[16,17], is known to fail when applied to annihilation (Table A2.I shows the result for three energies).

TABLE A2.I

Cross-sections for the MM-channels. A) observed; B) deduced from the magnitude of the leading channels by means of the statistical branching ratios.

	1.6 GeV/c		5.7 GeV/c		7 GeV/c	
	A	B	A	B	A	B
$2\pi^+2\pi^-$MM	12.0 ± 1.5	6.6 ± 0.5	8.3 ± 1.4	3.2 ± 0.15	10.5 ± 1.5	3.64 ± 0.20
$3\pi^+3\pi^-$MM	1.05± 0.25	0.2 ± 0.05	4.8 ± 0.15	1.95 ± 0.1	3.9 ± 0.5	2.87 ± 0.19
$4\pi^+4\pi^-$MM					0.96± 0.07	0.42 ± 0.08

The question is whether other assumptions for the branching ratios could avoid the discrepancy. We present in Table A2.II the branching ratios for multiplicities 4 to 10 in several assumptions. The first four columns refer to uncorrelated pion production with I-spin conservation in the statistical framework. Note that the method of averaging between the two I-states is not important for the conclusion that the leading channels are severely overestimated.

Columns 5-11 present the branching ratios obtained in several conditions of resonant production, calculated in the following way. The resonances are treated as stable particles in the statistical isospin distribution and then allowed to decay to build up the multipion states. It is seen that the general trend of ρ and ω mesons (but not of the tensors) is to deplete the leading channels; this is in the direction which is needed to bring closer model and experiment.

Other situations in uncorrelated pion production also lead to too large percentages for the leading channels. This is shown in the next columns. Column 12 corresponds to the simple statistical factor $(n_+!n_0!n_-!)^{-1}$; column 13 to multiperipheral production with incoherent graphs of nucleon exchange; column 14 the same with Δ exchange.

Isospin invariance does not at all necessarily imply that

$$\langle m(\pi^\pm)\rangle = 2\langle m(\pi^0)\rangle = \frac{2}{3} N \qquad (A2.1)$$

is obeyed in each multiplicity N (nor, of course, that $<n_c> = \frac{2}{3} <N>$ holds for the distribution). In the statistical framework for uncorrelated pion production (A2.1) would be generally true only if all isospins up to N were available; the restriction to definite isospins destroy the relation, except for the case I = 0 where all directions in isospace are on e equal footing.

However, in a variety of schemes, (A2.1) is almost satisfied, the better in general the larger N is. It can be verified by means of Table A2.II that the result for $<m(\pi^{\pm})>/N$ falls within a few percent of the value $\frac{2}{3}$ (except a few deviations at small N), in spite of large variations in the branching ratios between the charge configurations.

Finally, let us point out that $\bar{p}n$ annihilation being a pure I-state allows a more precise comparison between model and results. The presently available data are, to my knowledge, still scarce, but the future will certainly bring interesting information. As a general remark, let us urge the model builders to treat in parallel $\bar{p}p$ et $\bar{p}n$.

TABLE A2.II

Branching ratios in multipion states (N = 4 to 10), for various theoretical situations.

Column 1 - statistical distribution for I = 0
" 2 - statistical distribution for I = 1
" 3 - arithmetic average of 1 and 2
" 4 - average between 1 and 2 according to the isospin statistical weight for the multiplicity
" 5 - $(N - 2)\pi + \rho$
" 6 - $(N - 3)\pi + \omega$
" 7 - $(N - 4)\pi + 2\rho$
" 8 - $(N - 5)\pi + \rho + \omega$
" 9 - $(N - 6)\pi + 2\omega$
" 10 - $(N - 2)\pi + f$
" 11 - $(N - 3)\pi + A_2$
" 12 - proportional to the factor $\frac{1}{n_+!n_0!n_-!}$
" 13 - multiperipheral N exchange
" 14 - multiperipheral Δ exchange

n	+ 0 -	1	2	3	4	5	6	7	8	9	10	11	12	13	14
4	2 0 2	0.400	0.400	0.400	0.400	0.283	-	0.167	-	-	0.555	0.417	0.316	0.390	0.271
	1 2 1	0.533	0.600	0.566	0.578	0.717	1.000	0.833	-	-	0.389	0.583	0.632	0.585	0.728
	0 4 0	0.067	-	0.034	0.022	-	-	-	-	-	0.055	-	0.053	0.002	0.001
5	2 1 2	0.667	0.629	0.648	0.639	0.689	0.833	0.717	1.000	-	0.567	0.567	0.588	0.661	0.580
	1 3 1	0.333	0.343	0.338	0.340	0.311	0.167	0.283	-	-	0.383	0.433	0.392	0.331	0.420
	0 5 0	-	0.029	0.014	0.020	-	-	-	-	-	0.050	-	0.020	0.008	0.000
6	3 0 3	0.190	0.179	0.184	0.182	0.128	-	0.096	-	-	0.267	0.200	0.142	0.175	0.159
	2 2 2	0.629	0.643	0.636	0.639	0.716	0.850	0.807	1.000	1.000	0.519	0.633	0.638	0.675	0.587
	1 4 1	0.171	0.179	0.175	0.176	0.156	0.150	0.096	-	-	0.207	0.167	0.213	0.164	0.254
	0 6 0	0.010	-	0.005	0.003	-	-	-	-	-	0.007	-	0.007	0.003	0.000
7	3 1 3	0.417	0.403	0.410	0.407	0.395	0.400	0.358	0.283	-	0.426	0.384	0.356	0.410	0.396
	2 3 2	0.500	0.505	0.503	0.504	0.543	0.577	0.608	0.717	1.000	0.439	0.528	0.534	0.512	0.447
	1 5 1	0.083	0.088	0.086	0.087	0.062	0.022	0.034	-	-	0.126	0.088	0.107	0.077	0.157
	0 7 0	-	0.004	0.002	0.003	-	-	-	-	-	0.007	-	0.002	0.001	0.000
8	4 0 4	0.086	0.080	0.083	0.082	0.058	-	0.043	-	-	0.121	0.091	0.063	0.078	0.094
	3 2 3	0.537	0.535	0.536	0.535	0.565	0.639	0.593	0.689	0.883	0.487	0.517	0.506	0.546	0.487
	2 4 2	0.337	0.345	0.341	0.343	0.350	0.340	0.352	0.311	0.167	0.330	0.360	0.379	0.341	0.322
	1 6 1	0.039	0.040	0.040	0.040	0.027	0.020	0.012	-	-	0.061	0.032	0.051	0.034	0.097
	0 8 0	0.001	-	0.001	0.000	-	-	-	-	-	0.001	-	0.001	0.000	0.000
9	4 1 4	0.241	0.234	0.238	0.236	0.216	0.182	0.188	0.128	-	0.271	0.232	0.201	0.234	0.267
	3 3 3	0.534	0.536	0.535	0.535	0.573	0.639	0.620	0.716	0.850	0.472	0.534	0.535	0.546	0.448
	2 5 2	0.207	0.212	0.209	0.210	0.201	0.176	0.188	0.156	0.150	0.226	0.218	0.241	0.205	0.226
	1 7 1	0.017	0.018	0.018	0.018	0.011	0.003	0.004	-	-	0.031	0.015	0.023	0.015	0.059
	0 9 0	-	0.000	0.000	0.000	-	-	-	-	-	0.001	-	0.000	0.000	0.000
10	5 0 5	0.038	0.036	0.037	0.037	0.026	-	0.019	-	-	0.055	0.041	0.028	0.035	0.055
	4 2 4	0.390	0.385	0.388	0.387	0.388	0.407	0.385	0.395	0.400	0.384	0.376	0.352	0.390	0.385
	3 4 3	0.446	0.450	0.448	0.449	0.474	0.504	0.504	0.543	0.577	0.407	0.458	0.469	0.455	0.369
	2 6 2	0.118	0.120	0.119	0.120	0.107	0.087	0.091	0.062	0.022	0.141	0.121	0.141	0.114	0.156
	1 8 1	0.008	0.008	0.008	0.008	0.004	0.003	0.001	-	-	0.013	0.005	0.010	0.006	0.036
	0 10 0	0.000	-	0.000	0.000	-	-	-	-	-	0.000	-	0.000	0.000	0.000

REFERENCES

1) Cf. D. Cline, in Symposium on Nucleon-Antinucleon Interactions, Argonne (1968) ANL/HEP 6812

2) S. Devons et al., Phys. Rev. Letters 27 (1971) 1614

3) CERN-Collège de France results, presented by C. Ghesquière at the Aix-en-Provence Conference on Elementary Particles, Sept. 1970

4) B.P. Desai, Phys. Review 119 (1960) 1390

5) J. McConnel & J. Shapiro, Nuovo Cimento 28 (1963) 1272

6) D.Q. Lam-, Thesis, Liverpool (1969); Report to the Meeting on High Multiplicity Events, Ecole Polytechnique Paris (1970)

7) H. Muirhead & A. Poppleton, Phys. Letters 29B (1969) 448

8) Chan-Hong-Mo et al., Nuovo Cimento 57A (1968) 93

9) Fong-Ching Chen, Nuovo Cimento 62A (1969) 113

10) L. Bar-Nir et al., Nuclear Physics B20 (1970) 45

11) H.R. Rubinstein & H. Stern, Physics Letters 21 (1966) 447

12) J. Kirz, Physics Letters 22 (1966) 524; J. Harte et al., Nuovo Cimento 49 (1967) 555

13) This investigation was suggested by Prof. L. Van Hove in a private communication

14) H. Caprasse & J. Vandermeulen, Nuovo Cimento 6A (1971) 516

15) J.D. Hansen et al., Nuclear Physics B25 (1971) 605

16) F. Cerulus, Suppl. Nuovo Cimento 15 (1960) 402

17) K. Zalewski & J. Danysz, Nuclear Physics B2 (1967) 249

18) A. Pais, Annals of Physics 9 (1960) 548

19) R. Bizzarri, Nuovo Cimento 53A (1967) 956

FIGURE CAPTIONS

Fig. 1 Pion multiplicity distribution in $p\bar{p}$ annihilation at rest. Curve A : Poisson form with $\langle N \rangle = 5$. Curve B : statistical model with $\langle N \rangle = 5$. Curve C : statistical model with $\langle N_c \rangle = 3.06$.

Fig. 2 $\pi^+\pi^-$ cross-section (in microbarn) vs s. Both coordinates in logarithmic scale. Full curve : model of Ref. 14; Dashed curve : Lamb's result (Ref. 6).

Fig. 3 $\pi^+\pi^-\pi^o$ cross-section. Curve : Ref. 14.

Fig. 4 Averaged matrix element squared divided by s^4 (arbitrary units) vs s (logarithmic scales), for the 2π, 3π, 4π and 5π cross-sections.

Fig. 5 The same as in Fig. 8 for 6π and 7π. Also shown is the s^{-5} dependence.

Fig. 6 The same as in Fig. 9 for 8π, 9π, 10π and 11π.

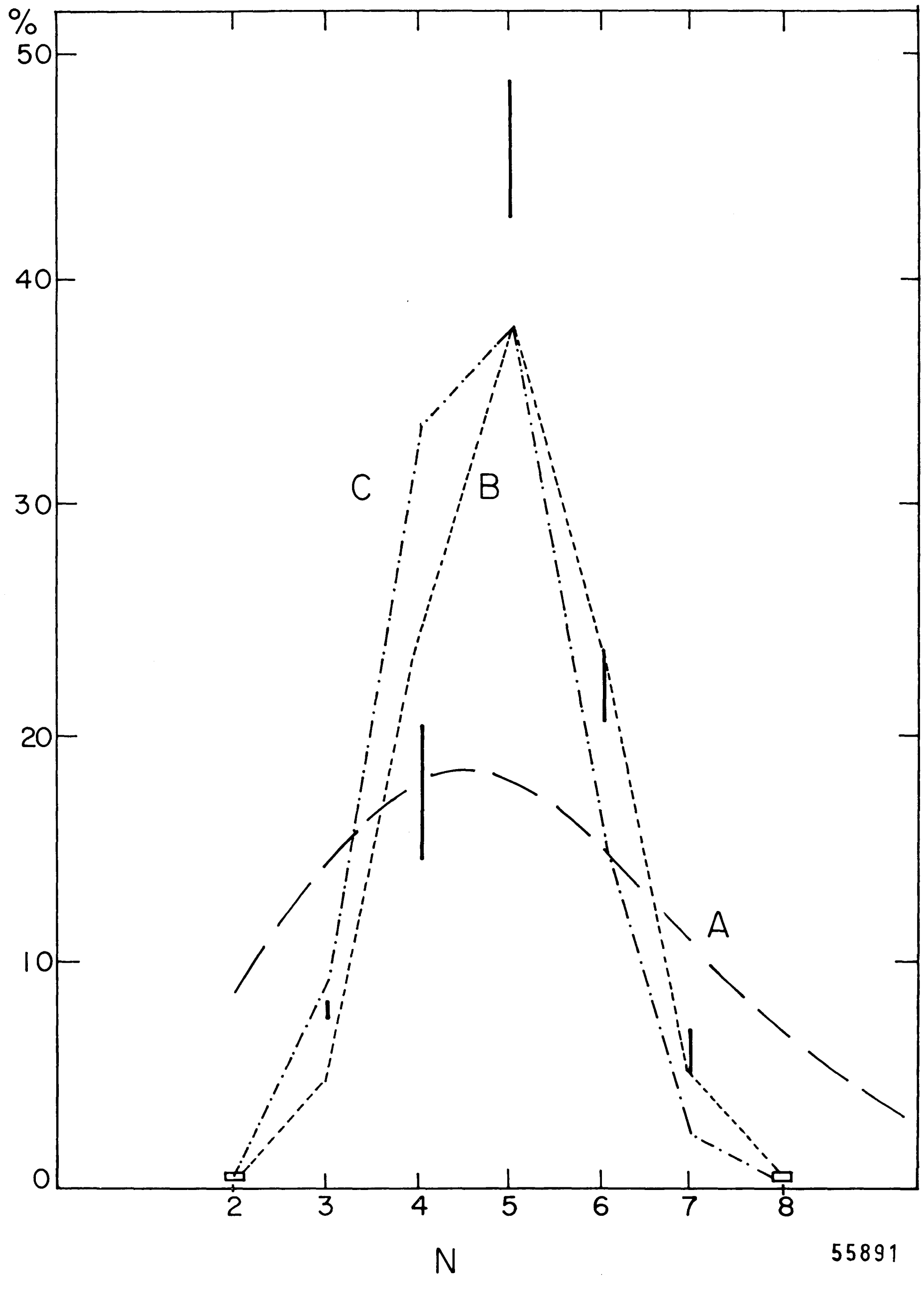


Fig. 1

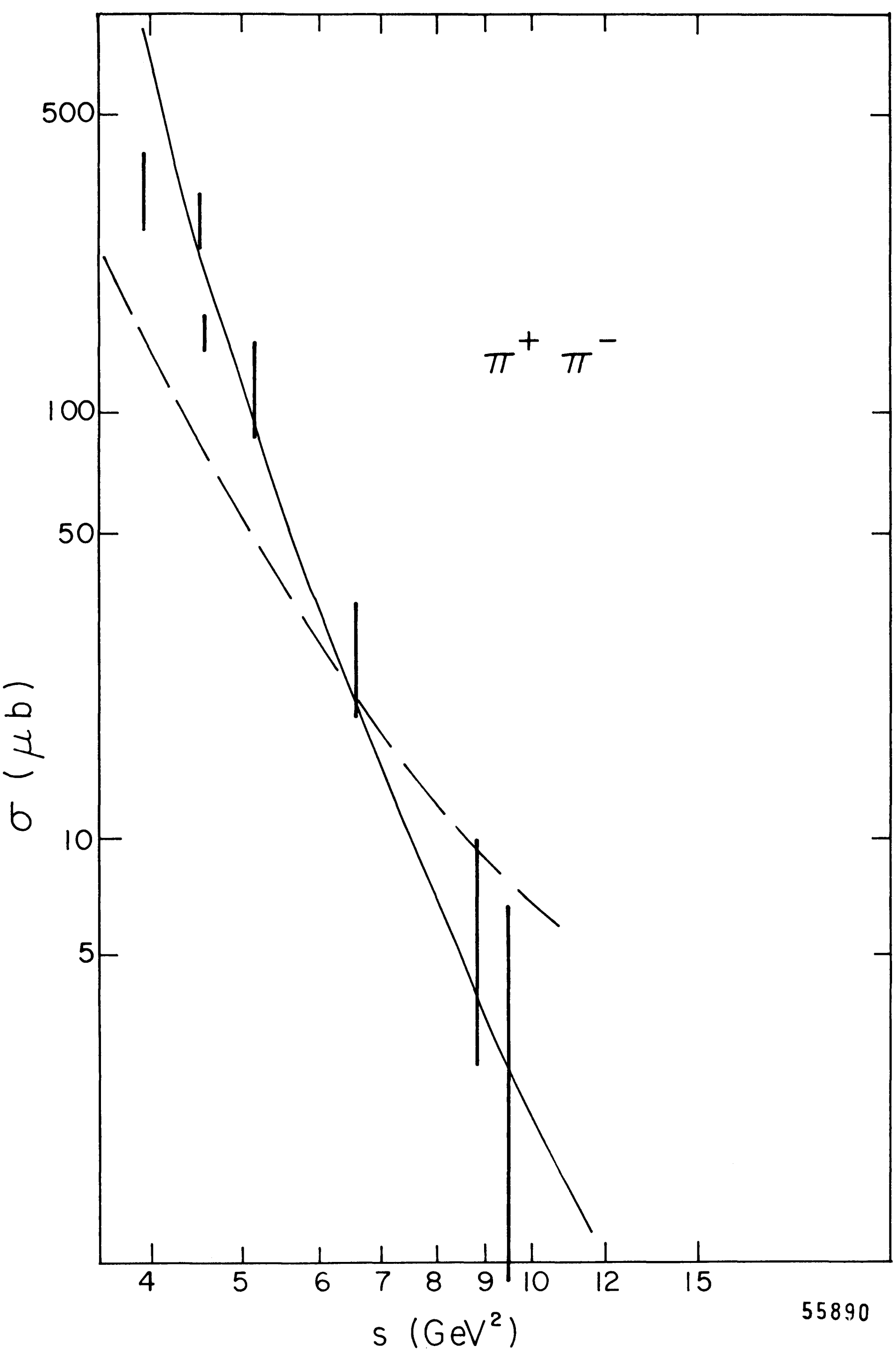


Fig. 2

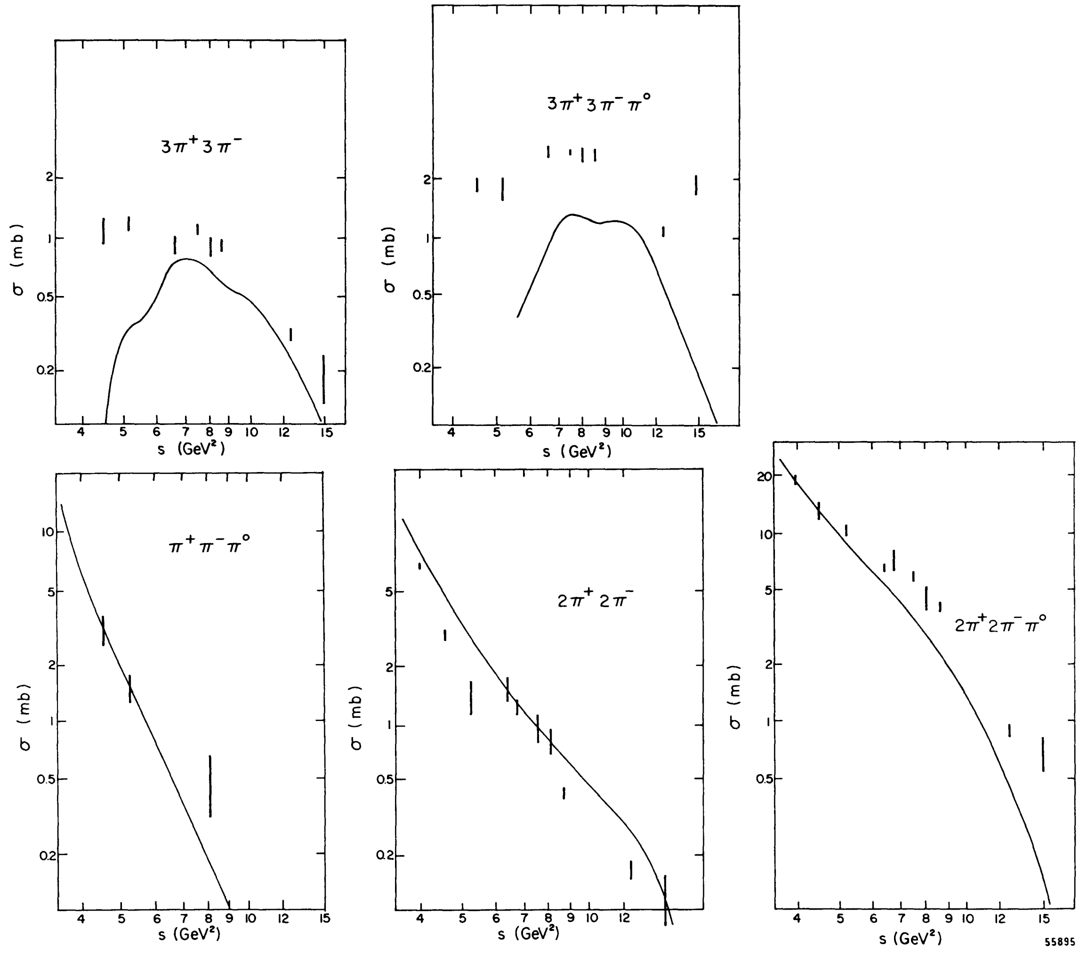
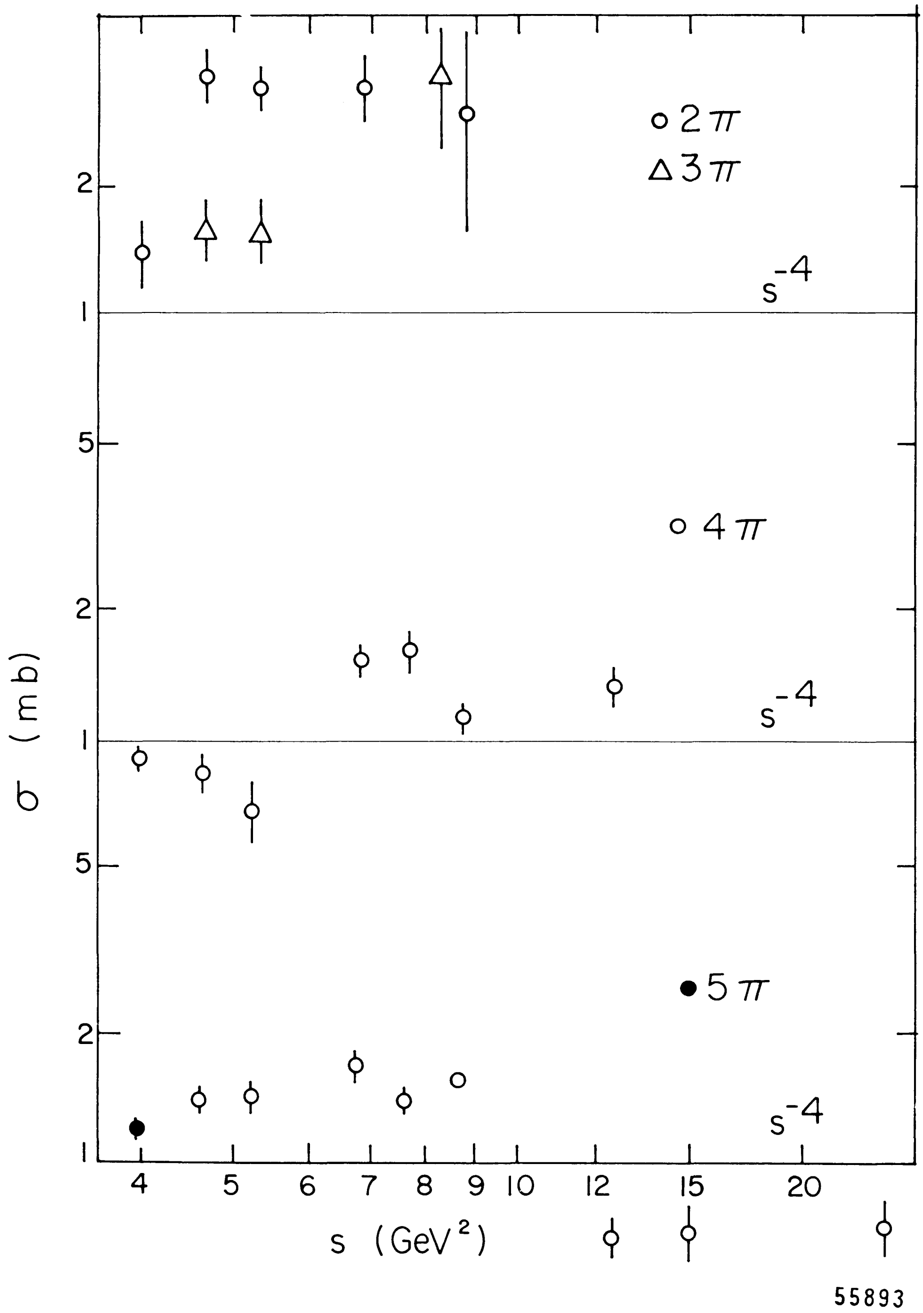


Fig. 3



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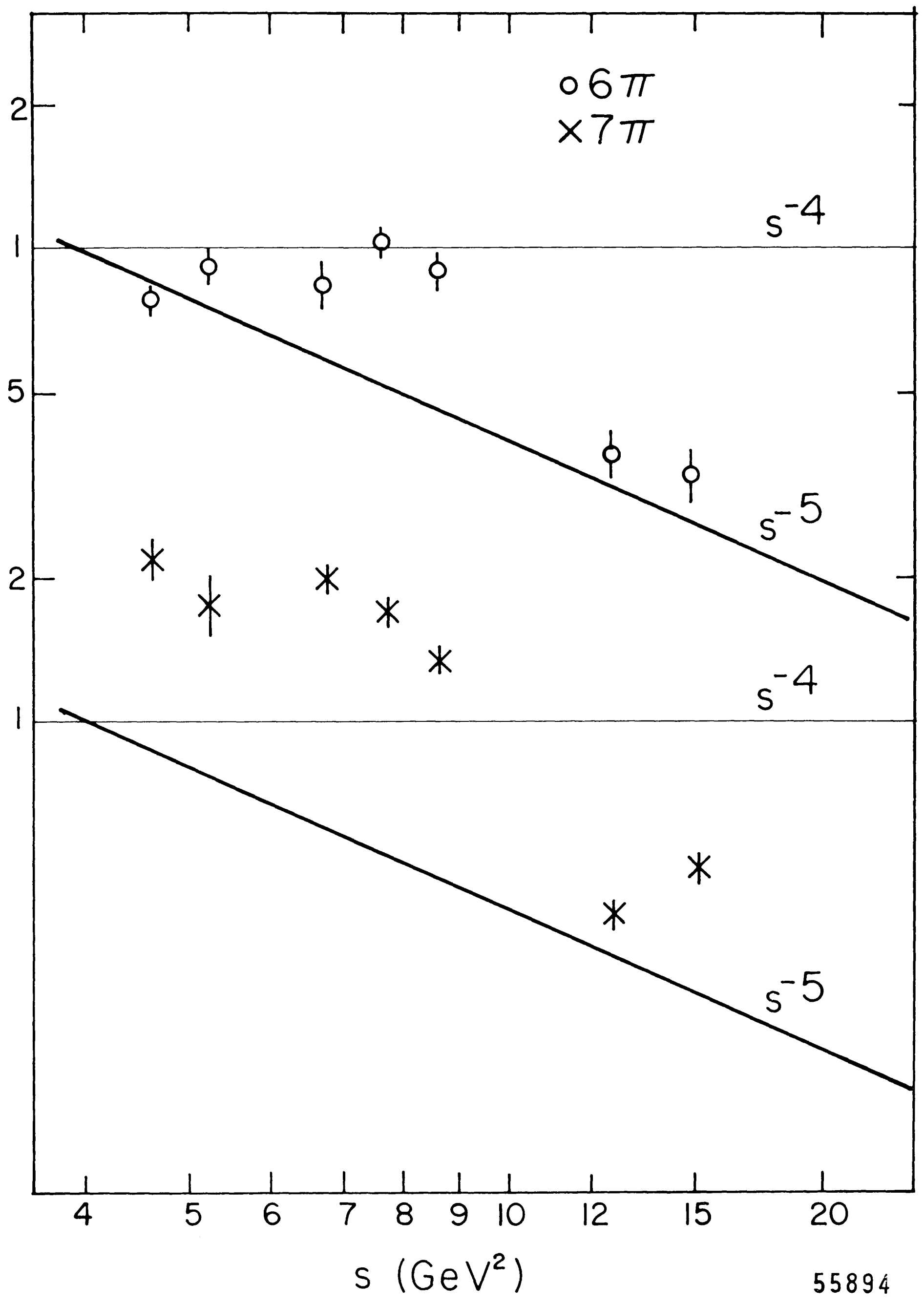

Fig. 5

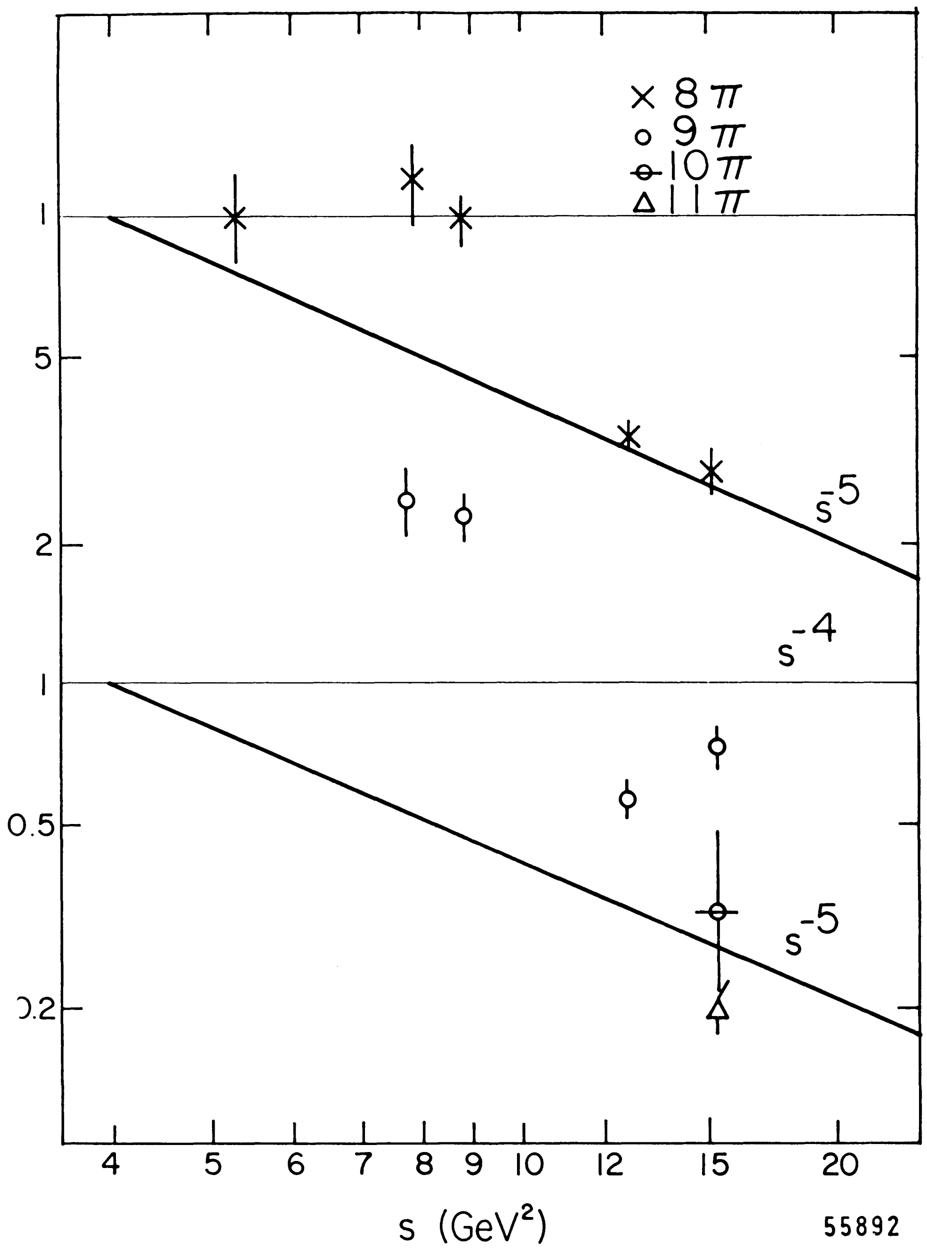

Fig. 6