

METHODS OF ANALYSIS - BROAD RESONANCES (CUTS AND RATES)

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In giving a survey of analysis methods for broad resonances I face a difficulty. Several aspects have already been dealt with - I refer you to the articles by Laloum and Lillestol in this report - and hence there is an overlap to be avoided. I shall make no reference to specific models used to analyse data. This does not mean that I regard the use of such models as of no value. Although my personal prejudice is that no model of strong interactions is at the same time sufficiently well grounded theoretically, and corresponds to reality in sufficient detail, to regard it as being a satisfactory description of experiment, it is necessary to compare experiment with models in order to provide a test of the theorist's ideas and I do not dispute this. Equally I regard it as important to provide accurate reliable data as a base for future theoretical ideas so I intend to use this brief survey to concentrate on this aspect of analysis, with all due acknowledgement to the necessity and value of the comparison between models and experiment.

This being so, I cannot avoid some discussion of the standard methods used to date. I refer you to Phys. Rev. Lett. 25, 783, 1970. This is a paper by George L. Trigg, an editor of Phys. Rev. Lett., in which he comments on the characteristics of some papers claiming new resonances which are submitted to the journal and in which he lays down minimum standards\* to be met by such papers. I detail these below

- 1) Is the data compatible with no peak?
- 2) Is it really the peak that causes the incompatibility?
- 3) A renormalisation of the phase space
- 4) Establishment that the peak is significant
- 5) Determination of the parameters by a fitting procedure.

I do not intend to dwell on such elementary considerations, but there are other problems to be touched on.

Surveying the literature one finds various ways of studying a suspected enhancement.

- 1) A smooth freehand background is drawn, a Breit-Wigner is superimposed, and a

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\* Criteria provided by Maglic

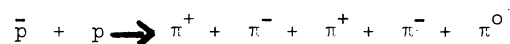
cross-section and significance is computed

2) A fit to a phase space background plus a Breit-Wigner is made by minimising  $\chi^2$  on the appropriate distribution

3) A fit is made to several distributions simultaneously by minimising the  $\chi^2$  computed by summing the  $\chi^2$  from the separate distributions

4) A maximum likelihood fit is made to the distribution of events in the appropriate multidimensional space

Methods 1) and 2) are based on the idea that the background under a peak is a smooth continuation of the shape on either side of it. This idea has a certain simple appeal but it is dangerous. Figure 1 is taken from Nuclear Physics B11, 551, 1969, and shows two things. Firstly the dashed curve (fig. 1a) is the background underneath the distribution  $\pi^+\pi^0$  combinations associated with a  $\rho$  meson (defined as a dipion falling in a given mass range) produced by the contribution of final states containing one resonance only. The bump in the background is a kinetic effect and the use of a smooth background would over estimate the cross-section for the final state  $\rho^0\rho^+\pi^-$ . Figure 1b shows a more remarkable effect. In the experimental distribution of  $\pi^+\pi^0$  combinations associated with a  $\rho^0$  there is a prominent enhancement which might be taken as evidence for the existence of a final state  $\rho^0\rho^0\pi^0$ . The full line is the prediction of the overall fit which does not contain any such contribution. Figure 2 shows an experimental distribution with a very prominent peak in the  $A_3$  region which again is totally misleading. The data is unpublished data from the final state



and the recipe to create the peak is as follows

- a) remove all events with a combination in the  $\rho^0$  region (suitably defined)
- b) plot all combinations  $\pi^+\pi^-\pi^0$  where the  $\pi^+\pi^-$  combination has a mass in the  $f^0$  region

The mechanism is quite clear; it is well known that  $\pi^+\pi^-$  from  $\omega^0$  mesons form a broad enhancement centred at about  $450 \text{ MeV}/c^2$ . If one replaces such a combination by a dipion external to the  $\omega^0$  which has a larger mass  $M$  restricted to some range  $M_1 \leq M \leq M_2$ , one, so to speak, moves the  $\omega^0$  meson up by approximately the difference between  $(M_1 + M_2)/2$  and  $450 \text{ MeV}/c^2$ , thus creating the observed peak.

Now, of course, this peak will not deceive the experimenter (it will not, for example,

produce an  $A_3^+$ ). However, this peak does lie buried in the background and reinforces my prejudice against any analysis based on the idea of backgrounds varying smoothly underneath a broad enhancement. In this context it is interesting to note that a few months ago a paper was published which contained elaborate methods of dealing with the background under peaks found in missing mass experiments. Hidden away in this paper is the crucial phrase "since the background has presumably a smooth behaviour we can safely extrapolate the expression derived for  $M > 2 \text{ GeV}/c^2$  into the resonance region."

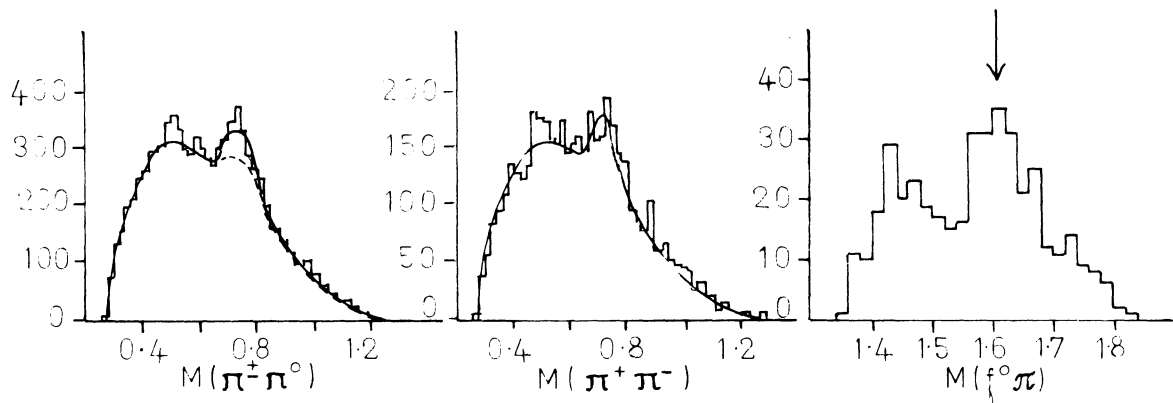


Fig. 1a

Fig. 1b

Fig. 2

Figure 2 is also perhaps an illustration of the dangers of enhancements which require cuts to make them visible.

Going back to the enumeration of some of the methods of analysis, the third method (evaluation of  $\chi^2$  on several distributions) suffers from obvious theoretical objections in that the distributions are not independent of each other. This leaves the Maximum Likelihood method of fitting, which is theoretically adequate, although subject to technical problems of computer usage and there may also be other problems.

I must now return to the main theme. The symposium programme specifies "Cuts and Rates" as the title. The literature is full of examples where groups have attempted to make a structure more obvious or more convincing. These cuts have broadly speaking two subdivisions. They are (without implying any value judgments)

a) Simple cuts, which exploit a knowledge of, or a hypothesis regarding, the decay mode of the resonance e.g. the study of  $\rho\pi$  combinations in studying the  $A_2$ . Also in this category are cuts which seek to eliminate some known major component of the background.

b) Intelligent cuts which seek to exploit a supposed understanding of the production mechanism. These include momentum transfer cuts and cuts based on exchange diagrams (see Nuclear Physics B35, 237, 1971 for some examples of these). These cuts often work but one finds that even in the same experiment they are successful in one final state, but not in another.

Also in this category comes selections based on well founded theoretical ideas. An example of this type is the use of the properties of Zemach spin coefficients to select resonances. This was very successful in the case of the CERN - College de France analysis of antiprotons at rest (Nuclear Physics B16, 239, 1970) and indeed led on to the impressive analysis explained by Lillestol in an earlier session. The draw back here is that this was a special case where the initial states are few and well known. When the same ideas are used in a case where we do not know the initial states of the  $\bar{p}p$  system (see Nuclear Physics B35, 237, 1971 again) they are not nearly so successful.

There does not seem to be any recipe for determining the best method of selecting data in order to isolate particular resonances. In this context the idea of "best" includes one knowledge of how the application of the cut distorts the distributions. However, recently there has been some interesting work which may help us. This is the work of Pless et al. (Phys. Rev. Lett. 27, 1481, 1971). If we have an unpolarised beam and target, an N-body final state requires  $3N-5$  independent parameters to describe that state completely. A 3 body final state thus requires 4 parameters, a 4 body final state, 7 parameters and so on. Thus if one analyses a 3 body state in terms of the Dalitz plot, not all the available information is being used. The suggestion of Pless et al. consists of a convenient choice of the parameters necessary to describe the state. For the 3 body state they use a 3 dimensional plot, produced by the kinetic energies of two particles and the Van Hove angle of the event. Fig. 3 taken from Pless et al. shows the definition of the Van Hove angle, and also that of the fourth parameter required  $R/R_{\max}$  ( $R_{\max}$  is the maximum value of the radius vector in Figure 3 at any given Van Hove angle  $\Theta$ ). In figure 3 the axes labelled  $p$ ,  $\pi^+$ ,  $\pi^0$  represent the longitudinal momenta of the 3 particles in the final state (here a proton and two pions). Different regions of  $\Theta$  correspond to different ordering of the magnitudes of these longitudinal momenta and it is this extra information which helps resolve overlapping resonances.

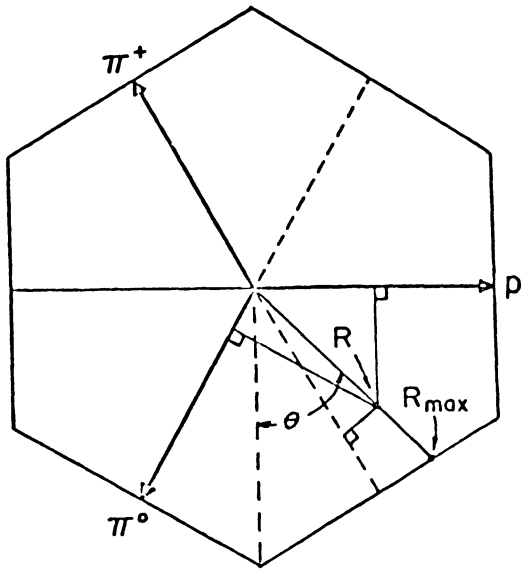


Fig. 3

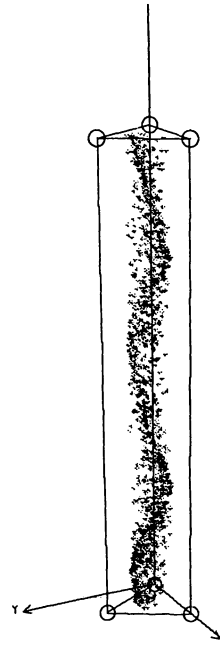


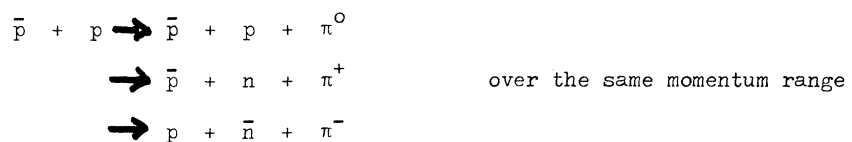
Fig. 4

The two kinetic energies and the Van Hove angle define a prism shown in fig. 4 where each event is represented by a point. The base of the prism is the well known Dalitz-Fabrizi plot. The data shown in fig. 4 are from



in the momentum range 1.5 - 2.0 GeV/c. There is a complicated structure as can be seen by viewing the prism from different points (fig. 5a and 5b). In the original work of Fless, they found very marked structures and selecting various regions of the prism plot gave remarkably total separation of the various resonances produced. The same was true of the corresponding 4 body final state.

Using the data of fig. 4 as an example, it is found that events with a  $\pi^+ \pi^-$  mass in the region of the  $\rho^0$  show strongly localised structures (fig. 6). Selection of events lying along the centre of the spiral like structure in this region yields fig. 7 with fig. 8 showing the unselected sample for comparison. Examining the events of fig. 7 as a function of  $R/R_{max}$  would then provide the most complete separation possible for these events. A more spectacular example is provided by the events of the type



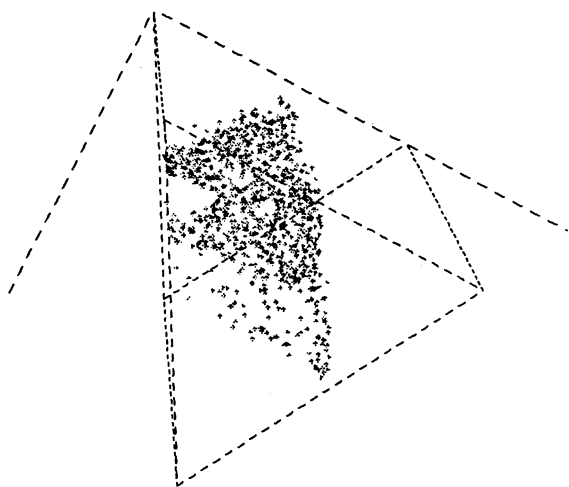


Fig. 5a

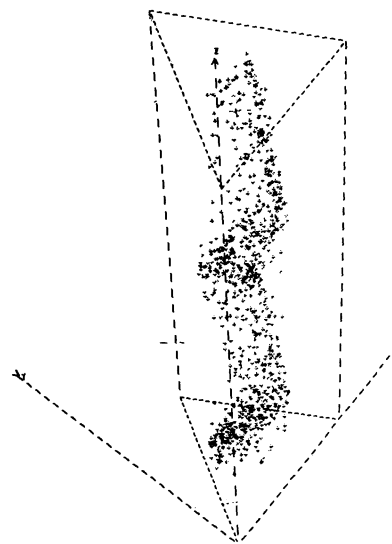


Fig. 5b

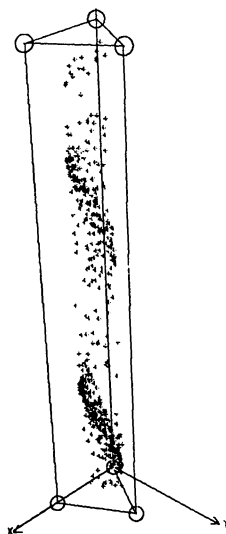


Fig. 6

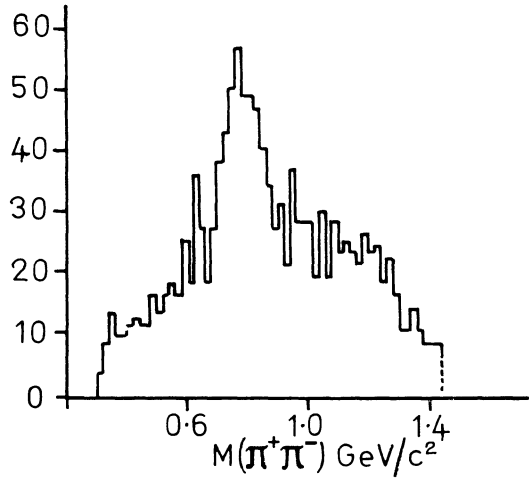


Fig. 7

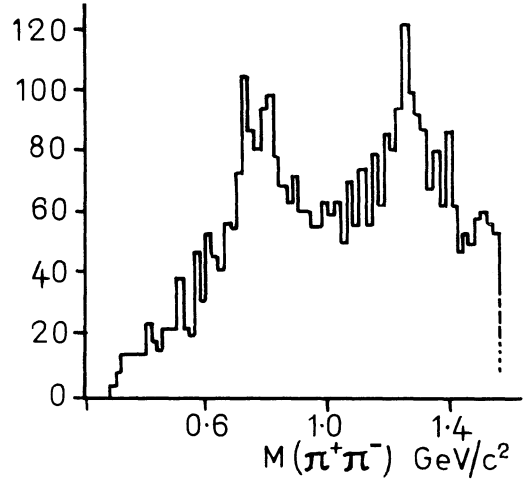


Fig. 8

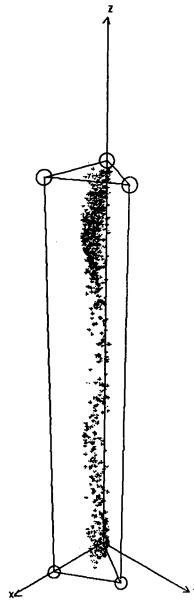


Fig. 9

These final states are dominated by the production of  $\Delta$  (1236). The prism plot is shown in fig. 9. Selection along the main structure of the spiral in the densely populated region at the top gives almost total separation of the  $\Delta$  (1236) events (fig. 10a, 10b).

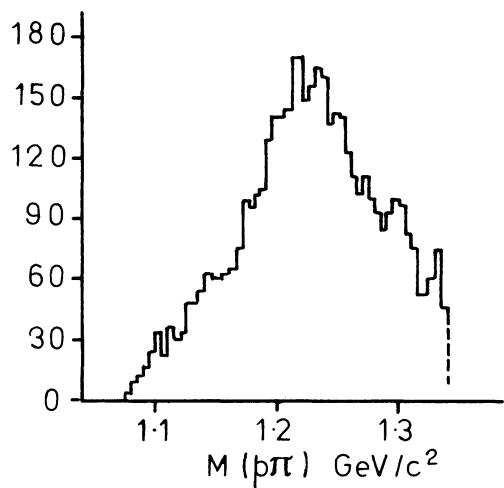


Fig. 10a

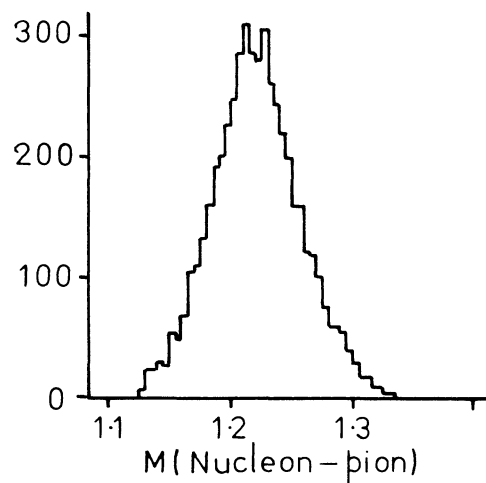


Fig. 10b

For a four body final state one uses the kinetic energy of 3 final state particles to form a tetrahedron which is a generalisation of the Dalitz-Fabrizi plots, two generalised Van Hove coordinates,  $R/R_{\max}$ , and one mass combination chosen to suit the experiment. Once again structures can be seen. Fig. 11a, 11b show examples of this, the data being from the 8.25 GeV/c  $K^- - p$  experiment of the Athens - Democritos - Liverpool - Vienna collaboration.

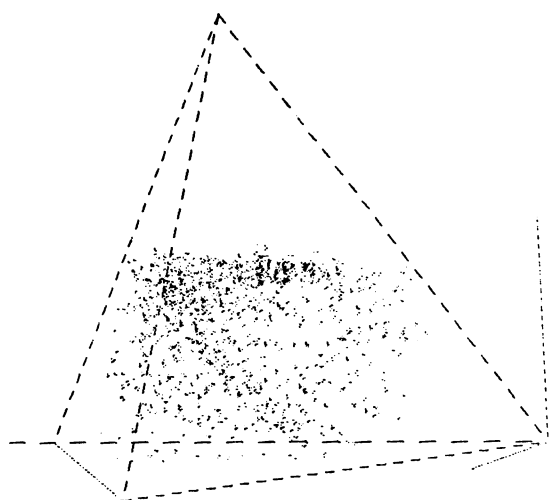


Fig. 11a

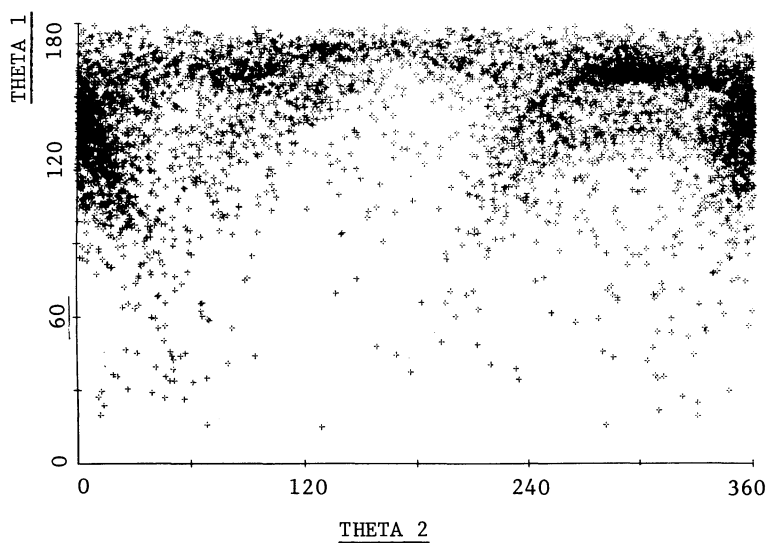


Fig. 11b



I suggest that, in any given experimental situation, it will be well worth while to examine the distributions in all of the  $3N - 5$  independent parameters. The structures found (if any) will suggest the most effective cuts possible. There is clearly scope for the ingenious physicist to devise alternative combinations of plotting 3 out of the  $(3N-5)$  parameters to suit particular experimental circumstances.

DISCUSSION AND COMMENTS

Mr. Bizzarri : In the study of  $\bar{p}p$  annihilations at rest, there is a clear trend to obtain an increasing percentage of resonance production as the statistical accuracy increases. This is presumably due to the fact that, as statistical accuracy increases, less events can be accounted for by phase space, but result from interference effects of the tail of the resonances.

In the in flight annihilations, such detailed studies cannot be made because of the lack of knowledge of the initial angular momentum state. In this situation I wonder which is the meaning of the rates for broad resonances production and if one should not rather try to invent new different parameters to characterize the annihilation properties ?