INELASTIC LEPTON SCATTERING

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INTRODUCTION

These lectures are devoted to a study of some aspects of inelastic lepton scattering and more precisely of inclusive reactions where only the final lepton is detected. Polarization effects are considered when the hadronic target has a spin $\frac{1}{2}$.

The inelastic scattering of leptons can be induced by charged lepton beams and it proceeds essentially via electromagnetic interactions or by neutrino and antineutrino beams and it then proceeds via weak interactions. For pedagogical purposes, we have separated both phenomena. The lessons treat only the electroproduction reactions and we have added to each lesson one appendix where the results for neutrino and antineutrino reactions are given in a parallel way.

The first lesson is devoted to kinematics with the definition of the hadronic tensor and of the structure functions. The differential cross sections and the asymmetries are computed in the one-photon exchange approximation for e.m. processes and in the local Fermi interactions for weak processes.

The second lesson begins with a study of the constraints due to positivity. Then we consider the limit in the forward laboratory direction at fixed incident energy. The third section gives some results about the current algebra sum rules with a derivation of the asymptotic sum rule for polarization in section 4. The last part gives an application of this last sum rule by considering the high energy limit at fixed q^2.

The third lesson is restricted more precisely to the deep inelastic scattering and the scaling for the structure functions is introduced with some of its consequences. In the second part of the lesson, we give the actual available experimental information about deep inelastic electron-proton and electron-deuterium scattering.

The parton model is considered in Lesson IV as a simple explanation and representation of the scaling. The quark parton model is then introduced as a particular case.

The comparison between theory and experiment is made in Lesson V in the framework of the quark parton model for the nucleon. So far, the agreement is satisfactory but more data, especially in neutrino and antineutrino scattering are needed to make it significant.

LESSON I

KINEMATICS

1. UNPOLARIZED CROSS SECTIONS

1°) We study the inelastic lepton scattering

$$\ell + p \Rightarrow \ell + \Gamma$$

where p is a target of energy momentum p, mass M, spin J and Γ an unspecified multiparticle final state of effective mass W

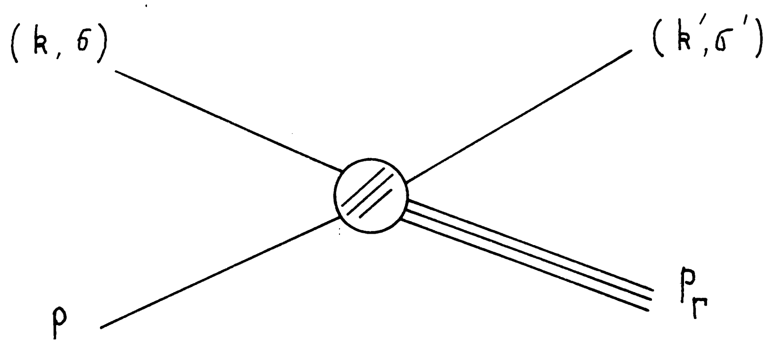

Fig. 1

The kinematics is indicated on Fig. 1 and, as usual, we introduce transfer of energy momentum between leptons q = k - k' and we define the scalar variables

$$s = -(p+k)^2 \qquad q^2 = (k-k')^2 \qquad W^2 = -p_\Gamma^2 = -(p+q)^2 \qquad M\nu = -p \cdot q$$

related by

$$W^2 = M^2 - q^2 + 2M\nu \tag{I.1}$$

It is useful to introduce also the laboratory variables

$$p \Rightarrow [\vec{o}, M] \qquad k \Rightarrow [\vec{k}, E] \qquad k' \Rightarrow [\vec{k}', E']$$

with the scattering angle θ given by $\vec{k}\cdot\vec{k}' = kk'\cos\theta$

The computation of the scalars s, q^2, W^2 and ν in terms of laboratory variables is straightforward

$$s = M^2 + m^2 + 2ME \qquad q^2 = 2(EE' - kk'\cos\theta - m^2)$$

$$W^2 = M^2 - q^2 + 2M(E-E') \qquad \nu = E - E'$$

where m is the lepton mass.

At a fixed incident energy E (or s) a measurement of the final lepton parameters E' and θ determines the invariants q^2 and W^2. When only the final lepton is detected the reaction will be called inclusive and the differential cross section has the form

$$\frac{d^2\sigma}{dq^2\,dW^2} = \frac{1}{[s-(M+m)^2][s-(M-m)^2]}\;\mathcal{T}$$

where the Lorentz invariant quantity $\mathcal{T}$ is given by

$$\mathcal{T} = \frac{1}{2\pi}\Big(\frac{N_p}{2J+1}\sum_{\substack{\mathrm{pol}\\ \mathrm{target}}}\Big)\Big(\frac{m^2}{2}\sum_{\sigma\sigma'}\Big)\, S_\Gamma\,(2\pi)^3\,\delta_4(p+q-p_\Gamma)\,\big|T(\ell+p\Rightarrow \ell+\Gamma)\big|^2$$

where N_p is a normalization factor for the target, $N_p = \frac{1}{2}$ when 2J is even and $N_p = M$ when 2J is odd . The operator S_Γ means a phase space integration and a summation over polarization for all the particles belonging to Γ

$$S_\Gamma \Rightarrow \sum_{\substack{\text{spins} \\ j \in \Gamma}} \int \prod_{j \in \Gamma} \frac{d_3 p_j}{E_j} \frac{N_j}{(2\pi)^3}$$

2°) Now comes the main assumption. To lowest order with respect to electromagnetic interactions only one photon is exchanged between the leptons and the hadrons as shown on Fig. 2

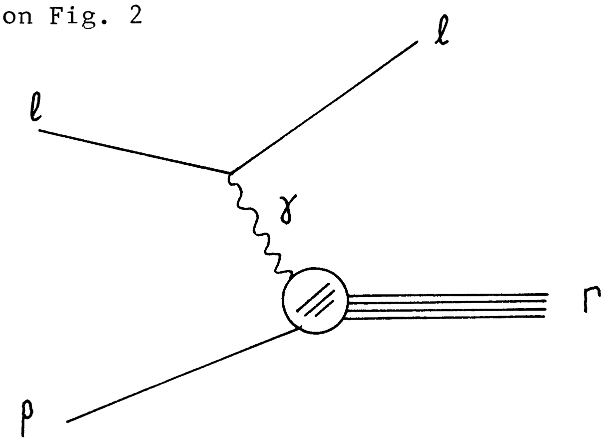

Fig. 2

It follows that the transition matrix element is factorized into the product of two matrix elements of the electromagnetic current, one for leptons known from quantum electrodynamics and one for hadrons we wish to study with the inelastic lepton scattering

$$T(l+p \Rightarrow l+\Gamma) = \frac{e^2}{q^2} \left[\bar{u}(k',\sigma')\, \gamma^\mu u(k,\sigma)\right] < \Gamma \mid J_\mu^{em}(0) \mid p > \tag{I.2}$$

where e is the electric charge normalized so that $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$,
u and $\bar{u}$ the lepton free Dirac spinors.

Therefore the invariant $\mathcal{T}$ is factorized as the product of a leptonic tensor $t^{\mu\nu}$ by a hadronic tensor $T_{\mu\nu}$

$$\mathcal{T} = \frac{8\pi\alpha^2}{q^4} t^{\mu\nu} T_{\mu\nu}$$

where

$$t^{\mu\nu} = \frac{m^2}{2} \sum_{\sigma} \sum_{\sigma'} \left[\bar{u}(k',\sigma') \gamma^{\mu} u(k,\sigma) \right] \left[\bar{u}(k',\sigma') \gamma^{\nu} u(k,\sigma) \right]^{*} \tag{I.3}$$

$$T_{\mu\nu} = \frac{N_p}{2J+1} \sum_{\substack{\text{pol.} \\ \text{target}}}' S_{\Gamma} (2\pi)^3 \delta_4 (p+q-p_{\Gamma}) \langle \Gamma | J^{em}_{\mu}(0) | p \rangle \langle \Gamma | J^{em}_{\nu}(0) | p \rangle^{*} \tag{I.4}$$

The computation of the leptonic tensor is straightforward

$$t^{\mu\nu} = \frac{1}{2} \left[k^{\mu} k'^{\nu} + k'^{\mu} k^{\nu} + \frac{1}{2} q^2 g^{\mu\nu} \right] \tag{I.5}$$

3°) The hadronic tensor $T_{\mu\nu}$ is the quantity we want to measure from experiment. Its structure is restricted by conservation laws. From Lorentz covariance it is a second rank tensor and it can be decomposed on the basis

$$p_{\mu} p_{\nu} \qquad g_{\mu\nu} \qquad p_{\mu} q_{\nu} \qquad q_{\mu} p_{\nu} \qquad q_{\mu} q_{\nu} \qquad \varepsilon_{\mu\nu\rho\sigma} p^{\rho} q^{\sigma}$$

The last covariant $\varepsilon_{\mu\nu\rho\sigma} p^{\rho} q^{\sigma}$ is excluded by parity conservation and we get relations due to the conservation of the electromagnetic current at the hadronic vertex

$$q^{\mu} T_{\mu\nu} = 0 = q^{\nu} T_{\mu\nu}$$

Finally we have only two independent covariants, the coefficients of which we call inelastic form factors or structure functions(*) depending

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(*) The structure functions V_1, V_2 are dimensionless. In the literature the symbol W is generally used for structure functions having the dimension of the inverse of a mass. The relations are

$$W_1 = \frac{V_2}{2M} \qquad\qquad W_2 = \frac{V_1}{M}$$

on two scalar variables we choose as q^2 and W^2

$$T_{\mu\nu} = \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) V_1(q^2, W^2) + \frac{1}{2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) V_2(q^2, W^2) \qquad (I.6)$$

The hadronic tensor $T_{\mu\nu}$ is hermitian by construction. Therefore the structure functions V_1 and V_2 are real functions of q^2 and W^2 in the physical region $q^2 \geq 0$ $W^2 \geq M^2$

4°) In order to compute the differential cross section we saturate the leptonic and the hadronic tensors. We then define two invariants I_1 and I_2 respectively associated to the structure functions V_1 and V_2

$$I_1 = \frac{p_\mu p_\nu}{M^2} t^{\mu\nu} = \frac{(p\cdot k)(p\cdot k')}{M^2} - \frac{1}{4} q^2$$
$$I_2 = \frac{1}{2} g_{\mu\nu} t^{\mu\nu} = \frac{1}{4}(q^2 - 2m^2) \qquad (I.7)$$

The differential cross section for inelastic lepton scattering takes the general form due to the one-photon exchange approximation

$$\frac{d^2\sigma}{dq^2 dW^2} = \frac{1}{[s-(M+m)^2][s-(M-m)^2]} \frac{8\pi\alpha^2}{q^4}\left\{ I_1 V_1(q^2, W^2) + I_2 V_2(q^2, W^2)\right\} \qquad (I.8)$$

In the laboratory frame the invariants I_1 and I_2 have the following expression

$$I_1 = \frac{1}{2}(EE' + kk'\cos\theta + m^2)$$
$$I_2 = \frac{1}{2}(EE' - kk'\cos\theta - 2m^2) \qquad (I.9)$$

Except in the exact forward direction, the lepton mass can be neglected in high energy scattering $\frac{m}{E} \ll 1$, $\frac{m}{E'} \ll 1$ and we get simplified expressions

$$I_1 \simeq EE'\cos^2\frac{\theta}{2} \qquad I_2 \simeq EE'\sin^2\frac{\theta}{2} \qquad (I.10)$$

with $q^2 = 4EE' \sin^2 \frac{\theta}{2}$

The differential cross section (I.8) takes the Rosenbluth form

$$\frac{d^2\sigma}{dq^2 dW^2} = \frac{2\pi\alpha^2}{M^2 q^4} \frac{E'}{E} \left\{ \cos^2\frac{\theta}{2} V_1(q^2,W^2) + \sin^2\frac{\theta}{2} V_2(q^2,W^2) \right\} \tag{I.11}$$

or equivalently

$$\frac{d^2\sigma}{dE' d\cos\theta} = \frac{\pi\alpha^2}{4ME^2} \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} \left\{ V_1(q^2,W^2) + \tan^2\frac{\theta}{2} V_2(q^2,W^2) \right\} \tag{I.12}$$

5°) In the particular case where Γ is a one-particle final state identical to the target, the scattering is called elastic. The variable W is now fixed to the value M and the structure functions have a trivial W dependence given by a Dirac delta distribution, the interesting part being contained in a function of q^2

$$V_j(q^2,W^2) \Rightarrow 2M^2 \delta(W^2-M^2) V_j^{el}(q^2) \qquad j=1,2$$

The fixed q^2 elastic cross section now takes the form

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E} \left\{ \cos^2\frac{\theta}{2} V_1^{el}(q^2) + \sin^2\frac{\theta}{2} V_2^{el}(q^2) \right\} \tag{I.13}$$

As a consequence of the equality $W = M$, from equation (I.1), we deduce the relation $q^2 = 2M\nu$ and in the laboratory frame the scattering angle is a function of the lepton energies E and E' given by

$$\sin^2\frac{\theta}{2} = \frac{M(E-E')}{2EE'}$$

The elastic structure functions $V_j^{el}(q^2)$ are computed using the hadronic tensor $T_{\mu\nu}$ of equation (I.4) which, in the elastic case reduces to

$$T_{\mu\nu} \Rightarrow 2M^2 \delta(W^2-M^2) T_{\mu\nu}^{el}$$

with

$$T^{el}_{\mu\nu} = \left(\frac{N_p}{M}\right)^2 \frac{1}{2J+1} \sum_{pol} \langle p' | J^{em}_{\mu}(0) | p \rangle \langle p' | J^{em}_{\nu}(0) | p \rangle^* \tag{I.14}$$

where p' is the energy momentum of the final hadron $p' = p + q$.

Let us now study the three cases $J = 0$, $J = \frac{1}{2}$ and $J = 1$.

In the spin zero case, the matrix element of the electromagnetic current is restricted by Lorentz covariance and the conservation of the electromagnetic current to the form

$$\langle p' | J^{em}_{\mu}(0) | p \rangle = (p+p')_{\mu} \; F(q^2) \tag{I.15}$$

From the hermiticity of the electromagnetic current, the elastic form factor $F(q^2)$ is real. At $q^2 = 0$, it is normalized to the electric charge Q of the spin zero particle (in units e).

$$F(0) = Q$$

The elastic structure functions are then computed from (I.14) and the result is

$$\begin{aligned} \mathcal{V}_1^{el}(q^2) &= F^2(q^2) \\ \mathcal{V}_2^{el}(q^2) &= 0 \end{aligned} \tag{I.16}$$

In the spin $\frac{1}{2}$ case, the matrix element of the electromagnetic current is restricted by Lorentz covariance, the conservation of the electromagnetic current and the invariance under space reflexion to the form

$$\langle p' | J^{em}_{\mu}(0) | p \rangle = i\, \bar{u}(p') \left\{ \gamma_{\mu} F_1(q^2) + \frac{i}{4M} [\gamma_{\mu}, \gamma_{\nu}] q^{\nu} F_2(q^2) \right\} u(p) \tag{I.17}$$

From the hermiticity of the electromagnetic current, the Dirac form factors $F_1(q^2)$ and $F_2(q^2)$ are real. At $q^2 = 0$, they are normalized to the electric charge Q (in units e) and to the anomalous moment κ (in units $e/2M$) of the spin $\frac{1}{2}$ particle

$$F_1(0) = Q \qquad\qquad F_2(0) = \kappa$$

A straightforward calculation of the structure functions gives

$$\begin{aligned} V_1^{el}(q^2) &= F_1^2(q^2) + \frac{q^2}{4M^2} F_2^2(q^2) \\ V_2^{el}(q^2) &= \frac{q^2}{2M^2}\left[F_1(q^2) + F_2(q^2)\right]^2 \end{aligned} \tag{I.18}$$

It is usual to introduce linear combinations of F_1 and F_2, the so-called Sachs form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2) \qquad , \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

normalized at $q^2 = 0$ to the electric charge Q and to the magnetic moment μ

$$G_E(0) = Q \qquad\qquad G_M(0) = \mu$$

In terms of G_E and G_M equations (I.18) become

$$\begin{aligned} V_1^{el}(q^2) &= \left(1 + \frac{q^2}{4M^2}\right)^{-1}\left[G_E^2(q^2) + \frac{q^2}{4M^2} G_M^2(q^2)\right] \\ V_2^{el}(q^2) &= \frac{q^2}{2M^2} G_M^2(q^2) \end{aligned} \tag{I.19}$$

In the general case, from the Lorentz covariance, the space and time reflexion invariances, the number of linearly independent electromagnetic form factors for a spin J particle is $2J + 1$ and these form factors are normalized at $q^2 = 0$ to the static moments of the spin J particle. The elastic structure functions $V_j^{el}(q^2)$ are sums of squares of the physical form factors. For instance, the spin 1 particle is described by three form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$ respectively normalized at $q^2 = 0$ to the electric charge (in units e) to the magnetic dipole moment (in units e/2M) and to the electric quadrupole moment(in units e/M^2). The elastic structure functions have the expression

$$
\begin{aligned}
-\mathcal{V}_1^{el}(q^2) &= F_0^2(q^2) + \frac{q^2}{6M^2} F_1^2(q^2) + \frac{q^4}{18M^4} F_2^2(q^2) \\
-\mathcal{V}_2^{el}(q^2) &= \left(1+\frac{q^2}{4M^2}\right)\frac{q^2}{3M^2} F_1^2(q^2)
\end{aligned}
\tag{I.20}
$$

2. POLARIZED CROSS SECTIONS

1°) We are now interested in the inelastic lepton scattering on a polarized target and we assume the incident lepton beam to be polarized. It is obviously equivalent —up to a statistical factor ½— to have an unpolarized lepton beam and to measure the polarization of the final lepton.

In the one-photon exchange approximation, the invariant is always the product of a leptonic tensor by a hadronic tensor. The leptonic tensor defined by

$$m^{\mu\nu} = m^2 \sum_{\sigma'} \left[\bar{u}(k',\sigma')\gamma^\mu u(k,\sigma)\right]\left[\bar{u}(k',\sigma')\gamma^\nu u(k,\sigma)\right]^* \tag{I.21}$$

is computed using the projection operator

$$u(k,\sigma)\bar{u}(k,\sigma) = \frac{m - i\gamma\cdot k}{2m} \cdot \frac{1 + i\gamma_5\,\gamma\cdot\sigma}{2}$$

where σ is a spacelike unit pseudo-vector orthogonal to k

$$\sigma^2 = 1 \qquad\qquad \sigma\cdot k = 0$$

The spin independent part of $m^{\mu\nu}$ is obviously $t^{\mu\nu}$ and the spin dependent part $s^{\mu\nu}$ is computed to be

$$s^{\mu\nu} = i\,\frac{m}{2}\,\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha \sigma_\beta \tag{I.22}$$

2°) The hadronic tensor is now defined by

$$M_{\mu\nu} = N_p\, S_r\, (2\pi)^3\, \delta_4(p+q-p_r) \langle r | J^{em}_\mu(0) | p,\lambda\rangle \langle r | J^{em}_\nu(0) | p,\lambda\rangle^* \tag{I.23}$$

where λ is the target polarization. The spin independent part of $M_{\mu\nu}$ is just $T_{\mu\nu}$ and we denote by $S_{\mu\nu}$ the spin dependent part $M_{\mu\nu} = T_{\mu\nu} + S_{\mu\nu}$

We shall discuss only the case of a spin ½ target and we introduce a polarization vector N which is a spacelike unit pseudo-vector orthogonal to p

$$N^2 = 1 \qquad\qquad N \cdot p = 0$$

The spin dependent part of the hadronic tensor must be linear in the vector N and the structure of $S_{\mu\nu}$ is obtained using, as previously, the Lorentz covariance, the invariance under space reflexion and the conservation of the electromagnetic current. We then define three independent covariants and therefore three structure functions(*) depending on the two scalar variables q^2 and W^2

$$S_{\mu\nu} = \frac{1}{2iM}\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha N^\beta\, X_1(q^2,W^2) + \frac{1}{2iM^2}\left[N_\mu\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) - \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)N_\nu\right] X_2(q^2,W^2) + $$
$$+ \frac{1}{2M^2}\left[N_\mu\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)N_\nu\right] Y(q^2,W^2) \tag{I.24}$$

(*) Again the structure functions X_1, X_2 and Y are dimensionless. The comparison with the structure functions defined by De Rafael and Doncel is

$$G_1 = \frac{\nu}{2M^2}(X_1 - X_2) \qquad\qquad G_2 = \frac{\nu}{2M^2} X_2$$

where the vector n is defined by

$$n_\mu = \frac{1}{M}\,\varepsilon_{\mu\alpha\beta\gamma}\,p^\alpha q^\beta N^\gamma$$

The hadronic tensor $M_{\mu\nu}$ is hermitian by construction. Therefore the structure functions X_1, X_2 and Y are real functions of q^2 and W^2 in the physical region $q^2 \geq 0$, $W^2 \geq M^2$. If the electromagnetic interactions are invariant under time reversal, we easily see that X_1 and X_2 are real and Y purely imaginary so that it vanishes. The structure function Y measures a violation of time-reversal invariance.

3°) Saturating now the leptonic tensor $m^{\mu\nu}$ with the hadronic tensor $M_{\mu\nu}$, we get the invariants I_1, I_2 and three new invariants we call as K_1, K_2 and L

$$K_1 = s^{\mu\nu}\frac{1}{2iM}\,\varepsilon_{\mu\nu\alpha\beta}\,q^\alpha N^\beta = -\frac{m}{2M}\left[q^2(\sigma\cdot N) - (\sigma\cdot q)(N\cdot q)\right] \tag{I.25}$$

$$K_2 = s^{\mu\nu}\frac{1}{iM^2}\,n_\mu p_\nu = \frac{m}{2M}\left[(\nu^2+q^2)(\sigma\cdot N) - (N\cdot q)\left(\sigma\cdot q - \frac{\nu}{M}\sigma\cdot p\right)\right] \tag{I.26}$$

$$L = t^{\mu\nu}\frac{1}{M^2}\,n_\mu p_\nu = \frac{1}{2M^3}\left[p\cdot(k+k')\right]\varepsilon_{\mu\nu\rho\sigma}\,k^\mu k'^\nu p^\rho N^\sigma \tag{I.27}$$

and the differential cross section is written as

$$\frac{d^2\sigma}{dq^2 dW^2} = \frac{d^2\sigma_{unp}}{dq^2 dW^2}\left\{1 + \frac{K_1 X_1 + K_2 X_2 + LY}{I_1 V_1 + I_2 V_2}\right\} \tag{I.28}$$

4°) The invariant L is independent of the lepton polarization and in the laboratory frame where the polarization pseudo-vector N has only space components, it takes the form :

$$L = -\frac{E+E'}{2M}\,\vec{N}\cdot\vec{k}\times\vec{k}' \tag{I.29}$$

When the lepton beam is unpolarized, we define an asymmetry Δ_N corresponding to a polarization vector $\vec{N}$ orthogonal to the scattering plane, e.g. parallel or antiparallel to $\vec{k} \times \vec{k}'$

$$\Delta_N = -\operatorname{tn}\frac{\theta}{2}\,\frac{\frac{E+E'}{M}\,Y}{V_1 + \operatorname{tn}^2\frac{\theta}{2}\,V_2} \qquad (I.30)$$

The asymmetry Δ_N is a measure of the violation of time reversal invariance in electromagnetic interactions.

5°) The invariants K_1 and K_2 contain a correlation between the lepton polarization and the target polarization.

It is convenient to introduce the polarization vector of the lepton in its rest frame $\vec{s}$ ($\vec{s}^{\,2} = 1$). The relation between σ and $\vec{s}$ is just a pure Lorentz transformation along the lepton momentum $\vec{k}$

$$m\sigma \Rightarrow \left[m\,\vec{s}_T + \vec{k}\,\frac{E}{k^2}(\vec{s}\cdot\vec{k})\ ,\ \vec{s}\cdot\vec{k} \right]$$

where $\vec{s}_T$ is orthogonal to $\vec{k}$.

In the zero lepton mass limit, only a longitudinal polarization for the lepton survives and we have

$$\lim_{m \Rightarrow 0} m\sigma_\mu = \frac{\vec{s}\cdot\vec{k}}{k}\,k_\mu \qquad (I.31)$$

In the laboratory frame, neglecting the lepton mass, we write the invariants K_1 and K_2 from equations (I.25) and (I.26) as

$$K_1 = -\frac{(\vec{s}\cdot\vec{k})}{k}\,\frac{q^2}{4M}\,\vec{N}\cdot(\vec{k}+\vec{k}') \qquad (I.32)$$

$$K_2 = \frac{(\vec{s}\cdot\vec{k})}{k}\,\frac{1}{2M}\left[(\vec{q}\cdot\vec{k})(\vec{N}\cdot\vec{k}') - (\vec{q}\cdot\vec{k}')(\vec{N}\cdot\vec{k})\right] \qquad (I.33)$$

The polarization vector $\vec{N}$ is now in the scattering plane and we have two degrees of freedom. We denote by $\Delta_{\parallel}$ the asymmetry when $\vec{N}$ is colinear to $\vec{k}$ and by $\Delta_{\perp}$ the asymmetry when $\vec{N}$ is orthogonal to $\vec{k}$. From (I.32) and (I.33) we obtain

$$\Delta_{\parallel} = \frac{-\sin^2\frac{\theta}{2}\,\frac{E+E'\cos\theta}{M}\,X_1 + \sin^2\theta\,\frac{E'}{2M}\,X_2}{\cos^2\frac{\theta}{2}\,\mathcal{V}_1 + \sin^2\frac{\theta}{2}\,\mathcal{V}_2} \tag{I.34}$$

$$\Delta_{\perp} = \frac{1}{2}\sin\theta\;\frac{\frac{E-E'\cos\theta}{M}\,X_2 - \sin^2\frac{\theta}{2}\,\frac{2E'}{M}\,X_1}{\cos^2\frac{\theta}{2}\,\mathcal{V}_1 + \sin^2\frac{\theta}{2}\,\mathcal{V}_2} \tag{I.35}$$

The polarization structure functions X_1 and X_2 are obtained from the asymmetries $\Delta_{\parallel}$ and $\Delta_{\perp}$ by linear combination. For instance, the asymmetry corresponding to a target polarization parallel or antiparallel to $\vec{q}$, Δ_q, gives directly the structure function X_1

$$\Delta_q = -\frac{\sin^2\frac{\theta}{2}\,\frac{E+E'}{M}\,\frac{\nu}{\sqrt{\nu^2+q^2}}\,X_1}{\cos^2\frac{\theta}{2}\,\mathcal{V}_1 + \sin^2\frac{\theta}{2}\,\mathcal{V}_2} \tag{I.36}$$

6°) We finally study the polarization effects in <u>elastic</u> lepton scattering on a spin ½ target. The two elastic form factors F_1 and F_2 have been introduced in equation (I.17) without any reference to the time-reversal operation. As a consequence of the one-photon exchange approximation the asymmetry Δ_N vanishes even if time-reversal invariance is violated.

We then have only two non-vanishing elastic structure functions

$$X_j(q^2,W^2) \Rightarrow 2M^2\,\delta(W^2-M^2)\,X_j^{el}(q^2) \qquad j=1,2$$

and a straightforward calculation gives their expressions in terms of the elastic form factors F_1 and F_2

$$X_1^{el}(q^2) = \left[F_1(q^2) + F_2(q^2)\right]^2 \tag{I.37}$$
$$X_2^{el}(q^2) = F_2(q^2)\left[F_1(q^2)+F_2(q^2)\right]$$

or in terms of the Sachs form factors G_E and G_M

$$X_1^{e\ell}(q^2) = G_M^2(q^2)$$
$$X_2^{e\ell}(q^2) = \left(1+\frac{q^2}{4M^2}\right)^{-1} G_M(q^2)\left[G_M(q^2) - G_E(q^2)\right] \tag{I.38}$$

The elastic form factors are known from the unpolarized differential cross section. As a consequence of the one-photon exchange approximation, the polarization effects can be completely predicted from the knowledge of the angular distribution. We then have a very interesting test of the one-photon exchange approximation.

For instance, in the backward direction, using helicity arguments we have the following prediction for the parallel antiparallel asymmetry $\Delta_{\parallel}^{e\ell}$

$$\lim_{\theta \Rightarrow \pi} \Delta_{\parallel}^{e\ell} = -1 \tag{I.39}$$

This result can be checked using equations (I.18), (I.34) and (I.37).

REFERENCES

G. Charpak and M. Gourdin, Lectures at the Cargèse Summer School 1962

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)

N. Dombey, Rev. Mod. Phys. 41, 236 (1969)

M.G. Doncel and E. de Rafael, Nuovo Cimento 4A, 363 (1971)

M. Gourdin, Nuovo Cimento 21, 1094 (1961)
Diffusion des Electrons de Haute Energie, Masson (1966)
Preprint Paris-LPTHE 1 (1972) to be published in Nucl. Phys. B

M. Gourdin and A. Martin, CERN TH.261 (1962)

T.D. Lee and C.N. Yang, Phys. Rev. Letters 4, 307 (1960)
Phys. Rev. 126, 2239 (1962)

APPENDIX I

The neutrino and antineutrino induced reactions

$$\nu_\ell + p \Rightarrow \ell^- + \Gamma \qquad \bar{\nu}_\ell + p \Rightarrow \ell^+ + \Gamma$$

are studied in the Fermi local theory where the transition matrix element is factorized into the product of two matrix elements of the weak current $J^{weak}(0)$

$$T^{(\pm)} = \frac{G}{\sqrt{2}} \left[\bar{u}_\ell(k',\sigma') \gamma^\mu (1 \pm \gamma_5) u(k) \right] \langle \Gamma | J^{(\pm)}_\mu(0) | p, \lambda \rangle$$

where G is the conventional Fermi coupling constant. The $\pm$ sign corresponds to the electric charge of the exchanged current so that $T^{(+)}$ describes neutrino reactions and $T^{(-)}$ antineutrino ones.

The double differential cross section can be written as the product of the leptonic tensor $m^{\mu\nu}$ by the hadronic tensor $M_{\mu\nu}$

$$\frac{d^2\sigma}{dq^2 dW^2} = \frac{1}{(s-M^2)^2} \frac{G^2}{4\pi} m^{(\pm)\mu\nu} M^{(\pm)}_{\mu\nu}$$

where

$$m^{(\pm)\mu\nu} = m^2 \sum_{\sigma'} \left[\bar{u}_\ell(k',\sigma') \gamma^\mu (1 \pm \gamma_5) u(k) \right] \left[\bar{u}_\ell(k',\sigma') \gamma^\nu (1 \pm \gamma_5) u(k) \right]^*$$

$$M^{(\pm)}_{\mu\nu} = N_p\, S_\Gamma\, (2\pi)^3 \delta_4(p+q-p_\Gamma) \langle \Gamma | J^{(\pm)}_\mu(0) | p,\lambda \rangle \langle \Gamma | J^{(\pm)}_\nu(0) | p,\lambda \rangle^*$$

The computation of the leptonic tensor is straightforward

$$m^{(\pm)\mu\nu} = 2 \left[k^\mu k'^\nu + k'^\mu k^\nu - k \cdot k' g^{\mu\nu} \pm \frac{1}{i} \varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right]$$

The hadronic tensor is written, using Lorentz covariance, for a spin ½ target and it depends on 20 structure functions

$$
\begin{aligned}
M_{\mu\nu} = {} & \frac{p_\mu p_\nu}{M^2}\left[V_1(q^2,W^2) + \frac{N\cdot q}{M} X_4(q^2,W^2)\right] + \frac{1}{2} g_{\mu\nu}\left[V_2(q^2,W^2) + \frac{N\cdot q}{M} X_5(q^2,W^2)\right] \\
& + \frac{1}{2iM^2}\,\varepsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta\, V_3(q^2,W^2) + \frac{q_\mu q_\nu}{M^2}\left[V_4(q^2,W^2) + \frac{N\cdot q}{M} X_6(q^2,W^2)\right] \\
& + \frac{p_\mu q_\nu + q_\mu p_\nu}{2M^2}\left[V_5(q^2,W^2) + \frac{N\cdot q}{M} X_7(q^2,W^2)\right] + i\,\frac{p_\mu q_\nu - q_\mu p_\nu}{2M^2}\left[V_6(q^2,W^2) + \frac{N\cdot q}{M} Y_3(q^2,W^2)\right] \\
& + \frac{1}{2iM}\,\varepsilon_{\mu\nu\alpha\beta}\, p^\alpha N^\beta\, X_1(q^2,W^2) + \frac{1}{2iM^2}\left(n_\mu p_\nu - p_\mu n_\nu\right) X_2(q^2,W^2) \\
& + \frac{1}{2iM^2}\left(n_\mu q_\nu - q_\mu n_\nu\right) X_3(q^2,W^2) + \frac{1}{2M^2}\left(n_\mu p_\nu + p_\mu n_\nu\right) Y_1(q^2,W^2) \\
& + \frac{1}{2M^2}\left(n_\mu q_\nu + q_\mu n_\nu\right) Y_2(q^2,W^2) + \frac{N_\mu p_\nu + p_\mu N_\nu}{2M} X_8(q^2,W^2) + \frac{N_\mu q_\nu + q_\mu N_\nu}{2M} X_9(q^2,W^2) \\
& + i\,\frac{N_\mu p_\nu - p_\mu N_\nu}{2M} Y_4(q^2,W^2) + i\,\frac{N_\mu q_\nu - q_\mu N_\nu}{2M} Y_5(q^2,W^2)
\end{aligned}
$$

The hadronic tensor is hermitian and the 20 structure functions are real functions of q^2 and W^2 in the physical range $q^2 \geq 0$, $W^2 \geq M^2$

It is interesting to classify these structure functions according to the properties of the associated covariants under space reflexion and time reflexion. The results are given in Table 1

	Parity conserving	Parity violating
Time-reversal invariant	V_1 V_2 V_4 V_5 X_1 X_2 X_3	V_3 X_4 X_5 X_6 X_7 X_8 X_9
Time-reversal violating	V_6 Y_1 Y_2	Y_3 Y_4 Y_5

Table 1

In order to compute the differential cross section, we saturate the leptonic and the hadronic tensors. We then define 20 invariants $I_j^{(\pm)}$ $K_m^{(\pm)}$ and $L_n^{(\pm)}$ respectively associated to the structure functions V_j $X_m^{(\pm)}$ and $Y_n^{(\pm)}$. In the limit where the lepton mass is neglected, we remain with only 10 invariants which, in the laboratory frame, have the following expression

$$I_1^{(\pm)} = 4EE'\cos^2\frac{\theta}{2} \qquad I_2^{(\pm)} = 4EE'\sin^2\frac{\theta}{2} \qquad I_3^{(\pm)} = \pm 4EE'\frac{E+E'}{M}\sin^2\frac{\theta}{2}$$

$$K_1^{(\pm)} = \pm\frac{4EE'}{M}\sin^2\frac{\theta}{2}\,\vec{N}\cdot(\vec{k}+\vec{k}') \qquad K_2^{(\pm)} = \pm\frac{2}{M}\left[(\vec{N}\cdot\vec{k})(\vec{q}\cdot\vec{k}') - (\vec{N}\cdot\vec{k}')(\vec{q}\cdot\vec{k})\right]$$

$$K_4^{(\pm)} = \frac{\vec{N}\cdot\vec{q}}{M}I_1^{(\pm)} \qquad K_5^{(\pm)} = \frac{\vec{N}\cdot\vec{q}}{M}I_2^{(\pm)} \qquad K_8^{(\pm)} = -2\left[(\vec{N}\cdot\vec{k})E' + (\vec{N}\cdot\vec{k}')E\right]$$

$$L_1^{(\pm)} = -\frac{2(E+E')}{M}\vec{N}\cdot(\vec{k}\times\vec{k}') \qquad L_4^{(\pm)} = \mp 2\,\vec{N}\cdot(\vec{k}\times\vec{k}')$$

The differential cross section for neutrino and antineutrino induced reactions on a polarized spin ½ target takes the form

$$\frac{d^2\sigma^{(\pm)}}{dq^2\,dW^2} = \frac{d^2\sigma^{(\pm)}_{unp}}{dq^2\,dW^2}\left[1 + \vec{N}\cdot\vec{\Delta}^{(\pm)}\right]$$

where the unpolarized cross section has the well-known expression

$$\frac{d^2\sigma^{(\pm)}}{dq^2\,dW^2} = \frac{G^2}{4\pi}\frac{1}{M^2}\frac{E'}{E}\left\{\cos^2\frac{\theta}{2}V_1^{(\pm)}(q^2,W^2) + \sin^2\frac{\theta}{2}\left[V_2^{(\pm)}(q^2,W^2) \pm \frac{E+E'}{M}V_3^{(\pm)}(q^2,W^2)\right]\right\}$$

The three asymmetries Δ_N, $\Delta_{\parallel}$ and $\Delta_{\perp}$ being defined as previously, we get

$$\Delta_N^{(\pm)} = -\tan\frac{\theta}{2}\,\frac{\frac{E+E'}{M}Y_1^{(\pm)} \pm Y_4^{(\pm)}}{V_1^{(\pm)} + \tan^2\frac{\theta}{2}\left[V_2^{(\pm)} \pm \frac{E+E'}{M}V_3^{(\pm)}\right]}$$

$$\Delta_{\parallel}^{(\pm)} = \frac{\cos^2\frac{\theta}{2}\left[\frac{E-E'\cos\theta}{M}X_4^{(\pm)} - X_8^{(\pm)}\right] + \sin^2\frac{\theta}{2}\left[\frac{E-E'\cos\theta}{M}X_5^{(\pm)} \pm \frac{E+E'\cos\theta}{M}X_1^{(\pm)}\right] \mp \frac{E'}{2M}\sin^2\theta\, X_2^{(\pm)}}{\cos^2\frac{\theta}{2}V_1^{(\pm)} + \sin^2\frac{\theta}{2}\left[V_2^{(\pm)} \pm \frac{E+E'}{M}V_3^{(\pm)}\right]}$$

$$\Delta_{\perp}^{(\pm)} = \frac{1}{2}\sin\theta \; \frac{\mp \frac{E - E'\cos\theta}{M} X_2^{(\pm)} - X_8^{(\pm)} - \frac{2E'}{M}\left[\cos^2\frac{\theta}{2} X_4^{(\pm)} + \sin^2\frac{\theta}{2}\left(X_5^{(\pm)} \mp X_1^{(\pm)}\right)\right]}{\cos^2\frac{\theta}{2} V_1^{(\pm)} + \sin^2\frac{\theta}{2}\left[-V_2^{(\pm)} \pm \frac{E+E'}{M} V_3^{(\pm)}\right]}$$

LESSON II

POSITIVITY AND CURRENT ALGEBRA SUM RULE

1. POSITIVITY CONDITIONS AND HELICITY CROSS SECTIONS

As noticed before, the tensor $M_{\mu\nu}$ is hermitian and semi-positive definite. We then get non trivial restrictions on the structure functions in the form of inequalities.

1°) For any complex four vector v_μ we must have

$$v^\mu v^{\nu *} M_{\mu\nu} \geqslant 0 \tag{II.1}$$

and this condition insures the positivity of the total cross sections for the absorption of the electromagnetic current of polarization α by a spin ½ target of polarization λ . The relation between the hadronic tensor and these cross sections is simply

$$\sigma_{\alpha\lambda}(q^2, W^2) = \frac{\pi}{M} \frac{1}{\sqrt{\nu^2+q^2}} \, e^\mu_\alpha(q) \, e^{\nu *}_\alpha(q) \, M_{\mu\nu}(q, p, \lambda) \tag{II.2}$$

where $e^\mu_\alpha(q)$ is a polarization vector for the electromagnetic current of energy momentum q and helicity α .

2°) Because of the conservation of the electromagnetic current

$$q^\mu e_{\mu\alpha}(q) = 0$$

and we can have only 3 different polarizations, 2 transverse $\alpha = +1, -1$ and 1 longitudinal $\alpha = L$. In the laboratory frame where

$$p \Rightarrow [0,0,0,M] \qquad N \Rightarrow [N_1, N_2, N_3, 0] \qquad q \Rightarrow [0,0,\sqrt{\nu^2+q^2}, \nu]$$

the polarization vectors have the following components

$$e_1 \Rightarrow \frac{1}{\sqrt{2}}[1,i,0,0] \qquad e_{-1} \Rightarrow \frac{1}{\sqrt{2}}[1,-i,0,0] \qquad e_L \Rightarrow \frac{1}{\sqrt{q^2}}[0,0,\nu,\sqrt{\nu^2+q^2}]$$

The target polarization is also measured in the $\vec{q}$ direction and we have

$\lambda = \pm\frac{1}{2}$ for $N_3 = \pm 1$

Because of the invariance under space reflexion, we have the relation

$$\sigma_{\alpha\lambda}(q^2, W^2) = \sigma_{-\alpha\,-\lambda}(q^2, W^2)$$

and there exist only three independent cross sections

$$\sigma_{+1\,+\frac{1}{2}} = \sigma_{-1\,-\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\,\frac{1}{2}\left[-V_2 - \frac{\nu}{M}X_1\right] \tag{II.3}$$

$$\sigma_{+1\,-\frac{1}{2}} = \sigma_{-1\,+\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\,\frac{1}{2}\left[-V_2 + \frac{\nu}{M}X_1\right] \tag{II.4}$$

$$\sigma_{L\,+\frac{1}{2}} = \sigma_{L\,-\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\left[\frac{\nu^2+q^2}{q^2}V_1 - \frac{1}{2}V_2\right] \tag{II.5}$$

From these expressions, we deduce positivity conditions

$$2\,\frac{\nu^2+q^2}{q^2}V_1 \geqslant -V_2 \geqslant \frac{\nu}{M}|X_1| \tag{II.6}$$

By inverting equations (II.3), (II.4) and (II.5) we can now express the structure functions V_1, V_2 and X_1 in terms of total cross sections. Defining

$$\sigma_{\alpha \frac{1}{2}} + \sigma_{\alpha -\frac{1}{2}} = 2\sigma_{\alpha} \qquad\qquad \sigma_{\alpha \frac{1}{2}} - \sigma_{\alpha -\frac{1}{2}} = 2\tau_{\alpha}$$

the conservation of parity implies

$$\sigma_{-1} = \sigma_{+1} = \sigma_{T} \qquad\qquad \tau_{-1} = -\tau_{+1} = \tau_{T}$$

and we get

$$\bar{V}_1 = \frac{M q^2}{\pi \sqrt{\nu^2+q^2}} \left[\sigma_T + \sigma_L \right] \qquad \bar{V}_2 = \frac{2M\sqrt{\nu^2+q^2}}{\pi} \sigma_T \tag{II.7}$$

$$X_1 = \frac{2M^2\sqrt{\nu^2+q^2}}{\pi \nu} \tau_T \tag{II.8}$$

3°) In the polarization space, the hadronic tensor is represented by a 6×6 matrix. The inequalities previously derived

$$\sigma_{\alpha\lambda}(q^2, W^2) \geq 0$$

express the positivity of the diagonal elements but we have, in addition, other restrictions due to the semi-positive definiteness of the hermitian matrix.

It can be observed that this 6×6 matrix is reducible according to the total helicity into

(a) 2 1×1 matrices for total helicity $\pm$ 3/2

(b) 2 2×2 matrices for total helicity $\pm$ 1/2

The positivity condition associated to part (a) is trivial

$$\sigma_{\pm 1 \pm \frac{1}{2}}(q^2, W^2) \geq 0$$

For part (b), we have to express the semi-positive character of a 2×2 hermitian matrix, e.g. to write that its trace and its determinant are non negative. These two constraints are

$$\sigma_L + \sigma_{\pm 1\ \mp\frac{1}{2}} \geq 0$$

already included in (II.6) and a new relation

$$\frac{q^2}{M^2}\left\{\left[X_1 - \frac{\nu^2+q^2}{q^2}X_2\right]^2 + \left(\frac{\nu^2+q^2}{q^2}Y\right)^2\right\} \leq \left[V_2 + \frac{\nu}{M}X_1\right]\left[\frac{\nu^2+q^2}{q^2}V_1 - \frac{1}{2}V_2\right] \qquad \text{(II.9)}$$

2. FORWARD DIRECTION

1°) For $q^2 = 0$, the virtual photon becomes real and it can have only transverse polarizations. The cross sections $\sigma_{\sigma\lambda}(q^2, W^2)$ become proportional to the photoabsorption cross sections by a polarized target and we have

$$\sigma_{1\lambda}(0,W^2) = \frac{1}{e^2}\,\sigma_\lambda^{\gamma_R}(W) \qquad \sigma_{-1\lambda}(0,W^2) = \frac{1}{e^2}\,\sigma_\lambda^{\gamma_L}(W)$$

$$\sigma_L(0,W^2) = 0 \qquad \text{(II.10)}$$

Because of the conservation of parity

$$\sigma_\lambda^{\gamma_R}(W) = \sigma_{-\lambda}^{\gamma_L}(W)$$

and we have only two independent photoabsorption cross sections that we can choose for instance as $\sigma_{1/2}^{\gamma_L}$ and $\sigma_{-1/2}^{\gamma_L}$. The unpolarized photoabsorption cross section will be called $\sigma^\gamma(W)$

$$2\sigma^\gamma(W) = \sigma_{1/2}^{\gamma_L} + \sigma_{-1/2}^{\gamma_L} = \sigma_{1/2}^{\gamma_R} + \sigma_{-1/2}^{\gamma_R}$$

2°) The limit at $q^2 = 0$ of the structure functions V_1, V_2 and X_1 is then deduced from equations (II.7) and (II.8)

$$\lim_{q^2 \Rightarrow 0} \frac{1}{q^2} V_1(q^2, W^2) = \frac{1}{\pi e^2} \frac{M}{\nu} \sigma^{\gamma}(W) \tag{II.11}$$

$$\lim_{q^2 \Rightarrow 0} V_2(q^2, W^2) = \frac{1}{\pi e^2} 2M\nu\, \sigma^{\gamma}(W) \tag{II.12}$$

$$\lim_{q^2 \Rightarrow 0} X_1(q^2, W^2) = \frac{1}{\pi e^2} M^2 \left[\sigma^{\gamma_L}_{1/2} - \sigma^{\gamma_L}_{-1/2} \right] \tag{II.13}$$

The other two structure functions X_2 and Y describing a longitudinal-transverse correlation, have a behaviour at $q^2 = 0$ which is restricted by the positivity inequality (II.9). As a consequence of the vanishing of the longitudinal cross section we get

$$\lim_{q^2 \Rightarrow 0} \frac{X_2}{\sqrt{q^2}} = 0 \qquad\qquad \lim_{q^2 \Rightarrow 0} \frac{Y}{\sqrt{q^2}} = 0 \tag{II.14}$$

3°) Let us now consider the kinematics of the forward direction in the laboratory system at fixed lepton energies. For $\theta = 0$ we have

$$q^2 = 2(EE' - kk' - m^2) \simeq m^2 \frac{(E-E')^2}{EE'}$$

or in terms of the scalar invariants s and W^2

$$q^2_{\theta=0} \simeq m^2 \frac{(W^2 - M^2)^2}{(s-M^2)(s-W^2)}$$

so that $q^2_{\theta=0}$ is generally of order m^2 and it vanishes only in the case of elastic scattering where $W^2 = M^2$. Nevertheless, in the forward direction the structure functions take their values at $q^2 = 0$ and from equations (II.11)-(II.14) we obtain the following limits for the unpolarized cross section and the asymmetries

$$\lim_{q^2 \Rightarrow 0} q^2 \frac{d^2\sigma}{dq^2 dW^2} = \frac{\alpha}{2\pi} \frac{1}{W^2-M^2} \left(1+ \frac{E'^2}{E^2}\right) \sigma^{\gamma}(W) \tag{II.15}$$

$$\lim_{q^2 \Rightarrow 0} \Delta_{\parallel} = - \frac{E^2-E'^2}{E^2+E'^2} \frac{\sigma^{\gamma_L}_{1/2} - \sigma^{\gamma_L}_{-1/2}}{\sigma^{\gamma_L}_{1/2} + \sigma^{\gamma_L}_{-1/2}} \tag{II.16}$$

$$\lim_{q^2 \Rightarrow 0} \Delta_N = 0 \qquad \lim_{q^2 \Rightarrow 0} \Delta_{\perp} = 0 \tag{II.17}$$

3. CURRENT ALGEBRA SUM RULE

1°) After summation over the final states Γ we can write the hadronic tensor as the Fourier transform of the matrix element of the commutator of two components of the electromagnetic current

$$M_{\mu\nu}(q,p,\lambda) = \frac{N_p}{2\pi} \int e^{-iq.x} \langle p,\lambda | [J^{em}_{\nu}(x) , J^{em}_{\mu}(0)] | p,\lambda \rangle \, d_4x \tag{II.18}$$

Beside some technical assumptions a sum rule for structure functions is related to the value of the equal time commutator of two particular components of the current.

2°) A nice current algebra sum rule has been derived by Adler for the difference of the structure function V_1 involved in neutrino and anti-neutrino induced reactions. We want to extract the part of this sum rule interesting for electroproduction.

We first use the equal time commutator of two time components of the isotopic spin vector current

$$[J^{I^+}_0(x) , J^{I^-}_0(0)] \delta(t) = 2 J^{I^3}_0(0) \delta_4(x) \tag{II.19}$$

Denoting by $V_1^{(+)}$ and $V_1^{(-)}$ the structure function V_1 associated to the charged component of the isotopic spin current, we obtain, at fixed q^2, the Adler sum rule

$$\int_{\frac{q^2}{2M}}^{\infty} \left[V_1^{(-)}(q^2,\nu) - V_1^{(+)}(q^2,\nu) \right] d\left(\frac{\nu}{M}\right) = 2\, I^3 \tag{II.20}$$

We introduce two 'charged' photons γ^+ and γ^- associated to the isotopic spin operators I^+ and I^- as the normal photon γ is associated to the electric charge operator Q. Using the equality (II.7) we express V_1 in terms of total cross sections. Restricting now our point to a proton target, we separate the elastic contribution from (I.19) and we write the Adler sum rule (II.20) as

$$F_{1V}^2(q^2) + \frac{q^2}{4M^2} F_{2V}^2(q^2) + \frac{q^2}{\pi^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\sqrt{\nu^2+q^2}} \left[\sigma_{T+L}^{\gamma^- p} - \sigma_{T+L}^{\gamma^+ p} \right] = 1 \tag{II.21}$$

where the F^V's are the isovector nucleon form factors and ν_0 the first inelastic threshold.

We now use an isotopic spin rotation in order to reintroduce the isovector part γ^V of the physical photon. In the process

$$\gamma^V + \text{proton} \Rightarrow \text{hadrons}$$

the total isotopic spin I can be 1/2 or 3/2. Using total cross sections of definite total isotopic spin σ^I we get

$$\sigma^{\gamma^- p} - \sigma^{\gamma^+ p} = \frac{4}{3}\left[\sigma^{\frac{1}{2}} - \sigma^{\frac{3}{2}}\right]$$

$$\sigma^{\gamma^V p} = \frac{1}{3}\left[\sigma^{\frac{1}{2}} + 2\sigma^{\frac{3}{2}}\right]$$

and the sum rule (II.21) is written as

$$F_{1V}^2(q^2) + \frac{q^2}{4M^2} F_{2V}^2(q^2) + \frac{4}{3}\,\frac{q^2}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\sqrt{\nu^2+q^2}} \left[\sigma_{T+L}^{\frac{1}{2}} - \sigma_{T+L}^{\frac{3}{2}} \right] = 1 \tag{II.22}$$

The sum rule (II.22) cannot be directly compared with experiment. We must use phenomenological descriptions of $\sigma^{\frac{1}{2}}$ and $\sigma^{\frac{3}{2}}$ consistent with our knowledge of electroproduction.

3°) At $q^2 = 0$ the sum rule (II.22) reduces to the identity $1 = 1$. Its derivative at $q^2 = 0$ is the Cabibbo-Radicati sum rule

$$\frac{(\kappa_p - \kappa_n)^2}{4M^2} + \frac{4}{3}\frac{1}{\pi e^2}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\left[\sigma^{\frac{1}{2}} - \sigma^{\frac{3}{2}}\right] = \frac{1}{3}\langle r_{1p}^2 - r_{1n}^2\rangle \tag{II.23}$$

where the root mean square radius $\langle r_1^2\rangle$ is defined by

$$\langle r_1^2\rangle = -6\left.\frac{dF_1(q^2)}{dq^2}\right|_{q^2=0}$$

Models for photoproduction including the most important multipole have been used to test the Cabibbo-Radicati sum rule and the agreement with experiment is good.

4. SUM RULE FOR POLARIZATION

1°) Let us consider the Compton scattering amplitude on a polarized spin ½ target

$$A_{\mu\nu} = i\int d_4x\, e^{-iq.x}\,\theta(t)\langle p,\lambda|\left[J_\nu^{em}(x), J_\mu^{em}(0)\right]|p,\lambda\rangle \tag{II.24}$$

where θ (t) is the usual step function.

Schwinger terms, irrelevant for our considerations have been disregarded.

From equation (II.18) the absorptive part of $A_{\mu\nu}$ is proportional to the hadronic tensor $M_{\mu\nu}$ for inclusive lepton scattering

$$\mathrm{ABS}\, A_{\mu\nu} = \frac{\pi}{M} M_{\mu\nu} \tag{II.25}$$

The basis used in Lesson I to expand $M_{\mu\nu}$ in covariants can also be used for $A_{\mu\nu}$. In particular for the part of $A_{\mu\nu}$ skew symmetric in the $\mu\nu$ indices $[\mu\nu]$ we can write

$$A_{[\mu\nu]} = \frac{1}{2iM} \varepsilon_{\mu\nu\alpha\beta} q^\alpha N^\beta A_1(q^2,W^2) + \frac{1}{2iM^2} \left[\eta_\mu \left(p_\nu - \frac{p.q}{q^2} q_\nu\right) - \left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\eta_\nu\right] A_2(q^2,W^2)$$

and from (II.25) we have

$$\mathrm{Im}\, A_{1,2} = \frac{\pi}{M} X_{1,2}$$

2°) We now work in the unphysical frame $q \Rightarrow [\vec{0}, q_0]$ where

$$q^2 = -q_0^2 \qquad\qquad \nu = -\frac{p_0 q_0}{M}$$

The space part of $A_{[\mu\nu]}$ is written in this frame

$$A_{[ij]} = \frac{q_0}{2iM} \left\{ \varepsilon_{ijk} N^k A_1 + \frac{1}{M^2} \left[\varepsilon_{irs} p_r N_s p_j - \varepsilon_{jrs} p_r N_s p_i \right] A_2 \right\} \tag{II.26}$$

We assume that, in the limit $q_0 \Rightarrow i\infty$, $\vec{p}$ fixed, the amplitude $A_{\mu\nu}$ can be expanded in powers of $1/q_0$

$$A_{\mu\nu} \Rightarrow -\frac{1}{q_0} \int d_4x \langle p,\lambda | [J_\nu^{em}(x), J_\mu^{em}(0)] | p,\lambda \rangle \delta(t) -$$

$$-\frac{i}{q_0^2} \int d_4x \langle p,\lambda | [\tfrac{\partial}{\partial t} J_\nu^{em}(x), J_\mu^{em}(0)] | p\lambda \rangle \delta(t) + \cdots \tag{II.27}$$

Equation (II.27) is known as the Bjorken-Johnson-Low expansion and it is obtained after successive integrations by parts over q_o.

We are interested in the quantity $A_{[ij]}$ and in order to compute the equal time commutator of two space components of the electromagnetic current, we use the quark model algebra

$$\left[J_j^{em}(x), J_i^{em}(0) \right] \delta(t) = i \, \varepsilon_{ijk} \, d^{QQ}_{\gamma} \, J_k^{A\gamma}(0) \, \delta_4(x) \qquad \text{(II.28)}$$

where $J_k^{A\gamma}$ (0) is the space component k of the axial vector current of U (3) index γ. The symmetric coefficient d^{QQ}_{γ} is computed in the fundamental representation of SU(3) for quarks

$$d^{QQ}_{\gamma} = \frac{2}{3} \left[2 \delta_{\gamma B} + \delta_{\gamma Q} \right] \qquad \text{(II.29)}$$

On the other hand, the axial vector coupling constant g_A^{γ} is defined by

$$\langle p, \lambda | \, J_\mu^{A\gamma}(0) \, | p, \lambda \rangle = - g_A^{\gamma} \, N_\mu \qquad \text{(II.30)}$$

Combining these results, we get

$$\lim_{q_0 \Rightarrow i\infty} q_0 \, A_{[ij]} = i \, \varepsilon_{ijk} \, N^k \, \frac{2}{3} \, g_A^{2B+Q} \qquad \text{(II.31)}$$

By comparison with the expansion (II.26) we deduce

$$\lim_{q_0 \Rightarrow i\infty} \frac{q_0^2}{2M} A_1 = - \frac{2}{3} g_A^{2B+Q} \qquad \text{(II.32)}$$

$$\lim_{q_0 \Rightarrow i\infty} \frac{q_0^2}{2M} A_2 = 0 \qquad \text{(II.33)}$$

3°) In order to transform the result (II.32) into a sum rule, we assume the validity for the amplitude A_1 of a dispersion relation in ν at fixed q^2 without subtraction

$$A_1\,(q^2,\nu) = \frac{2}{M}\int_{\frac{q^2}{2M}}^{\infty} \frac{\nu'\,X_1\,(q^2,\nu')}{\nu'^2-\nu^2-i\varepsilon}\,d\nu' \tag{II.34}$$

We then consider the quantity $\frac{q_0^2}{2M}A_1$ in the $q_0 \Rightarrow i\infty$ limit where

$$q^2 = -q_0^2 \Rightarrow +\infty \qquad \frac{\nu}{\nu'} < \frac{2p_0}{q_0} \Rightarrow 0$$

$$\lim_{q_0\Rightarrow i\infty} \frac{q_0^2}{2M}A_1 = -\lim_{q^2\Rightarrow+\infty}\int_{\frac{q^2}{2M}}^{\infty}\frac{d\nu}{\nu}\,\frac{q^2}{M^2}\,X_1(q^2,\nu) \tag{II.35}$$

Using the variable $W^2 = M^2 - q^2 + 2M\nu$ and the current algebra result (II.32), we finally obtain the Bjorken asymptotic sum rule

$$\lim_{q^2\Rightarrow+\infty}\int_{M^2}^{\infty} X_1(q^2,W^2)\,d\left(\frac{W^2}{M^2}\right) = \frac{2}{3}\,g_A^{2B+Q} \tag{II.36}$$

4°) Similar considerations on the structure function $X_2(q^2, W^2)$ will give the result

$$\lim_{q^2\Rightarrow\infty}\int_{M^2}^{\infty} X_2\,(q^2,W^2)\,d\left(\frac{W^2}{M^2}\right) = 0 \tag{II.37}$$

5. <u>HIGH ENERGY</u>

1°) We study the high-energy limit at fixed q^2 and W^2 of the differential cross section and of the asymmetries. For the laboratory variables E, E' and θ we have

$$E \simeq E' \simeq \frac{s}{2M} \qquad \text{with } E - E' = \nu \qquad \theta \simeq \frac{2M\sqrt{q^2}}{s}$$

By inspection of equations (I.30), (I.34) and (I.35) we get the following limits

$$\lim_{s\to\infty} \frac{d^2\sigma_{unp}}{dq^2 dW^2} = \frac{2\pi\alpha^2}{M^2 q^4} V_1(q^2,W^2) \tag{II.38}$$

$$\lim_{s\to\infty} \Delta_N = -\sqrt{\frac{q^2}{M^2}} \frac{Y(q^2,W^2)}{V_1(q^2,W^2)}$$

$$\lim_{s\to\infty} \Delta_{\parallel} = -\frac{q^2}{s} \frac{X_1(q^2,W^2)-X_2(q^2,W^2)}{V_1(q^2,W^2)} \tag{II.39}$$

$$\lim_{s\to\infty} \Delta_{\perp} = \frac{\nu\sqrt{q^2}}{s} \frac{X_2(q^2,W^2)}{V_1(q^2,W^2)}$$

2°) The fixed q^2 cross section is defined integrating over W^2

$$\frac{d\sigma}{dq^2} = \int_{M^2}^{W_M^2} dW^2 \frac{d^2\sigma}{dq^2 dW^2}$$

where the upper limit W_M^2 is given by

$$W_M^2 = s\left(1 - \frac{q^2}{s-M^2}\right)$$

Assuming now that the order between limit and integration can be inverted

$$\lim_{s\to\infty} \frac{d\sigma}{dq^2} = \int_{M^2}^{\infty} dW^2 \lim_{s\to\infty} \frac{d^2\sigma}{dq^2 dW^2}$$

we get from (II.38) and (II.39) the result

$$\lim_{s\to\infty} \left(\frac{d\sigma_P}{dq^2} - \frac{d\sigma_A}{dq^2}\right) = -\frac{4\pi\alpha^2}{q^2 s} \int_{M^2}^{\infty} \left[X_1(q^2,W^2) - X_2(q^2,W^2)\right] d\left(\frac{W^2}{M^2}\right) \tag{II.40}$$

where P(A) means a target polarization parallel (antiparallel) to the incident lepton longitudinal polarization.

For large q^2 the right-hand side of equation (II.40) is computed using the Bjorken sum rule

$$\lim_{q^2 \to \infty} \lim_{s \to \infty} \left(\frac{d\sigma_P}{dq^2} - \frac{d\sigma_A}{dq^2} \right) = - \frac{8\pi\alpha^2}{3q^2 s} \, g_A^{2B+Q} \qquad \text{(II.41)}$$

REFERENCES

S.L. Adler, Phys. Rev. 135B, 963 (1964) ; 143, 1144 (1966)

J.D. Bjorken, Phys. Rev. 148, 1467 (1966); Phys. Rev. D1, 1376 (1970)

J.D. Bjorken and E.A. Paschos, Phys. Rev. D1, 315 (1970)

N. Cabibbo and L. Radicati, Phys. Letters 19, 697 (1965)

C.G. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969)

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)

N. Dombey, Rev. Mod. Phys. 41, 236 (1969)

M.G. Doncel and E. De Rafael, Nuovo Cimento 4A, 363 (1971)

M. Gourdin, Ie Rencontre de Moriond 1966

Lectures at the Cargèse Summer School 1966

Preprint Paris LPTHE 1, (1972) to be published in Nucl. Phys. B

D.J. Gross and C.H. Llewellyn-Smith, Nucl. Phys. B14, 337 (1969)

K. Johnson and F.E. Low, Prog. Theor. Phys. Supp. 37-38, 74 (1966)

APPENDIX II

1. POSITIVITY

Parity is not conserved in weak interactions and we can define 8 total cross sections $\sigma^{(\pm)}_{\alpha\lambda}(q^2, W^2)$

$$\sigma^{(\pm)}_{+1\,+\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\ \frac{1}{2}\left\{ V_2^{(\pm)} - \frac{\sqrt{\nu^2+q^2}}{M} V_3^{(\pm)} - \frac{\nu}{M} X_1^{(\pm)} + \frac{\sqrt{\nu^2+q^2}}{M} X_5^{(\pm)} \right\}$$

$$\sigma^{(\pm)}_{+1\,-\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\ \frac{1}{2}\left\{ V_2^{(\pm)} - \frac{\sqrt{\nu^2+q^2}}{M} V_3^{(\pm)} + \frac{\nu}{M} X_1^{(\pm)} - \frac{\sqrt{\nu^2+q^2}}{M} X_5^{(\pm)} \right\}$$

$$\sigma^{(\pm)}_{-1\,+\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\ \frac{1}{2}\left\{ V_2^{(\pm)} + \frac{\sqrt{\nu^2+q^2}}{M} V_3^{(\pm)} + \frac{\nu}{M} X_1^{(\pm)} + \frac{\sqrt{\nu^2+q^2}}{M} X_5^{(\pm)} \right\}$$

$$\sigma^{(\pm)}_{-1\,-\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}}\ \frac{1}{2}\left\{ V_2^{(\pm)} + \frac{\sqrt{\nu^2+q^2}}{M} V_3^{(\pm)} - \frac{\nu}{M} X_1^{(\pm)} - \frac{\sqrt{\nu^2+q^2}}{M} X_5^{(\pm)} \right\}$$

$$\sigma^{(\pm)}_{L\,+\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}} \left\{ \frac{\nu^2+q^2}{q^2} V_1^{(\pm)} - \frac{1}{2} V_2^{(\pm)} + \frac{\sqrt{\nu^2+q^2}}{M}\left[\frac{\nu^2+q^2}{q^2} X_4^{(\pm)} - \frac{\nu M}{q^2} X_8^{(\pm)} - \frac{1}{2} X_5^{(\pm)} \right] \right\}$$

$$\sigma^{(\pm)}_{L\,-\frac{1}{2}} = \frac{\pi}{M\sqrt{\nu^2+q^2}} \left\{ \frac{\nu^2+q^2}{q^2} V_1^{(\pm)} - \frac{1}{2} V_2^{(\pm)} - \frac{\sqrt{\nu^2+q^2}}{M}\left[\frac{\nu^2+q^2}{q^2} X_4^{(\pm)} - \frac{\nu M}{q^2} X_8^{(\pm)} - \frac{1}{2} X_5^{(\pm)} \right] \right\}$$

$$\sigma^{(\pm)}_{S\,\lambda} = \frac{\pi}{M\sqrt{\nu^2+q^2}} \left\{ \left[\frac{\nu^2}{q^2} V_1^{(\pm)} + \frac{1}{2} V_2^{(\pm)} + \frac{q^2}{M^2} V_4^{(\pm)} - \frac{\nu}{M} V_5^{(\pm)} \right] + \right.$$
$$\left. + 2\lambda \frac{\sqrt{\nu^2+q^2}}{M}\left[\frac{\nu^2}{q^2} X_4^{(\pm)} + \frac{1}{2} X_5^{(\pm)} + \frac{q^2}{M^2} X_6^{(\pm)} - \frac{\nu}{M} X_7^{(\pm)} - \frac{\nu M}{q^2} X_8^{(\pm)} + X_9^{(\pm)} \right] \right\}$$

with the obvious positivity constraint

$$\sigma^{(\pm)}_{\alpha\lambda}(q^2, W^2) \geq 0$$

As a consequence of these inequalities for the transverse and longitudinal part we have, in terms of structure functions

$$\frac{\nu^2+q^2}{q^2}V_1^{(\pm)} + \eta\,\frac{\sqrt{\nu^2+q^2}}{M}\left[\frac{\nu^2+q^2}{q^2}X_4^{(\pm)} - \frac{\nu M}{q^2}X_8^{(\pm)}\right] \geq \frac{1}{2}\left[V_2^{(\pm)} + \eta\,\frac{\sqrt{\nu^2+q^2}}{M}X_5^{(\pm)}\right] \geq$$

$$\geq \frac{1}{2}\left|\frac{\sqrt{\nu^2+q^2}}{M}V_3^{(\pm)} + \eta\,\frac{\nu}{M}X_1^{(\pm)}\right|$$

for any real number η between -1 and +1. The case $\eta = 0$ corresponds to the conditions for unpolarized structure functions.

Conversely, the structure function V_1, V_2, V_3, $X_4 - \frac{\nu M}{\nu^2+q^2}X_8$, X_5 and X_1 can be expressed in terms of transverse and longitudinal total cross sections

$$V_1^{(\pm)} = \frac{Mq^2}{\pi\sqrt{\nu^2+q^2}}\left[\sigma_T^{(\pm)} + \sigma_L^{(\pm)}\right] \qquad X_4^{(\pm)} - \frac{\nu M}{\nu^2+q^2}X_8^{(\pm)} = \frac{M^2q^2}{\pi(\nu^2+q^2)}\left[\tau_T^{(\pm)} + \tau_L^{(\pm)}\right]$$

$$V_2^{(\pm)} = \frac{2M\sqrt{\nu^2+q^2}}{\pi}\sigma_T^{(\pm)} \qquad X_5^{(\pm)} = \frac{2M^2}{\pi}\tau_T^{(\pm)}$$

$$V_3^{(\pm)} = \frac{M^2}{\pi}\left[\sigma_{-1}^{(\pm)} - \sigma_{+1}^{(\pm)}\right] \qquad X_1^{(\pm)} = \frac{M^2\sqrt{\nu^2+q^2}}{\pi\nu}\left[\tau_{-1}^{(\pm)} - \tau_{+1}^{(\pm)}\right]$$

where

$$\sigma_{\alpha\,+\frac{1}{2}}^{(\pm)} + \sigma_{\alpha\,-\frac{1}{2}}^{(\pm)} = 2\sigma_\alpha^{(\pm)} \qquad \sigma_{\alpha\,+\frac{1}{2}}^{(\pm)} - \sigma_{\alpha\,-\frac{1}{2}}^{(\pm)} = 2\tau_\alpha^{(\pm)}$$

$$\sigma_{+1}^{(\pm)} + \sigma_{-1}^{(\pm)} = 2\sigma_T^{(\pm)} \qquad \tau_{+1}^{(\pm)} + \tau_{-1}^{(\pm)} = 2\tau_T^{(\pm)}$$

The inequalities previously derived

$$\sigma_{\alpha\lambda}^{(\pm)}(q^2, W^2) \geq 0$$

express the positivity of the diagonal elements but we have, in addition, other restrictions due to the semi-positive definiteness of the hermitian matrix.

It can be observed that this 8×8 matrix is reducible according to the total helicity into

(a) 2 $\quad 1 \times 1$ matrices for total helicity $\pm 3/2$

(b) 2 $\quad 3 \times 3$ matrices for total helicity $\pm 1/2$.

The positivity conditions associated to part (a) are trivial

$$\sigma^{(\pm)}_{+1\ +\frac{1}{2}}(q^2, W^2) \geqslant 0 \qquad\qquad \sigma^{(\pm)}_{-1\ -\frac{1}{2}}(q^2, W^2) \geqslant 0$$

For part (b), we have to express the semi-positive character of a 3×3 hermitian matrix, e.g. to write that its determinant of order 1 and 2 are non negative. Neglecting now the amplitudes carrying a scalar polarization for the weak current, we find the 3×3 matrices to reduce to 2×2 matrices and as a consequence of positivity we first recover the well-known constraints

$$\sigma^{(\pm)}_{+1\ -\frac{1}{2}}(q^2, W^2) \geqslant 0 \qquad\qquad \sigma^{(\pm)}_{L\ +\frac{1}{2}}(q^2, W^2) \geqslant 0$$

$$\sigma^{(\pm)}_{-1\ +\frac{1}{2}}(q^2, W^2) \geqslant 0 \qquad\qquad \sigma^{(\pm)}_{L\ -\frac{1}{2}}(q^2, W^2) \geqslant 0$$

and we obtain two new inequalities associated to transverse-longitudinal correlations

$$\frac{\pi^2}{M^2(\nu^2+q^2)}\,\frac{q^2}{2M^2}\left\{\left[X_1^{(\pm)} - \frac{\nu^2+q^2}{q^2}X_2^{(\pm)} + \frac{M\sqrt{\nu^2+q^2}}{q^2}X_8^{(\pm)}\right]^2 + \left[\frac{\nu^2+q^2}{q^2}Y_1^{(\pm)} - \frac{M\sqrt{\nu^2+q^2}}{q^2}Y_4^{(\pm)}\right]^2\right\} \leqslant \sigma^{(\pm)}_{+1\ -\frac{1}{2}}\,\sigma^{(\pm)}_{L\ +\frac{1}{2}}$$

$$\frac{\pi^2}{M^2(\nu^2+q^2)}\,\frac{q^2}{2M^2}\left\{\left[X_1^{(\pm)} - \frac{\nu^2+q^2}{q^2}X_2^{(\pm)} - \frac{M\sqrt{\nu^2+q^2}}{q^2}X_8^{(\pm)}\right]^2 + \left[\frac{\nu^2+q^2}{q^2}Y_1^{(\pm)} + \frac{M\sqrt{\nu^2+q^2}}{q^2}Y_4^{(\pm)}\right]^2\right\} \leqslant \sigma^{(\pm)}_{-1\ +\frac{1}{2}}\,\sigma^{(\pm)}_{L\ -\frac{1}{2}}$$

2. FORWARD DIRECTION

The strangeness-conserving part of the weak current is the isotopic spin current. Using CVC and PCAC we can obtain the limit of the neutrino and antineutrino cross sections in the forward direction where

$q^2 \Rightarrow 0$ when the lepton mass is neglected. The result due to Adler is, for an inelastic final state

$$\lim_{q^2 \Rightarrow 0} \frac{d^2\sigma^{(\pm)}}{dq^2\, dW^2} = \frac{G^2}{4\pi M^2} \frac{E'}{E} \left\{ \frac{F_\pi^2}{\pi} \frac{M\sqrt{\nu^2 - m_\pi^2}}{\nu^2} \sigma_{TOT}^{(\pm)\Gamma}(W)_0 \right\}$$

where F_π is the $\pi^\pm$ meson weak decay coupling constant and $\sigma_{TOT}^{(\pm)\Gamma}(W)_0$ is the integrated cross section for the reaction $\pi^\pm + p \Rightarrow \Gamma$ extrapolated for zero mass incident $\pi^\pm$ meson.

Because of the conservation of parity for strong interaction, it can be shown that the neutrino and antineutrino cross section are spin independent in the forward direction for an inelastic final state

$$\lim_{q^2 \Rightarrow 0} \left[\frac{\nu}{M} X_4^{(\pm)} - X_8^{(\pm)} \right] = 0$$

For elastic scattering, we have a very interesting result for the limit of the asymmetry $\Delta_{\parallel}$ in the forward direction in the two reactions on a nucleon target

$$\bar{\nu}_\ell + p \Rightarrow \ell^+ + n \qquad\qquad \nu_\ell + n \Rightarrow \ell^- + p$$

In both cases the asymmetry is very large

$$\lim_{\theta \Rightarrow 0} \Delta_{\parallel}^{elas} = \frac{2 g_A}{1 + g_A^2} \simeq .98$$

Let us notice that the parallel asymmetry for elastic scattering in the backward direction has the limit expected from helicity arguments

$$\lim_{\theta \Rightarrow \pi} \Delta_{\parallel}^{\bar{\nu}_\ell p \Rightarrow \ell^+ n} = -1 \qquad\qquad \lim_{\theta \Rightarrow \pi} \Delta_{\parallel}^{\nu_\ell n \Rightarrow \ell^- p} = +1$$

3. CURRENT ALGEBRA SUM RULE

We give the classification of the sum rules in the laboratory frame where $\vec{q}$ is along the third direction

	$[J_0^{(\mp)}, J_0^{(\pm)}]$	$[J_1^{(\mp)}, J_1^{(\pm)}]$	$[J_1^{(\mp)}, J_2^{(\pm)}]$
Parity conserving	$V_1^{(\pm)}$	$V_2^{(\pm)}$	$X_1^{(\pm)}$
Parity violating	$\frac{\nu}{M} X_4^{(\pm)} - X_8^{(\pm)}$	$X_5^{(\pm)}$	$V_3^{(\pm)}$

Table 2

The two sum rules of the first column are the Adler sum rule and a sum rule for polarization both based on identical assumptions

$$\frac{1}{2}\int_{M^2}^{\infty}\left[V_1^{\bar{\nu}}(q^2,W^2) - V_1^{\nu}(q^2,W^2)\right] d\left(\frac{W^2}{M^2}\right) = 4\left[\cos^2\theta_c\, g_V^{I^3} + \sin^2\theta_c\, g_V^{V^3}\right]$$

$$\frac{1}{2}\int_{M^2}^{\infty}\left\{\left[\frac{\nu}{M}X_4 - X_8\right]^{\bar{\nu}} - \left[\frac{\nu}{M}X_4 - X_8\right]^{\nu}\right\} d\left(\frac{W^2}{M^2}\right) = 4\left[\cos^2\theta_c\, g_A^{I^3} + \sin^2\theta_c\, g_V^{V^3}\right]$$

where the g_V 's are vector coupling constants and the g_A 's axial vector coupling constants. As usual θ_c is the Cabibbo angle.

The other four sum rules are based on the validity of the Bjorken-Johnson-Low expansion. It is then convenient to introduce the scalar variable $\xi = q^2/2M\nu$ and to define new structure functions

$$V_2^{(\pm)}(q^2,W^2) \equiv 2\, F_T^{\nu,\bar{\nu}}(q^2,\xi) \qquad \frac{\nu}{M} V_3^{(\pm)}(q^2,W^2) \equiv F_-^{\nu,\bar{\nu}}(q^2,\xi) - F_+^{\nu,\bar{\nu}}(q^2,W^2)$$

$$\frac{\nu}{M} X_5^{(\pm)}(q^2,W^2) \equiv 2\, P_T^{\nu,\bar{\nu}}(q^2,\xi) \qquad \frac{\nu}{M} X_1^{(\pm)}(q^2,W^2) \equiv P_-^{\nu,\bar{\nu}}(q^2,\xi) - P_+^{\nu,\bar{\nu}}(q^2,\xi)$$

and the lower index means the weak current helicity.

When the space-space commutators are assumed to be as for free quarks, we get the following asymptotic sum rules

(a) The Bjorken sum rule for V_2

$$\lim_{q^2 \to \infty} \int_0^1 \left[F_T^{\bar{\nu}}(q^2,\xi) - F_T^{\nu}(q^2,\xi) \right] d\xi = 2 \left[\cos^2\theta_c \, g_V^{I^3} + \sin^2\theta_c \, g_V^{V^3} \right]$$

(b) A polarization sum rule for X_5

$$\lim_{q^2 \to \infty} \int_0^1 \left[P_T^{\bar{\nu}}(q^2,\xi) - P_T^{\nu}(q^2,\xi) \right] d\xi = 2 \left[\cos^2\theta_c \, g_A^{I^3} + \sin^2\theta_c \, g_A^{V^3} \right]$$

(c) The Gross-Llewelyn-Smith sum rule for V_3

$$\lim_{q^2 \to \infty} \int_0^1 \left[F_-^{\bar{\nu}} - F_+^{\bar{\nu}} + F_-^{\nu} - F_+^{\nu} \right] d\xi = 2 \left[\cos^2\theta_c \, g_V^{2B+Y} + \sin^2\theta_c \, g_V^{2B+Q-Y} \right]$$

(d) An extension to neutrino and antineutrino reactions of the Bjorken polarization sum rule for X_1 in electroproduction

$$\lim_{q^2 \to \infty} \int_0^1 \left[P_-^{\bar{\nu}} - P_+^{\bar{\nu}} + P_-^{\nu} - P_+^{\nu} \right] d\xi = 2 \left[\cos^2\theta_c \, g_A^{2B+Y} + \sin^2\theta_c \, g_A^{2B+Q-Y} \right]$$

4. HIGH ENERGY

The fixed q^2, W^2 high energy limit of the cross section and asymmetries is given by

$$\lim_{s \to \infty} \frac{d^2\sigma^{(\pm)}_{unp.}}{dq^2\, dW^2} = \frac{G^2}{4\pi} \frac{1}{M^2} \, \bar{V}_1^{(\pm)}(q^2, W^2)$$

$$\lim_{s \to \infty} \Delta_{\parallel}^{(\pm)} = \frac{\frac{\nu}{M} X_4^{(\pm)}(q^2,W^2) - X_8^{(\pm)}(q^2,W^2)}{\bar{V}_1^{(\pm)}(q^2,W^2)}$$

$$\lim_{s \Rightarrow \infty} \Delta_N^{(\pm)} = -\sqrt{\frac{q^2}{M^2}} \; \frac{Y_1^{(\pm)}(q^2, W^2)}{V_1^{(\pm)}(q^2, W^2)}$$

$$\lim_{s \Rightarrow \infty} \Delta_\perp^{(\pm)} = -\sqrt{\frac{q^2}{M^2}} \; \frac{-X_4^{(\pm)}(q^2, W^2)}{V_1^{(\pm)}(q^2, W^2)}$$

The high-energy limit of the fixed q^2 differential cross section is assumed to be given by an integration over W^2 of the high-energy limit of the double differential cross section. We then get

$$\lim_{s \Rightarrow \infty} \frac{d\sigma^{(\pm)}(N_3)}{dq^2} = \frac{G^2}{4\pi} \int_{M^2}^{\infty} \left\{ V_1^{(\pm)}(q^2, W^2) + N_3 \left[\frac{\nu}{M} X_4^{(\pm)}(q^2, W^2) - X_8^{(\pm)}(q^2, W^2) \right] \right\} d\left(\frac{W^2}{M^2}\right)$$

where $N_3 = +1, -1$ for a target polarization parallel or antiparallel to the incident neutrino momentum $\vec{k}$.

The fixed q^2 cross section can be written as

$$\frac{d\sigma^{(\pm)}(N_3)}{dq^2} = \frac{d\sigma^{(\pm)}_{unp}}{dq^2} \left[1 + N_3 \, \delta^{(\pm)} \right]$$

By using now the Adler sum rule for V_1 and the polarization sum rule for $\frac{\nu}{M} X_4 - X_8$ we obtain the following limits

$$\lim_{s \Rightarrow \infty} \left[\frac{d\sigma^{\bar{\nu}}_{unp}}{dq^2} - \frac{d\sigma^{\nu}_{unp}}{dq^2} \right] = \frac{2G^2}{\pi} \left[\cos^2\theta_c \, g_V^{I^3} + \sin^2\theta_c \, g_V^{V^3} \right]$$

and for the asymmetry δ we get

$$\lim_{s \Rightarrow \infty} \left[\delta^{\bar{\nu}} - \delta^{\nu} \right] = \frac{\cos^2\theta_c \, g_A^{I^3} + \sin^2\theta_c \, g_A^{V^3}}{\cos^2\theta_c \, g_V^{I^3} + \sin^2\theta_c \, g_V^{V^3}}$$

The essential feature of these results is that the high-energy limits become independent of q^2.

LESSON III

SCALING AND EXPERIMENTAL DATA

1. SCALING

1°) By studying the properties of the matrix element

$$\langle p \mid [J_\nu^{em}(x) , J_\mu^{em}(0)] \mid p \rangle$$

in the frame of reference where the hadron momentum goes to infinity, Bjorken suggests that the structure functions $V_1(q^2, W^2)$ and $V_2(q^2, W^2)$ tend to simple limits for large q^2

$$\begin{aligned} \lim_{\substack{q^2, W^2 \Rightarrow \infty \\ \xi = \frac{q^2}{2M\nu} \text{ fixed}}} \frac{\nu}{M} \bar{V}_1(q^2, W^2) &= \bar{F}_1(\xi) \\ \lim_{\substack{q^2, W^2 \Rightarrow \infty \\ \xi = \frac{q^2}{2M\nu} \text{ fixed}}} \bar{V}_2(q^2, W^2) &= \bar{F}_2(\xi) \end{aligned} \tag{III.1}$$

These properties imply, for the total cross sections $\sigma_T(q^2, W^2)$ and $\sigma_L(q^2, W^2)$, a very simple behaviour for large q^2

$$\lim_{\substack{q^2, W^2 \Rightarrow \infty \\ \xi = \frac{q^2}{2M\nu} \text{ fixed}}} \frac{1}{\pi} \nu M \sigma_{T,L}(q^2, W^2) = \bar{F}_{T,L}(\xi) \tag{III.2}$$

and we shall assume the same property for any of the cross sections

$\sigma_{\alpha\lambda}(q^2, W^2)$

2°) The relation between the structure functions V_1, V_2, X and the total cross sections $\sigma_{\alpha\lambda}$ can be found in equations (II.7) and (II.8).

Defining

$$\begin{aligned} F_{\pm 1 \; \pm\frac{1}{2}}(\xi) &= \bar{F}_T(\xi) - P_{\parallel}(\xi) \\ F_{\pm 1 \; \mp\frac{1}{2}}(\xi) &= \bar{F}_T(\xi) + P_{\parallel}(\xi) \end{aligned}$$

we get(*)

$$\mathrm{LIM}\ \frac{\nu^2}{q^2} V_1(q^2, W^2) = F_T(\xi) + F_L(\xi) \qquad \mathrm{LIM}\ V_2(q^2, W^2) = 2F_T(\xi) \tag{III.3}$$

$$\mathrm{LIM}\ \frac{\nu}{M} X_1(q^2, W^2) = 2P_{\parallel}(\xi) \tag{III.4}$$

where LIM is the Bjorken limit defined in equation (III.1). The positivity inequalities (II.6) take the very simple form

$$F_T(\xi) + F_L(\xi) \geqslant F_T(\xi) \geqslant |P_{\parallel}(\xi)| \tag{III.5}$$

3°) If a scaling à la Bjorken holds for the other structure functions X_2 and Y, it is restricted by the positivity constraints to be

$$\mathrm{LIM}\ \frac{\nu^2}{M\sqrt{q^2}} X_2(q^2, W^2) = 2P_{\perp}(\xi) \qquad \mathrm{LIM}\ \frac{\nu^2}{M\sqrt{q^2}} Y(q^2, W^2) = 2P_N(\xi) \tag{III.6}$$

and the restriction (II.9) due to positivity takes a particularly simple form in the scaling region

$$P_{\perp}^2(\xi) + P_N^2(\xi) \leqslant \frac{1}{2}\left[F_T(\xi) + P_{\parallel}(\xi)\right] F_L(\xi) \tag{III.7}$$

The precise meaning of this statement is the following : if the unpolarized structure functions V_1 and V_2 scale, as suggested by Bjorken, then the positivity gives severe constraints on the high q^2 behaviour at fixed ξ of the polarized structure functions X_1, X_2 and Y. We assume that

---

(*) The relation between equations (III.1) and (III.3) is simply

$$F_1(\xi) = 2\xi\left[F_T(\xi) + F_L(\xi)\right] \qquad F_2(\xi) = 2F_T(\xi)$$

the scaling is as strong as allowed by positivity. Models like the light-cone algebra can predict, under definite assumptions the type of scaling for the polarized structure functions but in any model the positivity constraints must be fulfilled. For instance, if the electromagnetic current has no longitudinal polarization in the scaling limit, $F_L = 0$ and as a consequence of the positivity unequality (III.7) the scaling functions $P_\perp(\xi)$ and $P_N(\xi)$ will vanish and instead of equations (III.6) one must expect a faster decreasing with q^2 at fixed ξ as for instance

$$\mathrm{LIM}\ \frac{\nu^2}{q^2}\frac{\nu}{M}X_2(q^2,W^2) = 2P_2(\xi) \qquad \mathrm{LIM}\ \frac{\nu^2}{q^2}\frac{\nu}{M}Y(q^2,W^2) = 2Q(\xi)$$

4°) We introduce a second scaling variable $\rho = E'/E$ and the high-energy limit in the scaling region is defined for s, q^2, W^2 large with ρ and ξ fixed.

The unpolarized cross section is written as

$$\frac{d\sigma_{unp}}{dq^2 dW^2} \Rightarrow \frac{4\pi\alpha^2}{s}\frac{1}{(1-\rho)^2}\frac{1}{\xi}\left[(1+\rho^2)F_T(\xi) + 2\rho F_L(\xi)\right] \tag{III.8}$$

and for the three asymmetries Δ_N , $\Delta_{\parallel}$ and $\Delta_\perp$ we obtain high-energy limits independent of s

$$\Delta_N \Rightarrow -2\sqrt{\rho}\ \frac{(1+\rho)P_N(\xi)}{(1+\rho^2)F_T(\xi)+2\rho F_L(\xi)} \tag{III.9}$$

$$\Delta_\perp \Rightarrow \cdot 2\sqrt{\rho}\ \frac{(1-\rho)P_\perp(\xi)}{(1+\rho^2)F_T(\xi)+2\rho F_L(\xi)} \tag{III.10}$$

$$\Delta_{\parallel} \Rightarrow - \frac{(1-\rho^2)\,\mathcal{P}_{\parallel}(\xi)}{(1+\rho^2)\,\overline{F}_T(\xi) + 2\rho\,\overline{F}_L(\xi)} \qquad \text{(III.11)}$$

From the positivity condition (III.7), we can deduce bounds for the asymmetries Δ_N and $\Delta_\perp$ depending only on the scaling functions F_T and F_L measured in experiments on an unpolarized target

$$\left(\Delta_N^2 + \Delta_\perp^2\right)^{\frac{1}{2}} \leq \frac{2(1+\rho)\sqrt{\rho\,\overline{F}_T(\xi)\,\overline{F}_L(\xi)}}{(1+\rho^2)\,\overline{F}_T(\xi) + 2\rho\,\overline{F}_L(\xi)} \qquad \text{(III.12)}$$

If the longitudinal scaling function F_L is small as compared to the transverse one F_T, the asymmetries Δ_N and $\Delta_\perp$ will also be small as compared to $\Delta_{\parallel}$. This result is a consequence of positivity only.

2. EXPERIMENTAL RESULTS

1°) A systematic study of the electron deep inelastic scattering on hydrogen and deuterium is made at SLAC and at DESY. We show on Figs. 3 and 4 two examples of the differential cross section $\frac{d^2\sigma}{dE'd\Omega}$ for an incident energy of 7 GeV and a scattering angle of $6°$ in the Lab. system.

The region of the q^2, W^2 plane where measurements have been performed is represented on Fig. 5. Over a portion of this region, it is possible to determine separately the two structure functions V_1 and V_2. The region marked 'separation region' includes all points where data at three or more points exists. An example of such a separation is given on Fig. 6.

2°) The function $\frac{\nu}{M}\,\bar{V}_1^{ep}(q^2, W^2)$ has been plotted on Fig. 7 as a function of $\omega = \frac{1}{\xi} = \frac{2M\nu}{q^2}$. The result is compatible with a unique curve $\bar{F}_1(\xi)$ as suggested by the Bjorken scaling law (III.1). Fig. 8 shows the independence of $\frac{\nu}{M}\,\bar{V}_1^{ep}(q^2, W^2)$ in q^2 for a fixed value $\omega = 4$ and brings a strong support to scaling. (Remember that $\nu/M\ V_1 \equiv \nu W_2$).

3°) Other scaling variables have been proposed in order to extend the region where the experimental data scale. For instance, Bloom and Gilman have suggested to use the variable ω' related to ω by

$$\omega' = \omega + \frac{M^2}{q^2} = 1 + \frac{W^2}{q^2}$$

Of course, for large q^2, ω and ω' coincide.

Fig. 9 represents a plot of $\frac{\nu}{M}\,\bar{V}_1^{ep}(q^2, W^2)$ versus ω' ; the dispersion in q^2 is somewhat less important than on Fig. 7. Fig. 10 shows a plot of $\frac{\nu}{M}\,\bar{V}_1^{ep}(q^2, W^2)$ versus q^2 for $\omega = 1.66$. The scaling is not as beautiful as in the case $\omega = 4$ of Fig. 8. The solid line is a fit using a polynomial in $1 - \frac{1}{\omega'}$ and we see the improvement obtained going from ω to ω'.

4°) The ratio of the longitudinal to the transverse contributions

$$R^p = \frac{\sigma_L^p}{\sigma_T^p}$$

in electron-proton scattering is always smaller than 0.5. If R is assumed to be constant over the measured range, its value is $R = .18 \pm .10$ but the measured values are also compatible with $R = 0.031\,\frac{q^2}{M^2}$ and with $R \simeq \frac{q^2}{\nu^2}$.

5°) We give in the following table a résumé of the experimental scaling in the variable ω for the proton structure function $\frac{\nu}{M}\,\bar{V}_1^{ep}(q^2, W^2)$.

$1 < \omega < 4$	$W < 2.6$ GeV $W > 2.6$ GeV	Resonance region no scaling scaling
$4 < \omega < 12$	$W > 2$ GeV $q^2 > 1$ GeV^2	$\frac{\nu}{M} V_1^{ep} \simeq$ const $\approx .3$ scaling
$12 < \omega$	few points 2 GeV$^2 \gtrsim q^2 \gtrsim 1$ GeV^2	Difficulty in studying scaling R not measured

As an experimental observation, the range of scaling of $\frac{\nu}{M} V_1^{ep}$ in ω' is greater than in ω .

A breaking of the scaling for $q^2 < 1$ GeV^2 is shown on Fig. 11 where $\frac{\nu}{M} V_1^{ep}$ is plotted versus q^2 for various values of ω' .

6°) The electron-neutron scattering information is extracted from the measured electron deuterium cross sections essentially by difference.

The structure function $\frac{\nu}{M} V_1^{ed}$ for deuterium scales within uncertainties essentially as well as for hydrogen. Fig. 12 shows a comparison of $\frac{\nu}{M} V_1$ for hydrogen and deuterium as a function of ω' .

The value of R for neutron has not been measured but taken to be the same as for proton and the structure function $\frac{\nu}{M} V_1^{en}$ for neutron is obtained by simple difference and obviously it also scales.

The ratio of the neutron to the proton structure function V_1 is represented on Fig. 13. It exhibits a strong tendency to decrease when ω' tends to unity but the uncertainties remain very large.

REFERENCES

J.D. Bjorken, Lectures given at the Varenna Summer School 1967 ;
Phys. Rev. 179, 1547 (1969)

Experimental

Electroproduction

E.D. Bloom et al., Report at the Kiev Conference 1970, SLAC PUB 796

H. Kendall, Report at the Cornell Conference 1971

Neutrino production

I. Budagov et al., Phys. Letters 30B, 364 (1969)

G. Hyatt and D.H. Perkins, Phys. Letters 34B, 542 (1971).

APPENDIX III

1. SCALING

1°) We assume for the total cross sections $\sigma^{(\pm)}_{\alpha\lambda}(q^2, W^2)$ a scaling in the sense

$$\mathrm{LIM}\ \frac{1}{\pi}\,\nu M\,\sigma^{(\pm)}_{\alpha\lambda}(q^2,W^2) = F^{\nu,\bar\nu}_{\alpha\lambda}(\xi)$$

Defining

$$F_{\alpha\,+\frac{1}{2}} + F_{\alpha\,-\frac{1}{2}} = 2\,F_\alpha \qquad F_{\alpha\,+\frac{1}{2}} - F_{\alpha\,-\frac{1}{2}} = 2\,P_\alpha$$

$$F_+ + F_- = 2\,F_T \qquad P_+ + P_- = 2\,P_T$$

we get

$$\mathrm{LIM}\ \frac{\nu^2}{q^2} V_1^{(\pm)} = F_T^{\nu,\bar\nu} + F_L^{\nu,\bar\nu} \qquad \mathrm{LIM}\ \frac{\nu^2}{q^2}\left(\frac{\nu}{M} X_4^{(\pm)} - X_8^{(\pm)}\right) = P_T^{\nu,\bar\nu} + P_L^{\nu,\bar\nu}$$

$$\mathrm{LIM}\ V_2^{(\pm)} = 2\,F_T^{\nu,\bar\nu} \qquad \mathrm{LIM}\ \frac{\nu}{M} X_5^{(\pm)} = 2\,P_T^{\nu,\bar\nu}$$

$$\mathrm{LIM}\ \frac{\nu}{M} V_3^{(\pm)} = F_-^{\nu,\bar\nu} - F_+^{\nu,\bar\nu} \qquad \mathrm{LIM}\ \frac{\nu}{M} X_1^{(\pm)} = P_-^{\nu,\bar\nu} - P_+^{\nu,\bar\nu}$$

If a scaling à la Bjorken holds for the other polarization structure functions, it is restricted by the positivity constraints to be

$$\mathrm{LIM}\ \frac{\nu^2}{M\sqrt{q^2}} X_2^{(\pm)} = 2\,P_{\perp 1}^{\nu,\bar\nu} \qquad \mathrm{LIM}\ \frac{\nu^2}{M\sqrt{q^2}} X_4^{(\pm)} = \mathrm{LIM}\ \frac{\nu}{\sqrt{q^2}} X_8^{(\pm)} = 2\,P_{\perp 2}^{\nu,\bar\nu}$$

$$\mathrm{LIM}\ \frac{\nu^2}{M\sqrt{q^2}} Y_1^{(\pm)} = 2\,P_{N1}^{\nu,\bar\nu} \qquad \mathrm{LIM}\ \frac{\nu}{\sqrt{q^2}} Y_4^{(\pm)} = 2\,P_{N2}^{(\pm)}$$

and the new restrictions due to positivity take particularly simple form in the scaling region

$$(P_{\perp 1}-P_{\perp 2})^2+(P_{N1}-P_{N2})^2 \leq \frac{1}{2}(F_+-P_+)(F_L+P_L)$$

$$(P_{\perp 1}+P_{\perp 2})^2+(P_{N1}+P_{N2})^2 \leq \frac{1}{2}(F_++P_+)(F_L-P_L)$$

2°) Assuming that there exists in the high-energy limit a region where the local Fermi interaction is still applicable and where the scaling takes place, we write the unpolarized cross sections as

$$\frac{d^2\sigma^\nu}{d\rho\, d\xi} \Rightarrow \frac{G^2 s}{2\pi}\,\xi\left[\rho^2\,\overline{F}_+^{\nu}(\xi)+\overline{F}_-^{\nu}(\xi)+2\rho\,\overline{F}_L^{\nu}(\xi)\right]$$

$$\frac{d^2\sigma^{\bar\nu}}{d\rho\, d\xi} \Rightarrow \frac{G^2 s}{2\pi}\,\xi\left[\rho^2\,\overline{F}_-^{\bar\nu}(\xi)+\overline{F}_+^{\bar\nu}(\xi)+2\rho\,\overline{F}_L^{\bar\nu}(\xi)\right]$$

For the three symmetries Δ_N, $\Delta_{\parallel}$ and $\Delta_{\perp}$ we obtain limits independent of the incident energy E

$$\Delta_N^{\nu} \Rightarrow -2\sqrt{\rho}\;\frac{(1+\rho)P_{N1}^{\nu}+(1-\rho)P_{N2}^{\nu}}{\rho^2\,\overline{F}_+^{\nu}+\overline{F}_-^{\nu}+2\rho\,\overline{F}_L^{\nu}}$$

$$\Delta_N^{\bar\nu} \Rightarrow -2\sqrt{\rho}\;\frac{(1+\rho)P_{N1}^{\bar\nu}-(1-\rho)P_{N2}^{\bar\nu}}{\rho^2\,\overline{F}_-^{\bar\nu}+\overline{F}_+^{\bar\nu}+2\rho\,\overline{F}_L^{\bar\nu}}$$

$$\Delta_{\perp}^{\nu} \Rightarrow -2\sqrt{\rho}\;\frac{(1-\rho)P_{\perp 1}^{\nu}+(1+\rho)P_{\perp 2}^{\nu}}{\rho^2\,\overline{F}_+^{\nu}+\overline{F}_-^{\nu}+2\rho\,\overline{F}_L^{\nu}}$$

$$\Delta_{\perp}^{\bar\nu} \Rightarrow -2\sqrt{\rho}\;\frac{-(1-\rho)P_{\perp 1}^{\bar\nu}+(1+\rho)P_{\perp 2}^{\bar\nu}}{\rho^2\,\overline{F}_-^{\bar\nu}+\overline{F}_+^{\bar\nu}+2\rho\,\overline{F}_L^{\bar\nu}}$$

$$\Delta_{\parallel}^{\nu} \Rightarrow \frac{\rho^2 P_+^{\nu}+P_-^{\nu}+2\rho P_L^{\nu}}{\rho^2\,\overline{F}_+^{\nu}+\overline{F}_-^{\nu}+2\rho\,\overline{F}_L^{\nu}}$$

$$\Delta_{\parallel}^{\bar{\nu}} \Rightarrow \frac{\rho^2 P_-^{\bar{\nu}} + P_+^{\bar{\nu}} + 2\rho P_L^{\bar{\nu}}}{\rho^2 F_-^{\bar{\nu}} + F_+^{\bar{\nu}} + 2\rho F_L^{\bar{\nu}}}$$

From the positivity condition, we can deduce bounds for the asymmetries Δ_N and $\Delta_\perp$ depending only on the scaling functions F_α measured in experiments on an unpolarized target

$$\left[\Delta_N^2 + \Delta_\perp^2\right]_\nu^{\frac{1}{2}} \leqslant 2 \, \frac{\sqrt{2\rho(1+\rho)\, F_L^\nu (\rho F_+^\nu + F_-^\nu)}}{\rho^2 F_+^\nu + F_-^\nu + 2\rho F_L^\nu}$$

$$\left[\Delta_N^2 + \Delta_\perp^2\right]_{\bar{\nu}}^{\frac{1}{2}} \leqslant 2 \, \frac{\sqrt{2\rho(1+\rho)\, F_L^{\bar{\nu}} (\rho F_-^{\bar{\nu}} + F_+^{\bar{\nu}})}}{\rho^2 F_-^{\bar{\nu}} + F_+^{\bar{\nu}} + 2\rho F_L^{\bar{\nu}}}$$

If the longitudinal scaling functions F_L are small as compared to the transverse ones F_+ and F_- , the asymmetries Δ_N and $\Delta_\perp$ will also be small as compared to $\Delta_{\parallel}$. This result is a consequence of positivity only.

3°) The total cross sections are obtained integrating the differential ones in the region $0 \leqslant \xi \leqslant 1 \qquad 0 \leqslant \rho \leqslant 1$

We make the strong assumption that the result of this integration is not too different from the one obtained by using the scaling forms in the complete integration domain. We then deduce, following Bjorken, a linear rising with the incident energy of the total cross sections for neutrino and antineutrino reactions

$$\sigma_{TOT}^{\nu,\bar{\nu}}(s) \Rightarrow \frac{G^2 s}{2\pi} A^{\nu,\bar{\nu}}$$

where the constants A^{ν} and $A^{\bar{\nu}}$ are first moment distributions given by

$$A^{\nu,\bar{\nu}} = \int_0^1 \xi \, A^{\nu,\bar{\nu}}(\xi) \, d\xi$$

with as a result of the integration over

$$A^{\nu}(\xi) = \frac{1}{3} \bar{F}_{+}^{\nu}(\xi) + \bar{F}_{-}^{\nu}(\xi) + \bar{F}_{L}^{\nu}(\xi)$$

$$A^{\bar{\nu}}(\xi) = \frac{1}{3} \bar{F}_{-}^{\bar{\nu}}(\xi) + \bar{F}_{+}^{\bar{\nu}}(\xi) + \bar{F}_{L}^{\bar{\nu}}(\xi)$$

2. EXPERIMENTAL RESULTS

An experiment performed at CERN in a propane bubble chamber gives some indications for the scaling à la Bjorken and for a linear rising with energy of the total cross section. The results are

(a) A^{ν} (propane per nucleon) $= 0.52 \pm 0.13$

(b) $A^{\nu p} / A^{\nu n} = 1.8 \pm 0.3$

(c) No strangeness changing events observed

Another experiment is now performed in the Gargamelle chamber with freon.

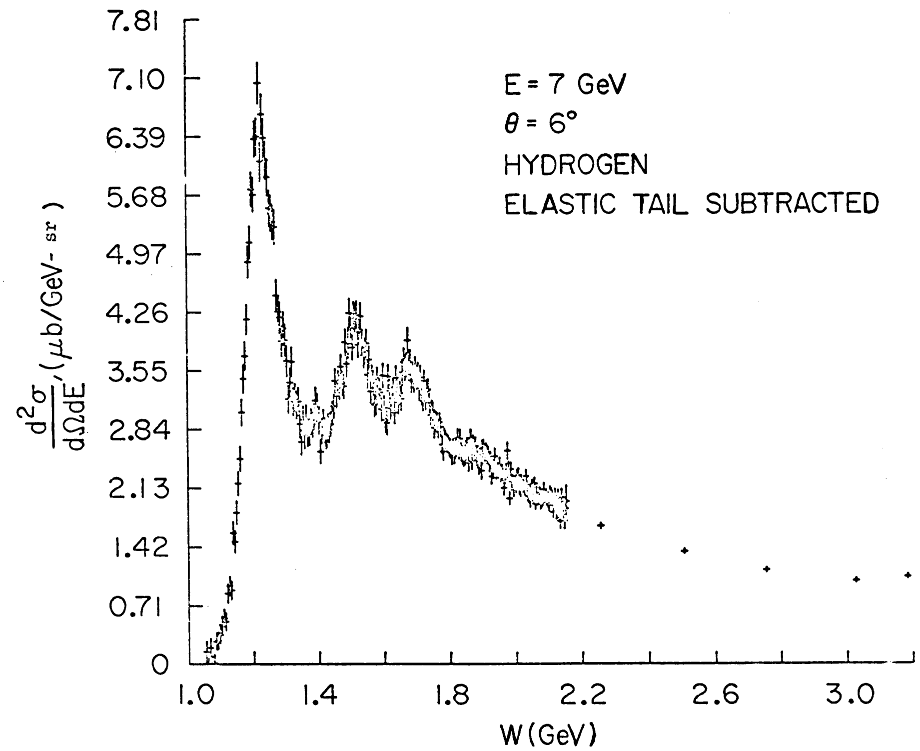

Fig. 3 - $d^2\sigma/d\Omega dE'$ on Hydrogen

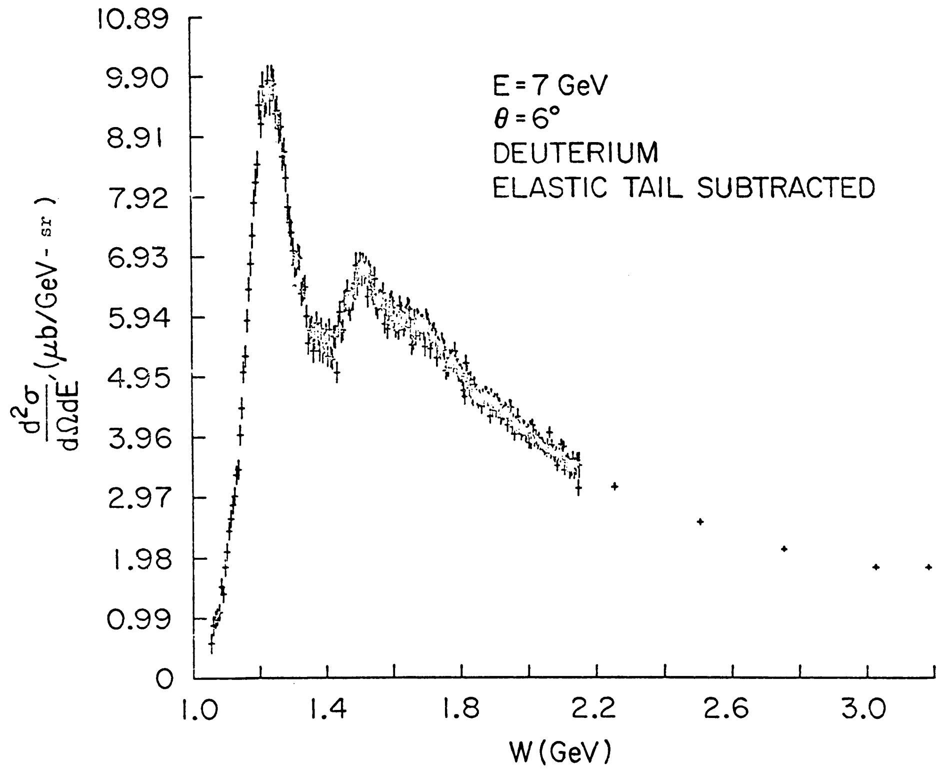

Fig. 4 - $d^2\sigma/d\Omega dE'$ on Deuterium

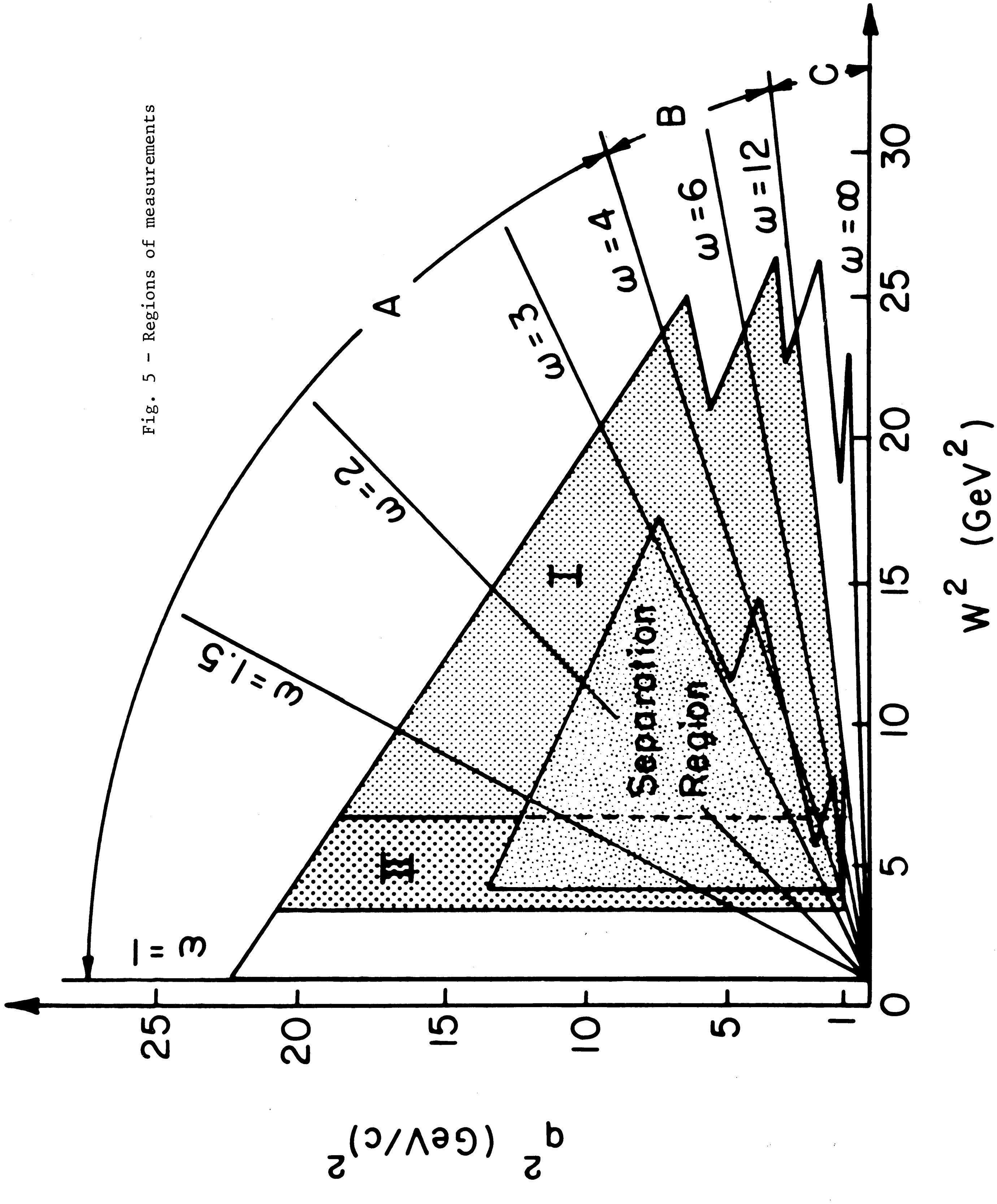

Fig. 5 - Regions of measurements

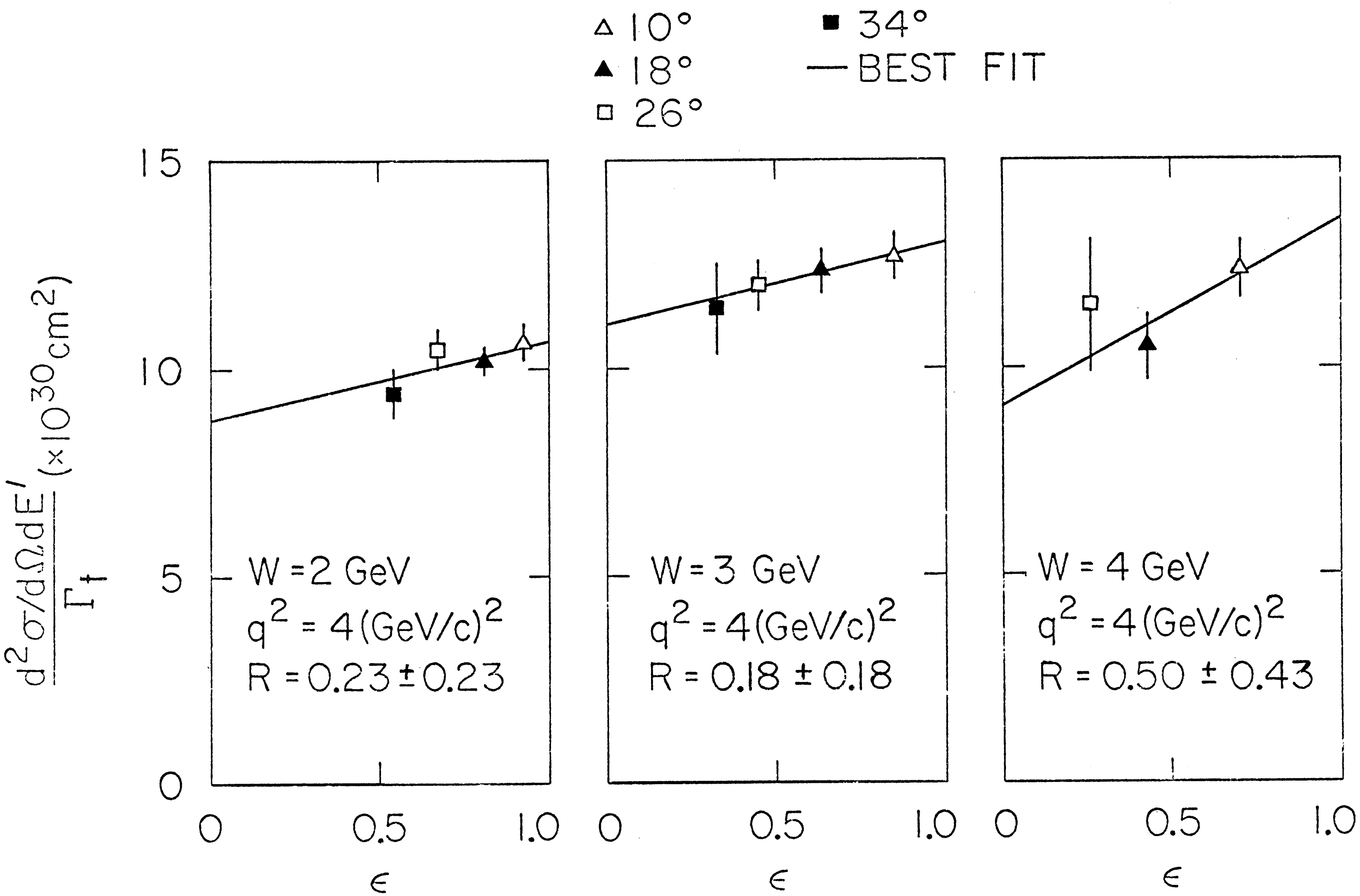

Fig. 6 - Separation of the structure functions

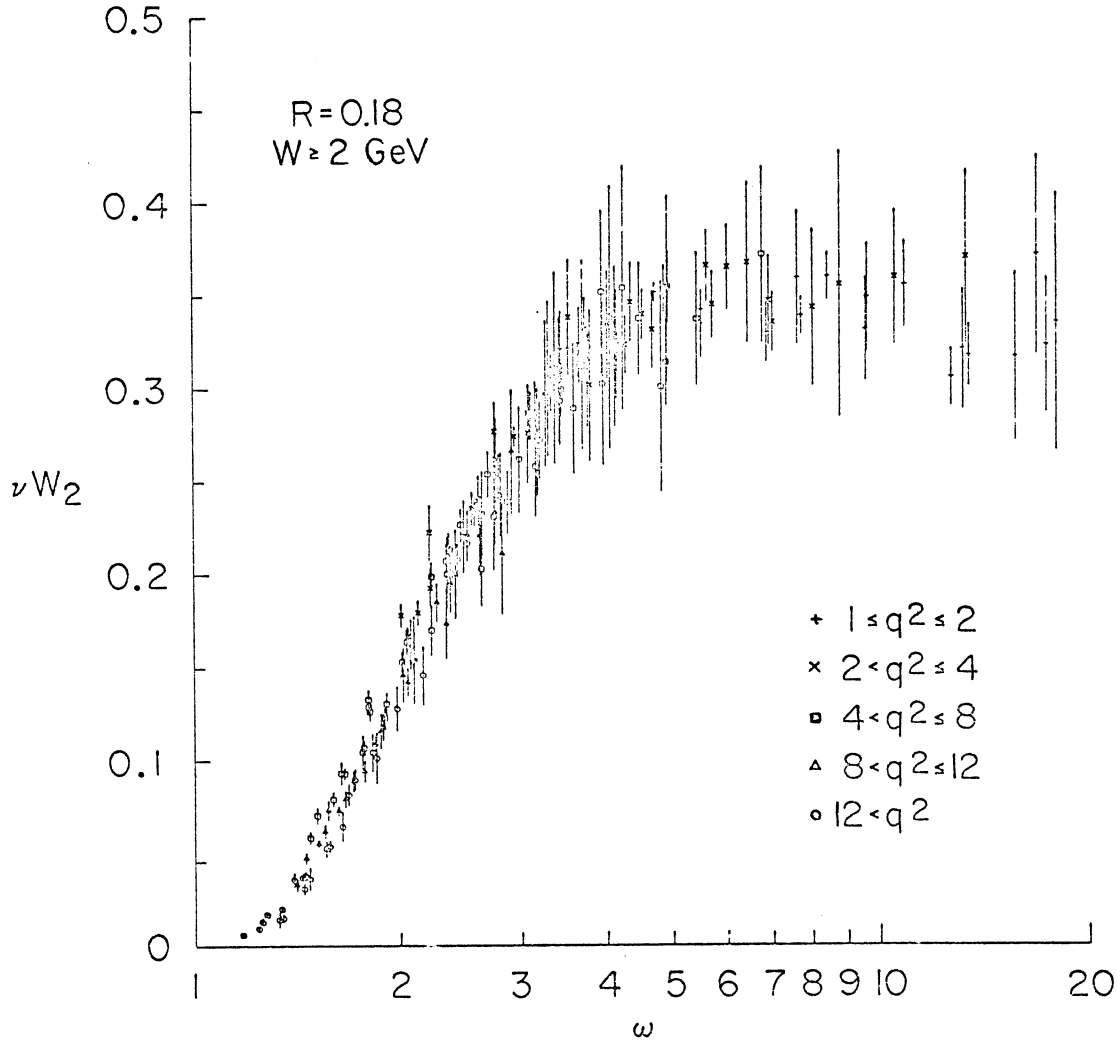

Fig. 7 - $\nu/M\ V_1^{ep}$ versus ω

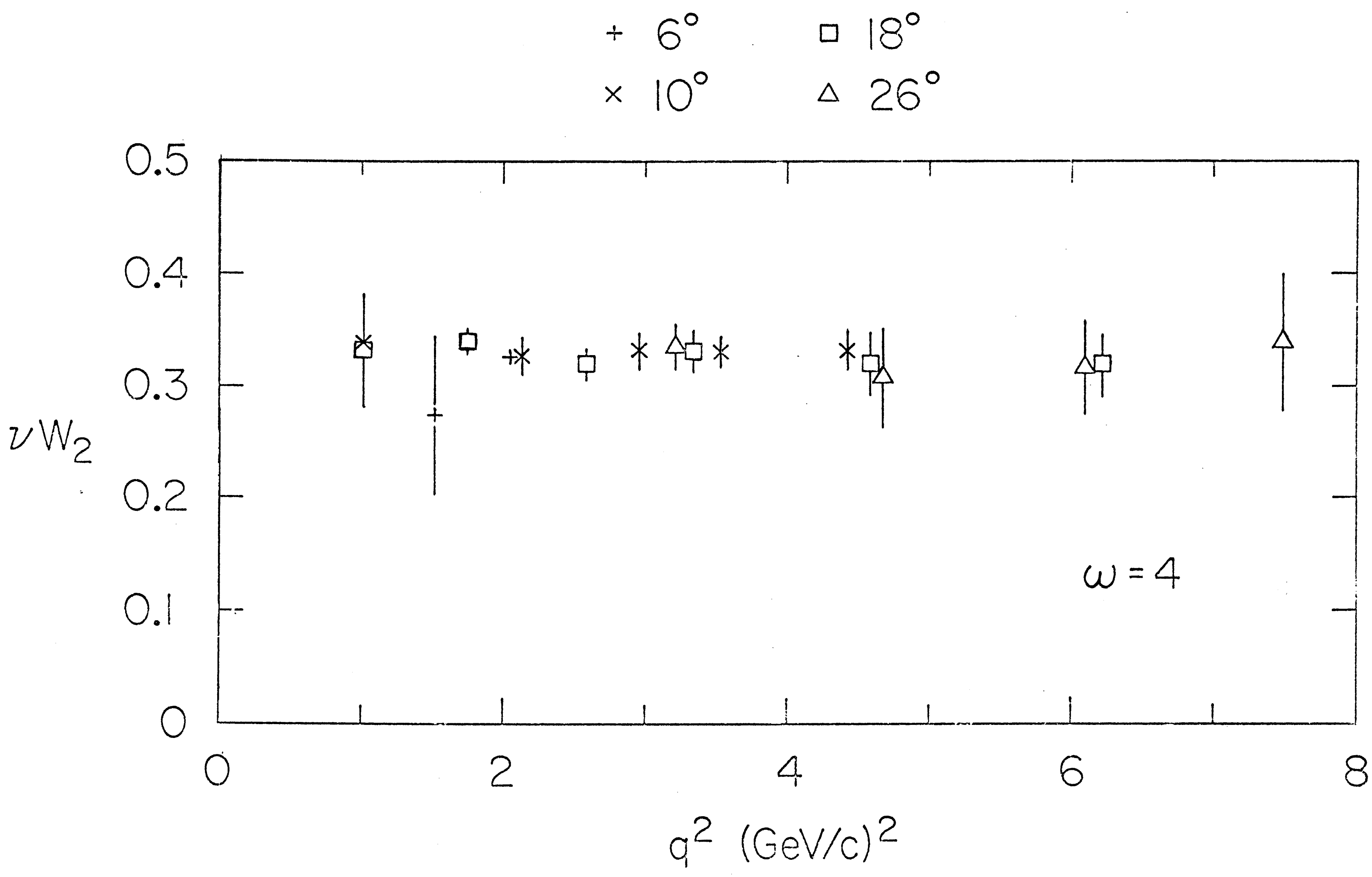

Fig. 8 - $\nu/M\ V_1^{ep}$ versus q^2 $\omega = 4$

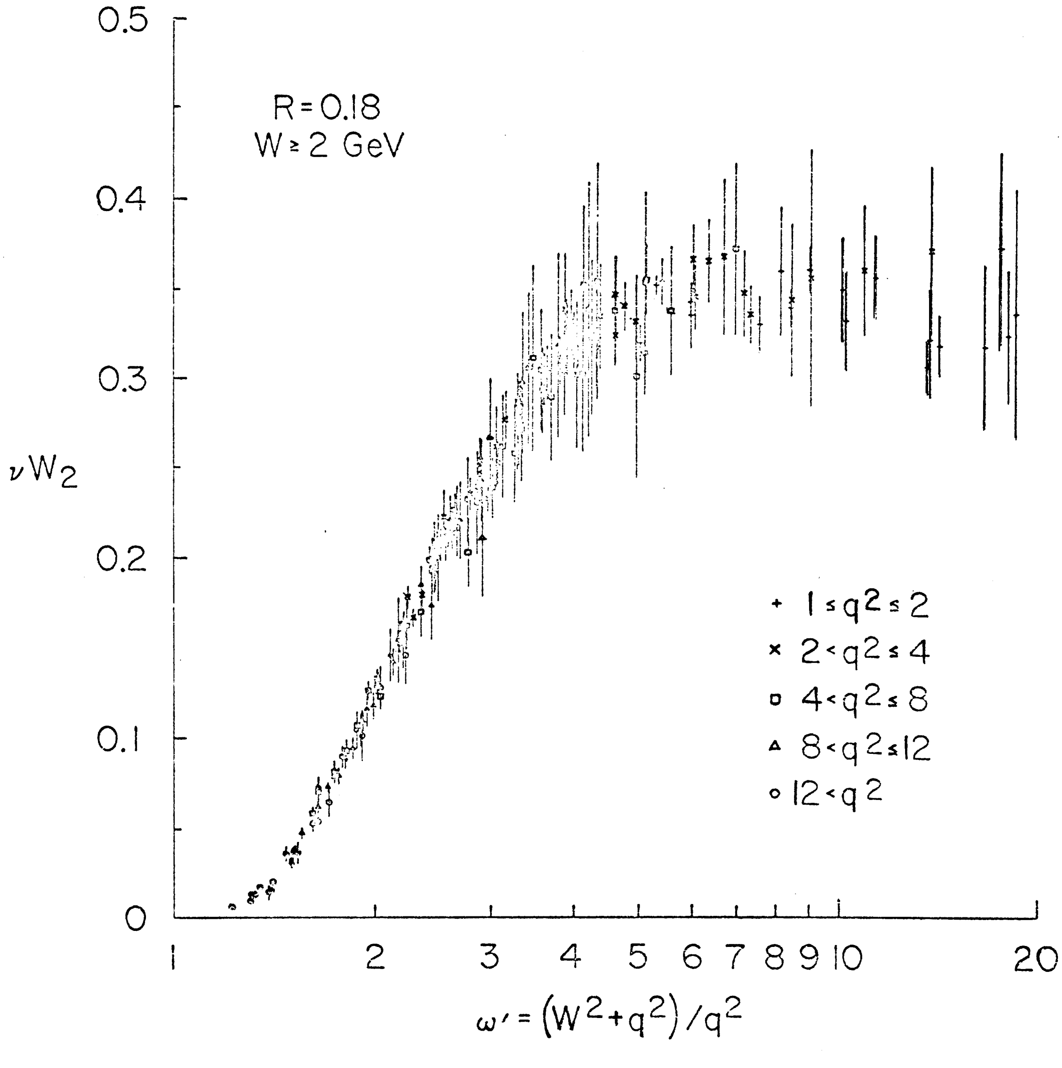

Fig. 9 - $\nu/M\ V_1^{ep}$ versus ω'

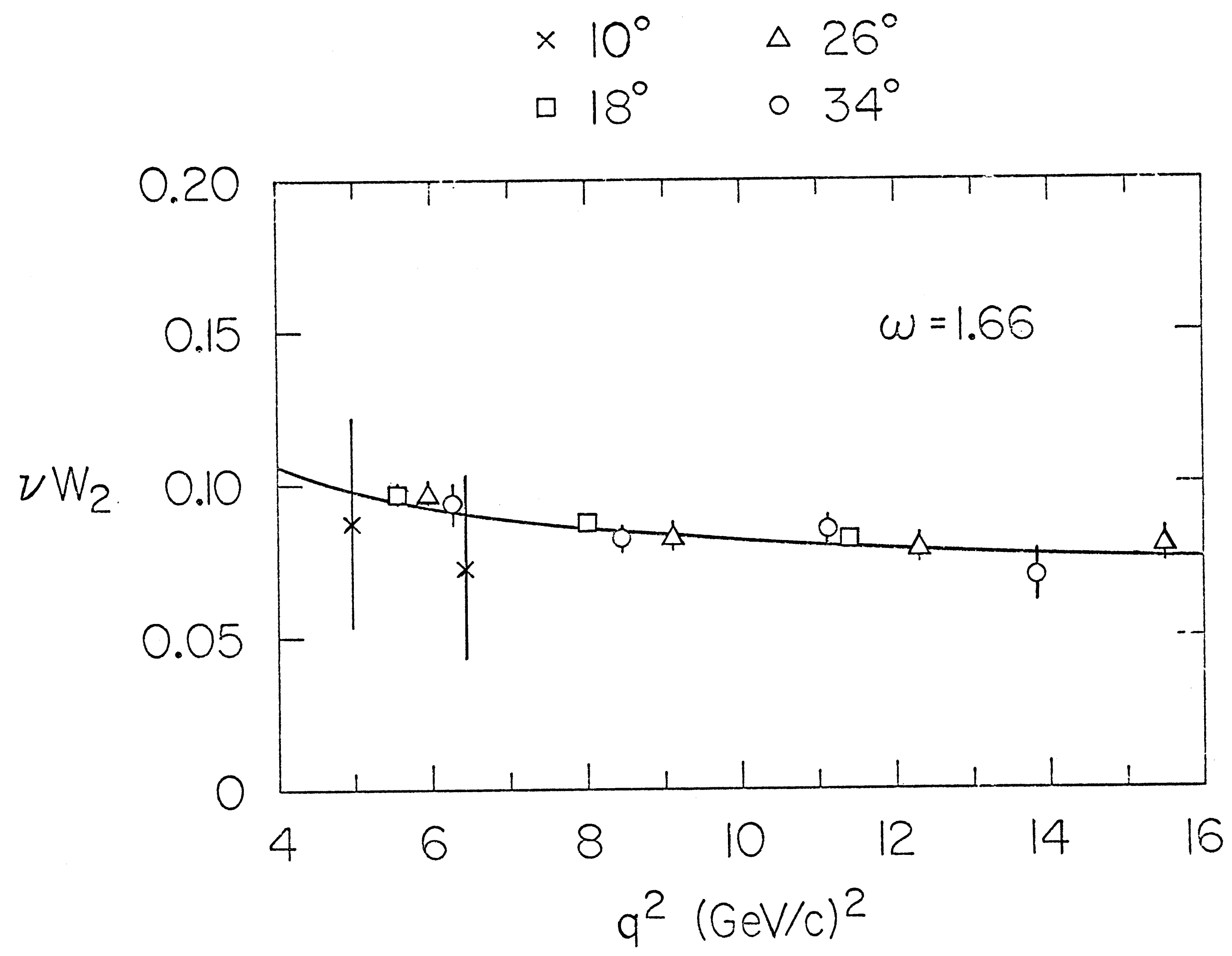

Fig. 10 - $\nu/M\ V_1^{ep}$ versus q^2 $\omega = 1.66$

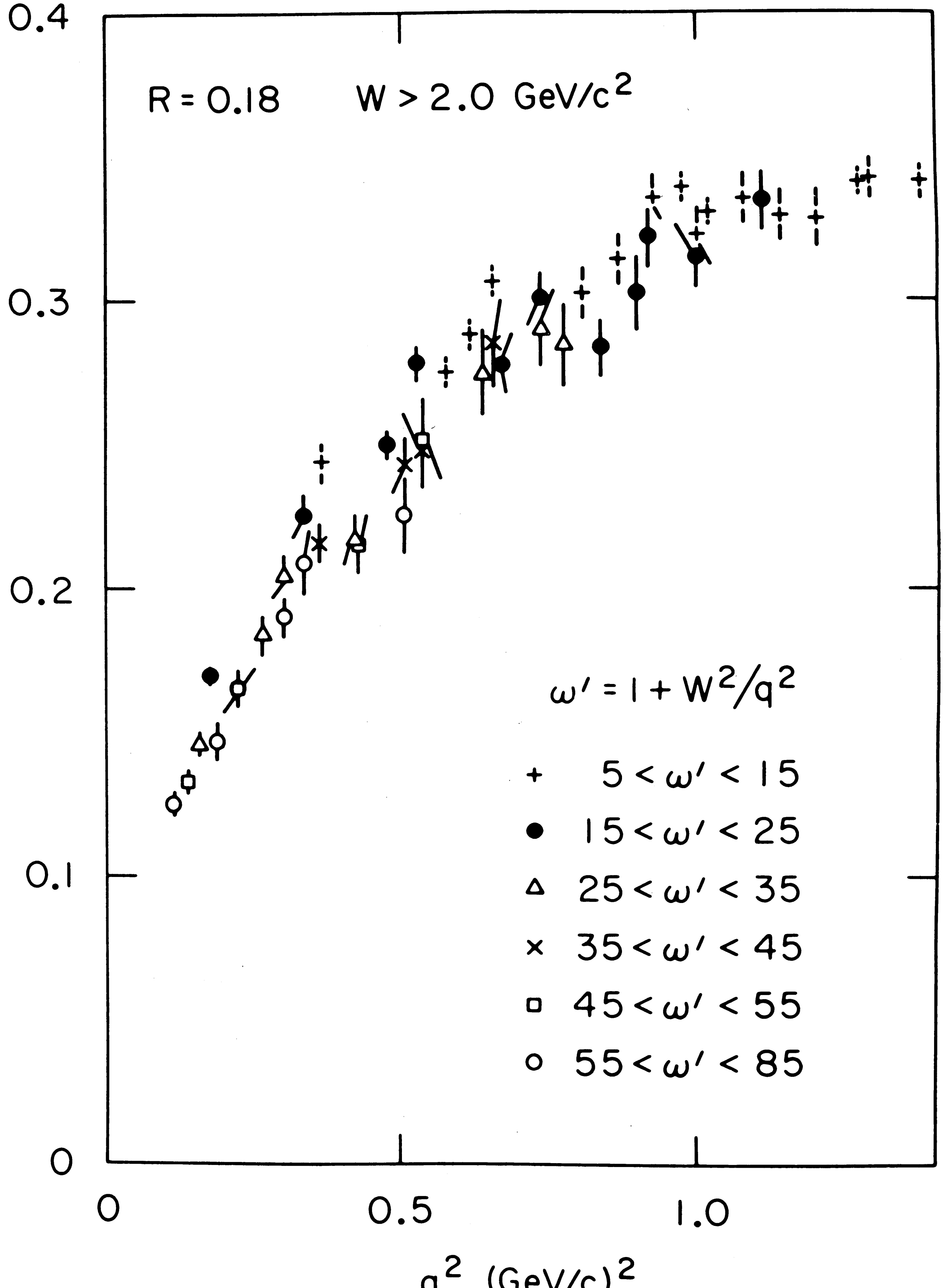

Fig. 11 - $\nu/M\ V_1^{ep}$ versus $q^2 < 1\ \mathrm{GeV}^2$

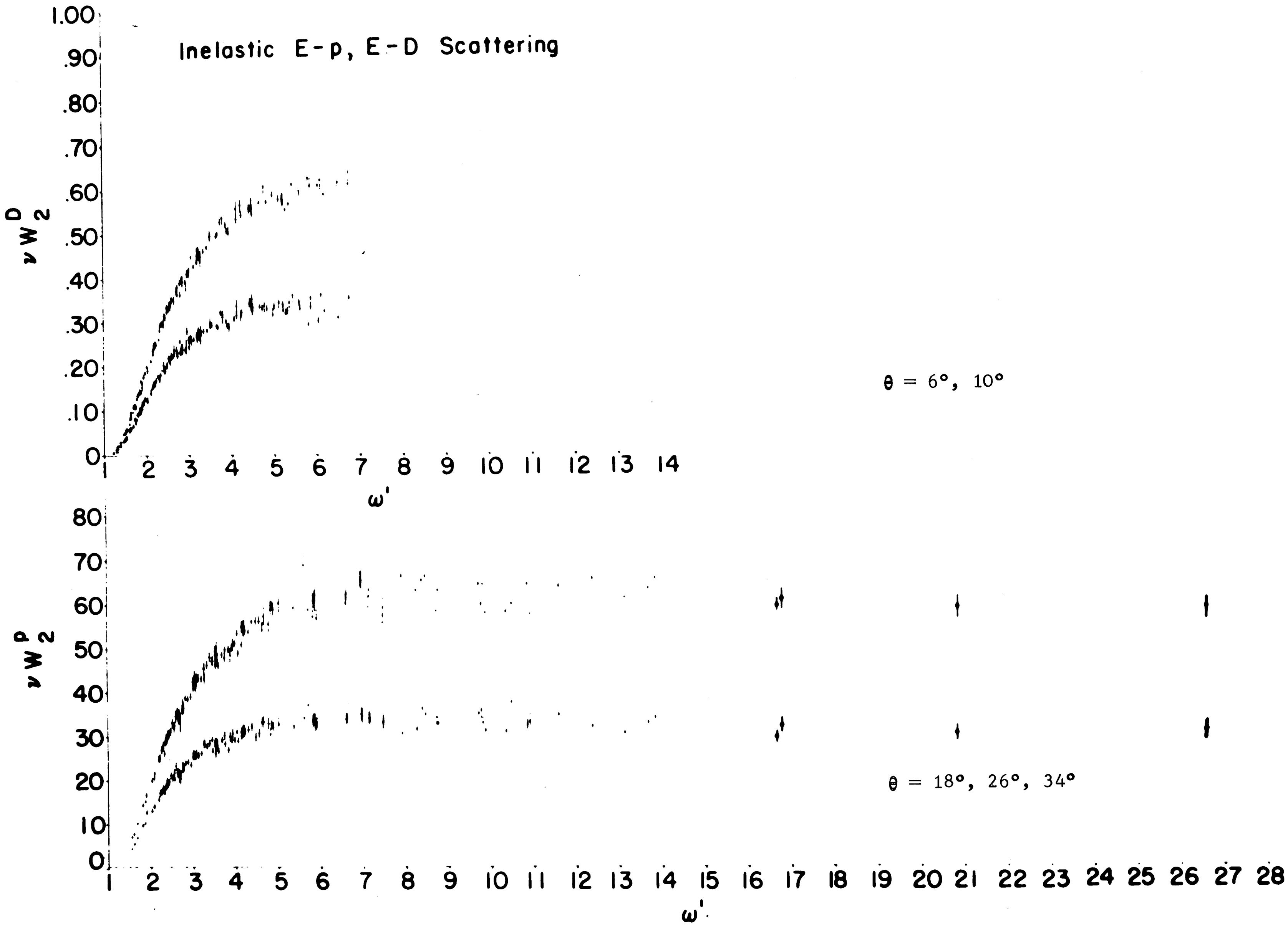

Fig. 12 - $\nu/M\ V_1^{eD}$ and $\nu/M\ V_1^{ep}$ versus ω'

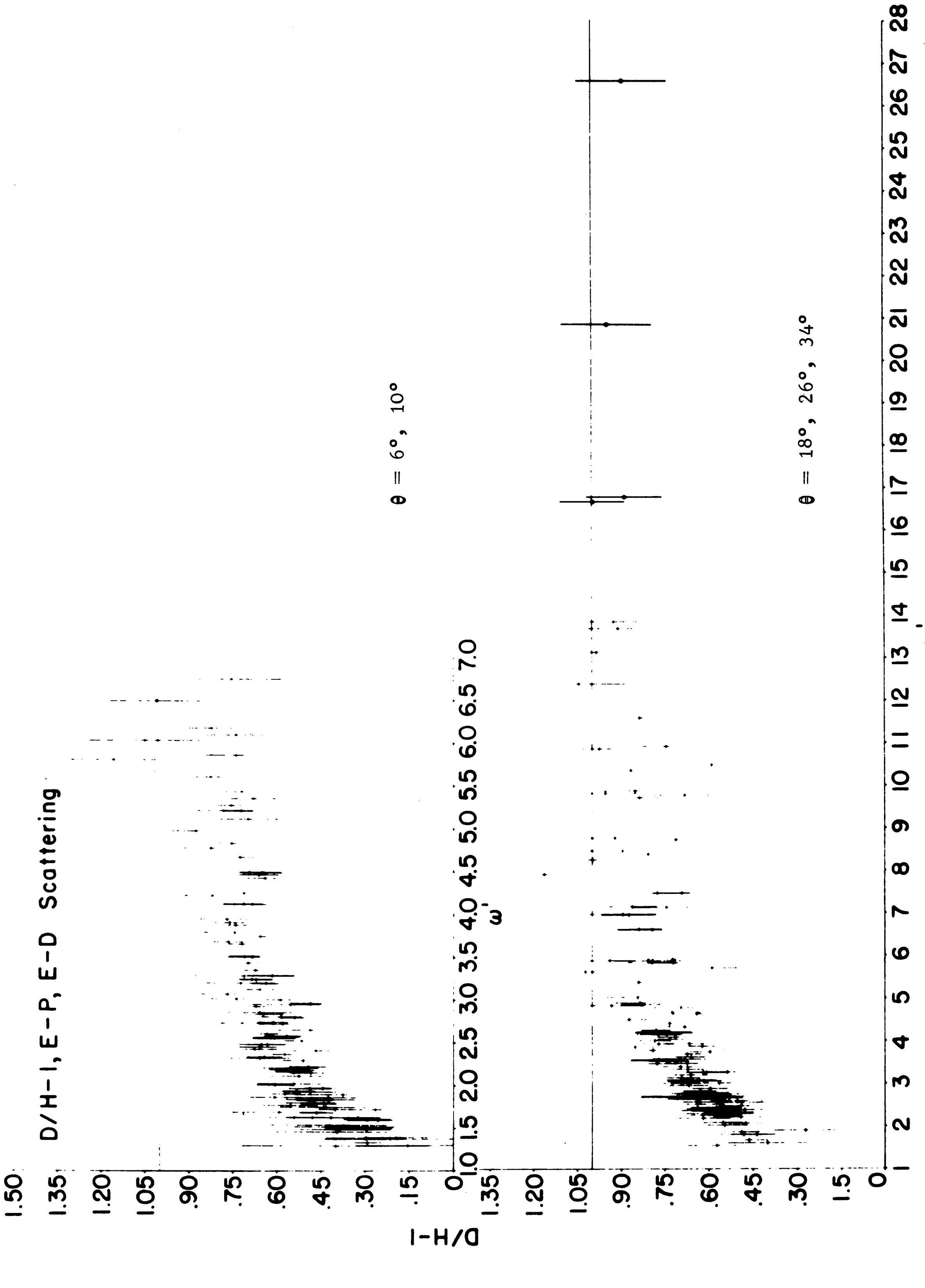

Fig. 13 - V_1^{en}/V_1^{ep} versus ω'

LESSON IV

PARTON MODEL

1. PARTON MODEL

1°) The hadron is assumed to be a composite system of elementary constituants called partons. The structure functions are Lorentz invariant quantities so that they can be computed in any frame of reference.

A simple description of the hadron occurs in the $P \Rightarrow \infty$ system where the hadron momentum P becomes very large as compared to the hadron mass M. The partons appear to be quasi free particles and we can use the impulse approximation for the interaction of the electromagnetic current with the hadron. The partons have an instantaneous interaction with the current which is point-like and only the parton electric charge can be seen by the current. After interaction the partons gain a transverse momentum $\sqrt{q^2}$ and they remain quasi free on mass shell.

2°) In the $P \Rightarrow \infty$ frame, each parton α moves along $\vec{P}$ with a transverse momentum very small as compared with its longitudinal momentum $\vec{p}_\alpha$ which is a fraction of $\vec{P}$

$$\vec{p}_\alpha = x_\alpha \vec{P} \qquad\qquad 0 \leqslant x_\alpha \leqslant 1 \tag{IV.1}$$

The conservation of momentum implies

$$\sum_\alpha x_\alpha = 1 \tag{IV.2}$$

To leading order in P the parton energy is also a fraction x_α of the hadron energy so that equation (IV.1) can be used for the energy momentum four vector

$$P_\alpha \simeq x_\alpha P \tag{IV.3}$$

After interaction with the electromagnetic current, the energy momentum of the parton becomes $P_\alpha + q$. The parton remaining on mass shell

$$P_\alpha^2 \simeq (P_\alpha + q)^2$$

or

$$q^2 + 2q \cdot P_\alpha = 0$$

With the relation (IV.3), we obtain

$$x_\alpha \simeq \frac{q^2}{2M\nu} = \xi \tag{IV.4}$$

so that the scaling variable ξ is associated, in the parton model, to the distribution of longitudinal momentum in the $P \Rightarrow \infty$ system.

3°) The main condition for the impulse approximation to be valid is that the time of interaction of the current with a parton must be small as compared with the typical lifetime of metastable states in the hadron. In other words, the effective mass W of the final hadronic system must be large as compared with a typical resonance energy W_R, $W \gg W_R$ so that the lepton scattering must be deeply inelastic.

2. STRUCTURE FUNCTIONS V_1 AND V_2

1°) Let us first consider a configuration of the hadron with N partons. The parton longitudinal momentum distribution is described by a N dimensional correlation function $f^N(x_1, x_2, \ldots, x_N)$. The normalization condition is written taking account of the conservation of momentum constraint (IV.2)

$$\iint \cdots \int dx_1\, dx_2 \ldots dx_N\, f^N(x_1, x_2, \ldots, x_N)\, \delta\left(\sum_\alpha x_\alpha - 1\right) = 1 \tag{IV.5}$$

The density of probability for the parton α to have the longitudinal momentum $x\vec{P}$ in the N parton configuration is simply obtained integrating f^N over all variables but x_α

$$f_\alpha^N(x) = \iint \cdots \int dx_1\, dx_2 \cdots dx_N\, f^N(x_1, x_2, \ldots, x_N)\, \delta(x - x_\alpha)\, \delta\left(\sum_\beta x_\beta - 1\right) \tag{IV.6}$$

and from (IV.5) we deduce the obvious normalization condition

$$\int_0^1 f_\alpha^N(x)\, dx = 1 \tag{IV.7}$$

Another interesting property concerns the first moment of the distribution $f_\alpha^N(x)$ defined by

$$\bar{x}_\alpha = \int_0^1 x\, f_\alpha^N(x)\, dx \tag{IV.8}$$

Using the definition (IV.6) of $f_\alpha^N(x)$ and the normalization condition (IV.5), it is straightforward to prove the equality

$$\sum_\alpha \bar{x}_\alpha = 1 \tag{IV.9}$$

2°) With these concepts and definitions, the computation of the unpolarized structure functions is straightforward. Using the impulse approximation we simply add incoherently the parton contributions

$$V_1(q^2,W^2) = \sum_N P_N \sum_\alpha \int_0^1 \frac{dx}{x} f_\alpha^N(x)\, V_1^\alpha(q^2,\nu;x) \qquad \text{(IV.10)}$$

where P_N is the probability of finding, in the hadron, a configuration with N partons normalized so that

$$\sum_N P_N = 1 \qquad \text{(IV.11)}$$

The parton α contribution $V_1^\alpha(q^2,\nu;x)$ is computed for a point-like interaction and therefore it is proportional to the squared charge Q_α^2 of the parton α.

3°) The function $V_1^\alpha(q^2,\nu;x)$ is independent of the parton spin and using, for instance, the result (I.16) we get

$$V_1^\alpha(q^2,\nu;x) = \frac{M}{\nu}\, x^2\, \delta(x-\xi)\, Q_\alpha^2 \qquad \text{(IV.12)}$$

Combining equations (IV.10) and (IV.12);we obtain the expression of the structure function V_1 in the deep inelastic region

$$\frac{\nu}{M} V_1(q^2,W^2) = \xi \sum_N P_N \sum_\alpha f_\alpha^N(\xi)\, Q_\alpha^2 \qquad \text{(IV.13)}$$

The combination $\frac{\nu}{M} V_1$ is a function of the variable ξ only as predicted by the Bjorken scaling. Using the scaling limit (III.3) we get

$$2\left[F_T(\xi)+F_L(\xi)\right] = \sum_N P_N \sum_\alpha f_\alpha^N(\xi)\, Q_\alpha^2 \qquad \text{(IV.14)}$$

4°) The calculation of $V_2^\alpha(q^2,\nu;x)$ depends on the parton spin. For a spin zero parton $V_2^\alpha = 0$ and this result is trivially understood using helicity arguments. For a spin ½ parton V_2^α is computed from (I.17) to be

$$V_2^{\alpha}(q^2, \nu; x) = x\,\delta(x - \xi)\, Q_\alpha^2 \tag{IV.15}$$

and the spin ½ parton contribution to the structure function V_2 is written as

$$V_2(q^2, W^2) = \sum_N P_N \sum_{\alpha(\frac{1}{2})}' f_\alpha^N(\xi)\, Q_\alpha^2 \tag{IV.16}$$

In the deep inelastic region $V_2(q^2, W^2)$ becomes function of the variable ξ only as predicted by the Bjorken scaling. In what follows, we shall consider parton models with only spin 0 and spin ½ partons. Using the scaling limit (III.3) we get

$$2F_T(\xi) = \sum_N P_N \sum_{\alpha(\frac{1}{2})}' f_\alpha^N(\xi)\, Q_\alpha^2 \tag{IV.17}$$

Comparing now (IV.14) and (IV.17) we see that the spin 0 partons contribute only to the longitudinal scaling function $F_L(\xi)$ and the spin ½ partons contribute only to the transverse scaling function $F_T(\xi)$

3. POLARIZATION EFFECTS FOR A SPIN ½ HADRON

1°) As previously, we study the case of a spin ½ hadronic target. Obviously the spinless partons cannot contribute to the structure functions for polarization. Therefore, using the positivity constraints (III.7) we easily see that with spin ½ partons the scaling functions $P_N(\xi)$ and $P_\perp(\xi)$ vanish. We then have to consider only the structure function $X_1(q^2, W^2)$ in the deep inelastic region.

2°) Measuring the spins along the hadron momentum $\vec{P}$ we introduce a spin distribution $S_\alpha^N(\sigma)$ which is the probability for the parton α in the

N parton configuration to have a spin parallel $\sigma = +1$ or antiparallel $\sigma = -1$ to the hadron spin. The conservation of probabilities implies

$$S_\alpha^N(1) + S_\alpha^N(-1) = 1 \tag{IV.18}$$

Moreover, if we assume the conservation of spin

$$\sum_\alpha{}' \sigma_\alpha = 1 \tag{IV.19}$$

it can be shown, using the same techniques as in section 2 that the mean value of σ_α in the N parton configuration

$$\bar{\sigma}_\alpha = \sum_\sigma{}' \sigma S_\alpha^N(\sigma) \tag{IV.20}$$

satisfies a relation analogous to (IV.21)

$$\sum_\alpha{}' \bar{\sigma}_\alpha = 1 \tag{IV.21}$$

3°) Adding incoherently the parton contributions as before, we write the structure function $X_1(q^2, W^2)$ as

$$X_1(q^2, W^2) = \sum_N{}' P_N \sum_\alpha{}' \int_0^1 \frac{dx}{x} f_\alpha^N(x) \sum_\sigma{}' S_\alpha^N(\sigma) X_1^\alpha(q^2, \nu; x, \sigma) \tag{IV.22}$$

From equation (I.35) the parton α contribution is computed to be

$$X_1^\alpha(q^2, \nu; x, \sigma) = \sigma x \frac{M}{\nu} \delta(x - \xi) Q_\alpha^2 \tag{IV.23}$$

Combining now equations (IV.22) and (IV.23), we obtain the expression of the structure function X_1 in the deep inelastic region

$$\frac{\nu}{M} X_1(q^2, W^2) = \sum_N{}' P_N \sum_{\alpha(\frac{1}{2})}{}' f_\alpha^N(\xi) \bar{\sigma}_\alpha Q_\alpha^2 \tag{IV.24}$$

The combination $\frac{\nu}{M} X_1$ is a function of the variable ξ only as predicted by the scaling limit (III.1) and we get

$$2\mathcal{P}_{\parallel}(\xi) = \sum_N{}' \mathcal{P}_N \sum_{\alpha(\frac{1}{2})}{}' f_\alpha^N(\xi)\,\bar{\sigma}_\alpha\, Q_\alpha^2 \qquad \text{(IV.25)}$$

From equations (IV.18) and (IV.20) it is easy to prove that each individual mean value $\bar{\sigma}_\alpha$ is bounded in modulus by unity. The comparison of equations (IV.17) and (IV.25) gives immediately the positivity constraint (III.7)

$$|\mathcal{P}_{\parallel}(\xi)| \leqslant F_T(\xi)$$

4. <u>SUM RULES</u>

1°) The existence of the normalization integral (IV.7)

$$\int_0^1 f_\alpha^N(x)\,dx$$

implies a necessary condition at $x = 0$ for the distribution $f_\alpha^N(x)$

$$\lim_{x \Rightarrow 0} x f_\alpha^N(x) = 0 \qquad \text{(IV.26)}$$

We then derive two interesting consequences for the scaling functions

(a) If the number of terms in the sum $\sum_N$ is finite, the quantities $\xi F_T(\xi)$, $\xi F_L(\xi)$ and $\xi \mathcal{P}_{\parallel}(\xi)$ vanish at $\xi = 0$.

(b) If one of these quantities has a finite limit at $\xi = 0$, we must have, in the parton model, configurations with an arbitrary large number of partons.

2°) We now integrate over ξ the equality (IV.14) using the normalization condition (IV.7)

$$J = \int_0^1 2\left[F_T(\xi)+F_L(\xi)\right]d\xi = \sum_N P_N \sum_\alpha' Q_\alpha^2 \tag{IV.27}$$

The integral J measures the mean squared charge of the partons in the hadron. However, if the condition

$$\lim_{\xi\to 0} \xi\left[F_T(\xi)+F_L(\xi)\right] = 0 \tag{IV.28}$$

is not satisfied, the integral J diverges and the right-hand side of equation (IV.27) is an infinite sum of positive contributions.

3°) The experimental situation is quite inconclusive. We have shown on Fig. 7 and Fig. 12 the function $\frac{\nu}{M} V_1^{ep}(q^2,W^2)$ in the deep inelastic region. Its limiting behaviour at large $\omega = 1/\xi$ is not known and it can be finite or zero. More data are needed before concluding about the number of partons in the nucleon.

4°) We consider the first moment of the distribution $f_\alpha^N(\xi)$ and from the equality (IV.14) and the definition (IV.8) we introduce a second type of integrals

$$I = \int_0^1 2\xi\left[F_T(\xi)+F_L(\xi)\right]d\xi = \sum_N P_N \sum_\alpha' \bar{x}_\alpha Q_\alpha^2 \tag{IV.29}$$

The integral I is expected to be convergent now and if the parton electric charge Q_α is bounded in modulus by a value Q_M

$$|Q_\alpha| \leq Q_M \qquad \text{for all } \alpha\text{'s}$$

from the equality (IV.9) we derive a finite upper bound for I

$$I \leq Q_M^2 \tag{IV.30}$$

5. THE QUARK PARTON MODEL

1°) We are now interested in a specific model where the interacting partons are quarks (j = 1,2,3) and antiquarks (j = -1, -2, -3) labelled as shown on Fig. 14 where the two fundamental representations of SU(3) have been drawn.

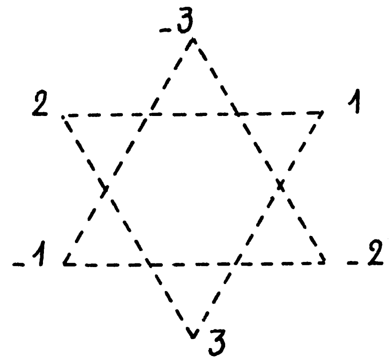

Fig. 14

Neutral particles called gluons might be present in the hadron for dynamical purpose and they have the only role of carrying a fraction of the hadron momentum, being neutral in all the other respects.

2°) Denoting by N_j the number of quarks or antiquarks of type j present in the N parton configuration, it is convenient to work with the following set of distribution functions $D_j(\xi)$ defined by

$$D_j(\xi) = \sum_N P_N \; N_j \; f_j^N(\xi) \tag{IV.31}$$

From the normalization condition (IV.7) the ξ integral of the distribution $D_j(\xi)$ is the mean value of the number of type j quarks or antiquarks in the hadron

$$D_j = \int_0^1 D_j(\xi)\, d\xi = \sum_N{}' P_N\, N_j = \langle N_j \rangle \qquad \text{(IV.32)}$$

As noticed in the previous lesson, the integrals D_j are divergent if the limit (IV.28) for the scaling functions is not satisfied. From the definition (V.1) the distributions $D_j(\xi)$ are non negative in the physical region $0 \leq \xi \leq 1$

$$D_j(\xi) \geqslant 0 \qquad \text{(IV.33)}$$

In particular the vanishing of an integral D_j implies the nonexistence of quark or antiquark of type j in the hadron. We shall call the <u>positivity</u> of the D_j 's (IV.33).

3°) The baryonic charge B, the electric charge Q and the hypercharge Y are additive quantum numbers. We know their values for quarks and antiquarks and the conservation of B, Q and Y implies three constraints on the mean number of quarks and antiquarks

$$\langle N_1 - N_{-1} \rangle = B + Q \qquad \langle N_2 - N_{-2} \rangle = B + Y - Q \qquad \langle N_3 - N_{-3} \rangle = B - Y \qquad \text{(IV.34)}$$

4°) In the parton quark model, the longitudinal scaling function $F_L(\xi)$ vanishes because of the absence of spin 0 interacting partons. We compute the transverse scaling function $F_T(\xi)$ inserting the values of the quark and antiquark electric charge into equation (IV.17). The result is

$$2 F_T(\xi) = \sum_j D_j(\xi)\, Q_j^2 \qquad \text{(IV.35)}$$

or by expliciting the Q_j^2 's

$$2F_T(\xi) = \frac{4}{9}\left[D_1(\xi) + D_{-1}(\xi)\right] + \frac{1}{9}\left[D_2(\xi) + D_{-2}(\xi) + D_3(\xi) + D_{-3}(\xi)\right] \qquad \text{(IV.36)}$$

REFERENCES

J.D. Bjorken, Lectures given at the Varenna Summer School 1967

S. Drell, Lectures given at the Erice Summer School (1969)

R.P. Feynman, Phys. Rev. Letters 23, 1415 (1969) ;

Proceedings Stony Brook Conference (1969)

M. Gourdin, Lectures given at the Erice Summer School 1971.

APPENDIX IV

1. SCALING FUNCTIONS

1°) The scaling functions are computed in the Cabibbo theory with an angle θ_c

$$F_{\pm}^{\nu}(\xi) = \sum_{N} P_N \sum_{\alpha(\frac{1}{2})}' f_{\alpha}^{N}(\xi)(1 \mp \varepsilon_\alpha)\left[\cos^2\theta_c\, I_\alpha^2 + \sin^2\theta_c\, V_\alpha^2\right]$$

$$F_{\pm}^{\bar\nu}(\xi) = \sum_{N} P_N \sum_{\alpha(\frac{1}{2})}' f_{\alpha}^{N}(\xi)(1 \mp \varepsilon_\alpha)\left[\cos^2\theta_c\, \bar I_\alpha^2 + \sin^2\theta_c\, \bar V_\alpha^2\right]$$

$$F_{L}^{\nu}(\xi) = \sum_{N} P_N \sum_{\alpha(0)}' f_{\alpha}^{N}(\xi)\left[\cos^2\theta_c\, I_\alpha^2 + \sin^2\theta_c\, V_\alpha^2\right]$$

$$F_{L}^{\bar\nu}(\xi) = \sum_{N} P_N \sum_{\alpha(0)}' f_{\alpha}^{N}(\xi)\left[\cos^2\theta_c\, \bar I_\alpha^2 + \sin^2\theta_c\, \bar V_\alpha^2\right]$$

where $\varepsilon_\alpha = +1$ for partons, $\varepsilon_\alpha = -1$ for antipartons. The weak charges are defined by the mean values of I spin and V spin operator products

$$I_\alpha^2 = \langle\alpha|\, I^- I^+ \,|\alpha\rangle \qquad V_\alpha^2 = \langle\alpha|\, V^- V^+ \,|\alpha\rangle$$

$$\bar I_\alpha^2 = \langle\alpha|\, I^+ I^- \,|\alpha\rangle \qquad \bar V_\alpha^2 = \langle\alpha|\, V^+ V^- \,|\alpha\rangle$$

2°) The Adler sum rule is a direct consequence of the I spin and V spin commutation relations

$$[I^+, I^-] = 2 I^3 \qquad [V^+, V^-] = 2 V^3$$

and in the scaling function language, it is simply written as

$$\int_0^1 \left\{ \left[F_+^{\bar\nu}(\xi) + F_-^{\bar\nu}(\xi) + 2F_L^{\bar\nu}(\xi) \right] - \left[F_+^{\nu}(\xi) + F_-^{\nu}(\xi) + 2F_L^{\nu}(\xi) \right] \right\} d\xi$$

$$= 4 \left[\cos^2\theta_c \, I^3 + \sin^2\theta_c \, V^3 \right]$$

2. QUARK PARTON MODEL

The weak charges are given in the following table

α	1	2	3	-1	-2	-3
I_α^2	0	1	0	1	0	0
$\bar I_\alpha^2$	1	0	0	0	1	0
V_α^2	0	0	1	1	0	0
$\bar V_\alpha^2$	1	0	0	0	0	1

It is convenient to separate the contributions coming from the strangeness-conserving and the strangeness-changing transitions

$$F_\pm^{\nu,\bar\nu}(\xi) = \cos^2\theta_c \, G_\pm^{\nu,\bar\nu}(\xi) + \sin^2\theta_c \, H_\pm^{\nu,\bar\nu}(\xi)$$

and the result is

$$G_+^{\nu}(\xi) = 2D_{-1}(\xi) \qquad H_+^{\nu}(\xi) = 2D_{-1}(\xi)$$

$$G_-^{\nu}(\xi) = 2D_2(\xi) \qquad H_-^{\nu}(\xi) = 2D_3(\xi)$$

$$G_+^{\bar\nu}(\xi) = 2D_{-2}(\xi) \qquad H_+^{\bar\nu}(\xi) = 2D_{-3}(\xi)$$

$$G_-^{\bar\nu}(\xi) = 2D_1(\xi) \qquad H_-^{\bar\nu}(\xi) = 2D_1(\xi)$$

For electroproduction, neutrino and antineutrino processes, one can measure nine structure functions. The number of different types of quarks and antiquarks being six we have only six distribution functions $D_{\lambda}(\xi)$ to our disposal so that the quark parton model predicts <u>three</u> relations one can, for instance, write as

$$H_+^{\nu}(\xi) = G_+^{\nu}(\xi) \qquad\qquad H_-^{\bar\nu}(\xi) = G_-^{\bar\nu}(\xi)$$

$$2F_T^{e}(\xi) = \frac{2}{9}\left[G_+^{\nu}(\xi) + G_-^{\bar\nu}(\xi)\right] + \frac{1}{18}\left[G_-^{\nu}(\xi) + G_+^{\bar\nu}(\xi) + H_-^{\nu}(\xi) + H_+^{\bar\nu}(\xi)\right]$$

These relations are strict tests of the most general quark parton model.

LESSON V

SCALING FUNCTIONS FOR A NUCLEON TARGET

1. NUCLEON SCALING FUNCTIONS FOR UNPOLARIZED SCATTERING

1°) When the target is a nucleon we use charge symmetries to relate the proton and neutron distributions as follows

$$D^n_{\pm1}(\xi) = D^p_{\pm2}(\xi) \qquad D^n_{\pm2}(\xi) = D^p_{\pm1}(\xi) \qquad D^n_{\pm3}(\xi) = D^p_{\pm3}(\xi) \tag{V.1}$$

We shall use the six proton distribution functions $D^p_j(\xi)$ to describe the proton and neutron scaling functions

$$\begin{aligned} 2F_T^{ep}(\xi) &= \frac{4}{9}\left[D^p_1(\xi)+D^p_{-1}(\xi)\right] + \frac{1}{9}\left[D^p_2(\xi)+D^p_{-2}(\xi)+D^p_3(\xi)+D^p_{-3}(\xi)\right] \\ 2F_T^{en}(\xi) &= \frac{4}{9}\left[D^p_2(\xi)+D^p_{-2}(\xi)\right] + \frac{1}{9}\left[D^p_1(\xi)+D^p_{-1}(\xi)+D^p_3(\xi)+D^p_{-3}(\xi)\right] \end{aligned} \tag{V.2}$$

As a consequence of charge symmetry and of the positivity of the D_j 's we have the inequality

$$\frac{1}{4} < \frac{F_T^{en}(\xi)}{F_T^{ep}(\xi)} < 4 \tag{V.3}$$

or in terms of structure functions

$$\frac{1}{4} < \frac{V_1^{en}(q^2,W^2)}{V_1^{ep}(q^2,W^2)} < 4 \tag{V.4}$$

in the deep inelastic region. Experimentally this ratio seems to decrease when ξ approaches unity. Its limit at $\xi = 1$ obtained by extrapolation of the data is compatible with zero or a small number. The lower bound 1/4 is certainly not excluded by the present data. If, however, it turns out that for one particular value of ξ between 0 and 1 the ratio V_1^{en}/V_1^{ep} is definitively less than 1/4 the quark parton model would have to be given up.

2°) We now integrate the scaling functions (V.7) over ξ assuming the integrals to be convergent. Using (V.2) we get

$$J^{ep} = \frac{4}{9} \langle N_1^{p} + N_{-1}^{p} \rangle + \frac{1}{9} \langle N_2^{p} + N_{-2}^{p} + N_3^{p} + N_{-3}^{p} \rangle$$
$$J^{en} = \frac{4}{9} \langle N_2^{p} + N_{-2}^{p} \rangle + \frac{1}{9} \langle N_1^{p} + N_{-1}^{p} + N_3^{p} + N_{-3}^{p} \rangle \qquad (V.5)$$

On the other hand, the mean numbers of quarks and antiquarks are related by (IV.34)

$$\langle N_1^{p} \rangle = 2 + \langle N_{-1}^{p} \rangle \qquad \langle N_2^{p} \rangle = 1 + \langle N_{-2}^{p} \rangle \qquad \langle N_3^{p} \rangle = \langle N_{-3}^{p} \rangle \qquad (V.6)$$

The positivity of the mean numbers of antiquarks implies the inequalities

$$J^{ep} \geqslant 1 \qquad J^{en} \geqslant \frac{2}{3} \qquad (V.7)$$

Obviously, the constraint (V.3) holds for the integrals J

$$\frac{1}{4} \leqslant \frac{J^{en}}{J^{ep}} \leqslant 4 \qquad (V.8)$$

The domain of allowed values of J^{ep} and J^{en} is shown on Fig. 15.

The experimental values of J^{ep} and J^{en} are very sensitive to the lower limit ξ_m used to compute the integral

$$\begin{aligned} J^{ep} &= 0.78 \pm 0.04 && \text{for } \xi_m = 0.05 \\ J^{ep} &= 1 \pm 0.15 && \text{for } \xi_m = 0.02 \\ J^{en} &= .6 - .8 && \text{depending on } \xi_m \end{aligned} \qquad (V.9)$$

There is no violation of the bounds (V.12) and the great sensitivity of the integrals to ξ_m might be an indication that the integrals diverge or, at least, converge slowly.

An interesting quantity, which must be convergent, is the difference $J^{ep} - J^{en}$. From (V.5) and (V.6) we have

$$J^{ep} - J^{en} = \frac{1}{3} + \frac{2}{3} \langle N_{-1}^{b} - N_{-2}^{b} \rangle \tag{V.10}$$

Experimentally this difference is possibly less than 1/3 and it does not exhibit an evident sensitivity to ξ_m.

3°) We study the first moment of the quark and antiquark distribution functions defined by

$$d_j = \int_0^1 \xi \, D_j(\xi) d\xi \tag{V.11}$$

These integrals are expected to be convergent so that we are in a more comfortable position to make useful statements.

The first moment relation (IV.9) is written in the quark parton language as

$$\sum_j{}' d_j + \sum_{gluons}{}' \bar{x}_\alpha = 1 \tag{V.12}$$

It is then convenient to introduce a parameter ε measuring, in some sense, the amount of gluons in the hadron

$$\varepsilon = \sum_{gluons}{}' \bar{x}_\alpha \qquad\qquad \sum_j{}' d_j = 1 - \varepsilon \tag{V.13}$$

From the positivity of all the first moments of distributions, we deduce the allowed range of variation of ε

$$0 \leqslant \varepsilon < 1 \qquad (V.14)$$

In particular, $\varepsilon = 0$ corresponds, in the quark parton model, to a hadron made only of quarks and antiquarks and $\varepsilon \neq 0$ implies the existence of gluons in this model.

The integrals I defined in (IV.29) have the following expression from (V.2)

$$\begin{aligned} I^{ep} &= \frac{1}{3}(d_1^p + d_{-1}^p) + \frac{1}{9}(1-\varepsilon) \\ I^{en} &= \frac{1}{3}(d_2^p + d_{-2}^p) + \frac{1}{9}(1-\varepsilon) \end{aligned} \qquad (V.15)$$

Using charge symmetry, we deduce the following inequality from the positivity of the d_j 's

$$\frac{2}{9}(1-\varepsilon) < I^{ep} + I^{en} \leqslant \frac{5}{9}(1-\varepsilon) \qquad (V.16)$$

The equality of the upper limit holds when no strange quarks and antiquarks $(j = \pm 3)$ are present in the nucleon.

Equations (V.14) and (V.16) imply the existence of an absolute bound

$$0 < I^{ep} + I^{en} \leqslant \frac{5}{9} \qquad (V.17)$$

and limits on ε when the sum $I^{ep} + I^{en}$ is known from experiment

$$0 \leqslant \varepsilon \leqslant 1 - \frac{9}{5}(I^{ep} + I^{en}) \qquad (V.18)$$

Again the constraint (V.3) holds for the integrals I

$$\frac{1}{4} \leqslant \frac{I^{en}}{I^{cp}} \leqslant 4 \qquad (V.19)$$

The domain of allowed values of I^{ep} and I^{en} is shown on Fig. 16.

The experimental situation is the following

$$I^{ep} \simeq 0.172 \pm 0.009 \qquad \text{with } \xi_m = 0.05$$
$$I^{en} \simeq 2/3\ I^{ep} \qquad (V.20)$$

so that good estimates of the complete integrals can be

$$I^{ep} \simeq 0.18 \pm 0.018$$
$$I^{en} \simeq 0.12 \pm 0.012 \qquad (V.21)$$
$$I^{ep} + I^{en} \simeq 0.30 \pm 0.03$$

The absolute upper bound (V.20) is easily satisfied and the upper limit obtained for ε in the one standard deviation limit is

$$\varepsilon < 0.52$$

2. POLARIZATION EFFECTS

1°) We define new distribution functions $D_{j\sigma}(\xi)$ by

$$D_{j\sigma}(\xi) = \sum_{N} P_N\, N_j\, f_j^N(\xi)\, S_j^N(\sigma) \qquad (V.22)$$

and the scaling function $P_{\parallel}(\xi)$ is given from (IV.25) by

$$2\, P_{\parallel}(\xi) = \sum_{j,\sigma} Q_j^2\, \sigma\, D_{j\sigma}(\xi) \qquad (V.23)$$

2°) Let us integrate the scaling function $P_{\parallel}(\xi)$ over ξ

$$Z = \int_0^1 z\, P_{\shortparallel}(\xi)\, d\xi \tag{V.24}$$

Using equations (V.22), (V.23) and the normalization condition (IV.7), we can write Z in the convenient form

$$Z = \sum_{j,\sigma} D_{j\sigma} \langle j,\sigma | \sigma_3 Q^2 | j,\sigma \rangle \tag{V.25}$$

where the $D_{j\sigma}$ can be interpreted as the mean values, in the hadron, of the number of quarks and antiquarks of type j having a spin parallel ($\sigma = +1$) or antiparallel ($\sigma = -1$) to the hadron spin. Therefore the integral Z is the average value, in the hadron, of the operator $\sigma_3 Q^2$

By definition of the symmetric coefficients of the quark model algebra, Z can be decomposed into

$$Z = \frac{1}{2} d^{QQ}{}_{\gamma}\, g_A^{\gamma} = \frac{2}{3} g_A^{B} + \frac{1}{3} g_A^{Q} \tag{V.26}$$

where g_A^{γ} defined by

$$g_A^{\gamma} = \sum_{j,\sigma}' D_{j\sigma}\, \varepsilon_j \langle j,\sigma | \sigma_3 F^{\gamma} | j,\sigma \rangle \tag{V.27}$$

is the average value of the U(6) algebra operator $\sigma_3 F^{\gamma}$. Here ε_j is +1 for quarks and -1 for antiquarks.

It follows that g_A^{γ} is the coupling constant of the axial vector current of U(3) index γ .

Equation (V.26) is identical to the Bjorken sum rule for polarization already derived in Lesson II from the quark model algebra.

3°) The coupling constant g_A^{B} is the axial vector baryonic coupling constant and it cannot be directly measured. The octuplet coupling constant g_A^{Q} depends in general on two reduced matrix elements

$$g_A^Q = f_a\, Q + f_s\, \Omega_Q^{(8S)} \qquad (V.28)$$

where $\Omega_Q^{(8S)}$ is the symmetric isometry of weight Q.

In the case of the baryon octuplet $J^P = \frac{1}{2}^+$ to which the nucleon belongs, the combination $f_a + f_s$ is the axial vector part for neutron β decay when the vector part is normalized to unity. Various combinations of f_a and f_s describe the hyperon β decay axial vector amplitudes and from an overall fit we get

$$f_a + f_s = 1.23 \qquad\qquad f_s \simeq 0.6\,(f_a + f_s)$$

4°) The integrals Z for a nucleon target are given from equations (V.26) and (V.28) by

$$\begin{aligned} Z^{ep} &= \frac{2}{3}\, g_A^B + \frac{1}{3}\left(f_a + \frac{1}{3} f_s\right) \\ Z^{en} &= \frac{2}{3}\, g_A^B - \frac{2}{9}\, f_s \end{aligned} \qquad (V.29)$$

Considering the difference $Z^{ep} - Z^{en}$ we get a prediction independent of g_A^B

$$Z^{ep} - Z^{en} = \frac{1}{3}\,(f_a + f_s) \simeq 0.41 \qquad (V.30)$$

The constant g_A^B being associated to the baryonic axial vector current is given from equation (V.27) by

$$g_A^B = \frac{1}{3} \sum_{j,\sigma} \sigma\, D_{j\sigma} \qquad (V.31)$$

If we assume the conservation of spin without correlation between the gluon spin and the nucleon spin, we obtain, from (IV.21) the value 1/3 for g_A^B . It follows the two predictions

$$Z^{ep} \simeq 0.47 \qquad\qquad Z^{en} \simeq 0.06 \tag{V.32}$$

so that Z^{ep} and Z^{en} differ by essentially one order of magnitude.

3. ISOTOPIC SPIN SYMMETRY

1°) In order to study the consequences of symmetries it is convenient to consider a forward scattering amplitude with different U(3) indices for the current and for the hadrons

$$\gamma^{\ell_1 m_1} + \alpha_1 \Rightarrow \gamma^{\ell_2 m_2} + \alpha_2$$

In the quark-antiquark representation the system ℓm is associated to one of the nine U(3) generators (ℓ, m = 1,2,3)

The scaling function generalizing (V.2) is given by

$$F_T^{\ell_2 m_2, \alpha_2 ; \ell_1 m_1, \alpha_1}(\xi) = \sum_{k,j}' D_{kj}^{\alpha_2 \alpha_1}(\xi) < k | F^{m_2 \ell_2} F^{\ell_1 m_1} | j > \tag{V.33}$$

where now $D_{kj}^{\alpha_2 \alpha_1}(\xi)$ is a matrix distribution of quarks and antiquarks in the hadron multiplet.

The $F^{\ell m}$ are the infinitesimal generators of the U(3) Lie algebra and in the two three-dimensional representations of quarks and antiquarks they have the explicit matrix form

$$F^{\ell m} = | \ell > < m | \qquad \text{for quarks}$$

$$F^{\ell m} = - | m > < \ell | \qquad \text{for antiquarks}$$

Substituting now in equation (V.33) we exhibit the quark and antiquark contributions to F_T

$$F_T^{l_2 m_2, \alpha_2; l_1 m_1, \alpha_1}(\xi) = \delta_{l_2 l_1} D^{\alpha_2 \alpha_1}_{m_2 m_1}(\xi) + \delta_{m_2 m_1} D^{\alpha_2 \alpha_1}_{-l_2 -l_1}(\xi) \tag{V.34}$$

The positive definite character of the hermitian matrix F_T implies an analogous property for the quark distribution matrix $D^{\alpha_2 \alpha_1}_{m_2 m_1}(\xi)$ and the antiquark distribution matrix $D^{\alpha_2 \alpha_1}_{-l_2 -l_1}(\xi)$.

2°) We consider, for simplicity, only the case of the symmetry group $SU(2) \otimes U(1)$ of isotopic spin and hypercharge. The strange quarks and antiquarks ($j = \pm 3$) are decoupled from the doublets of non-strange quarks and non-strange antiquarks ($j = \pm 1, \pm 2$).

Using the well-known product of representations in SU(2)

$$D(\tfrac{1}{2}) \otimes D(\tfrac{1}{2}) = D(1) \oplus D(0)$$

we apply the Wigner-Eckart theorem and we introduce two reduced matrix elements for the quark distribution

$$D^{\alpha_2 \alpha_1}_{m_2 m_1}(\xi) = \delta_{\alpha_2 \alpha_1} \delta_{m_2 m_1} D_S(\xi) + \langle \alpha_2 | F^{m_2 m_1} | \alpha_1 \rangle D_V(\xi) \tag{V.35}$$

and two reduced matrix elements for the antiquark distribution

$$D^{\alpha_2 \alpha_1}_{-l_2 -l_1}(\xi) = \delta_{\alpha_2 \alpha_1} \delta_{l_2 l_1} \overline{D}_S(\xi) + \langle \alpha_2 | F^{l_1 l_2} | \alpha_1 \rangle \overline{D}_V(\xi) \tag{V.36}$$

The four distributions $D_S(\xi)$, $D_V(\xi)$, $\overline{D}_S(\xi)$ and $\overline{D}_V(\xi)$ are real functions of ξ in the physical range $0 \leq \xi \leq 1$.

3°) It is very easy now to apply these results to the nucleon isotopic spin doublet. From (V.35) and (V.36) we get

$$D^{\alpha_2\alpha_1}_{m_2m_1}(\xi) = \delta_{\alpha_2\alpha_1}\delta_{m_2m_1} D_s(\xi) + \delta_{\alpha_2 m_2}\delta_{\alpha_1 m_1} D_v(\xi)$$

$$D^{\alpha_2\alpha_1}_{-\ell_2-\ell_1}(\xi) = \delta_{\alpha_2\alpha_1}\delta_{\ell_2\ell_1} \overline{D}_s(\xi) + \delta_{\alpha_2\ell_1}\delta_{\alpha_1\ell_2} \overline{D}_v(\xi)$$

These matrices are reducible and the positivity inequalities are simply written as

$$D_s(\xi) \geqslant 0 \qquad\qquad \overline{D}_s(\xi) \geqslant 0$$

$$D_s(\xi) + D_v(\xi) \geqslant 0 \qquad\qquad \overline{D}_s(\xi) + \overline{D}_v(\xi) \geqslant 0$$

$$D_s(\xi) + 2D_v(\xi) \geqslant 0 \qquad\qquad \overline{D}_s(\xi) - \overline{D}_v(\xi) \geqslant 0$$

In terms of the proton distribution functions $D^p_j(\xi)$ previously used the constraints due to isotopic spin symmetry are

$$2\,D^p_1(\xi) \geqslant D^p_2(\xi) \geqslant 0 \tag{V.37}$$

$$2\,D^p_{-2}(\xi) \geqslant D^p_{-1}(\xi) \geqslant 0 \tag{V.38}$$

Of course for the strange quark and antiquark part we remain with

$$D^p_3(\xi) \geqslant 0 \qquad\qquad D^p_{-3}(\xi) \geqslant 0$$

The consideration of isotopic spin symmetry adds very little to that of charge symmetry for electroproduction and the constraints on the structure functions, the integrals J and I already obtained remain the same.

As an example of a specific consequence of isotopic spin symmetry, let us consider the $\langle N^p_{-1}\rangle$, $\langle N^p_{-2}\rangle$ plane. From charge symmetry and positivity the allowed domain was a quarter of plane $\langle N^p_{-1}\rangle \geqslant 0$, $\langle N^p_{-2}\rangle \geqslant 0$. It is now somewhat smaller as shown on Fig. 17.

REFERENCES (A highly incomplete list)

J.D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969) ; D1, 3151 (1970)

M. Gourdin, Nucl. Phys. B29, 601 (1971) ; B38, 418 (1972)
Lectures given at the Intern. School of Physics, Ettore Majorana 1971 (CERN TH. 1384)

L. Heiko and J. Pestieau, Preprint Mexico (1971)

R.L. Jaffe, SLAC-PUB 999 (1972)

J. Kuti and V.F. Weisskopf, Preprint MIT 211 (1971)

P.V. Landshoff and J.C. Polkinghorne, Nucl. Phys. B28, 240 (1971)

C. Llewellyn-Smith, Talk given at the Naples meeting 1970 (CERN TH.1188) ; Phys. Rev. D4, 2392 (1971)

C. Nash, Nucl. Phys. B31, 419 (1971)

A. Suri and D.R. Yennie, SLAC-PUB 954 (1971)

Light-Cone Approach

R. Brandt, Phys. Rev. Letters 23, 1260 (1969) ; Phys. Rev. D1, 2808 (1970)

R. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1970)

L. Brown, Boulder lectures 1969.

Y. Frishman, Phys. Rev. Letters 25, 966 (1970) ; Ann. Phys. 66, 373 (1971)

H. Fritzsch and M. Gell-Mann, Preprint CALT 68-297

R. Jakiw, R. Van Royen and G. West, Phys. Rev. D2, 2473 (1970)

H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970)

APPENDIX V

SCALING FUNCTIONS FOR UNPOLARIZED TARGET

1°) As seen in Appendix IV, the scaling functions $G_\pm$ for $\Delta S = 0$ transitions and $H_\pm$ for $\Delta S = \pm 1$ transitions depend only on one type of quark or antiquark. Taking account of the charge symmetry between proton and neutron, we get

$$G_+^{\nu p} = G_+^{\bar\nu n} = 2\, D_{-1}^{p}(\xi) \qquad H_+^{\nu p} = 2\, D_{-1}^{p}(\xi) \qquad H_+^{\nu n} = 2\, D_{-2}^{p}(\xi)$$

$$G_-^{\nu p} = G_-^{\bar\nu n} = 2\, D_{2}^{p}(\xi) \qquad H_-^{\nu p} = H_-^{\nu n} = 2\, D_{3}^{p}(\xi)$$

$$G_+^{\bar\nu p} = G_+^{\nu n} = 2\, D_{-2}^{p}(\xi) \qquad H_+^{\bar\nu p} = H_+^{\bar\nu n} = 2\, D_{-3}^{p}(\xi)$$

$$G_-^{\bar\nu p} = G_-^{\nu n} = 2\, D_{1}^{p}(\xi) \qquad H_-^{\bar\nu p} = 2\, D_{1}^{p}(\xi) \qquad H_-^{\bar\nu n} = 2\, D_{2}^{p}(\xi)$$

In particular, we can derive interesting relations between electroproduction on proton and neutron and the neutrino and antineutrino production

$$F_T^{ep} + F_T^{en} = \frac{5}{18}\left[G_T^{\nu p} + G_T^{\bar\nu p}\right] + \frac{1}{18}\left[H_+^{\bar\nu p} + H_-^{\nu p}\right]$$

$$F_T^{ep} - F_T^{en} = \frac{1}{12}\left[G_+^{\nu p} - G_-^{\nu p} - G_+^{\bar\nu p} + G_-^{\bar\nu p}\right]$$

Of course, equivalent relations can be obtained using the previous equalities between proton and neutron scaling functions for neutrino and antineutrino induced reactions. For instance the first relation gives an upper limit for $\Delta S = 0$ scaling functions and a lower limit for $\Delta S = \pm 1$ scaling functions in terms of the electroproduction scaling functions

$$\left[G_T^{\nu p} + G_T^{\nu n}\right] \leq \frac{18}{5}\left[F_T^{ep} + F_T^{en}\right] \leq \left[H_T^{\nu p} + H_T^{\nu n} + H_T^{\bar\nu p} + H_T^{\bar\nu n}\right]$$

The three quantities become equal when strange quarks and antiquarks are absent from the nucleon.

2°) We now integrate the scaling function $F_{\pm}^{\nu,\bar{\nu}}$ over ξ assuming, as previously, the integrals to be convergent

$$J_{\pm}^{\nu,\bar{\nu}}(\xi) = \int_0^1 F_{\pm}^{\nu,\bar{\nu}}(\xi)\, d\xi$$

The positivity of the mean numbers of antiquarks implies lower bounds for these integrals

$$J_+^{\nu p} \geqslant 0 \qquad J_-^{\nu p} \geqslant 2 \cos^2\theta_c \qquad J_+^{\nu n} \geqslant 0 \qquad J_-^{\nu n} \geqslant 4 \cos^2\theta_c$$

$$J_+^{\bar{\nu} p} \geqslant 0 \qquad J_-^{\bar{\nu} p} \geqslant 4 \qquad J_+^{\bar{\nu} n} \geqslant 0 \qquad J_-^{\bar{\nu} n} \geqslant 2$$

On the other hand, linear combinations of J's can be convergent even if the J's are not. This can be the case for the Adler sum rule

$$J_T^{\bar{\nu} p} - J_T^{\nu p} = \cos^2\theta_c + 2 \sin^2\theta_c$$
$$J_T^{\bar{\nu} n} - J_T^{\nu n} = -\cos^2\theta_c + \sin^2\theta_c$$

for the Gross-Llewelyn-Smith sum rule

$$J_-^{\bar{\nu} p} - J_+^{\bar{\nu} p} + J_-^{\nu p} - J_+^{\nu p} = 6 \cos^2\theta_c + 4 \sin^2\theta_c$$
$$J_-^{\bar{\nu} n} - J_+^{\bar{\nu} n} + J_-^{\nu n} - J_+^{\nu n} = 6 \cos^2\theta_c + 2 \sin^2\theta_c$$

and for the proton-neutron differences one can relate to electroproduction integrals

$$J_{+}^{\nu p} - J_{+}^{\nu n} = 3\left[J^{ep} - J^{en}\right] - 1$$

$$J_{-}^{\nu p} - J_{-}^{\nu n} = -\cos^2\theta_c\left[1 + 3\left(J^{ep} - J^{en}\right)\right]$$

$$J_{+}^{\bar{\nu} p} - J_{+}^{\bar{\nu} n} = \cos^2\theta_c\left[1 - 3\left(J^{ep} - J^{en}\right)\right]$$

$$J_{-}^{\bar{\nu} p} - J_{-}^{\bar{\nu} n} = 3\left[J^{ep} - J^{en}\right] + 1$$

Unfortunately, the experimental information is not available to allow a test of these relations.

3°) Let us study the first moments of the quark and antiquark distributions d_j. In Appendix III we have shown that the total cross sections for neutrino and antineutrino induced reactions are expected to increase linearly with the incident energy

$$\sigma_{TOT}^{\nu,\bar{\nu}}(s) \Rightarrow \frac{G^2 s}{2\pi} A^{\nu,\bar{\nu}}$$

With the decomposition into strangeness conserving and strangeness violating parts

$$A^{\nu,\bar{\nu}} \equiv \cos^2\theta_c \, B^{\nu,\bar{\nu}} + \sin^2\theta_c \, C^{\nu,\bar{\nu}}$$

we obtain

$$B^{\nu p} = \frac{2}{3} d_{-1}^{p} + 2 d_{2}^{p} \qquad B^{\nu n} = \frac{2}{3} d_{-2}^{p} + 2 d_{1}^{p}$$

$$B^{\bar{\nu} p} = \frac{2}{3} d_{1}^{p} + 2 d_{-2}^{p} \qquad B^{\bar{\nu} n} = \frac{2}{3} d_{2}^{p} + 2 d_{-1}^{p}$$

$$C^{\nu p} = \frac{2}{3} d_{-1}^{p} + 2 d_{3}^{p} \qquad C^{\nu n} = \frac{2}{3} d_{-2}^{p} + 2 d_{3}^{p}$$

$$C^{\bar{\nu} p} = \frac{2}{3} d_{1}^{p} + 2 d_{-3}^{p} \qquad C^{\bar{\nu} n} = \frac{2}{3} d_{2}^{p} + 2 d_{-3}^{p}$$

Some consequences of positivity and charge symmetry are the following

$$B^{\nu p} + B^{\nu n} \leq 2(1-\varepsilon) \qquad B^{\bar{\nu} p} + B^{\bar{\nu} n} \leq 2(1-\varepsilon)$$

$$B^{\nu p} + B^{\nu n} + B^{\bar{\nu} p} + B^{\bar{\nu} n} \leq \frac{8}{3}(1-\varepsilon)$$

$$C^{\nu p} + C^{\nu n} \leq 4(1-\varepsilon) \qquad C^{\bar{\nu} p} + C^{\bar{\nu} n} \leq 4(1-\varepsilon)$$

$$\frac{2}{3}(1-\varepsilon) \leq C^{\nu p} + C^{\nu n} + C^{\bar{\nu} p} + C^{\bar{\nu} n} \leq 4(1-\varepsilon)$$

Absolute bounds are simply derived by putting $\varepsilon = 0$ in the upper limits of the previous inequalities. Limits on ε are obtained from an experimental knowledge of the constants B and C as for instance

$$\varepsilon \leq 1 - \frac{B^{\nu p} + B^{\nu n}}{2}$$

Using our information on ε deduced from electroproduction data, we obtain a lower limit on the total strangeness changing transitions

$$C^{\nu p} + C^{\nu n} + C^{\bar{\nu} p} + C^{\bar{\nu} n} > 0.32$$

The experimental data on the propane bubble chamber experiment have been quoted in Appendix III. Taking account of the particular structure of the propane in protons and neutrons, we obtain the experimental figure

$$B^{\nu p} + B^{\nu n} = 1{,}15 \pm 0.29$$

This result satifies the absolute upper bound of 2 and it implies for an upper limit which is $\varepsilon < 0.57$, e.g. of the same magnitude as that deduced from electroproduction.

4°) We now combine the neutrino and electroproduction integrals in order to obtain the maximum of information from experiment. As an immediate consequence of general relations between structure functions quoted in the first paragraph, we get the simple inequality

$$B^{\nu p} + B^{\nu n} \leq \frac{18}{5} \left(I^{ep} + I^{en} \right)$$

From electroproduction data this inequality implies

$$B^{\nu p} + B^{\nu n} < 1.19$$

so that this upper limit is consistent with the present neutrino data.

A more detailed study of the integrals $B^{\nu p} + B^{\nu n}$ and $I^{ep} + I^{en}$ allows us to establish a double inequality leading to an improved lower limit for ε

$$1 - \frac{9}{2} \left(I^{ep} + I^{en} \right) + \frac{3}{4} \left(B^{\nu p} + B^{\nu n} \right) \leq \varepsilon \leq 1 - \frac{9}{5} \left(I^{ep} + I^{en} \right)$$

With the experimental data inserted in the one-standard deviation limit we obtain a range for

$$0.25 < \varepsilon < 0.52$$

so that gluons must be present in the nucleon to make the quark parton model consistent with experiment. Let us notice that because of the large errors in neutrino data, the value $\varepsilon = 0$ is excluded only in the two-standard deviation limit so that improved data are fully necessary to make definite statements.

5°) Predictions can now be made for antineutrino rections and proceeding as previously we deduce, for instance,

$$\frac{1}{3}\left(B^{\nu p}+B^{\nu n}\right) \leq B^{\bar{\nu} p}+B^{\bar{\nu} n} \leq \frac{24}{5}\left(I^{ep}+I^{en}\right)-\left(B^{\nu p}+B^{\nu n}\right)$$

and inserting the experimental data

$$\frac{1}{3} \leqslant \frac{B^{\bar{\nu} p}+B^{\bar{\nu} n}}{B^{\nu p}+B^{\nu n}} \leqslant 0.86$$

Let us notice that the lower limit 1/3 is a strict condition, a consequence of positivity and charge symmetry, whereas the upper limit involves experimental data. Fig. 18 shows the present situation in the $B^{\nu p} + B^{\nu n}$, $B^{\bar{\nu} p} + B^{\bar{\nu} n}$ plane. The large triangle corresponds to absolute bounds ; the oblique line is the restriction due to electroproduction with the one-standard deviation errors ; the vertical lines are the neutrino propane data and the dashed triangle is the resulting region compatible with experiments, charge symmetry and positivity.

6°) Finally, isotopic spin symmetry gives supplementary restrictions that are consequences of the inequalities

$$2\, D_1^{p}(\xi) \geqslant D_2^{p}(\xi) \qquad\qquad 2\, D_{-2}^{p}(\xi) \geqslant D_{-1}^{p}(\xi)$$

An immediate trivial consequence is

$$2\, G_{\pm}^{\bar{\nu} p}(\xi) \geqslant G_{\pm}^{\nu p}(\xi)$$

or equivalently, using charge symmetry

$$2\, G_{\pm}^{\nu n}(\xi) \geqslant G_{\pm}^{\nu p}(\xi)$$

In practice, the new constraints due to isotopic spin symmetry are not very useful because they occur in domains far from the present experimental data and their predictive power is extremely weak. Nevertheless, they must be fulfilled and as a second example for the total cross section, we get

$$2\,B^{\nu n} \geq B^{\nu p} \qquad\qquad 2\,B^{\bar{\nu} p} \geq B^{\bar{\nu} n}$$

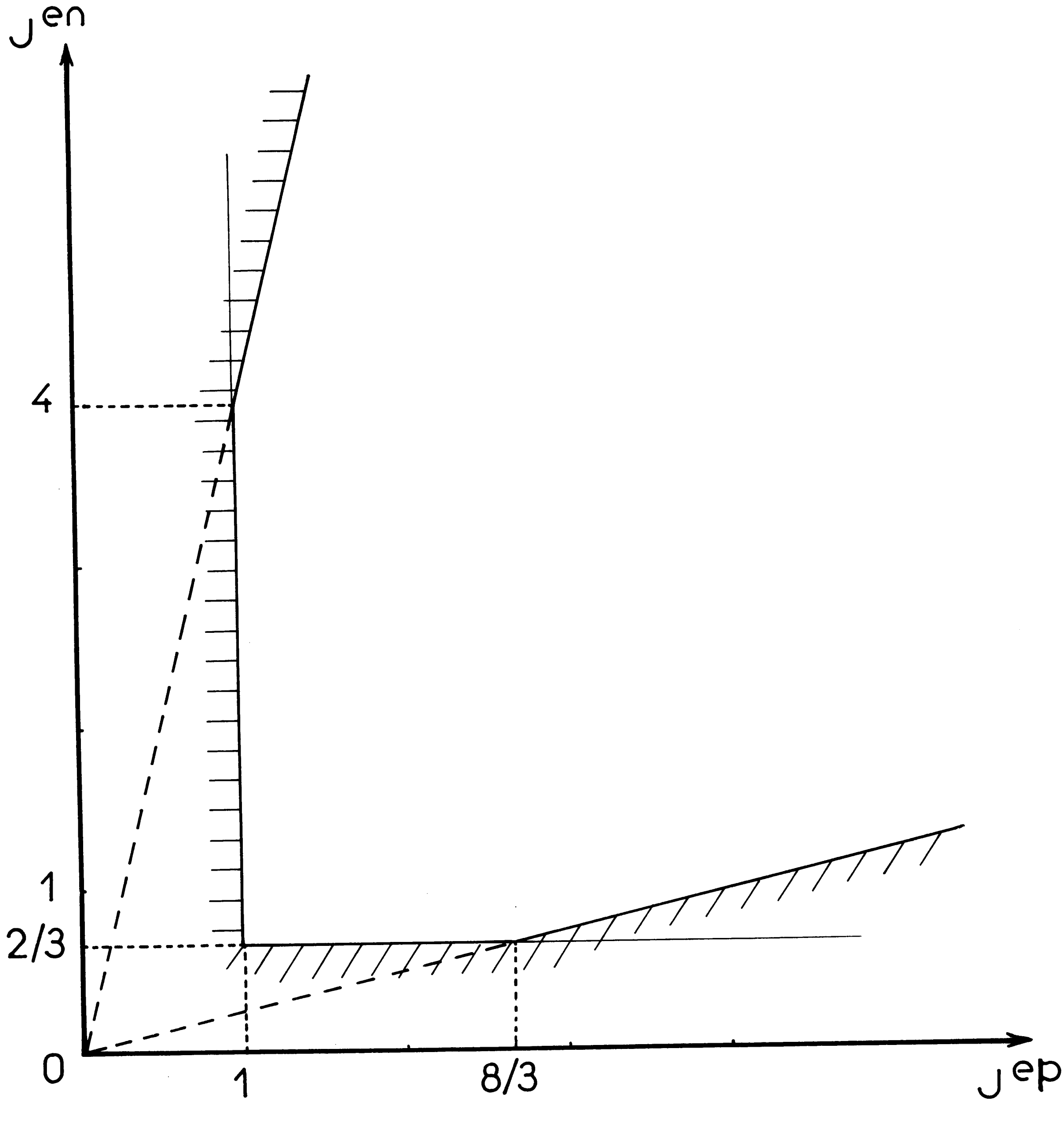

Fig. 15 - Domain for J^{ep} and J^{en} determined by positivity and charge symmetry

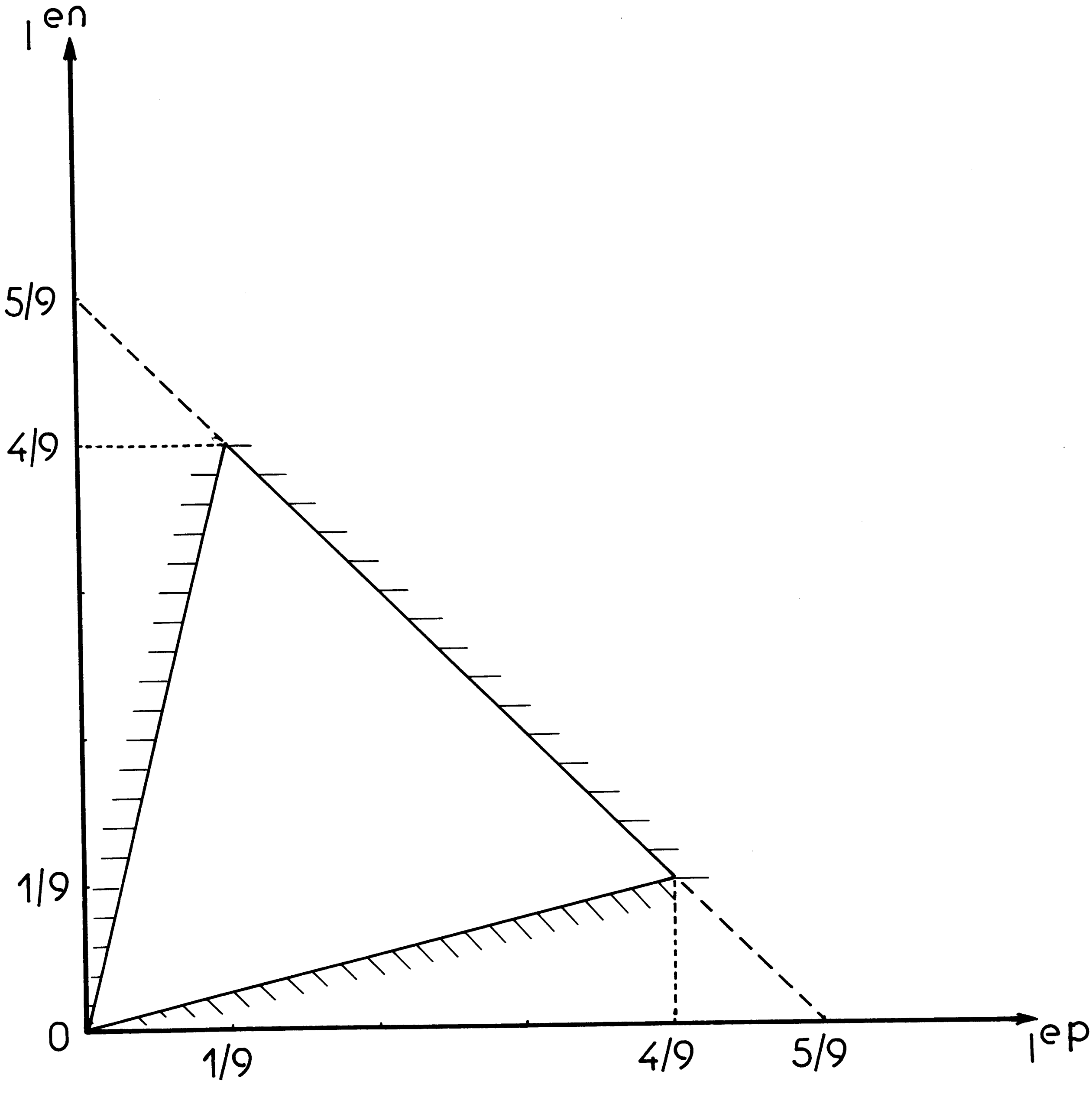

Fig. 16 - Domain for I^{ep} and I^{en} determined by positivity and charge symmetry

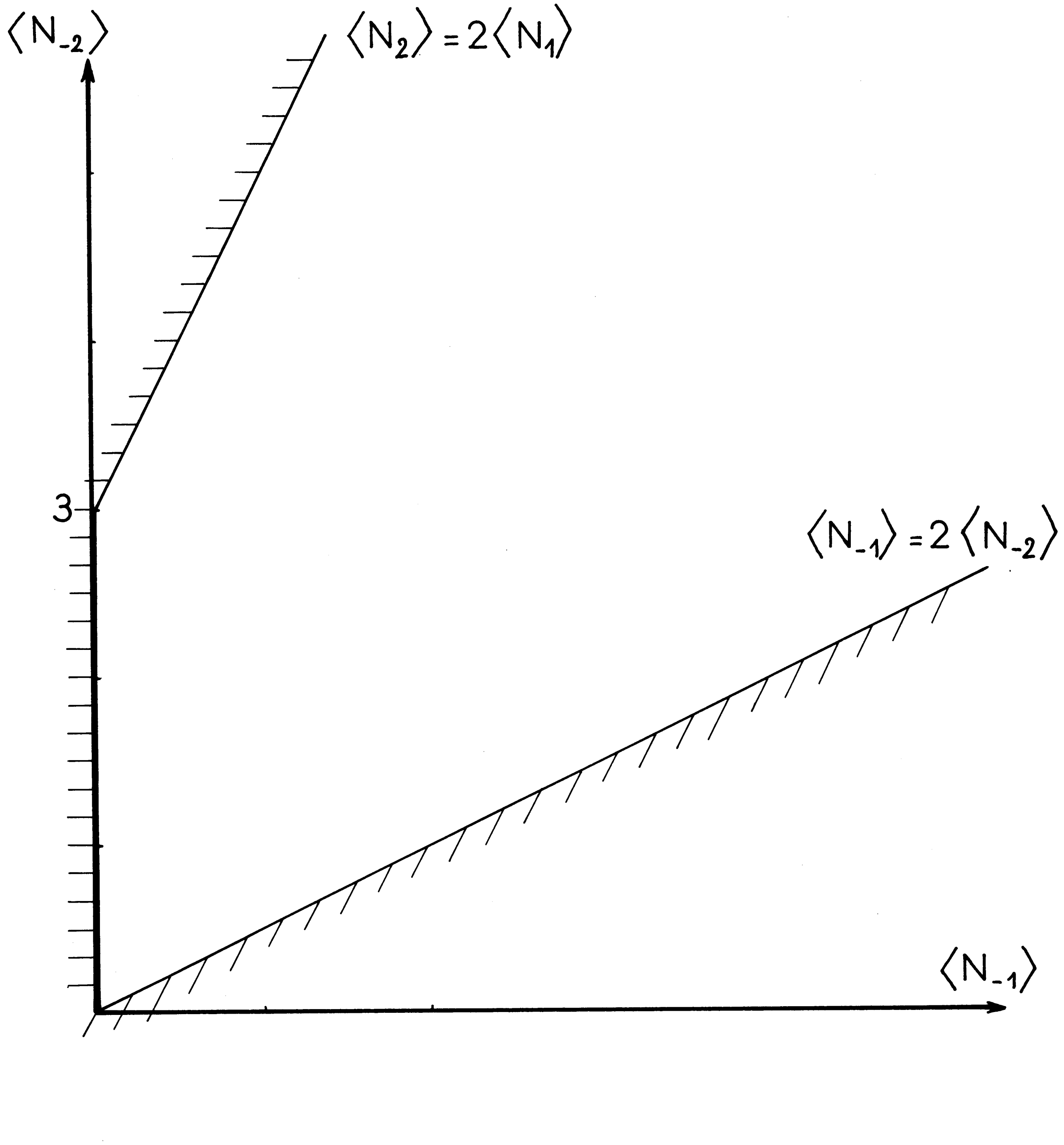

Fig. 17 - Positivity and isotopic spin symmetry

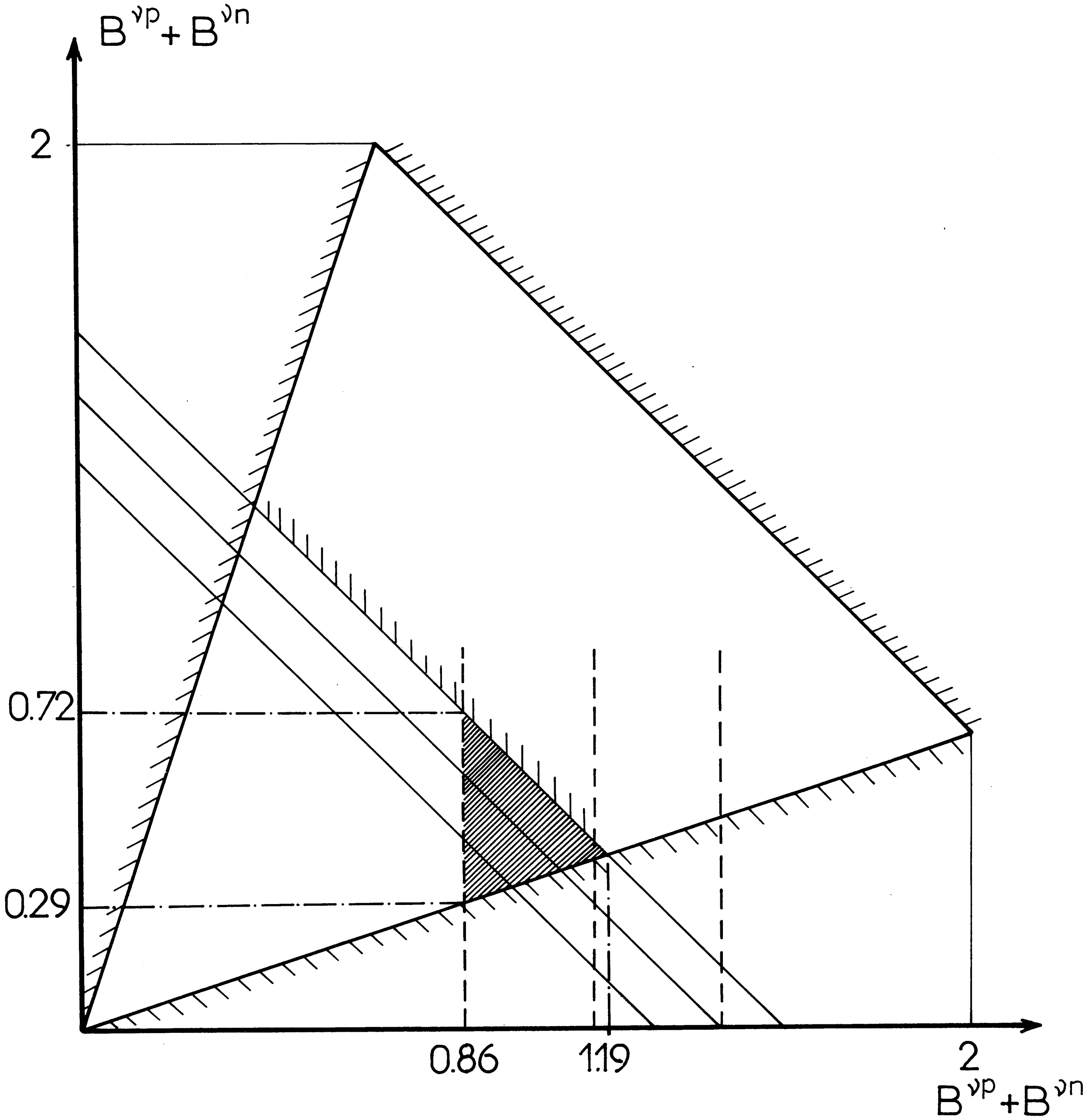

Fig. 18 - The neutrino antineutrino total cross sections

CONCLUSION

We hope we have, in these lectures, given some ideas about inclusive lepton scattering, scaling in the deep inelastic region and a tentative interpretation of the data with the quark parton model. Many other related topics have been disregarded because of the lack of time and we only briefly comment on them.

Obviously, other theoretical ways of interpreting the data have been proposed : the light-cone algebra approach, the Regge-type model, the vector dominance model... The light-cone approach turns out to be very close to the parton model and for deep inelastic scattering of leptons it gives equivalent results. In its general form the parton model gives the scaling in the deep inelastic region and this fundamental feature is also insured assuming the leading singularities of the current commutators near the light-cone to be the same as those given by a free field theory. The hadronic tensor is the Fourier transform of a matrix element of the commutator of two current components in the Bjorken limit we pick out in the integration essentially the region near the light cone and scaling can be achieved in this way. Moreover, the quark parton model makes precise assumptions about the nature of the constituents in the hadron and therefore allows the possibility to derive sum rules, to relate electroproduction to neutrino and antineutrino induced reactions and to make predictions. In the light-cone approach equivalent results are obtained assuming a particular algebra for the current operators namely the quark model algebra with chiral symmetry so that the parallelism between the two models is really complete.

The hadronic tensor can be considered as the absorptive part of a forward Compton scattering amplitude. In a simple Regge model this amplitude is described at fixed q^2 and large ν by a finite sum of poles (R) whose residues are functions of q^2. For instance the total photo-absorption cross section σ_T and σ_L can be written in this region as

$$\sigma_{T,L}(q^2,\nu) \simeq \sum_R{}' \beta^R_{T,L}(q^2)\left(\frac{\nu}{M}\right)^{\alpha_R(0)-1}$$

If the Regge limit (q^2 fixed, large ν) and the Bjorken limit (q^2 and ν both large) are governed by the same leading terms, the existence of scaling implies the following high q^2 behaviour of the residue functions

$$\beta^R_{T,L}(q^2) \simeq \left(\frac{M^2}{q^2}\right)^{\alpha_R(0)} \beta^R_{T,L}$$

The scaling functions F_T and F_L have then a Regge expansion for small ξ

$$F_{T,L}(\xi) = \frac{M^2}{\pi} \sum_R \beta^R_{T,L}\left(\frac{1}{2\xi}\right)^{\alpha_R(0)}$$

In particular with a dominant Pomeron exchange $\alpha_P(0) = 1$, we get the constancy in ν of the total cross sections in the Regge limit and a pole at $\xi = 0$ for the scaling functions. Therefore the structure function $\nu/M\, V_1$ in the deep inelastic region will remain finite at $\xi = 0$ and the limit is the same for a proton and a neutron, the Pomeron being iso-scalar. In the parton language we know that this implies the existence in the hadron of configurations with an arbitrary large numbers of partons. To conclude with the Regge model let us emphasize two points : first, the connexion between the Regge limit and the Bjorken limit is model dependent and it can break down ; secondly, the presence of Regge cuts can modify the low ξ behaviour of the scaling functions.

A naive application of the vector meson dominance model with only ρ , ω , ϕ meson contributions seems to disagree with experiments. In particular the variation with q^2 at fixed ν of the cross section $\sigma_T(q^2, \nu)$ is less rapid than expected with a vector meson propagator so that more elaborate models must be introduced. In fact the vector meson dominance model cannot be used in the complete q^2, ν plane but only in a region where the virtual photon is expected to behave like a hadron ; this implies a restriction of the type

$$\nu M \gg 5 q^2 + 3 M^2$$

and the experimental data are not very far in the allowed domain.

Recently on the spirit of duality, two-component theories of the virtual photon have been proposed. The diffractive component is determined by the vector meson dominance model and the non-diffractive component is determined by resonance contributions expressed in terms of the parton model. Obviously, such a model is consistent with experiment and the transition region will be a crucial test when experimental data become available.

The last point we wish to stress is the interest of semi-inclusive and exclusive reactions induced by leptons. The experimental material is rapidly increasing and we can take advantage of the existence of a continuous mass spectrum for the projectile. Let us consider as an example an electroproduction experiment where, in the final state, a hadron has been detected in coincidence with the lepton

$$e + p \Rightarrow e + p_1 + \text{anything}$$

We expect to have a target fragmentation region very similar to that observed in inclusive reactions induced by real hadrons. But the projectile fragmentation region and the pionization region will crucially depend on q^2 in extension and in shape. Also the determination of the q^2 dependence of the multiplicity is an interesting question closely related to scaling. Up to now we have theoretical speculations but in a near future the problems must be solved at least phenomenologically.