Zedology

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1. Introduction

It is clear that the Z^o peak will be a very important landmark in e^+e^- collisions[1,2,3,4,5]. It will be a rich cornucopia of all conceivable particles with masses $\lesssim m_Z$ or $m_{Z/2}$. If its properties resemble at all the predictions of simpler gauge models such as the Weinberg-Salam model[6], it will produce about an event per second for a LEP luminosity of the order of 10^{32} cm^{-2} sec^{-1}. As far as the weak interactions are concerned, Z^o decays present the opportunity to measure in detail the neutral weak couplings of all lepton and quark flavours with masses $\lesssim m_{Z/2}$. As for the strong interactions, one can imagine experiments with $0(10^6$ or $10^7)$ hadronic events at $Q^2 = 0(10^4)$ GeV^2, for more than are conceivable in any experiments at space-like Q^2 [7]. These events will provide a unique window into the dynamics of quarks and gluons at short distances. Strong interaction studies with LEP are discussed later in this report[8]: the rest of this section will concentrate on weak interaction studies at the Z^o pole[9]. Two main aspects will be emphasized: the fact that high statistics enable precision measurements which bear on the fundamental structure of the theory, and the possibility to search for very rare decay modes involving exotic particles.

2. General Features of the Z^o

We will be mainly concerned with the couplings of the Z^o to fundamental fermions (quarks, leptons) which we parametrize[2,5] in the following form:

$$\mathcal{L}_{Z^o\bar{f}f} = -m_Z \left(\frac{G_F}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{f}\,\gamma_\mu \left[\frac{v_f - a_f\,\gamma_5}{\sqrt{2}}\right] f\, Z^\mu \qquad (1)$$

Constant factors have been removed to the front of equation (1) so that the reduced couplings v_f and a_f are expected to be of order unity in any unified gauge model for which the coupling constant would be of order e. As an example, we can consider the standard SU(2) x U(1) Weinberg-Salam model[6] in which

$$\left.\begin{array}{lll} v_e = v_\mu = v_e = -1 + 4\sin^2\theta_W & , & a_e = a_\mu = a_\tau = -1 \\ v_d = v_s = v_b = -1 + \frac{4}{3}\sin^2\theta_W & , & a_d = a_s = a_b = -1 \\ v_\nu = 1 & & a_\nu = 1 \\ v_u = v_c = v_t = 1 - \frac{8}{3}\sin^2\theta_W & , & a_u = a_c = a_t = 1 \end{array}\right\} \qquad (2)$$

Before discussing the size of the Z^o peak implied by the couplings (1) and (2) we need a standard reference cross-section, which we take to be:

$$\sigma_{pt} \equiv \sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi}{3}\frac{\alpha^2}{Q^2} \simeq \frac{87}{Q^2\ (GeV2)}\ nb \qquad (3)$$

Relative to this cross-section we define, for any $f\bar{f}$ final state,

$$R_f = \frac{\sigma(e^+e^- \to f\bar{f})}{\sigma_{pt}} \tag{4}$$

Also used often will be the integrated forward-backward asymmetries

$$A_f \equiv \frac{\int_0^1 d\,(\cos\theta)\,\frac{d\sigma}{d\,(\cos\theta)}\,(e^+e^- \to f\bar{f}) - \int_{-1}^0 d\,(\cos\theta)\frac{d\sigma}{d\,(\cos\theta)}(e^+e^- \to f\bar{f})}{\int_0^1 d\,(\cos\theta)\,\frac{d\sigma}{d(\cos\theta)}\,(e^+e^- \to f\bar{f}) - \int_{-1}^0 d\,(\cos\theta)\,\frac{d\sigma}{d\,(\cos\theta)}\,(e^+e^- \to f\bar{f})} \tag{5}$$

Gauge theories generally expect that the mass of the Z^o will be

$$m_{Z^o} = 0(e/G_F) \simeq 0(100) \text{ GeV} \tag{6}$$

The simplest Weinberg-Salam model[6] in fact predicts:

$$m_Z = \frac{\pi\alpha}{\sqrt{2}G_F} / \sin\theta_W \cos\theta_W = \frac{37.4 \text{ GeV}}{\sin\theta_W \cos\theta_W} \tag{7}$$

Taking $\sin^2\theta_W \simeq 0.20$ consistent with the latest neutral current experiments[10], we find $m_Z \simeq 94$ GeV. In this energy region, equation (3) tells us that $\sigma_{pt} \sim 10^{-35}$ cm^2, corresponding to 3.6 events/hour at a luminosity of 10^{32} cm^{-2} sec^{-1}. Elementary considerations tell us that, neglecting radiative corrections, the cross-sections at the peak of the resonance are given by:

$$\frac{\sigma(e^+e^- \to Z^o \to X)}{\sigma_{pt}} = \frac{9}{\alpha^2} B(Z^o \to e^+e^-)\, B(Z^o \to X) \tag{8}$$

Taking three generations of each type of fermion (ν, ℓ^-, charge -1/3 quark, charge 2/3 quark) we are led to expect:

$$B(Z^o \to e^+e^-) = 0(\tfrac{1}{20} \text{ to } \tfrac{1}{30}) \tag{9}$$

Inserted into equation (8) this branching ratio suggests that:

$$\frac{\sigma(e^+e^- \to Z^o \to \text{all})}{\sigma_{pt}} \sim \text{few thousand} \tag{10}$$

at the resonance peak, corresponding to the order of ten thousand events/hour. In fact, in the simplest Weinberg-Salam model[6] with $\sin^2\theta_W = 0.20$, we find:

$$\left.\frac{\sigma(e^+e^- \to Z^o \to \text{all})}{\sigma_{pt}}\right|_{WS} \simeq 5100 \tag{11}$$

if there are just three generations, corresponding to about 5 events/second.

Before continuing, a word should be said about the reliability of estimates of the Z^o mass. The prediction[7] of the Weinberg-Salam model depends on the SU(2) symmetry being spontaneously broken by an isodoublet of Higgs fields[11]. More complicated Higgs sectors could alter the prediction[7], but they are already severely constrained by neutral current data. Since the couplings[1,2] are fixed in the Weinberg-Salam model, we see that the neutral to charged current cross-section ratio

$$\frac{\sigma(NC)}{\sigma(CC)} \propto \frac{1}{m_Z^4} \tag{12}$$

The present agreement of neutral current cross-sections with Weinberg-Salam constrains the Z mass within the $SU(2)_L \times U(1)$ framework[10]:

$$\frac{m_Z^2}{m_Z^2\ (WS)} = 1.02 \pm 0.05 \tag{13}$$

If one goes beyond the SU(2) x U(1) weak electromagnetic gauge model the restriction (13) is of course greatly relaxed. However, it can be argued that in a wide class of models with more than one Z^o boson, at least one of them must have a mass smaller than that predicted by Weinberg and Salam[12].

For an arbitrary Z^o, the formulae (1) and (2) correspond to decay widths

$$\Gamma(Z^o \to f\bar{f}) \approx \frac{G_F m_Z^3}{24\sqrt{2}\pi}(v_f^2 + a_f^2) \tag{14}$$

for $m_f << m_{Z/2}$. For the favoured range of values of m_Z and v_f, a_f of order unity, equation (14) implies that $\Gamma(Z^o \to f\bar{f}) = 0(100)$ MeV. Including 3 generations of fermions one would therefore expect a total Z^o decay width

$$\Gamma(Z^o \to all) = 0(2 \text{ to } 3)\ GeV \tag{15}$$

which is much wider than the expected machine energy resolution $0(10^{-3})m_Z = 0(100)$ MeV. In the simplest Weinberg-Salam model (since one expects decays into fermion-antifermion pairs to dominate) one finds[2,5] from equation (2) that

$$\Gamma(Z^o \to all) \approx \frac{G_F m_Z^3}{24\sqrt{2}\pi}\Big[2N_\nu + (1 + (1 - \tfrac{8}{3}\sin^2\theta_w)^2)\ N_{\ell^-} + 3(1 + (1 - \tfrac{8}{3}\sin^2\theta_w)^2)\ N_{2/3} + 3(1 + (1 - \tfrac{4}{3}\sin^2\theta_w)^2)\ N_{-1/3}\Big] \tag{16}$$

If we take $\sin^2\theta_w = 0.20$ we find that

$$\Gamma(Z^o \to \nu\bar{\nu}) : \Gamma(Z^o \to \ell^+\ell^-) : \Gamma(Z^o \to u\bar{u}) : \Gamma(Z^o \to d\bar{d}) \approx 2 : 1.04 : 3.63 : 4.67 \tag{17}$$

with the decay rate

$$\Gamma(Z^o \to e^+e^-) \approx 90\ MeV \tag{18}$$

Combining the results (17) and (18) we see that if there are N_G generations of fundamental fermions

$$\Gamma(Z^o \to \text{all}) \sim 1.0\ N_G\ \text{GeV} \tag{19}$$

and

$$B(Z^o \to e^+e^-) \sim 1/11\ N_G \tag{20}$$

resulting in $\Gamma(Z^o \to \text{all}) \sim 3$ GeV, $B(Z^o \to e^+e^-) \sim 3\%$ for the minimal case of 3 generations.

3. Determining the Fermion Spectrum

The above results are encouraging, in the sense that the Z^o peak is large and dramatic, as long as there are not too many generations of fermions. Is it conceivable that there might be so many fermions as to wash out the Z^o peak? The "established" fermions are the three generations:

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix} \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix} \begin{pmatrix}\nu_e \\ \tau^-\end{pmatrix} \quad \begin{pmatrix}u \\ d\end{pmatrix} \begin{pmatrix}c \\ s\end{pmatrix} \begin{pmatrix}t \\ b\end{pmatrix} \tag{21}$$

and the question arises whether we have any constraints on the total number of other as yet undiscovered fermions. Neutrinos are a particular headache because they seem to have negligibly small masses, while charged fermion masses increase sufficiently rapidly that not too many of them can be reasonably expected to have masses $< m_{Z/2}$.

What limits do we have on the number of unobserved neutrinos? The best limit from high energy physics at the present time may come[5] from the upper limit on the decay $K \to \pi\ \nu\bar{\nu}$:

$$B(K \to \pi\nu\bar{\nu}) < 6 \times 10^{-7} \tag{22}$$

which when compared with the theoretical branching ratio

$$B(K \to \pi\nu\bar{\nu}) \sim O(10^{-10})\ N_\nu \tag{23}$$

suggests[5] that $N_\nu < 6000$ – not a very stringent limit! It has been proposed that one might establish a good limit on N_ν from decays of heavy quark-onia into neutrinos[13]. One finds[5] that

$$\frac{\Gamma(V \to Z^o \to \nu\bar{\nu})}{\Gamma(V \to \gamma^* \to e^+e^-)} \simeq \frac{G_F^2}{64\ \pi^2\alpha^2}\ \frac{m_V^4}{e_q^2}\ (1 - 4\ |\ e_q\ |\ \sin^2\theta_w)^2 \tag{24}$$

$$\approx 0.2 \times 10^{-8}\ m_V^4\ N_\nu \ \text{for}\ e_q = \frac{2}{3} \tag{25}$$

Putting in $V = J/\psi$: $m_V \sim 3$ GeV, and assuming an upper limit $\Gamma(J/\psi \to \nu\bar{\nu})/\Gamma(J/\psi \to e^+e^-) \lesssim 1$, one finds[5] $N_\nu < 5 \times 10^6$, an even less stringent limit! However, if there happens to be toponium with a mass of 30 GeV, the strong mass dependence in (25) would enable a much more stringent limit to be set on N_ν. The decay $V \to \nu\bar{\nu}$ could be looked for by looking for events of the type $e^+e^- \to V' \to V + \pi\pi$, $V \to$ nothing visible.

There are some limits on neutrinos and other neutral, heavy leptons which come from cosmology. The standard big-bang cosmology is only consistent with the present astrophysical density of Helium if there are at most 3 or 4 "light" neutrinos with masses $\lesssim m_e$[14]. For heavier neutral leptons, there are no very strong constraints on unstable species, but stable neutral leptons are constrained[14] by the large scale dynamics of the universe and of galaxies to have masses $\gtrsim$ 10 GeV.

To limit the number of unstable massive neutral leptons or neutrinos, we fall back on the observation that generally $m_{\nu_c} << m_c$, for any given $\begin{pmatrix}\nu_c\\ \ell^-\end{pmatrix}$ doublet. From the results of PLUTO and SPEAR, we believe that any new heavy lepton must have a mass $\gtrsim$ 5 GeV. However, the number of such heavy leptons is theoretically constrained. In the simplest Weinberg-Salam model the result $m_Z = m_W/\cos\theta_W$ which underlies equation (7) is subject[15] to radiative corrections from all doublets containing massive fermions:

$$\frac{M^2_{W^\pm}}{m^2_{Z^o}} = \cos^2\theta_W \left[1 + \begin{pmatrix}1 \text{ for leptons}\\ 3 \text{ for quarks}\end{pmatrix} \frac{G_F}{8\sqrt{2}\pi^2} \left(\frac{2\, m_1^2 m_2^2}{m_1^2 - m_2^2}\, \ell n \frac{m_2^2}{m_1^2} + (m_1^2 + m_2^2)\right)\right] \qquad (26)$$

The experimental constraint (13) already means that for heavy leptons L with masses >> their associated neutrinos

$$\sum_L M_L^2 < 0(500 \text{ GeV})^2 \qquad (27)$$

Since present experimental limits tell us that any such heavy lepton has m_L > 5 GeV, the restriction (27) means there are less than $0(10^4)$ such heavy leptons, and so $N_\nu < 0(10^4)$. PETRA can soon improve the lower limit on m_L to ~ 15 GeV, in which case N_ν would be $< 10^3$. LEP could eventually improve the lower limit on m_L to about 100 GeV, corresponding to $N_\nu < 0(25)$. It is clear from equation (8) that $0(10^4)$ neutrinos would be required if the Z^o were washed out to the extent that $\sigma(e^+e^- \to Z^o \to X)/\sigma_{pt} = 0(10)$. Therefore either PETRA and LEP find vast numbers of heavy leptons, or the Z^o will be a large peak[16].

Assuming that the Z^o peak is indeed big and not very wide, one can then imagine a precision determination of the Z^o mass. A precision measurement of the W^+W^- threshold to get the W mass, and detailed neutral current measurements (see part 4 of this report) would then enable the radiative corrections in equation (26) to be severely restricted, so that the bound (27) could be improved. In this way, one could perhaps exclude the possible existence of any heavy lepton with mass > 100 GeV (and hence outside the mass range accessible to LEP), and of course detect any heavy lepton with mass < 100 GeV. A check that the studies of fundamental fermion spectroscopy were indeed completed by LEP would be furnished by measuring $\Gamma(Z^o \to \sum_\nu \nu\bar{\nu})$. Several ways of doing this come to mind. One possibility is looking for the decay chain $e^+e^- \to V' \to V\pi\pi$, $V \to \nu\bar{\nu}$ mentioned earlier[5,13]. Another is a precision measurement of the Z^o width, which increases by $0(5$ to $10)\%$ for each neutrino in addition to the canonical three. Another possibility is to look for the reactions $e^+e^- \to \nu\bar{\nu} + \gamma$, where the only particle visible in the final state would be an energetic large angle γ[17]. The rate for this seems prohibitively small at PETRA energies, but the experiment may be feasible[18] above the Z^o mass, where the dominant contribution to the cross-section is the radiative correction reaction $e^+e^- \to Z^o + \gamma$, $Z^o \to \nu\nu$. Depending of course on the total number of neutrinos, this process may have[18] a cross-section of the same order as σ_{pt} for centre-of-mass energies between 120 and 200 GeV.

4. Detailed Measurements Near the Z^o Peak

We will now survey the different neutral current measurements that can be made by observations at and near the Z^o peak. Radiative corrections will not be taken into account, because a complete calculation of these is only just becoming available[19], but we do not think they will make qualitative changes in the classes and qualities of measurements that can be made.

The Shape of the Total Cross-Section

If we consider an arbitrary fermion-antifermion pair (with the exception of e^+e^-) then

$$R_f \equiv \frac{\sigma(e^+e^- \to \gamma, Z \to f\bar{f})}{\sigma_{pt}}$$

$$\approx Q_f^2 - \frac{2s\ \rho\ Q_f\ V_e V_f}{\left(\left(\frac{s}{m_Z^2} - 1\right) + \Gamma^2_{Z/s-m_Z^2}\right)} + \frac{s^2\rho^2\ (v_e^2 + a_e^2)\ (v_f^2 + a_f^2)}{\left(\left(\frac{s}{m_Z^2} - 1\right)^2 + \Gamma^2_{Z/m_Z^2}\right)^2} \tag{28}$$

where $\rho \equiv (G_F/8\sqrt{2}\pi\alpha)$. (29)

We notice that the total cross-section shape is sensitive to the products of vector (or of axial) weak couplings. If we specialize to $\mu^+\mu^-$, assume μ-e universality:

$$\left.\begin{aligned} v_e &= v_\mu \equiv v \\ a_e &= a_\mu \equiv a \end{aligned}\right\} \tag{30}$$

and neglect Γ_Z, we find

$$R_\mu = 1 + 2v^2\chi + (v^2 + a^2)\chi^2 \tag{31}$$

$$\text{where } \chi \equiv m_Z^2\rho\left(\frac{s}{s - m_Z^2}\right) \quad \left[\approx 0.39\left(\frac{s}{s - m_Z^2}\right) \text{ if } m_Z = 94 \text{ GeV}\right] \tag{32}$$

In general, R_μ exhibits a minimum at

$$\frac{s}{m_Z^2} = \frac{\delta}{1+\delta} = \left(\frac{1}{m_Z^2\rho}\right) \frac{v^2}{(v^2+a^2)^2} \tag{33}$$

corresponding to $\sqrt{s}$ = 29 GeV if we take the Weinberg-Salam model with $\sin^2\theta_W = 0.20$. The value of R_μ at its minimum is

$$R_\mu^{min} = 1 - \frac{v^4}{(v^2+a^2)^2} \tag{34}$$

which is not very exciting if $\sin^2\theta_W = 0.20$:

$$R_\mu^{min} = 0.9985 \tag{35}$$

The general shapes of R_μ for different choices of v and a, and a Z^o mass of 83 GeV, are shown in Fig. 1. It is clear that if a = 0, which is not expected in the Weinberg-Salam model and is indeed disfavoured by experiments finding parity violation in deep inelastic electron scattering and atoms, then $R_\mu^{min} = 0$ at $\sqrt{s} \sim 0.85\ m_Z$ - rather dramatic!

Forward-Backward Asymmetry

The angular distributions for the processes $e^+e^- \to f\bar{f}$ ($f\bar{f} \neq e^+e^-$) have the following form near the Z^o peak:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to f\bar{f}) = \frac{\pi\alpha^2}{2s}\Big\{Q_f^2\,(1+\cos^2\theta) - 2Q_f\chi\,(v_e v_f\,(1+\cos^2\theta) + 2a_e a_f\cos\theta)$$
$$+ \chi^2\left[(v_e^2+a_e^2)\,(v_f^2+a_f^2)\,(1+\cos^2\theta) + 8v_e a_e v_f a_f\cos\theta\right]\Big\} \tag{36}$$

if we neglect the decay width Γ_Z compared with m_Z. If we define the integrated forward-backward asymmetry

$$A_f \equiv \frac{\int_o^1 \frac{d\sigma}{d\cos\theta}\,d(\cos\theta) - \int_o^1 \frac{d\sigma}{d\cos\theta}\,d(\cos\theta)}{\int_{-1}^1 \frac{d\sigma}{d\cos\theta}\,(\cos\theta)} \tag{37}$$

it is found from the angular distribution (36) to be

$$A_f = \frac{\frac{3}{2}\chi\left[-Q_f a_e a_f + 2v_e a_e v_f a_f \chi\right]}{\left[Q_f^2 - 2Q_f\chi v_e v_f + \chi^2(v_e^2+a_e^2)\,(v_f^2+a_f^2)\right]} \tag{38}$$

Since the angular distribution is only quadratic in $\cos\theta$, there is a trivial bound $|A_f| \leqslant 3/4$.

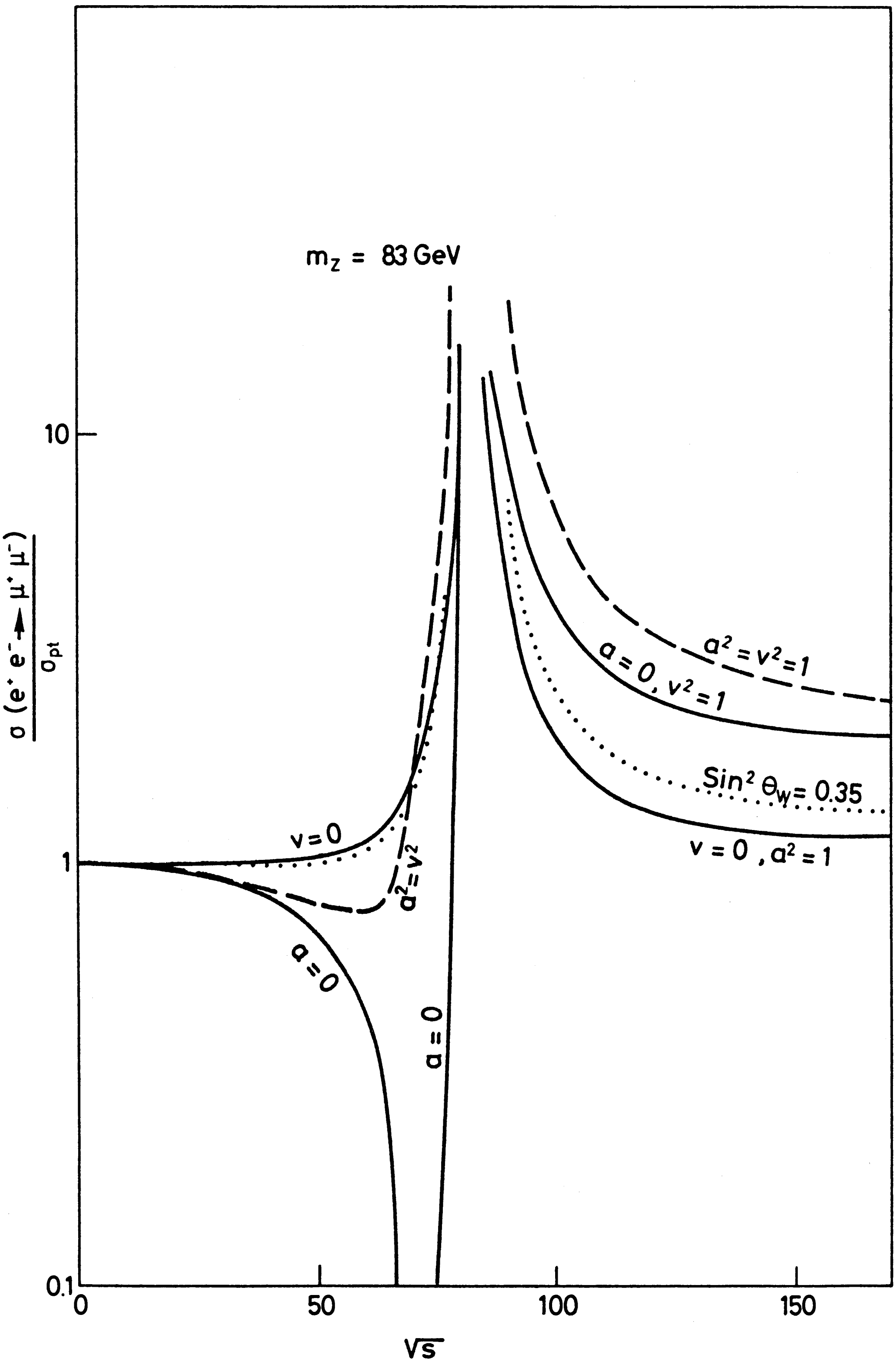

Figure 1 : The ratio R_μ of $(e^+e^- \to \mu^+\mu^-)$ relative to σ_{pt} (3), plotted for different values of the vector and axial couplings of the e and μ.

We see from equations (36) and (38) that the forward-backward asymmetry is non-zero if a_e and $a_f \neq 0$, as expected in the Weinberg-Salam model. Notice however that it does not permit a determination of the relative signs of v and a couplings (a limitation shared of course by the total cross-section formula (28)). Even at low (PETRA-PEP) energies, the asymmetry (38) can become quite large. When $\sqrt{s} \ll m_Z$,

$$a_f \approx \frac{-3}{2} \chi \frac{a_e a_f}{Q_f} \tag{39}$$

which at $\sqrt{s} = 40$ GeV is already

$$\left.\begin{array}{l} \sim 10\% \text{ for } \mu^+\mu^-,\ \tau^+\tau^- \\ \sim 14\% \text{ for } u\bar{u},\ c\bar{c},\ t\bar{t} \\ \sim 28\% \text{ for } d\bar{d},\ s\bar{s},\ b\bar{b} \end{array}\right\} \tag{40}$$

In the particular case of $e^+e^- \to \mu^+\mu^-$ or $\tau^+\tau^-$, and assuming charged lepton universality $a_e = a_\mu = a_\tau \equiv a$, $v_e = v_\mu = v_\tau \equiv v$, we find

$$A = \frac{3}{2}\chi \frac{(a^2 + 2v^2a^2\chi)}{(1 + 2\chi v^2 + \chi^2(v^2 + a^2)^2)} \tag{41}$$

which goes through a minimum at

$$\frac{s}{m_Z^2} = \frac{1}{1 + (m_Z^2\rho)\ (a^2 + 3v^2)} \tag{42}$$

($\sqrt{s} = 78$ GeV for $\sin^2\theta_w = 0.20$). At this point A takes the value

$$A^{min} = -\frac{3}{4} \frac{1}{\left(1 + \frac{2v^2}{a^2}\right)} \tag{43}$$

and in fact attains the kinematic bound of −0.75 when v = 0 corresponding to $\sin^2\theta_w = 0.25$, while $\sin^2\theta_w = 20$ would yield A = −0.69. The asymmetry also goes through a maximum at

$$\frac{s}{m_Z^2} = \frac{1}{1 - (\rho m_Z^2)\ (a^2 - \nu^2)} \tag{44}$$

($\sqrt{s} = 118$ GeV for $\sin^2\theta_w = 0.20$) at which point it takes the limiting value $A^{max} = +0.75$. At the peak of the resonance, $\sin^2\theta_w = 0.20$ would imply an asymmetry $A \simeq +0.11$ at $\sqrt{s} = 94$ GeV. General forms of the asymmetry for muons are shown in Fig. 2, corresponding to different choices of the values of v and a.

The above analysis does not apply to $e^+e^- \to e^+e^-$ because there are also crossed-channel γ and Z^o exchange diagrams. The cross-section formulae therefore become more complicated[20)], and will not be reproduced here. We will just make the qualitative observation that the behaviour of the forward-backward asymmetry

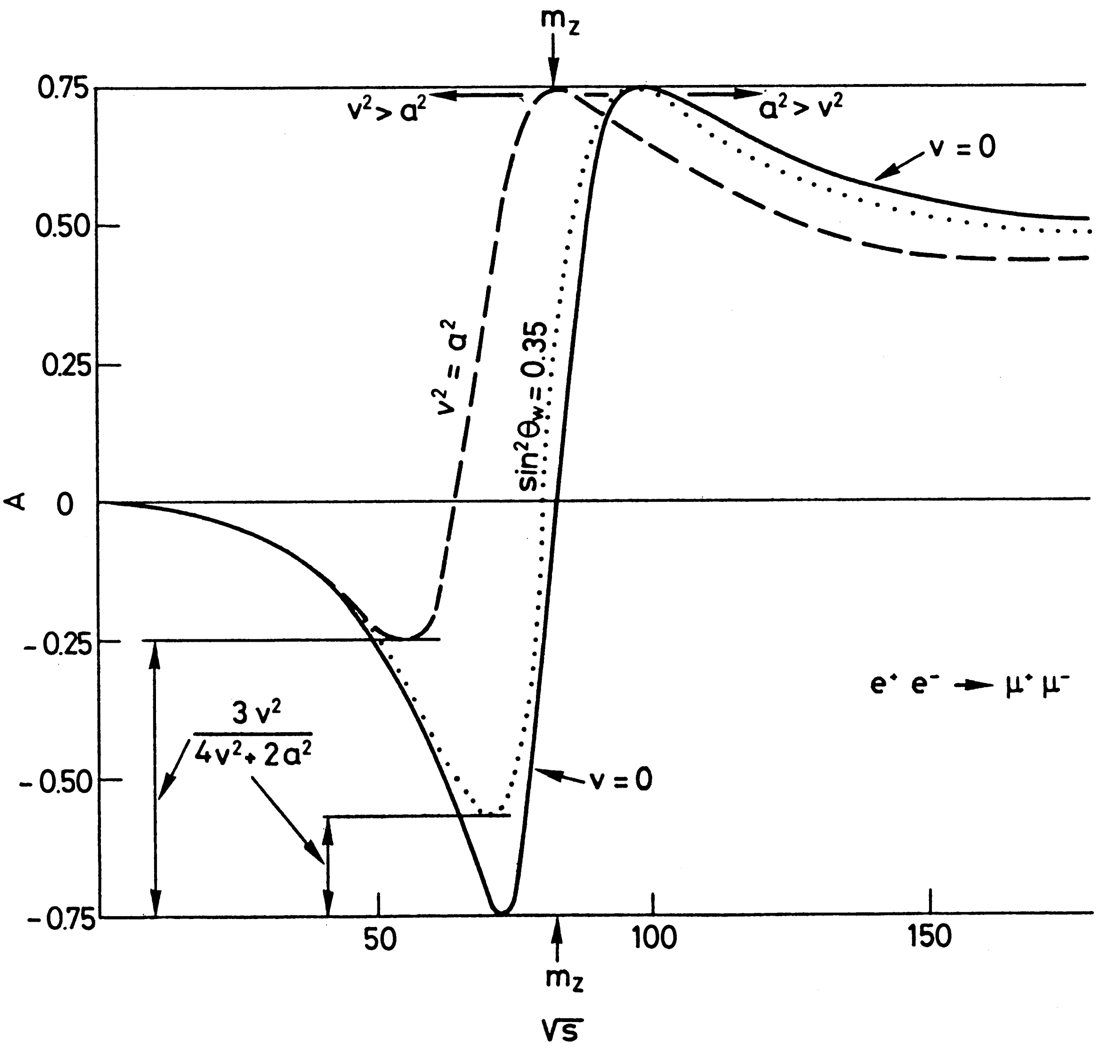

Figure 2 : The forward-backward asymmetry A (37) for $e^+e^- \to \mu^+\mu^-$, plotted for different values of the vector and axial couplings of the e and μ.

parameter is very different in this case from the reaction $e^+e^- \to \mu^+\mu^-$. In the case of $e^+e^- \to e^+e^-$ there is a large positive asymmetry, of the order of 3/4 off resonance, which may be drastically reduced on resonance - see the asymmetry in a restricted angular range $30^o < \theta < 150^o$ plotted in Fig. 3. For θ sufficiently small the well understood crossed channel γ exchange will always be at low enough Q^2 to dominate the cross-section and provide a reliable luminosity monitor even near the top of the Z^o peak.

<u>Helicity Measurements</u>

Another observable which is potentially interesting in the reaction $e^+e^- \to f\bar{f}$ is the helicity of the outgoing fermion. If we specialize for the moment to the case of $e^+e^- \to \mu^+\mu^-$ or $\tau^+\tau^-$, the helicity is maximal in the forward direction

$$H^{\ell^-}(s,\ \cos\theta = +1) \approx \frac{-4\chi av\ (1 + \chi(a^2 + v^2))}{1 + 2\chi(v^2 + a^2) + \chi^2\ \left[(a^2 + v^2)^2 + 4a^2v^2\right]} \tag{45}$$

so that on the resonance peak itself

$$H^{\ell^-}(m_Z^2,\ \cos\theta = +1) \approx \frac{-4av\ (a^2 + v^2)}{(a\ \ + v\ )\ \ + 4a\ v} \tag{46}$$

If $\sin^2\theta_w = 0.20$, this value on the peak is +0.13. The variation with s if $\sin^2\theta_w = 0.35$ is shown in Fig. 4. We notice in (45) (46) that the helicity is sensitive to the product of a and v, enabling their relative sign to be measured, which was not possible with the cross-section and angular asymmetry measurements discussed earlier.

So helicities are interesting and non-zero in general. Can they be measured? An early suggestion was to stop muons produced on resonance in a polarimeter, and determine their polarization from observations of the decay electrons. Such an experiment would be very cumbersome and difficult[20)], and a better idea may be to use the decays of the $\tau \to e\nu\nu$ or $\pi\nu$ as convenient polarization analyzers. Either of these seems possible, with $\tau \to \pi\nu$ measurements perhaps more sensitive to H^τ [21)]. Could one perform polarization measurements on quarks? In the absence of e-fermion universality, the numerator in (45) becomes

$$4av(a^2 + v^2) \to v_f a_f\ (a_e^2 + v_e^2)\ (1 + \cos^2\theta) + 2a_e f_e\ (a_f^2 + v_f^2)\ \cos\theta \tag{47}$$

and we have sensitivity to $a_f v_f$ as well as $a_e v_e$. The only problem is to find a quark helicity analyzer. It has been suggested[22)] to look at correlations of the type $\underline{p}_{jet}\cdot(\underline{p}_1 \times \underline{p}_2)$, where $\underline{p}_1$ and $\underline{p}_2$ are the momenta of the two fastest particles in a quark jet. Unfortunately, neutrino data suggest[23)] that any such correlations are in fact washed out. A related suggestion is to look at the polarization of

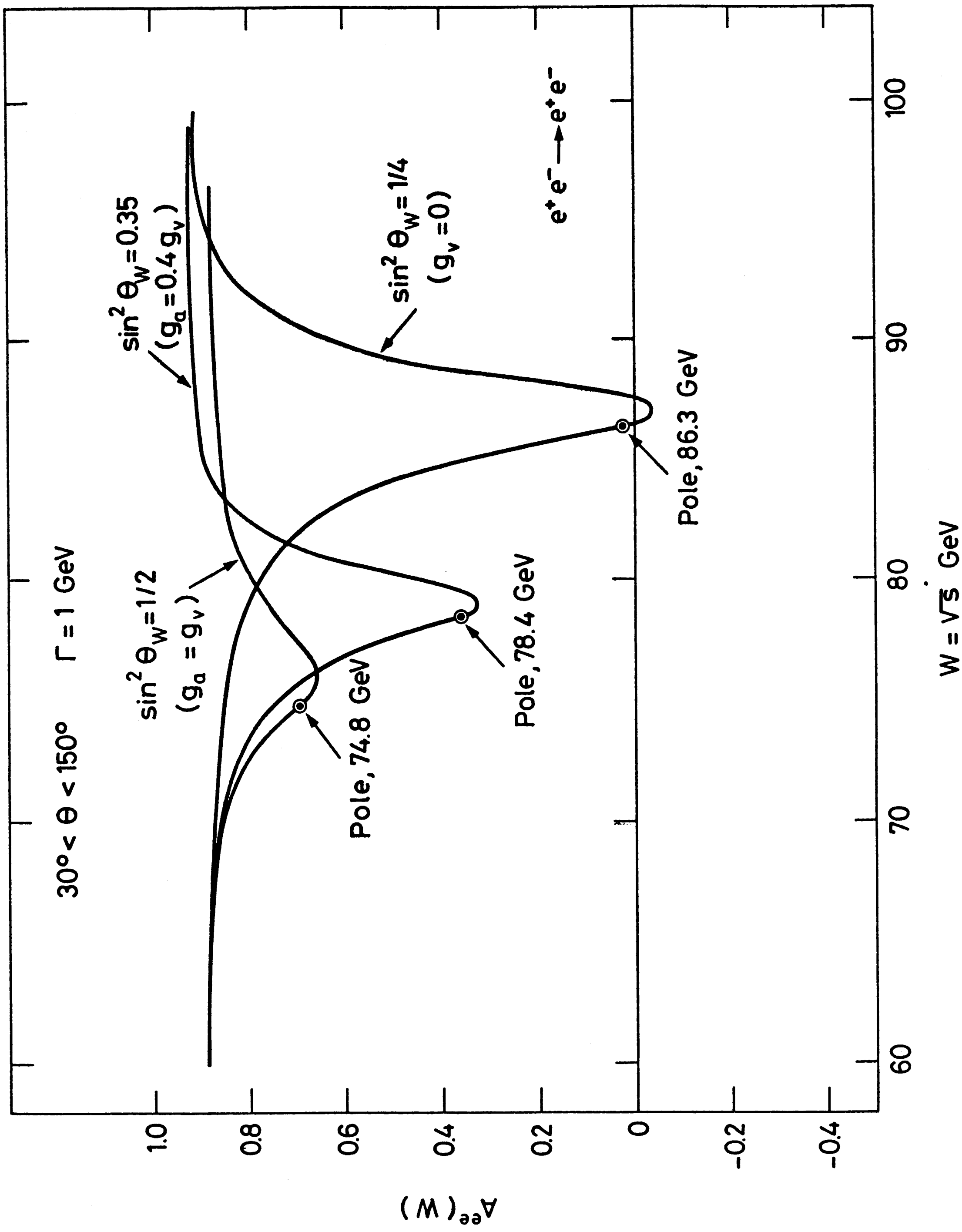

Figure 3 : The corresponding asymmetry for $e^+e^- \to e^+e^-$ in the Weinberg-Salam model with different values of $\sin^2\theta_W$.

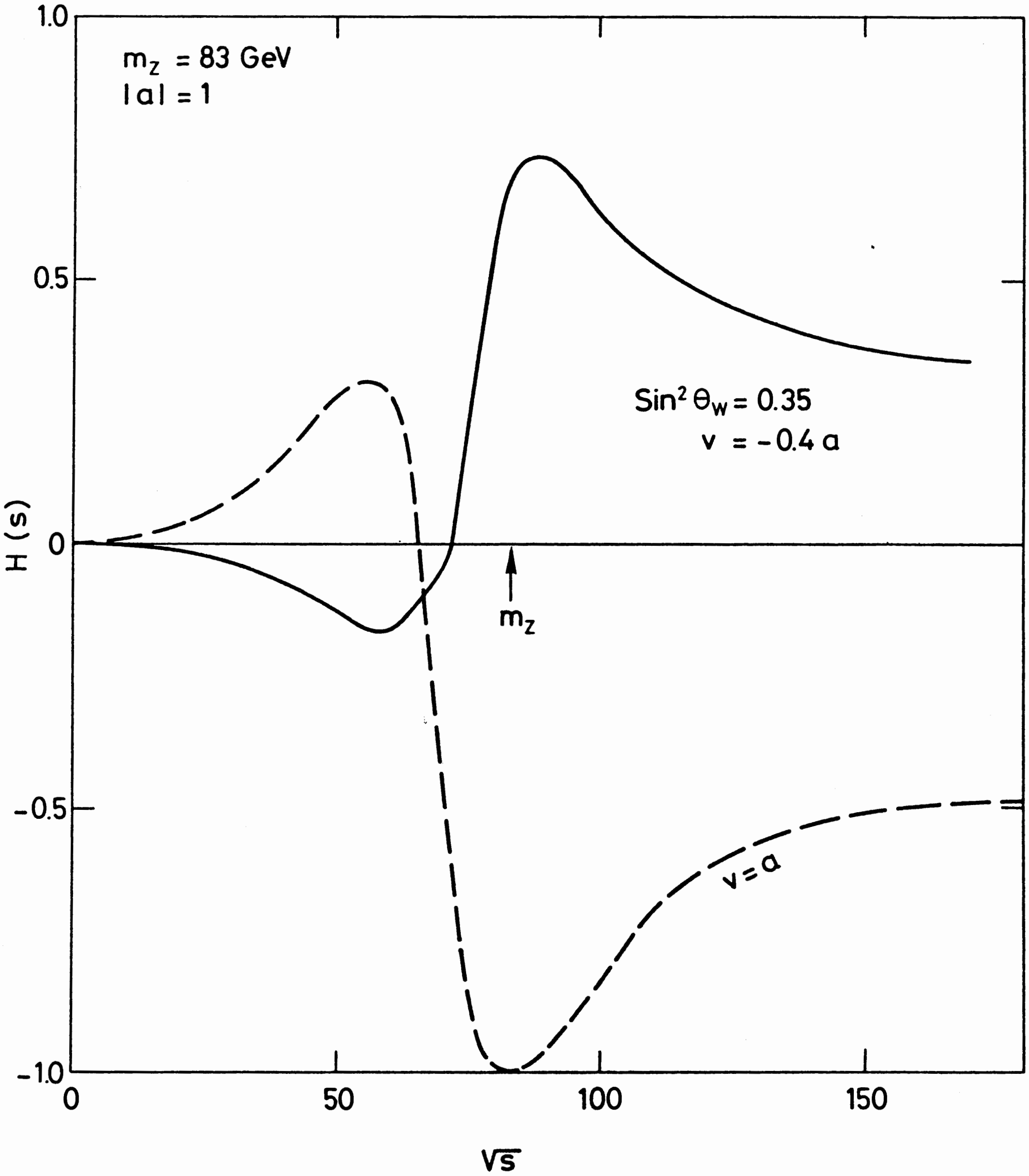

Figure 4 : The helicity of a muon or τ, for different values of the vector and axial couplings.

final state hadrons with non-zero spin, such as ρ, K^* and Λ[24]. A guaranteed quark polarimeter has yet to emerge, but we note that in a sense none is needed. Cross-section and angular asymmetry measurements enable the relative signs of all the v_f to be determined, and also those of all the a_f. The τ helicity experiment would then determine the signs of the a_f relative to the v_f, and any further information would in principle be redundant.

Polarized Beams

These would enable the extraction of physics similar to that obtainable from helicity measurements. If the incoming e^+ and e^- have helicities h^+, h^- respectively, then[5]

$$\frac{d\sigma_{f\bar{f}}}{d\cos\theta}(h^+, h^-) = \tfrac{1}{2}(1 - h^+h^-)\frac{d\sigma}{d\cos\theta}\text{ unpolarized}$$

$$+ (h^- - h^+)\frac{\pi\alpha^2}{2s}\chi\left\{Q_f\left[v_e a_f (1+\cos^2\theta) + 2a_e v_f \cos\theta\right]\right. \qquad (48)$$

$$\left. - \chi\left[v_f a_f (a_e^2 + v_e^2)(1+\cos^2\theta) + 2a_e v_e (a_f^2 + v_f^2)\cos\theta\right]\right\}$$

Equation (48) again exhibits sensitivity to the relative signs of v and a couplings. If polarized beams were freely available they would probably be more powerful probes of these signs than the helicity measurements. Certainly, the prospect of being able to turn on, or off, an $e^+e^- \to f\bar{f}$ cross-section by adjusting the beam polarizations seems very attractive. On the other hand, it should be emphasized that from a logical point of view, within the standard gauge theoretical point of view no new information is gained thereby. It is not clear how seriously this should be taken into account when considering the cost of developing polarized beams. They would certainly be invaluable analyzers if the standard gauge picture were wrong.

$Z^o \to$ Heavy $f\bar{f}$

The Z^o decays democratically into all fermions with essentially equal rates (if $m_f < m_{Z/2}$). In fact the event rates may be larger than those close to the associated thresholds. If $m_q \ll m_Z$, then for equal luminosities one finds

$$\frac{\text{Rate }(Z^o \to f\bar{f})}{\text{Rate (threshold} \to f\bar{f})} \approx 1000\left(\frac{m_q}{m_Z}\right)^2 \qquad (49)$$

which is promising for studies of $t\bar{t}$ and $b\bar{b}$ final states. Measurements of these final states may be the only way to determine heavy quark neutral current couplings. At present we know in principle the neutral current couplings of u,d,e,ν_μ and to some extent ν_e. Those for s,c,t,b,τ and even μ are still essentially unknown.

The problem resides in finding ways to detect heavy quark decays of the Z^o. One might look for

- events with many final state leptons[25] ($>$ 3 $e^{\pm}$ and $\mu^{\pm}$ + hadrons, or events with 2 identically charged leptons in the same jet);
- fat jets[2,5], because one expects[25,26] a heavy quark to decay into three light quarks and antiquarks: $Q \rightarrow qq\bar{q}$ as in Fig. 5. The transverse momenta of the associated hadrons would then probably be quite large;

$$\sum_{\text{hadrons}} |p_t| \approx \sum_{\text{quarks}} |p_t| \sim \frac{2}{3} m_Q \tag{50}$$

- long-lived heavy mesons? In the conventional six-quark extension of the Weinberg-Salam model, the decay rates of heavy quarks into light quarks are suppressed[25]. Careful analysis[27] suggests bounds on the lifetimes of bottom mesons: $B^o \equiv b\bar{d}$, $B^- \equiv b\bar{u}$:

$$10^{-11} \text{ sec} > \tau_B > 10^{-14} \text{ sec} \tag{51}$$

If the lifetime is in the upper half of this range, it might be observable in e^+e^- collisions at high enough energies to give a useful relativistic dilation of the decay track length. One might therefore look for "staggered" events of the type shown in Fig. 6, where there is a set of particles produced in the initial e^+e^- annihilation, and two other sets of tracks converging on separate B and $\bar{B}$ decay points.

In this section we have chiefly discussed relatively common Z^o decays with branching ratios $\gtrsim 10^{-2}$. The large event rates available at the Z^o peak should enable detailed studies of these channels. It may be worth emphasizing again the interest of such studies. For example, there are theories which purport to calculate $\sin^2\theta_W$ with a precision better than 0.01[28]. It would be nice to know if these theories were correct. It would also be nice to reduce the errors on the combination $m_W^2/m_Z^2\cos^2\theta$ to see how close it is to 1, and thereby (recall[15] equation (26)) get a useful constraint on the fermion mass spectrum. For this purpose, detailed measurements at the Z^o peak must be combined with precise measurements of the $e^+e^- \rightarrow W^+W^-$ threshold.

5. Decays Involving Higgs Bosons

The previous section dealt with the relatively common decays of the Z^o into fermion pairs. What other important decays of the Z^o are expected? Other members of the elementary particle zoo include the $W^{\pm}$ and the Higgs particles. Decays into the $W^{\pm}$ are expected to be very rare: in the Weinberg-Salam model with $\sin^2\theta_W = 0.35$ so that m_W = 62 GeV, m_Z = 80 GeV, it was found[29,2] that

$$\Gamma(Z^o \rightarrow W^-e^+\nu) \sim 3 \times 10^{-7} \text{ GeV} \tag{52}$$

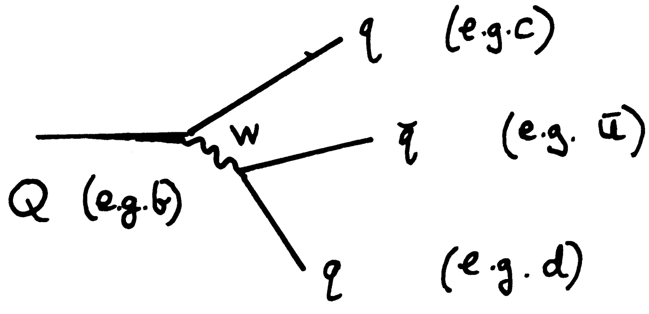

Figure 5 : The expected dominant decay mode $Q \to q q \bar{q}$ of a heavy quark.

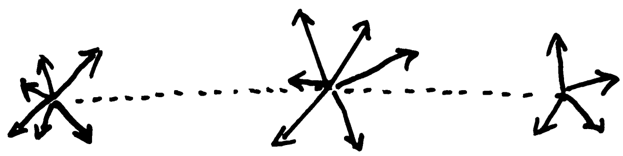

Figure 6 : The possible "staggered" event structure of a decay $Z^o \to Q\bar{Q}$ into a heavy quark-antiquark pair with lengthy lifetimes.

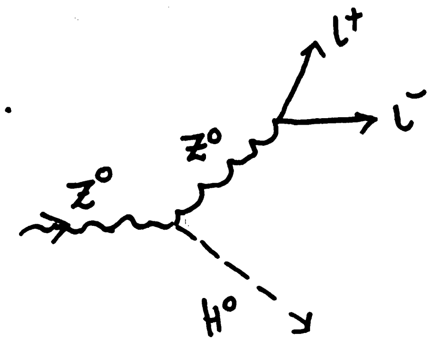

Figure 7 : The dominant diagram for $Z^o \to H^o l^+ l^-$.

and the presently preferred value of $\sin^2\theta_W = 0.20$ would imply a still smaller decay rate. So we turn to the detection of Higgs bosons. Their importance has been adequately stressed in the literature[30]. Higgs bosons play the essential rôle in generating the spontaneous symmetry breaking necessary to realistic renormalizable weak interaction models. Verification of their existence is therefore a crucial test of the entire gauge theory approach to weak interactions. Other promising ways of looking for Higgs bosons have been proposed which involve either lower e^+e^- centre-of-mass energies (e.g. the decay of a vector meson $V \to H + \gamma$[31]) or higher centre-of-mass energies (e.g. the reaction $e^+e^- \to Z^o + H$[30]). What are the possibilities in Z^o decays?

$Z^o \to H^o + \mu^+\mu^-$ or e^+e^-

This decay would proceed via the diagram shown in Fig. 7. In terms of the variable $x = 2E_{Higgs}/m_Z$ the decay spectrum has been computed[3] to be:

$$\frac{1}{\Gamma(Z \to \ell^+\ell^-)} \frac{d\Gamma(Z^o \to H^o\ \ell^+\ell^-)}{dx} = \frac{\alpha}{4\pi\sin^2\theta_W\ \cos^2\theta_W} \frac{\left[1 - x + \frac{x^2}{12} + \frac{2}{3}\frac{m_H^2}{m_Z^2}\right]\left(x^2 - \frac{4m_H^2}{m_Z^2}\right)^2}{\left(x - \frac{m_H^2}{m_Z}\right)^2} \tag{53}$$

The resulting total branching ratio is plotted in Fig. 8. We see that for $m_H < 40$ GeV, the branching ratio $B(Z^o \to H^o\ \ell^+\ell^-)$ is $> 3 \times 10^{-6}$. This may give an acceptable rate if one can indeed do experiments with tens of millions of Z^o decays, though more thought about backgrounds is required. The signature for Higgs decays is its propensity for decaying into the heaviest fermions available: $H \to Q\bar{Q}$, which should mean that its final states will contain an unusually high fraction of prompt decay leptons, and tend to have fatter jets on average than in the e^+e^- continuum.

$Z^o \to H^o + \gamma$

The branching ratio for this process has recently been calculated[32]. It was found that

$$\frac{\Gamma(Z^o \to H^o + \gamma)}{\Gamma(Z^o \to \mu^+\mu^-)} \simeq 8 \times 10^{-5}\left(1 - \frac{m_H^2}{m_Z^2}\right)^3 \left(1 + 0.17\frac{m_H^2}{m_Z^2}\right) \tag{54}$$

for $\sin^2\theta_W = 0.20$. We therefore see that $B(Z^o \to H + \gamma) \leq B(Z^o \to H^o\ \ell^+\ell^-)$ for $m_H \lesssim 0.6\ m_Z$ (see Fig. 8), with a total branching ratio $B(Z^o \to H\ \gamma) \sim (1 \text{ to } 2) \times 10^{-6}$. The final state may be cleaner than in the $H^o\ell^+\ell^-$ case, which could be polluted by decays involving heavy quarks and their subsequent semileptonic decays. On the other hand, the $H^o\gamma$ final state may be confused with the radiative reaction

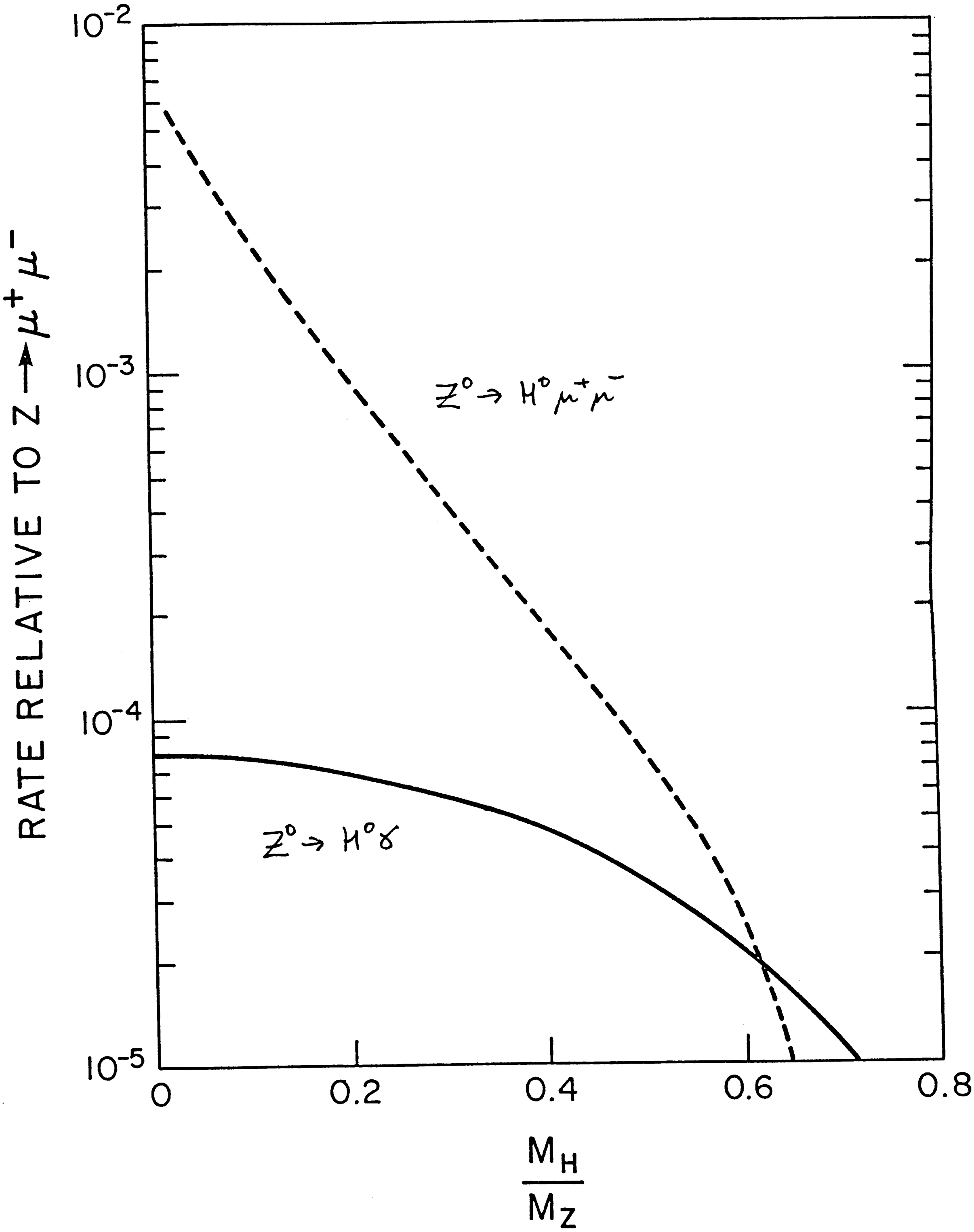

Figure 8 : The ratios $\Gamma(Z^o \to H^o\mu^+\mu^-)/\Gamma(Z^o \to \mu^+\mu^-)$ and $\Gamma(Z^o \to H^o\gamma)/\Gamma(Z \to \mu^+\mu^-)$ as functions of m_H/m_Z for $\sin^2\theta_w = 0.20$[32].

$Z^o \to q\bar{q}\gamma$ which occurs in lower order in α than $Z^o \to qq\ \ell^+\ell^-$. This background can be reliably computed in QCD[33]) and should enable a reliable assessment of the gravity of this potential background.

$Z^o \to H^+H^-$

The previous two reactions involved the single neutral Higgs boson found in the minimal Weinberg-Salam model where symmetry breaking is obtained from just one Higgs multiplet. If there are more than one Higgs multiplet, there will be additional physical charged Higgs bosons, as well as extra neutral ones. The decay rate

$$\Gamma(Z^o \to H^+H^-) \simeq \frac{G_F m_Z^3}{96\sqrt{2}\pi} \tag{55}$$

for charged Higgs particles with $m_H << m_{Z/2}$, corresponding to a branching ratio at the percent level. One might guess that each charged Higgs particle may like to decay into pairs of heavy quarks: $H^+ \to Q\bar{Q}'$? resulting in distinctive final states which should be detectable if $H^\pm$ exist.

$Z^o \to H_1^o\bar{H}_2$

If there is only one Higgs multiplet and hence only one neutral Higgs boson, the decay $Z^o \to H^oH^o$ is forbidden by Bose symmetry. On the other hand, if there is more than one Higgs multiplet and hence more than one neutral Higgs boson, decays like $Z^o \to H_1^oH_2^o$ become possible, and might have branching ratios up to the percent level. As in the case of $Z^o \to H^+H^-$, decays into heavy quarks might provide a useful signature for such final states.

6. Summary

Theoretical studies of the Z^o peak in e^+e^- annihilation suggest the following conclusions:

a) Large event rates can be expected near the Z^o peak, which should enable precision measurements of important fundamental parameters like m_Z, Γ_Z, the neutral current couplings of fermions, $\sin^2\theta_W$, etc.

b) One should also be able to search for rare decay modes of great interest, such as the Higgs boson processes $Z^o \to H^o\ell^+\ell^-$, $Z^o \to H^o\gamma$.

c) Combining Z^o width and mass measurements one should be able to determine the complete fermion spectrum: the number of neutrinos, the possible existence of massive fermions, copious decays into fermions with masses $< m_{Z/2}$.

d) A topic not emphasized here, but discussed elsewhere[8]), is the possibility of detailed strong interaction studies with tens of millions of events at $Q^2 \sim 10^4$ GeV^2 - a cornucopia not available in any other way.

Of course we hope for and expect surprises, but even the above minimal shopping list of predictable physics suggests that the "Z^o factory" aspect of LEP should be a copious source of new physics.

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