Comments and Addenda

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Comment on calculations of the $K_L \to \mu\bar{\mu}$ decay rate in gauge theories

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We reexamine the calculations of the $K_L \to \mu\bar{\mu}$ decay rate and the K_L-K_S mass difference in the Weinberg-Salam model with the Glashow-Iliopoulos-Maiani mechanism incorporated. We consider both the free-quark model and corrections due to strong interactions in an asymptotically free theory, and compare our results with those of other recent calculations. Our conclusions are basically identical to those drawn from our original free-quark calculation: The decay amplitude for $K_L \to \mu^+\mu^-$ is dominated by the conventional two-photon exchange, and the decay rate places no useful limit on the charmed-quark mass, whereas the K_L-K_S mass difference constrains this mass by $m_c \lesssim$ a few GeV, as noted previously.

I. INTRODUCTION

A number of authors have computed the decay rate $K_L \to \mu\bar{\mu}$ in the Weinberg-Salam model of electromagnetic and weak interactions which incorporates the so-called GIM (Glashow-Iliopoulos-Maiani) mechanism, and commented on the mass scale of the fourth quark, i.e., the charmed quark, which appears in this scheme. More recently, several authors discussed the effects of strong interactions on these amplitudes, assuming that strong interactions of hadrons are described by an asymptotically free gauge theory.

One of the purposes of this note is to clarify the discrepancy between Ref. 1 and those of Russian workers,[2] especially of Flambaum, on $K_L \to \mu\bar{\mu}$. We find that the result of paper I is in error, and our correct result agrees with Ref. 2. However, we disagree with Flambaum regarding the Ward-Takahashi identity for the Zds vertex. Under the circumstances, we feel it necessary to describe our calculation in some detail. The second subject we wish to discuss here has to do with estimating the size of strong interaction effects in the processes. We obtain results which are not completely in accord with previous authors,[3] including Vainshtein, Zakharov, Novikov and Shifman.

II. WARD-TAKAHASHI IDENTITY

The effective Zsd vertex discussed in Appendix B of I is a sum of diagrams depicted in Fig. 1, where black circles represent one-loop corrections. These diagrams are separately divergent, but the sum is not. To extract the finite sum, we make use of the Ward-Takahashi identity described below. We shall also outline the direct calculation of Fig. 1.

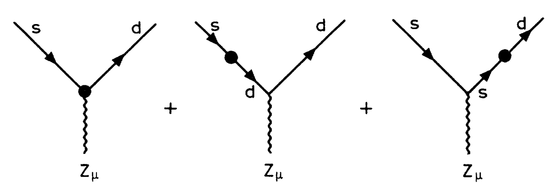

FIG. 1. Diagrams contributing to the effective $Z\bar{d}s$ vertex.

It was shown elsewhere[4] that in a gauge specified by

$$F^a[\phi]=f_i^a\phi_i \tag{1}$$

the Ward-Takahashi identity for the generating functional for proper vertices may be written as

$$\Gamma_i^a[\phi]\frac{\delta}{\delta\phi_i}\Gamma_0[\phi]=0, \tag{2}$$

where $\Gamma=\Gamma_0-\frac{1}{2}F_a{}^2$ is the generating functional of proper vertices with external ϕ lines, and $\bar{c}^a f_i^a\Gamma_i^b[\phi]c^b$ is the generating functional of proper vertices with two external ghost lines and an arbitrary number of external ϕ lines. The index a refers to the adjoint representation of the gauge group in question.

We consider a special case in which the index a in Eq. (2) refers to the transformation which leaves the photon field invariant but changes the Z_μ field by a translation:

$$\begin{aligned}&\delta A_\mu=0,\\ &\delta Z_\mu=\partial_\mu\lambda(x).\end{aligned} \tag{3a}$$

Under this transformation, the d and s fields and the Higgs field ϕ_1 change by

$$\delta\begin{pmatrix}d\\ s\end{pmatrix}=iT\begin{pmatrix}d\\ s\end{pmatrix}\quad, \tag{3b}$$

where

$$\begin{aligned}&T=\{-\tfrac{1}{2}-(Q-1)\sin^2\theta_W L+[-(Q-1)\sin^2\theta_W]R\}\\ &\quad\times(g^2+g'^2)^{1/2},\\ &L=\tfrac{1}{2}(1-\gamma_5),\ R=\tfrac{1}{2}(1+\gamma_5),\end{aligned} \tag{3c}$$

and

$$\delta\phi_1=i(-\tfrac{1}{2})(g^2+g'^2)^{1/2}(i\phi_2). \tag{3d}$$

In one-loop approximation, we consider

$$\frac{\delta}{\delta\bar{d}}\frac{\delta}{\delta s}\left[\Gamma_i^a[\phi]\frac{\delta\Gamma_0[\phi]}{\delta\phi_i}\right]\Bigg|_{\phi=v}=0. \tag{4}$$

We need to concentrate on the case in which $\Gamma_i^a[\phi]$ takes its bare form, and Γ_0 is given by the one-loop approximation. To this order, Eq. (4) is a statement that Γ_0 is invariant under the gauge transformations (3a)–(3d). Taking into account Eqs. (3a)–(3d) we obtain the expression (B2) in I:

$$\begin{aligned}&(q-p)^\mu\Gamma_\mu^{(Z)}(q,p)-[\Sigma(q)T-T^*\Sigma(p)]\\ &\quad=i\tfrac{1}{2}(g^2+g'^2)^{1/2}v\Gamma_2(q,p).\end{aligned} \tag{5}$$

As explained in I, to lowest order in $(m_c/m_W)^2$, the effective vertex $E_\mu^{(Z)}$ can be written as

$$E_\mu^{(Z)}=\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)E, \tag{6}$$

where the on-shell value of E is related to Γ_2 through Eq. (4):

$$\tfrac{1}{2}i(g^2+g'^2)^{1/2}v\bar{d}(q)\Gamma_2(q,p)s(p)\big|_{\not p=m_s,\ \not q=m_d}=\bar{d}(q)(m_dL-m_sR)s(p)E. \tag{7}$$

It is important to observe here that the Ward-Takahashi identity (5) is correct in all linear gauges of the form (1), and to all orders in strong interactions if they are invariant under the gauge symmetry of electromagnetic and weak interactions. We note that this Ward-Takahashi identity disagrees with that of Flambaum.[2] It will be useful for the calculation of $E_\mu^{(2)}$ to recall the following facts from I. Again to lowest order in $(m_q/m_W)^2$, and assuming $m_u\simeq m_d\simeq m_s\ll m_c$, Σ and $\Gamma_\mu^{(z)}$ have the forms

$$\begin{aligned}&\Gamma_\mu^{(Z)}(q,p)=\gamma_\mu Lx,\\ &\Sigma(p)=\not pLa+bL+cR.\end{aligned} \tag{8}$$

$E_\mu^{(Z)}$ can thus be written

$$\begin{aligned}i\bar{d}(q)E_\mu^{(Z)}s(p)&=\bar{d}(q)\left(i\Gamma_\mu^{(Z)}+i\gamma_\mu T\frac{i}{\not p-m_d}i\Sigma(p)+i\Sigma(q)\frac{i}{\not q-m_s}i\gamma_\mu T\right)s(p)\\ &=i\bar{d}(q)\gamma_\mu Ls(p)[x+\{\tfrac{1}{2}+(Q-1)\sin^2\theta_W](g^2+g'^2)^{1/2}a\}.\end{aligned} \tag{9}$$

III. $K_L\to\mu\bar{\mu}$

We proceed to the computation of Γ_2 and of $E_\mu^{(Z)}$. To ascertain gauge independence of our result, we carry out the computation in the R_ξ gauge, wherein the $W^\pm$ and $\phi^\pm$ propagators are, respectively,

$$i\Delta_{\mu\nu}(k)=\left[-ig_{\mu\nu}+i\frac{k_\mu k_\nu(1-1/\xi)}{k^2-m_W{}^2/\xi}\right]\frac{1}{k^2-m_W{}^2}$$

and

$$i\Delta(k)=i(k^2-m_W{}^2/\xi)^{-1}.$$

Dimensional regularization is used for the treatment of divergent integrals.

There are altogether five diagrams which contribute to Γ_2 to order $G_F\alpha(m_c/m_W)^2$, shown in Fig. 2. [Henceforth we neglect m_u, but we cannot neglect the external masses as the Ward identity, Eq. (7), is valid only on the mass shell.] The graph of Fig. 2(a) is convergent, but the graphs 2(d) and 2(e) have a logarithmically divergent term proportional to the external mass. This divergence cancels when the u and c contributions are added. Explicit evaluation of the diagrams 2(a), 2(d), 2(e) yields

$$i\Gamma_2^{2(a)} = \left(\ln\frac{m_W^2}{m_c^2} - 1 - \tfrac{1}{4}\ln\xi\right)C, \tag{10}$$

$$i\Gamma_2^{2(d)+2(e)} = -\left[\frac{3}{2}\,\frac{\xi\ln\xi}{\xi-1} + \frac{1}{2}\right]C, \tag{11}$$

where the common factor

$$C = g^2\frac{\sin\theta_c\cos\theta_c}{v(16\pi^2)}\,\frac{m_c^2}{m_W^2}\,\bar{d}(\not{q} - \not{p})Ls \tag{12}$$

has been extracted. The contributions of graph 2(b) are exactly canceled by the counterterm 2(c) [see Eqs. (24) and (25) below]:

$$i\Gamma_2^{2(b)} = -i\,\Gamma_2^{2(c)} = \frac{1}{2}\left(\frac{2}{4-n} + 1 + \ln\xi\right)C \tag{13}$$

in the limit $n\to 4$, where n is the dimension of space-time. Combining Eqs. (10)–(13), we find

$$i\Gamma_2(q,p) = g^2\frac{\sin\theta_c\cos\theta_c}{v(16\pi^2)}\,\frac{m_c^2}{m_W^2} \times\left[\ln\frac{m_W^2}{m_c^2} - \frac{3}{2} - \frac{(7\xi-1)}{4(\xi-1)}\ln\xi\right] \times\bar{d}(q)(\not{q}-\not{p})Ls(p). \tag{14}$$

The Zds vertex may be extracted from Eq. (14)

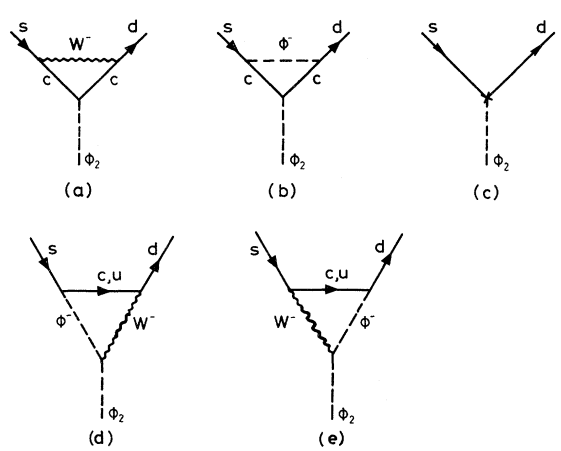

FIG. 2. Diagrams contributing to the effective $\phi_2\bar{d}s$ vertex to order $G_1 = \alpha(m_c/m_W)^2$.

using the Ward identity of Eq. (7). However, as we wish to demonstrate explicitly the validity of Eq. (6) we shall proceed to the direct evaluation of $E_\mu^{(Z)}$. The relevant diagrams are those of Fig. 1, which are made explicit in Figs. 3 and 4.

Graph 3(a) gives the following contribution to the (nondiagonal) self-energy:

$$\Sigma(p)_{3(a)} = g^2\frac{\sin\theta_c\cos\theta_c}{2(16\pi^2)}\,\frac{m_c^2}{m_W^2}(1 - \tfrac{3}{2}\ln\xi)\not{p}L. \tag{15}$$

Graphs 4(a) and 4(c)–4(e) yield

$$\Gamma_\mu^{4(a)} = \left[\left(1 - 2\ln\frac{m_W^2}{m_c^2} + 2Q\sin^2\theta_W\right) + (2 - 3Q\sin^2\theta_W)\ln\xi\right]K_\mu, \tag{16}$$

$$\Gamma_\mu^{4(c)} = 3\cos^2\theta_W\left(1 + \frac{\ln\xi}{\xi-1}\right)K_\mu, \tag{17}$$

$$\Gamma_\mu^{4(d)+4(e)} = \sin^2\theta_W\left(1 + \frac{3\xi\ln\xi}{\xi-1}\right)K_\mu, \tag{18}$$

where, for brevity of notation we have extracted the common factor

$$K_\mu = \frac{-g^2(g^2+g'^2)^{1/2}}{4(16\pi^2)}\sin\theta_c\cos\theta_c\frac{m_c^2}{m_W^2}\gamma_\mu L. \tag{19}$$

Substituting these contributions to Σ and $\Gamma_\mu^{(Z)}$ into Eq. (9), we obtain

$$E_\mu^{(Z)} = \frac{g^2(g^2+g'^2)^{1/2}}{2(16\pi^2)}\sin\theta_c\cos\theta_c\frac{m_c^2}{m_W^2} \times\left[\ln\frac{m_W^2}{m_c^2} - \frac{3}{2} - \frac{(7\xi-1)}{4(\xi-1)}\ln\xi\right]\gamma_\mu L. \tag{20}$$

This is in fact the final result for $E_\mu^{(Z)}$, as will now be shown.

The cancellation of logarithmic divergences which occurs for the above graphs does not occur for graphs 3(b), 4(b) or 4(f), since the divergent parts of these diagrams are proportional to m_c^2. The counterterms required to render Σ and $\Gamma_\mu^{(Z)}$ finite are given by

$$\bar{d}\{i\gamma\cdot\partial - (g^2+g'^2)^{1/2}[\tfrac{1}{2} + (Q-1)\sin^2\theta_W]\gamma\cdot Z\} \times Ls\sin\theta_c\cos\theta_c(Z_L - Z_L') + \text{H.c.}, \tag{21}$$

where, as in I, Z_L and Z_L' are the wave-function renormalization constants for the left-handed quark doublets. The same renormalization constants

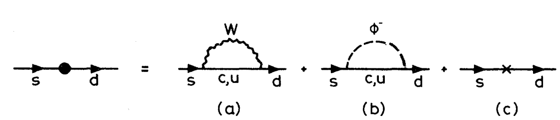

FIG. 3. Lowest-order diagrams contributing to Fig. 1(a).

render Σ and $\Gamma_\mu^{(Z)}$ finite, and can be chosen to cancel the sd transition on mass shell. Z_L and Z'_L can be determined from graph 3(b):

$$\Sigma(p)_{3(b)} = g^2 \frac{\sin\theta_c \cos\theta_c}{2(16\pi^2)} \frac{m_c^2}{m_W^2} \times \left[\frac{1}{2}\not{p}L\left(\frac{2}{4-n}+\frac{1}{2}+\ln\xi\right) - (m_d L + m_s R)\left(\frac{2}{4-n}+1+\ln\xi\right)\right], \tag{22}$$

where the expression represents the limit as $n \to 4$, with n the dimension of space-time. Thus,

$$(Z_L - Z'_L) = -g^2 \frac{\sin\theta_c \cos\theta_c}{4(16\pi^2)} \frac{m_c^2}{m_W^2} \times \left(\frac{2}{4-n}+\frac{1}{2}+\ln\xi\right). \tag{23}$$

It is readily checked that the corresponding Zds counterterm cancels the sum of the contributions of graphs 4(b) and 4(f) to $\Gamma_\mu^{(Z)}$. As is evident from Eqs. (9) and (21) the sd and Zsd counterterms give equal and opposite contributions to $E_\mu^{(Z)}$. Thus, alternatively, one could simply add all of the graphs in Figs. 3 and 4 together directly, without adding the counterterms in graphs 3(c) and 4(g), and the result would be that the divergences arising from Σ would exactly cancel those from $\Gamma_\mu^{(Z)}$. Both procedures are equivalent as regards the final answer for $E_\mu^{(Z)}$.

The second term in Eq. (22) represents a direct sd transition, as does the first, and must be canceled by another counterterm in the Lagrangian. This term is of the form

$$A\bar{d}Ls(\phi_1+v+i\phi_2)+B\bar{d}Rs(\phi_1+v-i\phi_2)+\text{H.c.}, \tag{24}$$

where ϕ_1 is the physical Higgs scalar and

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}.$$

The renormalization constants A and B are given by

$$A = \frac{m_d}{m_s} B = g^3 \frac{\sin\theta_c \cos\theta_c}{4(16\pi^2)} m_d \frac{m_c^2}{m_W^3} \left[\frac{2}{4-n}+1+\ln\xi\right]. \tag{25}$$

This is just the counterterm for the $\phi_2 sd$ vertex as shown in Fig. 2(e).

Now we may compare the expressions obtained for $E_\mu^{(Z)}$, Eq. (20), and for Γ_2, Eq. (14), and we see that the Ward identity of Eq. (7) is indeed satisfield. Our corrected result agrees with that of Ref. 2.

To conclude our discussion, the Z-exchange contribution to the process $s+\bar{d} \to \mu+\bar{\mu}$, relevant to K_L decay, is given by

$$-i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}\cos\theta_c\,\sin\theta_c\left(\frac{m_c}{38\text{ GeV}}\right)^2 \frac{1}{4}(\bar{\mu}\gamma_\alpha\gamma_5\mu)(\bar{d}\gamma^\alpha Ls)\left[\ln\left(\frac{m_W}{m_c}\right)^2 - \frac{3}{2} - \frac{7\xi-1}{4(\xi-1)}\ln\xi\right], \tag{26}$$

whereas the W^+W^- contribution to the process is

$$-i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}\cos\theta_c\,\sin\theta_c\left(\frac{m_c}{38\text{ GeV}}\right)^2 \frac{1}{4}(\bar{\mu}\gamma_\alpha\gamma_5\mu)(\bar{d}\gamma^\alpha Ls)\left[-\ln\left(\frac{m_W^2}{m_c^2}\right) - \frac{1}{2} + \frac{7\xi-1}{4(\xi-1)}\ln\xi\right]. \tag{27}$$

The sum of Eqs. (26) and (27) is gauge independent; while the sum is not equal to zero, as asserted in I, it is nevertheless small, being of order $G_F\alpha(m_c/38\text{ GeV})^2$ without any logarithmic factor. If $m_c \lesssim$ a few GeV as the K_L-K_S mass difference implies, then the dominant mechanism for $K_L \to \mu\bar{\mu}$ would be the conventional one of $K_L \to 2\gamma(\text{virtual}) \to \mu\bar{\mu}$.

IV. STRONG-INTERACTION EFFECTS ON $K_L \to \mu\bar{\mu}$

We treat the strong interactions by an asymptotically free gauge theory—specifically color

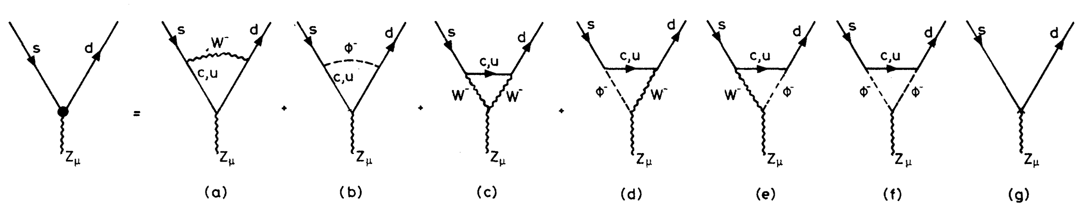

FIG. 4. Lowest-order diagrams contributing to the s-d transition [Figs. 1(a), 1(b)].

SU(3). We quantize the gluons in the Lorentz gauge: $\alpha = 1/\xi = 0$. In this gauge, there is no renormalization of the gauge parameter α, and no quark wave-function renormalization to lowest order.

The box diagram $s+\bar{d} \to W^+ + W^- \to \mu + \bar{\mu}$ is cut off at $p \approx m_W$, where p is the integral loop momentum, for removal of both W propagators renders the integral logarithmically divergent. For this diagram, therefore, the analysis of Nanopoulos and Ross is correct.

The operator-product expansion relevant to this amplitude is

$$j_\mu(x) j_\nu^\dagger(0) \approx \cdots + C_{\mu\nu\lambda}(x) m_c^2 J^\lambda(0) + \cdots , \tag{28}$$

where j_μ is the standard charged weak current,

$$j_\mu = \bar{u}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)(d\cos\theta_c + s\sin\theta_c) + \bar{c}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)(-d\sin\theta_c + s\cos\theta_c),$$

and

$$J_\lambda = \bar{d}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)s.$$

Since both j_μ and J_λ have zero anomalous dimensions,

$$\begin{aligned} m_c^2 C_{\mu\nu\lambda}(\xi^{-1}x, g) &= m_c^2(\xi) C_{\mu\nu\lambda}(x, g(\xi))\xi \\ &\simeq m_c^2 C_{\mu\nu\lambda}(x,0)\xi^{-1} \\ &\quad\times\left\{\exp\left[-2\int_{m_c/\mu}^{\xi} \gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}\right\}, \end{aligned} \tag{29}$$

where $g(\xi)$ is the running gluon coupling constant, $g(1)=g$. μ is the momentum subtraction point, and the anomalous dimension associated with the quark mass operator, γ_Θ, is given by

$$\gamma_\Theta = \frac{g^2}{8\pi^2}4. \tag{30}$$

Thus, the effect of strong interactions is to multiply the amplitude of Eq. (27) by the factor

$$\exp\left\{-2\int_{m_c/\mu}^{m_W/\mu} \gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}\right\} = \left[\frac{\alpha_4(m_W)}{\alpha_4(m_c)}\right]^{-24/25}, \tag{31}$$

where

$$\alpha_n(m_W) = 1 + \frac{g^2}{8\pi^2} b_n \ln\frac{m_W}{\mu},$$

$$b_n = \frac{33-2n}{3}$$

and n is the number of quark degrees of freedom in ordinary symmetry space SU(n) (i.e., flavors).

Equation (31) takes into account the fact that in the momentum range $m_c \lesssim |p| \lesssim m_W$, all four kinds of quark degrees of freedom are excited. Strictly speaking, we should add to Eq. (29) a contribution from the momentum region $\mu^2 \lesssim |p'|^2 \le m_c^2$, but this gives a correction factor

$$\exp\left\{-2\int_1^{m_c/\mu}\frac{d\xi'}{\xi'}\gamma_\Theta[g(\xi')]\right\} = \left[\frac{\alpha_3(m_c)}{\alpha_3(\mu)}\right]^{-24/27} \simeq 1.$$

Furthermore, Eq. (31) ignores any complications that might arise from the breakdown of perturbation theory near the charm-particle threshold.

For the Z-exchange diagrams, we note again that the relation (7) is valid in the presence of strong interactions. The bare $\phi_2\bar{c}c$ vertex is of the form

$$\mathcal{L}_{\phi_2\bar{c}c} = i\frac{m_c}{v}\bar{c}\gamma_5 c\phi_2$$

and we are led to consider the operator-product expansion

$$j_\mu(x) j_\nu^\dagger(y) p(0) \simeq \cdots + C'_{\mu\nu\lambda}(x,y) J_\lambda(0) + \cdots , \tag{32}$$

where

$$p = \bar{c}\gamma_5 c$$

is a pseudoscalar density. Since we have set the external masses to zero, the relevant operator on the right-hand side is necessarily a $V-A$ current operator. One c mass insertion is necessary to obtain a nonvanishing result since p is a right-left transition operator and j_μ is a left-left operator.

The analysis of this contribution differs from the above in that the operator $p(0)$ on the left-hand side has anomalous dimensions. We find

$$\gamma_p = \gamma_\Theta = \frac{4g^2}{8\pi^2}$$

Solution of the Callan-Symanzik equation then gives

$$m_c C'_{\mu\nu\lambda}(\xi^{-1}x, \xi^{-1}y, g) = m_c(\xi)\xi^5 C_{\mu\nu\lambda}[x,y,g(\xi)] \times\exp\left\{-\int_{m_c/\mu}^{\xi}\gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}\right\},$$

with $m_c(\xi)$ scaling as before. Then the expression on the right-hand side is modified with respect to the free-field case by a factor

$$\exp\left\{-2\int_{m_c/\mu}^{\xi}\gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}\right\}. \tag{33}$$

This is equivalent to the result of Nanopoulos and Ross, who instead considered the expansion of the product of three currents with two mass insertions. This corresponds to the effective $\bar{d}sZ$ vertex which one is actually trying to determine. However, our analysis is more straightforward as it is free of the complications entailed by the sum over individually divergent diagrams and the wavefunction renormalization arising from the weak $s \to d$ transition.

Our result therefore reduces to that of Nanopoulos and Ross if the momentum cutoff is effectively m_W, i.e., $\xi \to m_W/\mu$ in Eq. (33). This is precisely the point which is contested by Vainshstein *et al.*[3] and which is crucial if the cancellation of the leading contributions $[\sim\ln(m_W^2/m_c^2)]$ is to be maintained in the presence of strong interactions. Vainshtein *et al.* argue that while the effective distance $x-y$ is determined by the W propagator [see Fig. 5(a)],

$$|x-y|^2 \sim 1/m_W^2,$$

the effective distances x, y are determined by the charmed-quark propagator,

$$|x|^2,\ |y|^2 \sim 1/m_c^2.$$

In such a case the short-distance behavior of the coefficient function $C_{\mu\nu\lambda}(x,y)$ must be treated more carefully, as is in fact the case for the K_L-K_S mass difference.

In the free quark case, to lowest order in $1/m_W^2$, the hadronic weak vertex reduces to a local current-current interaction by the replacement

$$(m_W^2 - k^2)^{-1} \to m_W^{-2}.$$

This replacement is legitimate in a loop diagram [Fig. 5(b)] if the remaining integral is convergent; then the effective cutoff of the integral is clearly independent of the W mass. However, for the effective $\bar{d}s\phi_2$ vertex considered here, removal of the W propagator [which is equivalent to taking first the limit $x-y\to 0$ in Eq. (32)] leads to a divergent integral. Thus the momentum cutoff must be provided by the W mass, and we conclude that the result of Nanopoulos and Ross is the correct one.

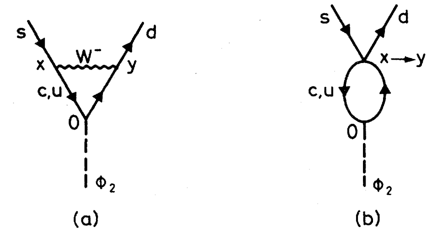

FIG. 5. Diagram (a) contributing to the effective operator of Eq. (32); the same diagram (b) in the limit $|x-y|\to 0$.

A further argument supporting this conclusion is the following. We saw in Sec. III that the WW and Z contributions to $K_L \to \mu\mu$ are not separately gauge invariant with respect to the gauge group of the weak interactions. Since the hadronic operators appearing in the Wilson expansion are the same for the gauge-dependent and gauge-independent pieces of each contribution, gauge invariance cannot be maintained unless both contributions have the same scaling behavior. Thus we conclude that the cancellation of $\ln(m_W^2/m_c^2)$ terms must be maintained in the presence of strong interactions.

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