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RECIPROCAL SPACE-TIME AND MOMENTUM-SPACE SINGULARITIES IN THE NARROW RESONANCE APPROXIMATION*)

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A B S T R A C T

A general scheme is proposed which makes explicit the relationship between the singularities of off-shell amplitudes in position-space and momentum-space in the narrow resonance approximation. In some ways this may be viewed as a duality scheme for amplitudes involving external quarks, in which narrow resonances in certain channels build the Fourier transform of power singularities in x^2 (x^μ being a position vector). This scheme is made precise by dual string off-shell amplitudes. As well as highlighting possible connections between the general dual framework and the structure of confined field theories we are able to pinpoint certain grave shortcomings of present dual models.

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1. INTRODUCTION

In any quantum field theory that builds extended particle states out of confined quarks the space-time properties of products of currents will be exceedingly complicated even if their short distance behaviour can be understood in perturbation theory (as in asymptotically free theories). Thus, for example, the space-time singularity structure of the current propagator must be sufficiently elaborate so that when viewed in momentum space it generates only the physical spectrum of particle states (with no quark thresholds). In the narrow resonance approximation this would consist of a sum of pure poles in momentum space with positive residues.

The nature of the relationship between position-space and momentum-space singularities can be studied in the dual string model which provides an explicit framework for understanding certain features of amplitudes involving off-shell states or currents[1-4]. We have been able to isolate a strikingly simple relationship between the space-time and momentum-space structure of off-shell dual amplitudes which suggests a general scheme for any theory which has, in first approximation, a narrow-resonance spectrum.

The crucial feature of this scheme is that, in quark-antiquark amplitudes, the direct channel sequence of resonance poles is built up from the Fourier transform of an infinite sequence of discrete singularities in the x^2 plane (where x^μ is the position separation of the ends of the resonance), inside as well as on the light cone. It may be that discrete singularities inside the light cone occur only as an approximation, the reflection of the narrow resonance approximation, and are smoothed out in the real world. It is this correspondence between x^2 singularities and the narrow resonance approximation that we shall see is made precise in the dual model.

Our point of view in describing these results will be to highlight features that we feel may be quite general as well as to clarify the possible relationship between ordinary quantum field theory and dual string theory. The main results have a simple diagrammatic representation, analogous to conventional duality diagrams but involving confined "quark" lines which couple to the external currents. From the point of view of the dual model the appearance of confined structure in the description of off-shell amplitudes is perhaps the most fascinating part of the scheme. This will be discussed more precisely in the latter part of the paper.

In Section 2 we will state the rules that are abstracted from the dual model calculations. We will concentrate mainly on those features which seem likely to be true in any dual model and may be appropriate for any narrow resonance approximation. In Section 3 two explicit string model calculations will be presented, in which the relation between momentum-space and position-space singularities will become apparent as the result of a Jacobi transformation. The reformulation of the string model amplitudes in terms of states which have the properties of the quarks described above will be outlined in Section 4. In present off-shell dual models there is one singularity outside the light-cone which is highly undesirable. In order to show that this is not a crucial feature of any conceivable narrow resonance scheme we give an example in Section 4 of a function satisfying all the positivity constraints together with an infinite set of x^2 singularities starting on the light-cone. In conclusion, in Section 5 we will make contact between the $\alpha' \to \infty$ limit (where α' is the Regge slope) of models of the type considered in this paper and usual field theory.

2. RECIPROCAL SINGULARITIES IN POSITION AND MOMENTUM SPACE

The simplest diagram that we can consider is the current propagator. For convenience only scalar currents will be considered in this paper, but there is no difficulty in extending these results to off-shell states of arbitrary angular momentum (these states are easily defined in the string picture[5)]).

The propagator may be viewed as the sum of resonances in q^2, coupling to currents at either end via quark-antiquark pairs (Fig. 1a). It turns out that the propagator may also be written as the Fourier transform of the sum over a discrete set of singularities in $x^2 (x^\mu = x_1^\mu - x_2^\mu)$ represented by the dashed line in Fig. 1b. The quark lines do not propagate in space-time and thus have no momentum-space singularities corresponding to normal thresholds. We emphasize that this is not an ad hoc assumption but is, of course, a crucial property of dual models. It is made precise in the operatorial method of constructing current amplitudes discussed in Section 4 and based on methods introduced in Ref. 2). Our diagrams are therefore <u>not</u> to be viewed as normal Feynman diagrams since, in the space-time representation (such as Fig. 1b) the whole length of each quark line is at fixed x. In the momentum space pictures (such as Fig. 1a) <u>both</u> ends of each quark in a pair always occur together and the momentum is considered to be transmitted by the pair as a whole. The dashed line in Fig. 1b carries a definite value of $(x_1^\mu - x_2^\mu)$, the variable in which poles occur.

The equivalence of a sum of momentum-space poles to the Fourier transform of a sum of discrete position-space singularities for amplitudes involving off-shell states compliments the usual duality relationship between resonance poles in crossed channels for on-shell amplitudes.

We find the following structure

$$F(q^2)=\sum_{n=n_0}^{\infty}\frac{c_n}{(\alpha' q^2-2n+i\epsilon)} = \int e^{iq\cdot x}\,\tilde{F}(x^2+i\epsilon)\,d^D x \qquad (1)$$

$$\tilde{F}(x^2)=\sum_{n=n_1}^{\infty}\frac{\bar{c}_n}{\left(\frac{x^2}{4\pi^2\alpha'}-2n\right)^{a_0}} \qquad (2)$$

where we are using the metric (+,-,-,-). As usual α' is the Regge trajectory slope. We will return to the derivation of Eq. (1) in the context of the dual model in the next section, but for the present we would like to emphasize certain features of this picture which are possibly general properties.

a) The singularities in x^2 are discrete and narrow. As noted before, this is probably a consequence of the narrow-resonance approximation. If this is to be a good approximation, the real world must be some sort of average of Eq. (1), with a cut starting at $x^2 = 0$, taking the place of the line of discrete power singularities.

b) The spacing of the singularities in x^2 is $8\pi^2\alpha'$ and is, therefore, determined by the inverse of the spacing of resonances in q^2.

c) The left-most singularity in x^2 ought to be on the light-cone ($x^2 = 0$) for obvious causal reasons. We thus expect $n_1 = 0$ in any realistic dual model. The ordinary Veneziano model has $n_1 = -\frac{1}{2}$ which must be interpreted as a sickness of the model. Likewise, absence of tachyons requires $n_0 \geq 0$ (as is well known, this condition is difficult to satisfy in presently known dual models).

d) All singularities in x^2 are of the same nature, determined by a_0. The exact value of a_0 depends on the particular model under consideration, but it is a simple integer or half-integer in all models considered. The high q^2 behaviour of the propagator is determined by the left-most singularity in x^2

if we make the usual dual model smoothing assumption by moving along a small angle ray in q^2. Thus, putting $n_1 = 0$, the short distance structure plays its usual role in determining the high q^2 behaviour with the leading correction terms contributing exponentially damped oscillations of the form $\exp\{\pi i \sqrt{8\alpha' |q^2|} \cos \theta/2 - \pi \sqrt{8\alpha' |q^2|} \sin \theta/2\}$ (where θ is the angle chosen for the ray in the q^2 plane). If there were to be an underlying connection with asymptotically-free field theories we would expect a_0 to take the appropriate free-field value*).

e) The coefficients, c_n, are all positive in dual theories. This is analogous to the positivity constraint on the spectral function in field theories. However, we also find that the coefficients of the x^2 singularities, $\bar{c}_n$, are all positive. This seems to be a highly non-trivial constraint in the dual model as well as in the mathematical example given in Section 4.

f) Singularities inside the light-cone are somewhat unpopular in quantum field theory due to constraints on the spectral representation. It may well be that the complete theory will not have such singularities but they only occur at the same level as the usual narrow-resonance approximation. Since in the actual dual models considered in Section 3 we cannot avoid having a pole outside the light-cone it is important to know if such a sickness is an inevitable result of the narrow-resonance approximation. In Section 4 we will show by means of a mathematical example that this is not the case.

The next process we consider is the forward scalar Compton scattering process of Fig. 2a (the diagram corresponding to the other time ordering of the current interactions is omitted). Fig. 2a may be viewed as a tree diagram for four quark-two particle scattering in which the q_1^2, q_2^2 and M^2 resonances are explicit. Once again the diagram may be expressed as the Fourier transform of a sum of poles in x^2 which can be drawn as Fig. 2b. This diagram can be thought of as a tree diagram illustrating the singularity structure of the amplitude.

We now meet internal quark lines for the first time. These carry excitations in much the same way as internal resonance lines are excited (this statement has a precise definition in the dual model formulation of Section 4) but as far as the diagrammatic rules are concerned, the main point is that they do not have momentum space singularities. All quark lines occur instantaneously once again, in the sense that each line corresponds to one value of the space-time label, x.

*) In this point of view the logarithmic cuts in the short distance behaviour of the field theory will arise as a higher order correction to the pure power behaviour of Eq. (2).

Our amplitude takes the form:

$$T(q_1^2, q_2^2, M^2, t) = \int e^{iq_2 \cdot x}\, \tilde{T}(x^2 + i\epsilon, p_1 \cdot x, p_2 \cdot x, t)\, d^D x \tag{3}$$

with

$$T(q_1^2, q_2^2, M^2, t) = \sum_{\ell, m, n = n_0}^{\infty} \frac{C_{\ell,m,n}(q_1^2, q_2^2, M^2, t)}{(\alpha' q_1^2 - 2\ell + i\epsilon)(\alpha' M^2 - 2m + i\epsilon)(\alpha' q_2^2 - 2n + i\epsilon)} \tag{4}$$

$$\tilde{T}(x^2, p_1 \cdot x, p_2 \cdot x, t) = \sum_{n=n_1}^{\infty} \sum_i \frac{d_n(x^2, p_1 \cdot x, p_2 \cdot x, t)}{\left(\frac{x^2}{4\pi^2 \alpha'} - 2n\right)^{a_0}} \tag{5}$$

where $t = (p_1 - p_2)^2$ and $M^2 = (p_1 + q_1)^2 = (p_2 + q_2)^2$ and we have omitted a momentum conservation δ function that sets $q_1 = p_2 + q_2 - p_1$. Likewise we have set $x^\mu = x_2^\mu - x_1^\mu = x_2^\mu$ by translation invariance.

Once again we leave the derivation of the precise structure of this relation in the dual model until the next section, but note these points:

a) the parameter n_1 which determines the position of the left-most x^2 singularities is the same as occurred in the propagator function [Eq. (2)]. In the Bjorken limit ($q_1^2 \to -\infty$, $q_2^2 \to -\infty$, $M^2 \to \infty$ with $x_1 = -q_1^2/M^2$ and $x_2 = -q_2^2/M^2$ fixed) this singularity dominates the amplitude. This means that present dual models do not have scaling (since $n_1 = -\frac{1}{2}$) but any model with $n_1 = 0$ would have scaling behaviour. Again, these statements are meant in the usual dual model sense of smoothing poles on the real M^2 axis by taking the limit along a small-angle ray in M^2. The corrections are again exponentially damped oscillations due to the first singularity inside the light-cone whose exact form depends on the smoothing procedure used.

b) the natures of the singularities in x^2 are governed by the parameters a_i which have a simple connection with a_0 and also depend on the type of external particles. In the models considered the a_i are all integers or half-integers.

In order to generalize the diagrams to an arbitrary process involving M external particles and N currents, we first draw a tree diagram that displays all the resonance poles -- this is a momentum-space diagram and all quark lines come in pairs beginning and ending at the same point. There are M + N - 1 independent momenta flowing into the diagram. When the external currents are amputated, the resultant M particle-2N quark tree diagram will satisfy duality-like transformation rules in which the basic elements are four-point subamplitudes. For example, Fig. 3 illustrates the three current-two particle diagram in various configurations. The diagrams are to be understood by analogy with the previous cases, as being mixed position-space momentum-space diagrams according to which configuration is being exhibited. Whenever a dashed line appears it represents position-space singularities in the variable labelling the line, together with the understanding that the diagram is to be Fourier transformed with respect to all independent position labels. Notice that one label may always be fixed at 0 by translation invariance. This is equivalent to momentum conservation and thus there are always M + N - 1 independent external variables in our diagrams. There is clearly much structure in these diagrams that we do not yet understand since we have only done explicit calculations for two special examples (for instance, it is not clear how the $(x_1 - x_3)^2$ singularities in Fig. 3d are built out of Fig. 3c). However, we shall see in Section 4 that the kinds of transformations involved in passing from Fig. 3a to Fig. 3d can easily be visualized in terms of distortions of rather peculiar world sheets that arise in the covariant description of currents.

3. CALCULATIONS WITH THE STRING MODEL

The picture for the coupling of an off-shell string was introduced in Ref. 3) and has a close connection with the covariant approach of Refs. 1) and 2). In this picture the external source couples to a section of the string that has collapsed to a point. However, a crucial ingredient in obtaining sensible form factors is that the collapsing section of string can split away from the rest of the string at some time, τ_0, and this time must be integrated over in the sum over histories of the string. Thus (see Fig. 3a) the possibility of the string breaking plays a vital role in providing finite form factors.

In the case of the ordinary Veneziano model in D = 26 dimensions these off-shell amplitudes are not strictly Lorentz scalars but we shall take the point of view that this is not a crucial drawback. Indeed, it is possible to construct off-shell amplitudes[4)] for closed strings which have the same sort of physical point-like picture and which are Lorentz scalars. Although we shall not consider these in this paper (since they are somewhat complicated) it seems clear that our general results will hold for them. It is intriguing that the only known consistent description of off-shell behaviour involves coupling the string to point-like sources exactly like those that define the Green functions in usual quantum field theory.

In this picture the current propagator is just the amplitude for a point-like string at $\tau = \tau_1$ [where τ is the light-cone time variable $i(x^0 + x^{D-1})/\sqrt{2}$]† to propagate into a point-like state at $\tau = \tau_2$. [See Fig. 4b and Ref. 6) for the relevant conventions.] This is [see Ref. 3) for the derivation] *)

$$F(q^2) = c\int_0^\infty d(\tau/2q^+)\, e^{(q^2+1)\tau/2q^+} \prod_{n=1}^{\infty}\left(1 - e^{-2n\tau/2q^+}\right)^{-12} \tag{6}$$

where

$$q^\pm = (q^0 \pm q^{D-1})/\sqrt{2}\,,\quad q^2 = 2q^+q^- - \underline{q}^2\,,\quad \tau = \tau_2 - \tau_1$$

As expected, this amplitude which diverges at small τ, may formally be expanded as a sequence of resonance poles as in Eq. (1) [in the D = 16 covariant approach these are all the scalar poles of the theory[7]]. The constant c absorbs non-scalar factors in the off-shell states which we shall ignore from here on.

The first step in looking at the x space picture of this propagator is the same as that involved in examining the high negative q^2 behaviour of Eq. (6) which is to rewrite the integrand in terms of the Jacobi theta functions[8], $\theta_1'(0|\tau)$, giving

$$F(q^2) = c(2\pi)^4\int_0^\infty d(\tau/2q^+)\, e^{q^2\tau/2q^+}\left[\theta_1'\left(0\left|\frac{i\tau}{2\pi q^+}\right.\right)\right]^{-4} \tag{7}$$

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†) We are working with imaginary light-cone times as in Ref. 6). Furthermore we shall also use $x^- = -i\,(x^0 - x^{D-1})/\sqrt{2}$ so that $x^2 = 2\tau x^- - \underline{x}^2$.

*) In all subsequent expressions we shall set $\alpha' = 1$. It is easy to reinstate α' at any point since it is the only dimensional parameter in the theory.

and use the Jacobi transform to obtain

$$F(q^2) = \frac{c\,2^4}{\pi^2}\int_0^\infty d(\tau/2q^+)\,(\tau/2q^+)^6\, e^{q^2\tau/2q^+}\left[\Theta_1'(0|2\pi i q^+/\tau)\right]^{-4} \tag{8}$$

For large negative q^2 the small τ region of the integral dominates and the integrand of Eq. (8) can be expanded in a power series in $e^{-4\pi^2 q^+/\tau}$. The first term gives the nasty divergence at small τ. We shall see the origin of this divergence in a moment. Equation (8) can easily be transformed into x space by first undoing the τ integral (and dividing out $e^{\tau q^-}$), multiplying by $e^{i\underline{q}\cdot\underline{x}}\, e^{q^+x^-}$ and integrating with respect to $\underline{q}$ and q^+ in the appropriate regions. This gives:

$$\tilde{F}(x^2) = \frac{c\,(4\pi)^6}{2(2\pi)^{25}}\int_0^\infty d(q^+/2\tau)\,(q^+/2\tau)^5\, e^{(x^2+4\pi^2)\,q^+/2\tau}\prod_{n=1}^{\infty}\left(1-e^{-2n\,4\pi^2 q^+/2\tau}\right)^{-4} \tag{9}$$

The integrand now has a power series expansion in which a general term has the form:

$$\int_0^1 dy\, y^5 \exp\left\{\left(\frac{x^2}{4\pi^2}+1-2m\right)4\pi^2 y\right\} = \Gamma(6)\Big/\left(\frac{x^2}{4\pi^2}-2m+1\right)^6 \tag{10}$$

where the number 6 comes from (D-2)/4 with D = 26. This is clearly a model-dependent number which corresponds to a_0 in Eq. (2). The series expansion starts with the term with the value m = 0 in Eq. (10) which gives rise to a singularity outside the light-cone at $x^2 = -4\pi^2\alpha'$. [This was the cause of the divergence at small τ in Eq. (8).] There is also no singularity on the light-cone ($x^2 = 0$). These, of course, are bad features of the model and requiring their absence may be expected to provide powerful constraints on possible models (see Section 4). Notice that the integral defining $\tilde{F}(x^2)$ [Eq. (9)] is divergent due to the presence of a tachyon pole in the model at $\alpha' q^2 = -1$.

We have found that the general structure of the dual scalar propagator is of the form given by Eqs. (1) and (2). The presence of both a q^2 tachyon and a x^2 singularity outside the light-cone means that only the mixed τ-p^+ representation is really well defined.

In order to be sure that the general structure of Eqs. (1) and (2) can really be consistent it would be reassuring to have at least a mathematical example of a function without these sicknesses which possesses poles in q^2 with positive residues and isolated singularities in x^2. In Section 4 such an example will be presented. For the moment we shall proceed to extract the asymptotic form of Eq. (6) ignoring the fact that the leading term is divergent.

The high q^2 behaviour of the propagator can be obtained by expanding the integrand in Eq. (8) as a power series in exp $\{-4\pi^2 \alpha' q^+/\tau\}$ which gives a sum of terms of the form (valid only for $n \neq 0$):

$$\int_0^\infty dz \; e^{\alpha' q^2 z - (2n-1)\pi^2/z} \; (z)^6$$

$$= 2K_{-7}\left(\left(-4\pi^2(2n-1)\alpha' q^2\right)^{1/2}\right)\left(\frac{-\alpha' q^2}{(2n-1)\pi^2}\right)^{-7/2} \qquad (11)$$

where $K_\nu(x)$ is a modified Bessel function[8] and q^2 lies outside of a wedge along the positive real axis. The asymptotic form of K_ν gives the leading form of the general term at large $|q^2|$ as:

$$\sim \left(\frac{-\alpha' q^2}{(2n-1)\pi^2}\right)^{-7/2} \left(-\alpha' q^2 (2n-1)\right)^{-1/4} \exp\left\{i\sqrt{4\pi^2(2n-1)\alpha' q^2}\right\} \qquad (12)$$

The $n = 0$ term is the divergent piece mentioned above.

Assuming that the basic structure of our result will hold in more realistic situations with $n_1 = 0$, we see that the high q^2 behaviour (excluding a wedge along the real axis) will be dominated by the term with $n = \frac{1}{2}$ and will, therefore, be power behaved. The leading correction will come from the first x^2 singularity inside the light cone and will contribute exponentially damped oscillating behaviour as previously described.

Our second example was Compton scattering (Fig. 2) which is represented in the string picture by Fig. 4c. Since the expressions easily become very burdensome we have chosen a particularly simple situation in which the on-shell particles are the massless vectors of the theory and we are looking at the forward amplitude so that the kinematics may be simplified. We can choose:

$$\not{p}_1 = \not{p}_2 = 0 \tag{13}$$

and

$$p_1^+ = p_2^+ = 0 \tag{14}$$

with p_1^- and p_2^- arbitrary and so there are no singular components of momentum (recall $p_1^2 = p_2^2 = 0$). We then have

$$q_1^+ = q_2^+ = q^+ \ , \quad \not{q}_1 = \not{q}_2 = \not{q} \tag{15}$$

The scaling limit of interest is, therefore:

$$\begin{gathered} M^2 \to \infty, \ q_1^2, q_2^2 \to -\infty, \ t = 0 \\ x_1 = -q_1^2/M^2, \ x_2 = -q_2^2/M^2 \ \text{fixed} \end{gathered} \tag{16}$$

(with M^2 actually along a small angle ray), where

$$M^2 = q_1^2 + 2q_1^+ p_1^- = q_2^2 + 2q_2^+ p_2^- \tag{17}$$

The amplitude that arises in Fig. 4c has the currents coupling at times 0 and τ to the point-like string states. These interact with the external vector states at times τ_1 and τ_2 and the amplitude can be written as :

$$T^{ij}(q_1^2, q_2^2, M^2) = c \int d\tau/2q^+ \, d\tau_1/2q^+ \, d\tau_2/2q^+$$
$$\exp\{ q_1^- \tau_1 + q_3^- (\tau_2 - \tau_1) + q_2^- (\tau - \tau_2)\} \, M(\tau, \tau_1, \tau_2, q, q^+) \tag{18}$$

where:

$$M = \sum_{m,n} \langle c, q_2 | e^{-\mathcal{P}^-(\tau-\tau_2)} (a_n^i + a_n^{i+}) e^{-\mathcal{P}^-(\tau_2-\tau_1)} (a_m^j + a_m^{j+}) e^{-\mathcal{P}^- \tau_1} | c, q_1 \rangle \tag{19}$$

and $|c\rangle$ is the point-like current state satisfying:

$$a_m^i |c\rangle = a_m^{i+} |c\rangle \tag{20}$$

The operators a_m^i obey the commutation relations

$$[a_n^i, a_m^{j+}] = -n \delta_{m,n} \delta^{ij} \tag{21}$$

and $\mathcal{P}^-$ is the integral of the Hamiltonian over σ at a fixed time. In Appendix A we show that Eq. (18) becomes:

$$T^{ij}(q_1^2, q_2^2, M^2) = \frac{ic\pi}{2}\,\delta^{ij}\int d\phi_1\, d\phi_2\, d\gamma$$

$$\times \exp\left\{-i\pi\left((\phi_1+\phi_2)\,q_1^2 + (\gamma+\phi_2-\phi_1)\,q_2^2 - 2\phi_2 M^2\right)\right\} \tag{22}$$

$$\times \left[\frac{\partial}{\partial\phi_1}\left(\frac{\Theta_1'(\phi_1|\gamma)}{\Theta_1(\phi_1|\gamma)}\right) + \frac{\partial}{\partial\phi_2}\left(\frac{\Theta_1'(\phi_2|\gamma)}{\Theta_1(\phi_2|\gamma)}\right)\right]\left[\Theta_1'(0|\gamma)/2\pi\right]^{-4}$$

where $\tau/2q^+ = -i\pi\gamma$.

The form of this solution is reminiscent of that of Schwarz[9)] but considerably simpler. This is partly because we are working at $t = 0$ and with vector external states which eliminate many nasty factors. Also recall that the string picture often provides expressions for amplitudes in a simpler form than the covariant approach.

The integral in Eq. (22) diverges at $\phi_2 = 0$ (i.e., $\tau_1 = \tau_2$ in Fig. 4c) for two trivial reasons. Firstly, the integral representation is improperly defined for $t = 0$ which is above the tachyon pole in the t-channel. Clearly we could have worked at non-zero t and this would cause no problem. Second, at $t = 0$ we are on top of the t channel vector pole. This does not contribute to the fully signatured amplitude (which includes the other time ordering) or to the M^2 discontinuity. Since both these effects are quite usual for forward amplitudes and cause no problems we may safely ignore them.

There is also a nasty singularity at small τ. This will be seen once more to be related to the presence of a x^2 singularity outside the light-cone.

The asymptotic q^2 and M^2 behaviour will again be dominated by small τ (and hence τ_1 and τ_2) and so it will be isolated by performing a Jacobi transform on Eq. (22). This will also be the first step towards looking at the Fourier transform. This change of variables is given in Appendix B from which it is easy to see that the leading term for $\gamma' = -1/\gamma \to \infty$ is dictated by the leading term of the same partition function as that appearing in the current propagator. Thus the leading term has the form

$$\int d\tau/2q^+ \, d\tau_1/2q^+ \, d\tau_2/2q^+ \, \exp\left\{ \frac{M^2\tau}{2q^+}\left(\frac{\tau_2-\tau_1}{\tau} - \frac{\tau_1 x_1}{\tau} - \frac{(\tau-\tau_2)x_2}{\tau} \right)\right\} \left(\tau/2q^+\right)^4 e^{2\pi^2 q^+/\tau} \prod_{n=1}^{\infty} \left(1-e^{-2n\, 2\pi^2 q^+/\tau}\right)^{-4} f(\tau_1/\tau, \tau_2/\tau) \tag{23}$$

Once more we see that the leading term is divergent which is a sickness again related to a x^2 singularity outside the light-cone. Again we can consider the situation that would arise in a model with the structure of Eq. (23) but with $n_1 = 0$ [Eq. (2)] which would eliminate the positive power of $e^{2\pi^2 q^+/\tau}$ in Eq. (23). We can now see that the first term in the series expansion of Eq. (23) in powers of $e^{-4\pi^2 q^+/\tau}$ gives a scaling form. Thus, if we change variables to

$$u = \tau_1/\tau \quad , \quad v = (\tau-\tau_2)/\tau \quad , \quad z = \frac{M^2\tau}{2q^+}\left(\frac{\tau_2-\tau_1}{\tau} - \frac{\tau_1 x_1}{\tau} - \frac{(\tau-\tau_2)x_2}{\tau} \right) \tag{24}$$

we obtain an integral of the form

$$(M^2)^{-7} \int dz\,du\,dv \; e^z \; z^6 \; g(u,v,x_1,x_2) \tag{25}$$

for the leading behaviour. The integral scales and our result is similar in structure to the scaling function analyzed by Schwarz[9]. We shall omit discussion of the discontinuity of Eq. (25) which is also similar to that given by Schwarz. Note that non-leading corrections to Eq. (25) arise from the lower powers of γ' which differ by an integer from the leading term and will affect the power of M^2 in Eq. (25) but will still scale. Further non-scaling corrections arise from the power series expansion in $e^{-4\pi^2 q^+/\tau}$ which give oscillatory corrections that decrease exponentially when M^2 is taken along a small angle ray rather as in the case of the simple propagator.

It is straightforward to complete the Fourier transform of Eq. (22) by undoing the τ integral (and dividing by $e^{q_2^- \tau}$), multiplying by $e^{q^+ x^-}$ and $e^{i\underline{q}\cdot\underline{x}}$ and integrating with respect to q^+ and $\underline{q}$, (using the momentum conservation δ function to eliminate q_1). The Jacobi transform of Appendix B must first be carried out which changes ϕ_1 and ϕ_2 to ϕ_1' and ϕ_2' and we obtain

$$\tilde{T}^{ij}(x^2, p_1\cdot x, p_2\cdot x) = c'\,\delta^{ij}\int d\phi_1' d\phi_2'\, d(q^+/2\tau)\,(q^+/2\tau)$$

$$\exp\left\{x^2 q^+/2\tau - ip_1\cdot x(\phi_1'+\phi_2') + ip_2\cdot x(\phi_1'-\phi_2')\right\} \tag{26}$$

$$\cdot\left[\frac{\partial}{\partial\phi_1'}\left(\frac{\theta_1'(\phi_1'|\gamma')}{\theta_1(\phi_1'|\gamma')}\right) + \frac{\partial}{\partial\phi_2'}\left(\frac{\theta_1'(\phi_2'|\gamma')}{\theta_1(\phi_2'|\gamma')}\right)\right.$$

$$\left. + \frac{2\pi i}{\gamma'}\left(2 - \frac{2\pi i}{\gamma'}(\phi_1'^2 + \phi_2'^2)\right)\right]\left[\frac{\theta_1'(0|\gamma')}{2\pi}\right]^{-4}$$

where $\gamma' = 2\pi i q^+/\tau$ and we have absorbed all constant factors into c'. Notice that since $e^{i\pi\gamma'} = e^{-4\pi^2 q^+/2\tau}$, the integrand in Eq. (26) has a power series expansion analogous to that of the propagator. This leads to a series of x^2 singularities of the form of Eq. (5) with $a_0 = 6$. The lower powers of γ' in the last term in Eq. (26) give rise to milder singularities in x^2 with strengths $a_1 = 5$ and $a_2 = 4$. The leading singularity is, as previously found, outside the light-cone. [Once more we have to ignore a formal divergence in Eq. (26) at small q^+ related to the presence of a tachyon in the model.]

The kinematic situation considered in this example was specially chosen to give relatively simple expressions. To understand the structure of the amplitude completely it should be generalized to currents coupling to arbitrary numbers of external particles with arbitrary momenta. This problem seems to be of the same order of difficulty as conventional single loop calculations.

4. CONFINED STRUCTURE AND DUAL MODELS

The string picture used in the previous section has been shown[3] to give rise to the same form factors and current propagator as the covariant approach[1,2] based on satisfying the gauge algebra of the dual model, despite a mismatch in the dimensions of the two approaches. Furthermore, in the closed string sector, the two methods are exactly equivalent[4,5]. We therefore feel confident that the string results of the previous section will be reproduced in the covariant method. An elegant formulation of this method was found in Ref. 2) by introducing c mode states, where c modes satisfy:

$$[c_n^\mu, c_m^{+\nu}] = -g^{\mu\nu}\delta_{nm} \qquad (27)$$
$$n,m = \tfrac{1}{2}, \tfrac{3}{2}, \ldots$$

The "propagator" formed out of these modes takes the form $1/(L_{0c}-1)$, where

$$L_{0c} = -\sum_{n=1/2}^{\infty} n\, c_n^{+\mu} c_{n\mu} \qquad (28)$$

Since there are no zero modes in L_{0c} there is no momentum in this "propagator" and its interpretation is not immediately apparent.

However, we may proceed to draw formal diagrams for form factors (Fig. 5) which are associated with precise rules and in which the off-shell state is described by two c moded ground states coupling to a normal Reggeon tree via a well-defined coupling. It is these c moded ground states that we have been calling the external "quarks" in Figs. 1 and 2. With this identification these figures also have a precise formal definition in the dual model. The excited quarks just carry higher c modes.

The c modes may be thought of as normal modes for a world sheet with an abnormal boundary on which the "position" vector $X^\mu(z)$ is fixed, instead of its normal derivative. [This is the analogue model viewpoint[2] from which the c-modes were originally obtained.] We can therefore draw world sheets for arbitrary processes involving these quark states (but bearing in mind that they do not have a simple space-time string interpretation because of the unusual boundary conditions).

In Figs. 6a-6d we show the same sequence of diagrams as in Figs. 3a-3d and it is now apparent that these are just distortions of the same basic world sheet structure. We have emphasized the abnormal boundaries by thick lines. The abnormal boundary condition ensures that the whole of such a boundary is at the same space-time point and so the labelling of Fig. 3 is clarified. Note that the dashed lines of Fig. 3 appear as strips with two abnormal boundaries, fixing the space-time values along both edges to the value indicated. It is these lines that have singularities in the space-time variables shown in Fig. 3. Indeed, these strips have a form very similar to the normal string but with the roles of position-space and momentum-space interchanged[*)]. A further attraction of the terminology "quark" for the c moded objects is apparent if flavour is added to the string by means of Chan-Paton factors. This is accomplished most simply by making the c moded objects carry the fundamental representation (quark quantum numbers). This theory quite clearly confines these quark quantum numbers. Furthermore, in quark-antiquark scattering (Fig. 1a), it is apparent that the channel "dual" to the flavoured resonances is a flavour singlet. The dashed lines in Figs. 1-3 carry no flavour and thus behave in many respects like confined glue states.

These statements are summarized very naturally in terms of the world sheets of Fig. 6. They correspond to the rule that only the normal boundary in any diagram should carry the quark quantum number. (This quite obviously reproduces the usual Chan-Paton rule.)

The closed string currents of Refs. 4) and 5) may be represented by a closed c moded loop (Fig. 7a). It has been suggested[4)] that the insertion of arbitrary numbers of such currents at zero momentum into a propagator may modify the model in a physically sensible fashion since they can be identified with a reweighting of point-like modes of the string. In the present picture (Fig. 7b) these insertions take the form of quark-loop insertions.

An altogether different approach to modifying the dual model may be to follow an approach similar to that of Nahm[10)] and examine the constraints on the automorphic functions in the theory[+)]. If we were to take the view that the current propagator will inevitably be the integral of an automorphic function then our requirements on n_0 and n_1 [Eqs. (1) and (2)] would provide strong constraints on possible candidates. While we do not understand the nature of these assumptions it does provide us with simple candidates for propagators which do not have

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*) I am grateful to J.A. Shapiro for helpful discussions on this point.

+) I am grateful to W. Nahm for a useful discussion concerning the properties of automorphic functions.

tachyons or x^2 singularities outside the light cone. These serve to illustrate that the basic structure of Eqs. (1) and (2) is not sensitive to these diseases. For example consider the function

$$F_1(q^2) = \int_0^\infty d(\tau/2q^+)\, e^{q^2\tau/2q^+} \left[\Theta_3(0|\gamma)\right]^4 \tag{29}$$

where $\gamma = i\tau/2\pi q^+$ as before and

$$\begin{aligned}\Theta_3(0|\gamma) &= \prod_{n=1}^{\infty}\left(1-e^{-2n\tau/2q^+}\right)\left(1+e^{-(2n-1)\tau/2q^+}\right)^2 \\ &= 1+2\sum_{m=0}^{\infty} e^{-m^2\tau/2q^+}\end{aligned} \tag{30}$$

It is clear that $F_1(q^2)$ has a positive definite spectrum and no tachyons, the lowest pole being massless. Fourier transforming as before gives

$$\tilde{F}_1(x^2) = 16(\pi)^{1-\frac{D}{2}} \int_0^\infty d(q^+/2\tau)\, e^{x^2 q^+/2\tau} \left(\frac{q^+}{2\tau}\right)^{D/2} \left[\Theta_3(0|-1/\gamma)\right]^4 \tag{31}$$

which has singularities in x^2 starting <u>on</u> the light-cone of the form $(x^2/4\pi^2\alpha' - n)^{-(D+2)/2}$ (clearly the appropriate value of D is not known*)). The function $F_1(q^2)$ therefore constitutes an example of a function which transforms in the manner suggested in Eqs. (1) and (2) with $n_0 = n_1 = 0$.

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*) Strictly speaking Eq. (29) has a mild divergence at small $\tau/2q^+$ which looks very much like a normal field-theoretic short-distance divergence. This can be removed by two subtractions.

5. CONCLUSION

The narrow resonance approximation, in the form of the Born term of the dual model, represents the first approximation to the full dual theory. The presence of discrete singularities inside the light-cone which may be excluded in the full theory is therefore, in this approximation, not a disaster. We would expect that in the complete theory the higher-order corrections which serve to shield the resonance poles behind normal threshold cuts will also shield x^2 singularities. Our hope is that the structure of the complete theory will represent some sort of local average of the initial approximation. The way in which Eqs. (1) and (2) are realized in the dual model is very subtle. It is clear that any one singularity in x^2 (at $x^2 = 0$) will give rise to a singularity at $q^2 = 0$ and a non-positive-definite spectral function. The model therefore incorporates a delicate mechanism that balances an infinite set of x^2 singularities (with positive coefficients) against an infinite set of q^2 poles (also with positive coefficients). Since the actual dual model expressions are so divergent when expressed in terms of q^2 [Eqs. (8) and (22)] we must clearly be careful in deciding whether small x^2 singularities dominate usual light-cone-like kinematic limits. However, our analysis, which starts with well-defined amplitudes in the mixed $\tau - p^+$ representation, shows that these exponential divergences can be directly attributed to a x^2 singularity outside the light-cone. Given the usual dual model method of smoothing resonances (which is necessary in any case to discuss Regge behaviour) our disease-free mathematical example [Eq. (29)] suggests that the small x^2 limit will be achieved in the usual kinematic regime in a model with realistic structure. Although we have considered only scalar off-shell states in this paper it is clear from the manner of construction of higher spin states in the dual model[5)] that the basic relationship between x^2 and q^2 singularities will generalize to amplitudes involving states of any angular momentum.

Fig. 6 illustrates the trajectories of singularities of the propagator inside the light-cone [this is the structure of Eqs. (1) and (2) with $n_0 = n_1 = 0$ as well as our example in Eq. (31)]. If we take equal time slices starting at $t = 0$ we see that up until $t = t_1 = 2\pi\sqrt{2\alpha'}$ the only singularity is the one on the light-cone. At $t = t_1$ the first singularity inside the cone is intersected and subsequently spreads out on a hyperboloid. Further singularities are encountered at $t = t_n = 2\pi\sqrt{2n\alpha'}$ and similarly spread out as shown in Fig. 6. At no time do two singularities get further apart than $2\pi\sqrt{2\alpha'}$.

In the classical string picture the limit $\alpha' \to \infty$ corresponds to the vanishing of the tension in the string which ought to be the free field limit in any underlying field theoretic construction of realistic strings. We see from Fig. 6 that since the spacing of the singularities grows with $\sqrt{\alpha'}$ we recover a single singularity on the light-cone in the limit $\alpha' \to \infty$ *) of the form $1/(x^2 + i\varepsilon)^{a_0}$, which is suggestive of a massless free field theory. The nature of the field theory is correllated with the value of a_0. These observations provide a hint of correspondence between realistic strings and confining field theories.

Acknowledgements

I am grateful to J.A. Shapiro and C. Thorn for several patient discussions. I am also grateful to the CERN Theory Division for its hospitality.

*) We must emphasize that in the actual dual model discussed in Sections 3 and 4 there is no singularity on the light-cone, so the limit $\alpha' \to \infty$ is not very helpful.

APPENDIX A

DERIVATION OF EQ. (22)

The absence of transverse momentum for p_1 and p_2 greatly simplifies the operator algebra. Using the property, Eq. (20), of our point-like states we have:

$$M^{ij} = \sum_{m,n=1}^{\infty}\left(1+e^{-2m\tau_1/2q^+}\right)\left(1+e^{-2n(\tau-\tau_2)/2q^+}\right) \langle c,q_2|e^{-P^-(\tau-\tau_2)}\, a_n^{i+}\, e^{-P^-(\tau_2-\tau_1)}\, a_m^{j}\, e^{-P^-\tau_1}|c,q_1\rangle \tag{A.1}$$

where we have also made use of the standard relation:

$$[a_n^i, P^-] = (-n/2q^+)\, a_n^i \tag{A.2}$$

We can obtain a relation for

$$\begin{aligned} U^{ij}_{m,n} &= \langle c,q_2|e^{-P^-(\tau-\tau_2)}\, a_n^{i+}\, e^{-P^-(\tau_2-\tau_1)}\, a_m^{j}\, e^{-P^-\tau_1}|c,q_1\rangle \\ &= e^{-(n-m)\tau_1/2q^+}\, e^{(m-n)\tau_2/2q^+}\, e^{-2m\tau/2q^+} \\ &\times \langle c,q_2|e^{-P^-(\tau-\tau_2)}\, a_m^{j+}\, e^{-P^-(\tau_2-\tau_1)}\, a_n^{i}\, e^{-P^-\tau_1}|c,q_1\rangle \\ &+ m\,\delta_{m,n}\,\delta^{ij}\,\langle c,q_2|\, e^{-P^-\tau}\,|c,q_1\rangle \end{aligned} \tag{A.3}$$

It is clear that, unless $m = n$ and $i = j$, $U^{ij}_{m,n}$ will vanish. We therefore obtain an equation for $U^{ij}_{m,n}$:

$$U^{ij}_{mn} = \frac{\delta_{mn}\delta^{ij}\, m \langle c,q_2| e^{-\mathcal{P}^-\tau} |c,q_1\rangle\, e^{-m(\tau_2-\tau_1)/2q^+}}{(1-e^{-2m\tau/2q^+})} \tag{A.4}$$

We thus obtain

$$M^{ij} = \delta^{ij}\sum_{m=1}^{\infty} \frac{m(1+e^{-2m\tau_1/2q^+})(1+e^{-2m(\tau_2-\tau)/2q^+})}{(1-e^{-2m\tau/2q^+})} \times e^{-m(\tau_2-\tau_1)/2q^+} \langle c,q_2|\, e^{-\mathcal{P}^-\tau}\, |c,q_1\rangle \tag{A.5}$$

But the factor $\langle c,q_2|e^{-\mathcal{P}^-\tau}|c,q_1\rangle$ is exactly the same as that occurring in the current propagator which allows us to write

$$M = \delta^{ij}\sum_{m=1}^{\infty} \frac{4m\, e^{-m\tau/2q^+} \cosh(m\tau_1/2q^+)\cosh(m(\tau_2-\tau)/2q^+)}{(1-e^{-2m\tau/2q^+})} \times e^{(-q^2+1)\tau/2q^+} \prod_{n=1}^{\infty}(1-e^{-2n\tau/2q^+})^{-12} \tag{A.6}$$

Using

$$2\cosh(m\tau_1/2q^+)\cosh(m(\tau_2-\tau)/2q^+) = \cosh\left(\frac{m}{2q^+}(\tau-\tau_2-\tau_1)\right) + \cosh\left(\frac{m}{2q^+}(\tau-\tau_2+\tau_1)\right) \tag{A.7}$$

we can rewrite Eq. (A.6) in terms of Jacobi θ functions by using the relations[8]:

$$\frac{\partial}{\partial\phi}\left(\frac{\Theta_4'(\phi|\gamma)}{\Theta_4(\phi|\gamma)}\right) = 8\pi^2 \sum \frac{mq^m}{1-q^{2m}} \cos 2m\pi\phi$$

and

$$\Theta_4(\phi+\tfrac{1}{2}\gamma|\gamma) = i\, e^{-i\pi(\phi+\frac{1}{4}\gamma)}\, \Theta_1(\phi|\gamma) \tag{A.8}$$

(the prime denotes differentiation with respect to ϕ) and the variables:

$$q = e^{i\pi\gamma} = e^{-\tau/2q^+}$$

$$\text{i.e.}\quad \gamma = i\tau/2\pi q^+ \tag{A.9}$$

$$\phi_1 = i(\tau_1+\tau_2)/4\pi q^+ \tag{A.10}$$

$$\phi_2 = i(\tau_1-\tau_2)/4\pi q^+ \tag{A.11}$$

where

$$0 \leqslant -i(\phi_1+\phi_2) \leqslant -i(\phi_1-\phi_2) \leqslant -i\gamma$$

We obtain

$$M = \frac{\delta^{ij}}{4\pi^2}\left[\frac{\partial}{\partial\phi_1}\left(\frac{\Theta_1'(\phi_1|\gamma)}{\Theta_1(\phi_1|\gamma)}\right) + \frac{\partial}{\partial\phi_2}\left(\frac{\Theta_1'(\phi_2|\gamma)}{\Theta_1(\phi_2|\gamma)}\right)\right] \tag{A.12}$$

$$\times \left[\Theta_1'(0|\gamma)/2\pi\right]^{-4}\, e^{-q^2\tau/2q^+}$$

Making use of the kinematic relations of Eqs. (13)-(17) we have

$$\exp\{q_1^-\tau_1 + q_3^-(\tau_2-\tau_1) + q_2^-(\tau-\tau_2)\}$$

$$= \exp\{q_1^2\tau_1/2q^+ + q_2^2(\tau-\tau_2)/2q^+ + M^2(\tau_2-\tau_1)/2q^+ + q^2\tau/2q^+\} \tag{A.13}$$

Equation (22) follows by substituting Eqs. (A.12) and (A.13) into Eq. (18) and making the appropriate identification of variables [Eqs. (A.9)-(A.11)].

APPENDIX B

In order to evaluate the Fourier transform of Eq. (18) we need to consider the Jacobi transform of Eq. (A.12) by changing variables to

$$\gamma' = -1/\gamma = 2\pi i q^+/\tau \tag{B.1}$$

$$\phi_1' = \phi_1/\gamma = \left(\frac{\tau_1+\tau_2}{2\tau}\right) \tag{B.2}$$

$$\phi_2' = \phi_2/\gamma = \left(\frac{\tau_1-\tau_2}{2\tau}\right) \tag{B.3}$$

The range of integration becomes

$$0 \leqslant (\phi_1'+\phi_2') \leqslant (\phi_1'-\phi_2') \leqslant 1$$

With this transformation we have[8]

$$M = \frac{\delta^{ij}}{4\pi^2}\,(-i\gamma')^{-4}\left[\frac{\partial}{\partial\phi_1'}\left(\frac{\Theta_1'(\phi_1'|\gamma')}{\Theta_1(\phi_1'|\gamma')}\right)+\frac{\partial}{\partial\phi_2'}\left(\frac{\Theta_1'(\phi_2'|\gamma')}{\Theta_1(\phi_2'|\gamma')}\right)\right. \tag{B.4}$$
$$\left.+\frac{2\pi i}{\gamma'}\left[2-\frac{2\pi i}{\gamma'}(\phi_1'^2+\phi_2'^2)\right]\right]\left[\frac{\Theta_1'(0|\gamma')}{2\pi}\right]^{-4} e^{-q^2\tau/2q^+}$$

The integration measure changes from

$$d\phi_1\, d\phi_2\, d\gamma$$

to

$$d\phi_1'\, d\phi_2'\, d\gamma'\ \gamma'^{-4}$$

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FIGURE CAPTIONS

Fig 1 : Representations of the propagator. a) The momentum-space poles in q^2 are displayed by thick lines. b) The position-space singularities in x^2 are represented by a dashed line. Figs 1a) and 1b) are related by a Fourier transform. Since the quarks do not propagate in space-time there are no phase-space integrals in these diagrams.

Fig 2 : The forward "Compton" amplitude. a) The momentum space configuration shows the poles in q_1^2, q_2^2 and M^2. b) The position space diagram exhibits singularities in x^2.

Fig 3 : An example of a more complicated process. The diagrams exhibit the singularity structure in various momentum-space or position-space channels. The labelling indicates which variables have been Fourier transformed. Thick lines represent momentum-space poles, while dashed lines are x^2 singularities. The quark lines generate no discontinuities. Such diagrams indicate the dominant singularities contributing to generalized Regge-pole, resonance-pole or light-cone-like limits.

Fig 4 : a) Form factor in the string picture. Note that the breaking time, τ_0, is integrated over.
b) The current propagator in the string picture.
c) The forward Compton amplitude in the string picture. (The other time ordering is not illustrated.)

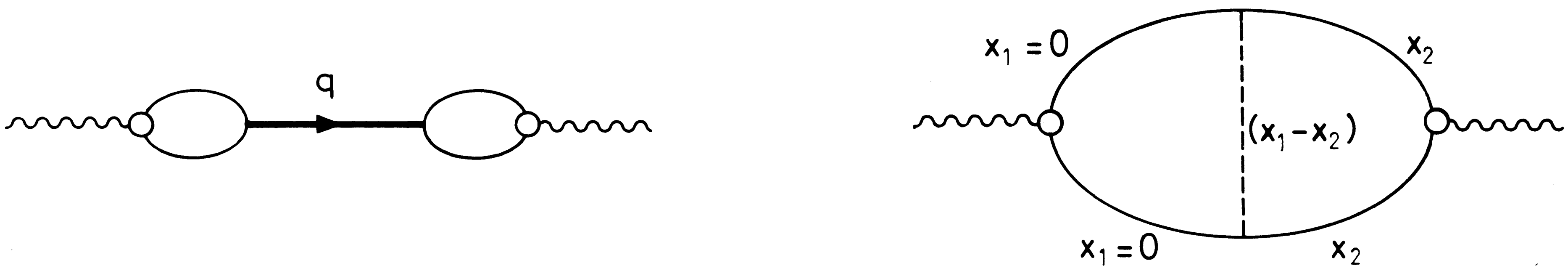
Fig 5 : The operatorial construction of the dual form factor. The c-moded states generate no normal thresholds. The circles represent the coupling of the current to c-mode ground states. Such diagrams provide the rules generating Figs 1-3.

Fig 6 : The world sheet configurations corresponding to Fig 3. The boundaries marked by thick lines satisfy the abnormal boundary condition $X^\mu(z)=0$. Points on the same abnormal boundary are at the same space-time point. The strips in b)-d) with two abnormal boundaries correspond to the dashed lines in Figs 3 b)-d).

Chan-Paton factors are incorporated by attaching quantum numbers to the normal boundaries (marked by arrows).

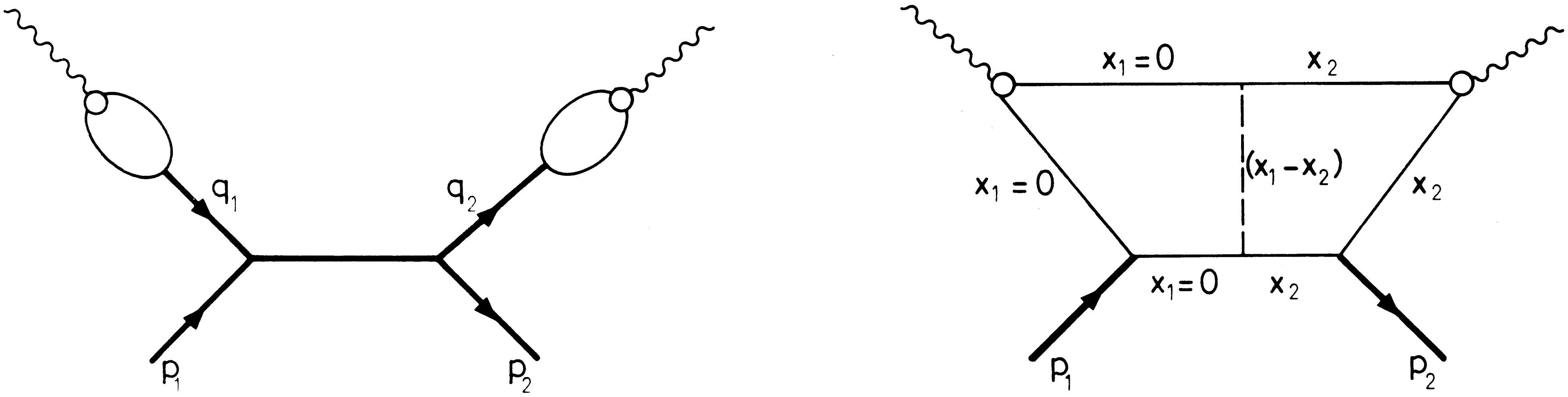
Fig 7 : a) An internal c-moded loop describing closed string currents.
b) A possible way of changing the model to reweight the internal quark structure (see refs 4), 5)).

Fig 8 : The singularity structure inside the light-cone in the narrow resonance approximation.



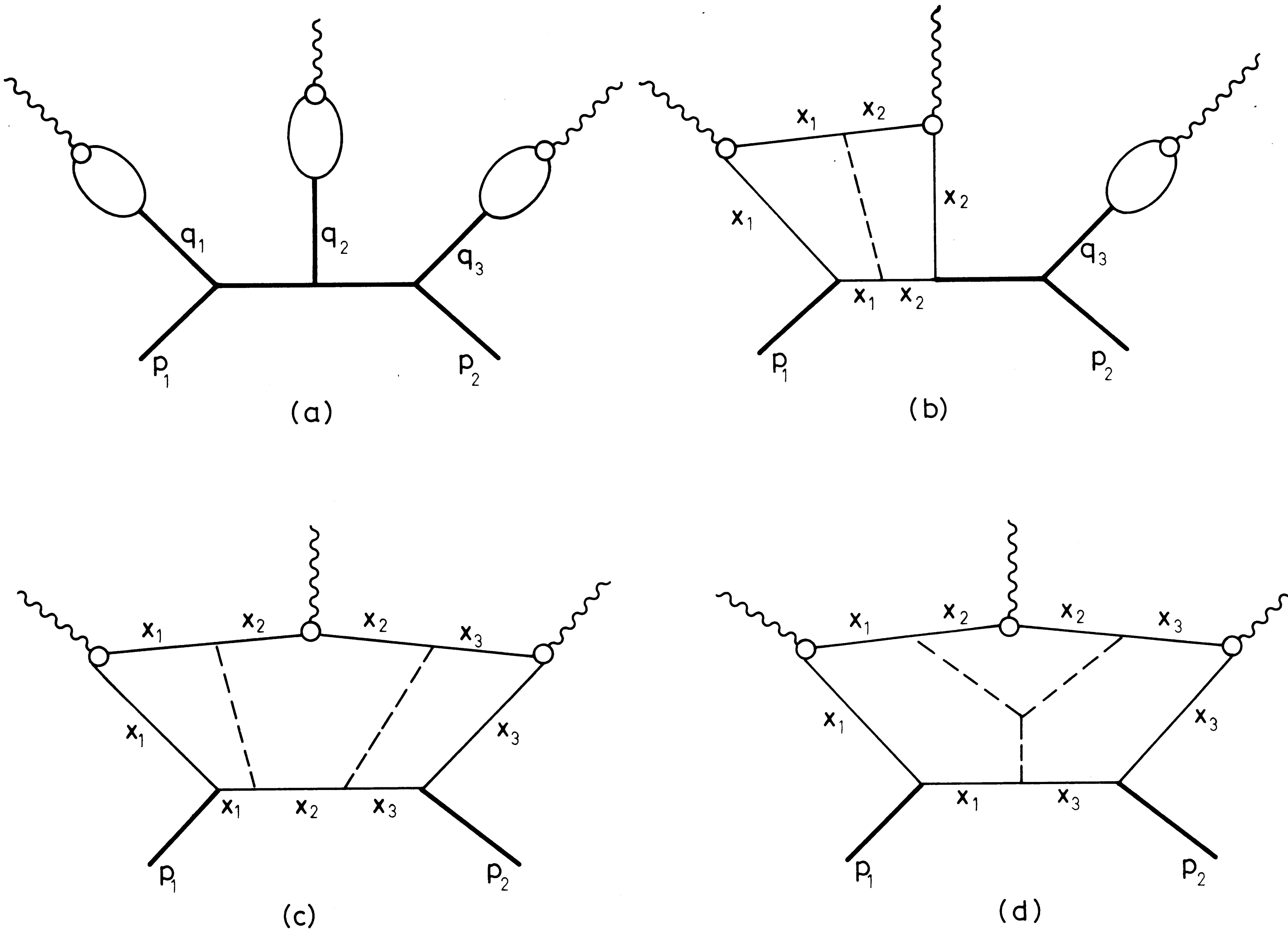
(a) (b)

FIG.1



(a) (b)

FIG.2



q1
q2
q3
p1
p2
(a)
x1
x2
x2
x1
x1
x2
q3
p1
p2
(b)
x1
x2
x2
x3
x1
x3
x1
x2
x3
p1
p2
(c)
x1
x2
x2
x3
x1
x3
x1
x3
p1
p2
(d)

FIG.3

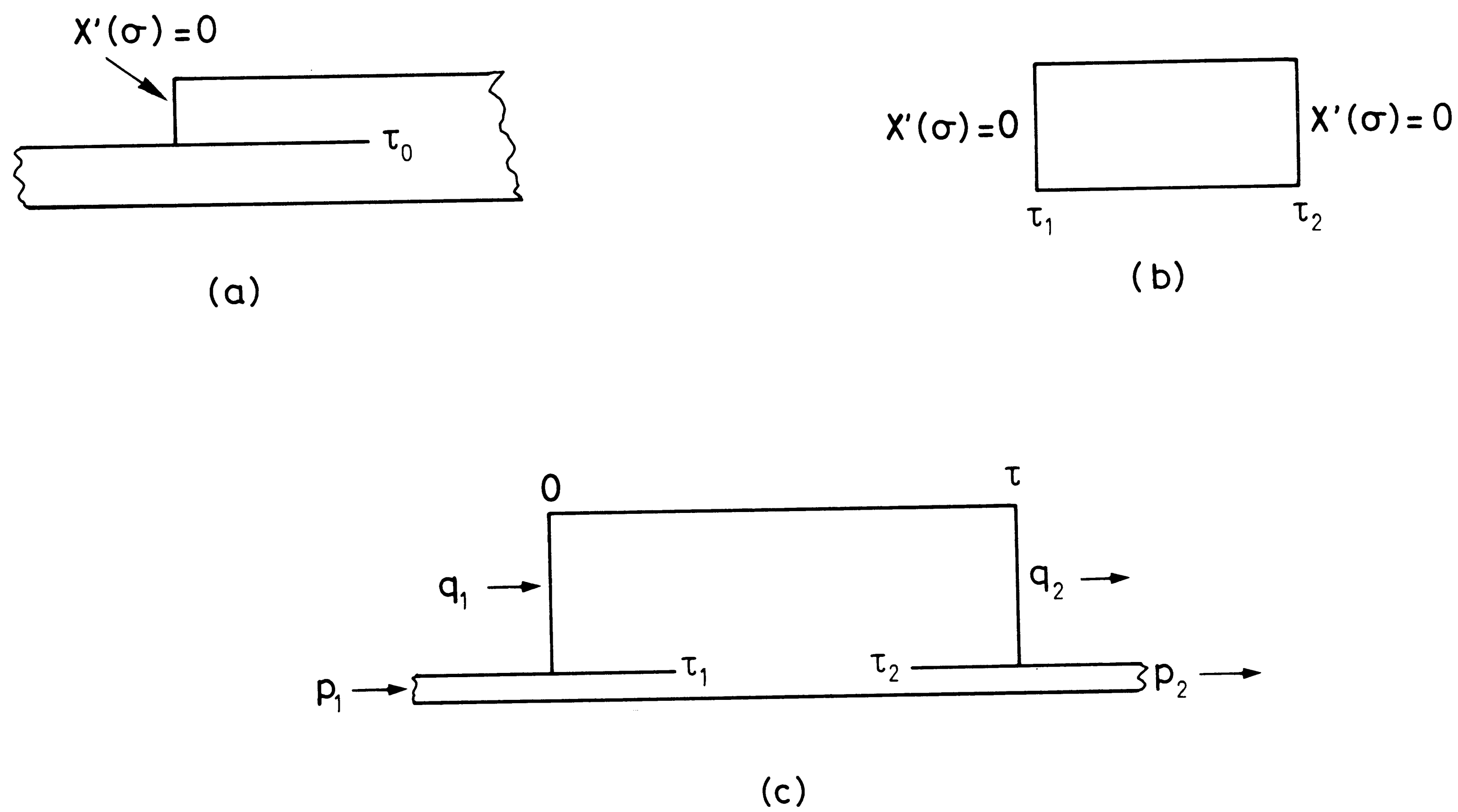


FIG. 4

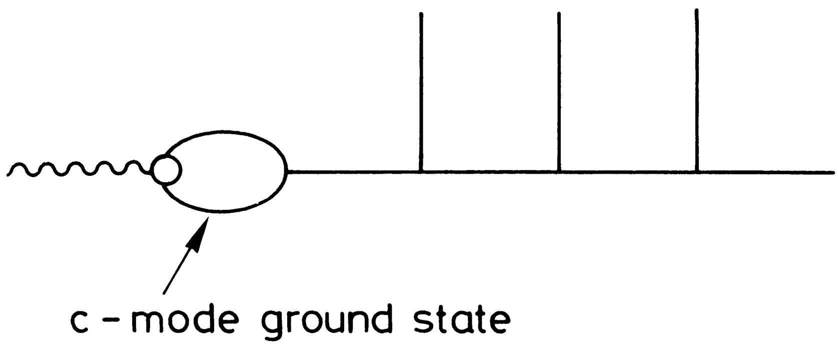


FIG.5

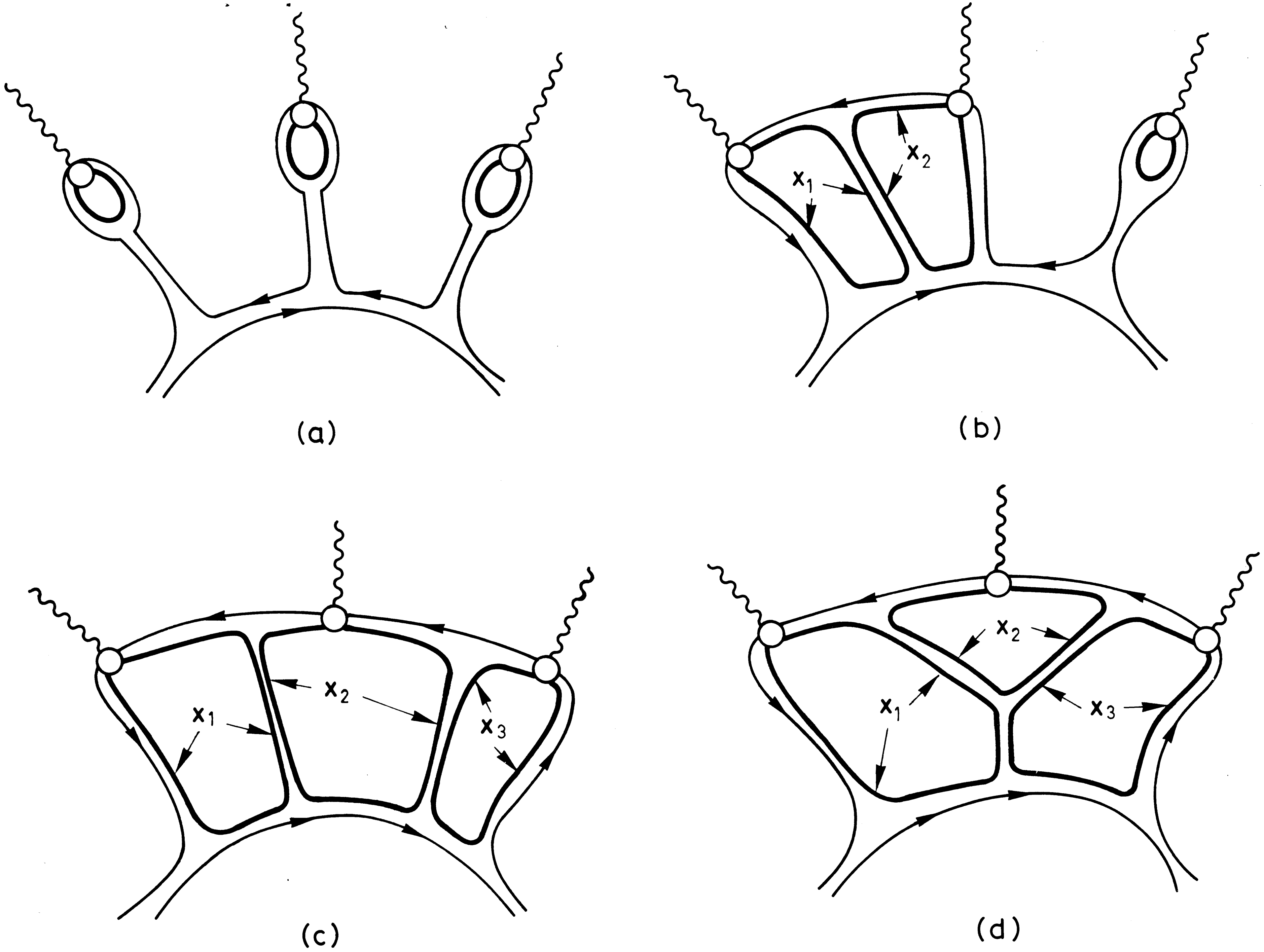


FIG. 6

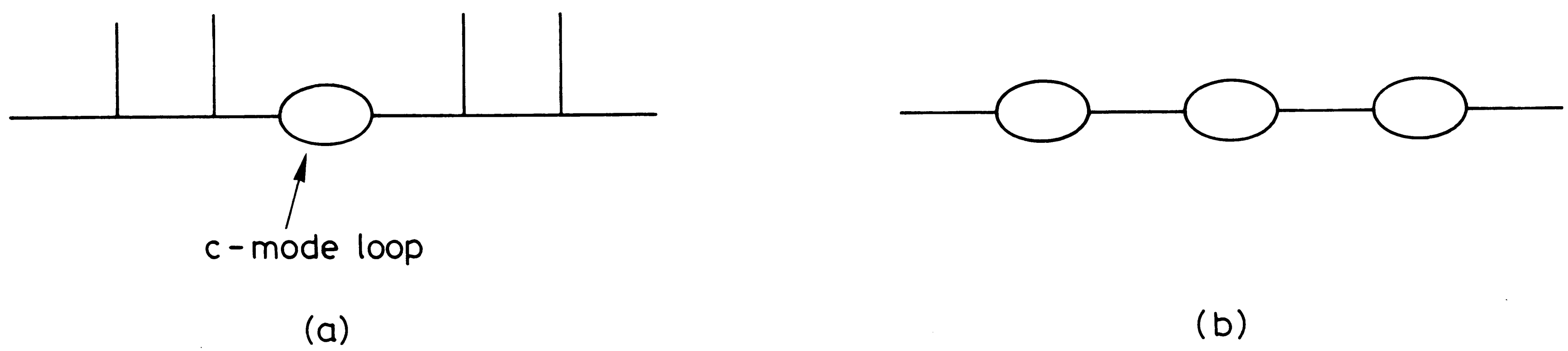


FIG.7

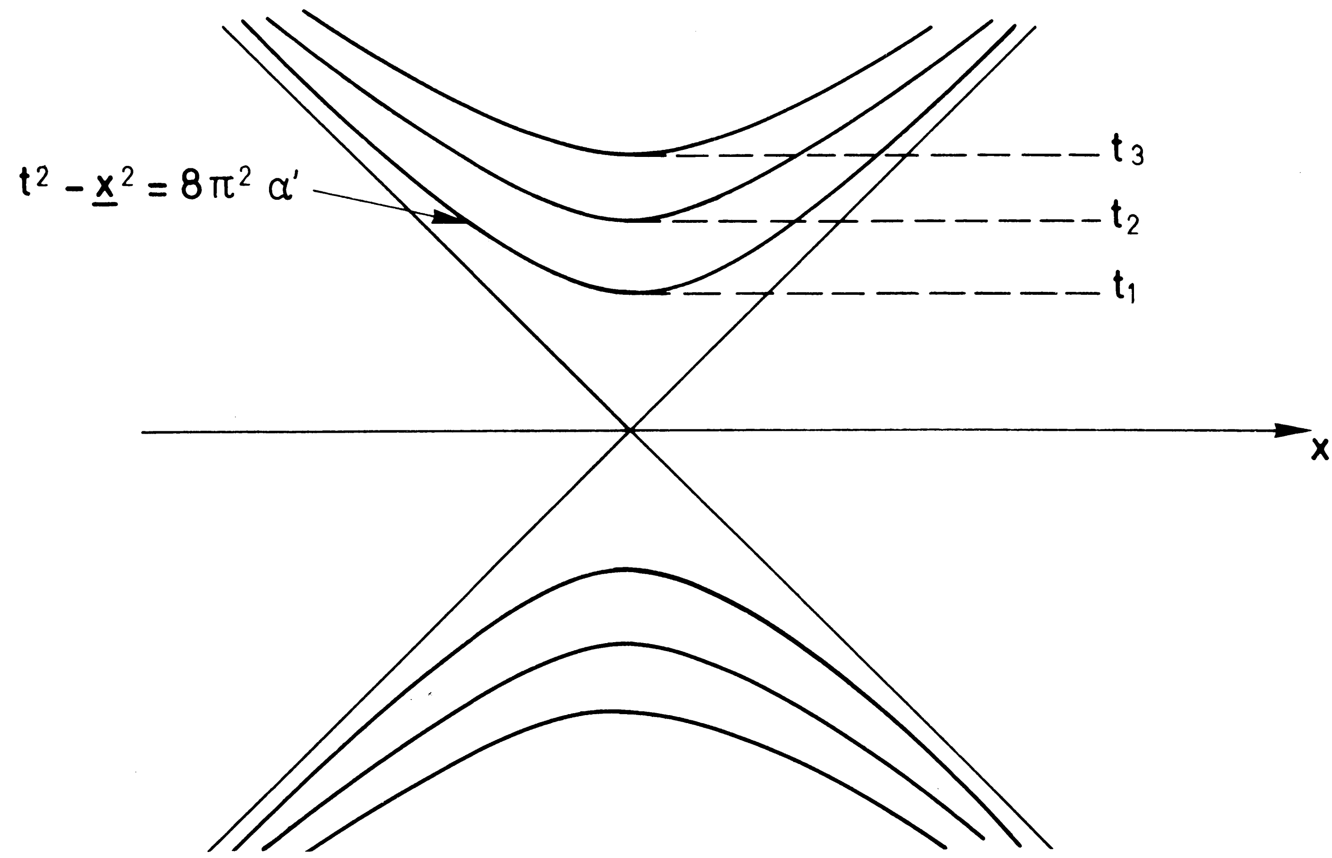


FIG. 8