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EXCHANGED NATURALITY CONTRIBUTIONS FROM HIGH ENERGY POLARIZATION MEASUREMENTS IN TWO-BODY INCLUSIVE AND EXCLUSIVE REACTIONS

J.P. Ader and C. Meyers
Laboratoire de Physique Théorique, Bordeaux

and

Ph. Salin
CERN - Geneva

ABSTRACT

In this paper, dealing with high energy quasi-two-body or multiparticle production, we focus on what can be learned about exchanged naturality amplitudes from final polarization measurements with polarized or unpolarized beam and/or target. The separation of t channel (boson exchange) and u channel (baryon exchange) exchanges into components of natural and unnatural parity and the measure of naturality interferences are extensively studied in all cases which are now or will soon be available with present experimental techniques. Special attention is paid to the transversity amplitudes which are shown to be always naturality conserving. In order to help in preparing or analyzing polarization experiments, we have considered in detail and taking each case separately many specific examples including reactions with unpolarized initial state or with initial polarized protons or photons.

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1. INTRODUCTION

Some years ago we have shown [1] that by considering given combinations of density matrix elements it was possible to separate the contributions of both natural and unnatural parity exchanged in some forward two-body reactions at high energy. Such a procedure, which allows one to extract from experiment dynamical information in a model-independent way, has been used in many places, and, we believe, has demonstrated its usefulness. The first attempts in this direction are in fact due to Gottfried and Jackson [2] and to Stichel [3] with his well-known theorem for single pion photoproduction with linearly polarized photons. Subsequent analyses of specific reactions can be found in the literature: vector meson photoproduction by Cooper [4] and by Schilling, Seyboth and Wolf [5], polarized photon reactions by Thews [6]. General bilinear relations between density matrix elements have been given by Ringland and Thews [7], but the most complete and general study of two-body reactions can be found in the work of Saenger and Schmidt [8], and many of the results contained in the present paper have been formally derived by them. Some results on diffractive multi-meson processes were also obtained by Meggs and Van Hove [9], and polarization effects in meson + nucleon → meson + meson + nucleon have been investigated by Kimel and Reya [10].

We have extended our results to multiparticle reactions [11] and to backward scattering [12] (baryon exchange). In this paper we give a complete survey of the subject by considering all possible types of reactions, two-body, quasi-two-body, exclusive and inclusive reactions, all possible types of exchanges, meson or baryon, and by studying explicitly and in detail, reactions with polarized initial states. More precisely we shall consider in this paper all scattering processes $a + b \to 1 + 2 + \ldots + n$, assuming that the polarization and momentum of particle 1 is measured, and that the initial particles are either unpolarized or polarized protons or photons (we do not consider lepton beams). Final systems of different mixed spin-parity states will not be studied here. Some model-independent tests of the absence of unnatural parity exchange have been recently derived by Sekulin [13] for the production of a mixed scalar-vector-tensor two-boson system.

We define, in each case, the observable quantities and the type of measurement which allows one to separate the contributions of exchanged states with given naturality in the crossed $a + \bar{1} \to \bar{b} + 2 + \ldots + n$ channel, or to compute interference contributions between the opposite naturalities. To take a middle course between a simple cross-section measurement and a complete amplitude analysis which requires measurement of all polarizations and all momenta of interacting particles, we try to find the simplest polarization experiments which provide valuable and immediate information on some dynamical mechanisms. It is clear that such information is particularly relevant for reactions which are supposed to be of a peripheral nature, i.e., high-energy two-body or quasi-two-body reactions, inclusive reactions in the triple Regge limit, etc., and we shall constantly use the words exchanged state or exchanged naturalities. In any case, our formalism is based on purely kinematical considerations and is a priori absolutely free of any dynamical assumption. It would be therefore more general but less convenient, to avoid everywhere the exchange notion and rather to speak only of the naturality of the state $|a\bar{1}\rangle$.

The paper is organized as follows. Section 2 first recalls the symmetry properties of scattering amplitudes and definitions of naturality conserving helicity amplitudes (which are well known) and transversity amplitudes (which we shall widely use). We emphasize the importance of Section 2.4 which gives a general classification of various types of experiments. This section is in fact the central part of our paper and presents in a rigorous, but we believe transparent and simple way, all results which will be developed throughout the paper in a more explicit but more technical manner. In Section 3 we consider the unpolarized initial state case. Initial states with polarized protons or photons are investigated, respectively, in Sections 4 and 5. In each of these last three sections we follow the same procedure, giving first very general expressions and definitions, then explicit applications.

We consider successively cases in which a produced particle of spin 0, $\frac{1}{2}$, 1, $\frac{3}{2}$ and 2 has its polarization measured. The case of both polarized beam and target reactions is discussed in Section 6 and G parity relations are finally studied in Section 7.

We have tried to organize this paper in such a way that many parts, in particular all applications, are independent of each other. This implies a certain amount of repetition, but, we hope, will help people interested only in a given type of reaction to find all necessary information without being forced to read everything preceding it.

In order to facilitate a first reading, we would like to emphasize two unusual notations or definitions used throughout our work.

a) Observable quantities $E_{mm'}$ and $O_{mm'}$

Anyone interested in polarization phenomena is familiar with the density matrix formalism $\rho_{mm'}$ and should be with the multipolar parameters or statistical tensors t^L_M. Unfortunately, it is not possible in general to measure all density matrix elements from experiment. We call $E_{mm'}$ and $O_{mm'}$ those combinations of density matrix elements, defined in Eqs. (3.2) and (3.3) which are actually accessible from experiment. When the polarization of a final-state particle is measured from its decay angular distribution, only the even part of the polarization $E_{mm'}$ can be measured from a parity-conserving decay, conversely measurement of the $O_{mm'}$'s requires a parity non-conserving decay. For two-body reactions the $E_{mm'}$ and $O_{mm'}$ reduce, respectively, to $\mathrm{Re}\,\rho_{mm'}$ and $\mathrm{Im}\,\rho_{mm'}$ a well-known result.

b) Reaction classification

The present work is not restricted to two-body reactions $a+b\to 1+2$, but includes all possible types of reactions, in particular inclusive reactions or multiparticle reactions $a+b\to 1+2+\ldots+n$. We found it necessary to classify reactions into two types, which we call Class 1 and Class 2.

i) Class 1

We call Class 1 reaction, a scattering process $a+b\to 1+2+\ldots+n$, in which none of the spin or momentum of the $2\ldots n$ final state particles are measured. Only the spin and momentum of particle 1 are observed. This case includes two-body reactions $(a+b\to 1+2)$, quasi-two-body reactions $(a+b\to 1^*+2^*)$, inclusive reactions $(a+b\to 1+X$, where X implies a summation over all possible $2+\ldots+n$ states, truly inclusive reactions, or a summation over only a given subset $2+\ldots+n$, for example $a+b\to 1+\text{neutrals}$ or $a+b\to 1+\text{pions}$, $a+b\to 1+p$ prongs, etc.) or exclusive reactions $a+b\to 1+2+\ldots+n$ in which the n final state particles are identified without measuring the momenta or the spins of the $2\ldots n$ particles (for example $\pi^+p\to\rho^o\pi^+p$ with measurement of the density matrix elements of ρ^o but no measurement related to π^+ or p). In all cases the general expressions related to the polarization measurement (decay angular distribution, properties of density matrix elements $\rho_{mm'}$ of multipolar parameters t^L_M, etc.) and to its interpretation (separation into natural or unnatural parity contributions, interferences, etc.) are just those known for two-body reactions.

ii) Class 2

We call Class 2 reaction, a scattering process $a+b\rightarrow 1+2+\ldots+n$ in which one measures some momenta of the packet of particles $2\ldots n$ $(n\geq 3)$.

The interesting point with Class 2 reactions is that they allow measurement of pseudoscalar quantities which is not possible with Class 1 reactions. Clearly these new quantities give a null result when integrated over the whole phase space. They are, in particular antisymmetric with respect to reflection in the scattering plane.

It is in general sufficient, in order to evaluate the pseudoscalar quantities mentioned above, to determine the emission direction of a particle (say particle 2) with respect to the scattering plane (a, b, 1). Therefore, one observes $a+b\rightarrow 1+2+\ldots+n$, measuring the polarization and momentum of particle 1, and separates experimental events into two sets according to whether particle 2 is emitted above or below the scattering plane. This Class 2 excludes two-body reactions (particle 2 always in the scattering plane) but includes both exclusive reactions $(a+b\rightarrow 1+2+\ldots+n$, n fixed) and quasi-inclusive reactions $a+b\rightarrow 1+(2+X)$, where, as in Class 1 reactions, X can represent a summation either over all possible states, or over given subsets $a+b\rightarrow 1+2+\text{neutrals}$, $a+b\rightarrow 1+2+\text{pions}$, $a+b\rightarrow 1+2+k$ prongs, etc.

The essential tools in our study are the parity relations among scattering amplitudes and symmetry properties with respect to reflection in the scattering plane of observable quantities. The general properties of observables for spin 1, 2, $\frac{1}{2}$ and $\frac{3}{2}$ particles are collected in four tables in the Appendix. We refer quite often to this Appendix, particularly for each application, and therefore suggest that the reader familiarizes himself with the use and content of the tables contained therein. We found it very convenient to use the label Bohr-symmetric or Bohr-antisymmetric [14] to characterize the transformation law, by reflection in the scattering plane, of the observable quantities $E_{m_1m'_1}$ and $O_{m_1m'_1}$, when other particles are polarized. This depends on the direction of the polarization vector $\vec{P}$ of the incident particles. For initial polarized protons the Bohr antisymmetric components of the polarization vector refer to $\vec{P}$ in the scattering plane, for initial polarized photons the Bohr symmetric components correspond to the parallel or perpendicular components of the electric polarization vector $\vec{\varepsilon}$.

Let us emphasize that one should not forget, in any polarization measurement, the important constraints due to positivity and rank conditions of the density matrix. Anyhow, as they require a specific study for each particular case they will not be mentioned in this work. For recent contributions to this subject see Ref. 14).

2. GENERALITIES

2.1 Definitions, notations, reference systems

We consider in this paper production processes of the type $ab \to 1 + \dots + n$ or $ab \to 1 + X$. We shall always assume that the production plane is defined by the three particle momenta p_a, p_b and p_1, and we specify the n-1 other final state particles by their polar co-ordinates $(p_i, \theta_i, \varphi_i)$. In the over-all c.m. system, we define a fixed frame of reference XYZ, such that $\vec{p}_a$ and $\vec{p}_b$ are along the Z axis and $\vec{p}_1$ in the XZ production plane according to Fig. 1.

We shall consider only two possible choices of quantization axis for the measurement of the spin of a given particle. The helicity system (z axis along $\vec{p}_i$, and for a particle in the production plane x axis in the production plane and y axis orthogonal to this plane) with corresponding scattering amplitudes denoted by M and helicity indices labelled by λ; the transversity system (z axis perpendicular to $\vec{p}_i$) with corresponding scattering amplitudes denoted by T and transversity incides labelled by τ. Our conventions for a two-particle state are those of Cohen-Tannoudji, Morel and Navelet [15] and for a multi-particle state $(n>2)$ have been given in Ref. 11). Our transversity frame of reference is defined in such a way that it is related to the helicity frame by the rotation $R = R(\pi/2, \pi/2, -\pi/2)$, following Kotanski's conventions [16].

We shall omit in general an explicit reference to kinematical variables in the argument of the M or T amplitudes, with the exception of the azimuthal angle dependence for multiparticle production reactions. In that case, we denote by $M(\varphi_i)$ or $T(\varphi_i)$ the corresponding amplitudes. The reason for such simplified notation is that we shall assume that we are working at fixed total energy s and fixed production angle θ, and that the only transformation to be considered is the reflection in the production plane in which case the azimuthal angles φ_i change into $-\varphi_i$.

In general we shall not specify whether the helicity or transversity system under consideration is the s channel c.m. one, $a+b \rightarrow 1+\ldots+n$, or the t channel c.m. (which is related to the Jackson frame), $a+\bar{1} \rightarrow \bar{b}+\ldots+n$, because the general properties of scattering amplitudes or experimentally accessible quantities can be expressed in a strictly equivalent way in any of these two systems. Furthermore, when the properties under consideration are even independent of the quantification system (helicity or transversity) we shall in that case denote the amplitudes by F and the spin indices by m.

2.2 Observable quantities

Once the reference frame has been chosen, it is necessary to specify a set of independent variables which fully describe all experimental measurements. When the polarization is deduced from observation of the angular distribution of the decay products of a given particle the most convenient choice is in terms of multipolar parameters [17]. The multipolar parameters t^L_M, or statistical tensors, are coefficients of the expansion in spherical harmonics $Y^L_M(\theta, \varphi)$ of the decay angular function $W(\theta, \varphi)$, which for two-body decay reads:

$$W(\theta,\varphi) = \sum_{L,M} C(L)\, t^L_M\, Y^{L\,*}_M(\theta,\varphi) \tag{2.1}$$

Only those parameters with even L can be measured in a parity conserving decay. More generally all experimental measurements can be expressed in terms of the density matrix

$$\rho^f = F \rho^i F^{\dagger} \tag{2.2}$$

whose matrix elements are related to the multipolar parameters by

$$t^L_M = \sum_{m,m'} \sqrt{2S+1} \begin{pmatrix} m' & L & S \\ S & M & m \end{pmatrix} \rho_{mm'} \tag{2.3}$$

where S denotes the spin of the decaying resonance.

Clearly not all density matrix elements can be measured in a parity conserving decay. Furthermore, they are subject to a number of important symmetry properties. Most of these properties are deduced from parity conservation in the production process, but their explicit formulation differs from one frame of reference to another and according to whether the initial state is or is not polarized. We shall therefore investigate each specific case separately in the following sections. To carry out such an investigation we shall make use of the symmetry properties of scattering amplitudes which we now briefly recall.

2.3 Scattering amplitudes

2.3.1 Parity relations and reflection in the scattering plane

Relations among scattering amplitudes due to parity conservation in the production reaction are well-known, but they remain the starting point and the main tool of our study. We therefore briefly recall that for a reaction $ab \rightarrow 1 + \ldots + n$, parity conservation implies the following relations:

i) In the helicity system, helicities change sign under reflection in the scattering plane and therefore

$$M_{\lambda_1 \ldots \lambda_n \lambda_a \lambda_b}(\varphi_i) = \varepsilon \, M_{-\lambda_1 \ldots -\lambda_n -\lambda_a -\lambda_b}(-\varphi_i) \tag{2.4}$$

with

$$\varepsilon = \eta_a \eta_b (-)^{s_a - \lambda_a} (-)^{s_b - \lambda_b} \prod_{i=1}^{n} \eta_i (-)^{s_i - \lambda_i} \tag{2.5}$$

In the two-body case the preceding expression reduces of course to the well-known relation

$$M_{\lambda_1 \lambda_2 \lambda_a \lambda_b} = \varepsilon \, M_{-\lambda_1 -\lambda_2 -\lambda_a -\lambda_b}$$

ii) In the transversity system, transversities do not change sign under reflection in the scattering plane, and one obtains

$$T_{\tau_1 \ldots \tau_n \tau_a \tau_b}(\varphi_i) = \varepsilon' \, T_{\tau_1 \ldots \tau_n \tau_a \tau_b}(-\varphi_i) \tag{2.6}$$

with

$$\varepsilon' = \eta_a \eta_b \, e^{i\pi(\tau_a + \tau_b)} \prod_{i=1}^{n} \eta_i \, e^{-i\pi \tau_i} \tag{2.7}$$

Such a relation expresses the fact that transversity amplitudes are either symmetric and antisymmetric under reflection in the scattering plane. Conversely for two-body reactions Eq. (2.6) implies that all transversity amplitudes such that $\varepsilon' = -1$ are identically zero:

$$T_{\tau_1 \tau_2 \tau_a \tau_b} \equiv 0 \qquad \text{when } \eta_a \eta_b \eta_1 \eta_2 (-)^{\tau_a + \tau_b - \tau_1 - \tau_2} = -1 \tag{2.8}$$

2.3.2 Naturality conserving amplitudes

i) In the helicity systems, one can define helicity amplitudes which are naturality conserving to leading order in $1/s$ and for fixed values of the momentum transfer t between particles a and 1 ($|t| \ll s$), by considering the following linear combinations

$$M_{\lambda_1 \{\lambda_j\} \lambda_a \lambda_b} = \frac{1}{\sqrt{2}} \left[M_{\lambda_1 \{\lambda_j\} \lambda_a \lambda_b} \pm \varepsilon(\lambda_1, \lambda_a) M_{-\lambda_1 \{\lambda_j\} -\lambda_a \lambda_b} \right] \tag{2.9}$$

with

$$\varepsilon(\lambda_1, \lambda_a) = \eta_1 \eta_a \, e^{i\pi(v + s_1 - \lambda_1 + s_a - \lambda_a)} \tag{2.10}$$

where $v = 0$ when $a + \bar{1}$ is a bosonic channel, $v = \frac{1}{2}$ when $a + \bar{1}$ is a baryonic channel *) and where $\{\lambda_j\}$ is a shorthand notation for $(\lambda_2 \ldots \lambda_n)$.

We recall that the only amplitudes which are truly naturality-conserving (at all orders in $1/s$) are the t channel $(a + \bar{1} \to \bar{b} + \ldots + n)$ helicity amplitudes with $\lambda_a = \lambda_1$. (For example $\pi N \to \rho N$ with ρ having helicity zero in the Jackson frame.)

ii) In the transversity systems we first write down the relation between helicity and transversity amplitudes given by

$$T_{\tau_1 \ldots \tau_n \tau_a \tau_b} = \sum_{\lambda} D^{s_a}(R)^{\lambda_a}_{\tau_a} \, D^{s_b}(R)^{\lambda_b}_{\tau_b} \left(\prod_{i=1}^{n} D^{s_i}(R^*)^{\lambda_i}_{\tau_i} \right) M_{\lambda_1 \ldots \lambda_n \lambda_a \lambda_b} \tag{2.11}$$

where the rotation R defined by its Euler angles $(\alpha, \beta, \gamma) = (\pi/2, \pi/2, -\pi/2)$ enjoys the following property

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*) One should pay attention to the fact that for baryonic channel, the naturality of the $a\bar{1}$ state is not the same as that of the $\bar{a}1$ one.

$$(-)^{s+\lambda} D^{s}(R)^{\lambda}_{\tau} = e^{i\pi\tau} D^{s}(R)^{-\lambda}_{\tau} \tag{2.12}$$

We now rewrite (2.11) as

$$T_{\tau_1 \ldots \tau_n \tau_a \tau_b} = \frac{1}{2} \sum_{\lambda} D^{s_b}(R)^{\lambda_b}_{\tau_b} \left(\prod_{i=2}^{n} D^{s_i}(R^*)^{\lambda_i}_{\tau_i} \right) \Big[D^{s_a}(R)^{\lambda_a}_{\tau_a} \times$$
$$\times D^{s_1}(R^*)^{\lambda_1}_{\tau_1} M_{\lambda_1 \lambda_2 \ldots \lambda_n \lambda_a \lambda_b} + D^{s_a}(R)^{-\lambda_a}_{\tau_a} D^{s_1}(R^*)^{-\lambda_1}_{\tau_1} M_{-\lambda_1 \lambda_2 \ldots \lambda_n -\lambda_a \lambda_b} \Big] \tag{2.13}$$

and using (2.12) finally obtain

$$T_{\tau_1 \ldots \tau_n \tau_a \tau_b} = \frac{1}{2} \sum_{\lambda} D^{s_a}(R)^{\lambda_a}_{\tau_a} D^{s_b}(R)^{\lambda_b}_{\tau_b} \left(\prod_{i=1}^{n} D^{s_i}(R^*)^{\lambda_i}_{\tau_i} \right)$$
$$\Big[M_{\lambda_1 \lambda_2 \ldots \lambda_n \lambda_a \lambda_b} + \sigma \eta_1 \eta_a e^{i\pi(\sigma + s_1 - \lambda_1 + s_a - \lambda_a)} M_{-\lambda_1 \lambda_2 \ldots \lambda_n -\lambda_a \lambda_b} \Big] \tag{2.14}$$

where

$$\sigma = \eta_1 \eta_a e^{i\pi(\sigma + \tau_1 - \tau_a)} \tag{2.15}$$

Since σ does not depend upon the helicity indices, relations (2.9) and (2.14) express the important fact that:

> transversity amplitudes are naturality conserving amplitudes *) both for boson and baryon exchange. The naturality associated with a transversity amplitude is given by (2.15), natural parity for $\sigma = +1$, unnatural parity for $\sigma = -1$.

The preceding property together with relation (2.6) due to reflection in the scattering plane show that transversity amplitudes enjoy much simpler properties than helicity amplitudes, and we shall in the following therefore prefer transversity amplitudes to helicity amplitudes in order to prove general theorems on observable quantities.

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*) We thank Dr. A. Kotanski for pointing out to us that this result for meson exchange has been independently obtained by A. Golemo, thesis, University of Krakcw 1972 (unpublished).

We would like to point out that it is not our purpose to recommend the use of one system of reference rather than another one. From an experimental point of view there is a priori no reason to prefer any system over other ones. In particular we shall show that the evaluation of natural and unnatural parity contributions can be achieved equally well in any system. One might be guided by possible selection rules (as for instance helicity conservation), but up to now no such general rule has been clearly established. From a theoretical point of view the choice may be directed by model-dependent considerations (such as geometrical properties of spin-dependent amplitudes), but the most general rule will be simplicity. In this respect, we recall that s channel helicity amplitudes are the easiest set of amplitudes one can find at high energy to avoid the cumbersome problems of kinematical singularities and constraint relations. Transversity amplitudes are difficult to deal with because of their complicated constraints at the boundary of the physical domain.

2.4 Classification of various types of experiments

It is now widely recognized that polarization measurements allow one to obtain in a model-independent way valuable information on the naturality of exchanged states in a production process, and in a number of cases to actually separate the natural and unnatural parity contributions. We intend in this section to give a classification of experiments which do achieve such a separation, or are able to exhibit interference contributions between opposite naturalities.

Quite general statements, which are valid for boson as well as for baryon exchanges, are obtained in a really straightforward way using the transversity formalism. Detailed demonstrations and examples will be given in Sections 3 and 4. Consider in fact a transversity density matrix element for particle 1 when the initial state is unpolarized:

$$\rho_{\tau_1\tau_1'} = \frac{1}{\mathcal{N}} \sum_{\tau_a,\{\tau_j\},\tau_b} T_{\tau_1\{\tau_j\}\tau_a\tau_b} \, T^*_{\tau_1'\{\tau_j\}\tau_a\tau_b} \tag{2.16}$$

i) <u>For two-body or quasi-two-body reactions, the only non-vanishing terms contributing to (2.16) are those for which $\tau_1 - \tau_1' = 2n$,</u> because otherwise due to the parity relation (2.8) on transversity amplitudes, one of the two amplitudes $T_{\tau_1\{\tau_j\}\tau_a\tau_b}$ or $T_{\tau_1'\{\tau_j\}\tau_a\tau_b}$ is identically zero.

Such a result applies also to inclusive reactions, because in that case after integration over all kinematical variables of the unobserved particles, only combinations of transversity amplitudes which are symmetric with respect to reflection in the scattering plane survive, i.e., such that $\tau_1 - \tau_1'$ is even. Recalling that transversity amplitudes are naturality conserving amplitudes, and that for $\tau_1 - \tau_1' = 2n$, both $T_{\tau_1 \{\tau_j\} \tau_a \tau_b}$ or $T_{\tau_1' \{\tau_j\} \tau_a \tau_b}$ are related to the same naturality, we can make the following statement:

Statement 1: <u>In two-body or inclusive reactions, with unpolarized initial state and measurement of only one polarization in the final state, any observable quantity can be expressed only in terms of incoherent sums over natural or unnatural parity, of the type $\sum (F^+F^{+*} + F^-F^{-*})$. In particular, observable quantities are insensitive to the relative phase between opposite naturality contributions</u>.

ii) When particle a has spin zero, then amplitudes $T_{\tau_1 \{\tau_j\} o \tau_b}$ and $T_{\tau_1' \{\tau_j\} o \tau_b}$ with $\tau_1 - \tau_1' = 2n$ correspond to exchange of the same naturality $\sigma = \eta_1 \eta_a e^{i\pi(v+\tau_1)}$ where $v = 0$ $(\frac{1}{2})$ for boson (baryon) exchanges respectively. This can be stated as follows:

Statement 2: <u>When particle a is spinless, then any density matrix element $\rho_{\tau_1 \tau_1'}$, in the transversity frame, is of the type $\sum F^+F^{+*}$ or $\sum F^-F^{-*}$, and therefore isolates one type of exchanged naturality in the $a\bar{1} \to \bar{b} + 2 + \ldots + n$ channel of any reaction</u>. In any other frame of reference the separation of observables into natural or unnatural contributions is also possible by combining various density matrix elements. In any case, for baryonic channels such an evaluation may require measurement of odd polarizations.

iii) When particle a is not spinless it is clearly not possible to extract from Eq. (2.16) contributions corresponding to a given exchanged naturality unless the transversities τ_a are fixed at some given values. This means that the initial particle a has to be polarized. When the initial particle a can be polarized in any direction, then it is possible to compute separately the contributions of each type of exchanged naturality, and for Class 2 reactions to get also an evaluation of interference terms between opposite naturalities. When only one type of initial polarization is available (for instance transverse polarization) then one has to consider in detail each specific case.

Such a study will be performed in Sections 4 and 5 for polarized nucleons and photons respectively.

<u>Statement 3: Separation of density matrix elements of particle 1 into natural or unnatural parity exchanged in the $a+\bar{1}\to\bar{b}+\ldots+n$ channel, requires measurement of both polarizations of particle a and 1. This means that either a is spinless or has to be polarized in a specific direction.</u>

iv) When $\tau_1 - \tau_1' = 2n+1$, then amplitudes $T_{\tau_1\{\tau_j\}\tau_a\tau_b}$ and $T_{\tau_1'\{\tau_j\}\tau_a\tau_b}$ are associated with opposite naturality exchanges, and their product is antisymmetric with respect to reflection in the scattering plane.

<u>Statement 4: When the difference $\tau_1 - \tau_1'$ is an odd integer, transversity density matrix elements $\rho_{\tau_1\tau_1'}$ are related to bilinear combinations of transversity amplitudes of the type $\sum T^+T^{-*}$, and therefore provide a measure of interference terms between natural and unnatural parity.</u>

According to statement 1 above, such elements are identically zero in two-body or inclusive reactions (class 1). Their evaluation is possible in multiparticle reactions and requires measurement of space coordinates of another particle in the final state (class 2).

v) From the former discussion it is clear that the evaluation in Class 1 reactions of interference contributions between opposite naturality exchanges implies the simultaneous measurement of at least two polarizations within the same reaction. Consider first the case where only the polarization of two particles (say particle a and 1) at the same vertex are measured, and let us define the correlation function

$$\rho^{\tau_a\tau_a'}_{\tau_1\tau_1'} = \frac{1}{N}\sum_{\tau_b,\{\tau_j\}} T_{\tau_1\{\tau_j\}\tau_a\tau_b}\, T^*_{\tau_1'\{\tau_j\}\tau_a'\tau_b}$$

The two transversity amplitudes occurring in this expression are related to opposite naturalities if and only if either $\tau_1 - \tau_1' = 2n$ and $\tau_a - \tau_a' = 2n'+1$ or $\tau_1 - \tau_1' = 2n+1$ and $\tau_a - \tau_a' = 2n'$. But then in each of these two cases one of the two amplitudes is either zero or antisymmetric with respect to the scattering plane, and therefore $\rho^{\tau_a\tau_a'}_{\tau_1\tau_1'} = 0$ for Class 1 reactions.

Assume that the two particles (say particles 1 and 2) are produced at different vertices, and consider the corresponding correlation function

$$\rho^{\tau_2 \tau'_2}_{\tau_1 \tau'_1} = \frac{1}{N} \sum_{\tau_a \{\tau_i\} \tau_b} T_{\tau_1 \tau_2 \{\tau_i\} \tau_a \tau_b} \, T^{*}_{\tau'_1 \tau'_2 \{\tau_i\} \tau_a \tau_b}$$

The condition $\tau_1 - \tau'_1 = 2n+1$ ensures that the two transversity amplitudes are related to opposite naturalities and a necessary condition to get a non-vanishing ρ in Class 1 reactions is $\tau_2 - \tau'_2 = 2n' + 1$.

Statement 5: Interferences between natural and unnatural parity exchanges can never be obtained in two-body or inclusive reactions from measurements of polarizations of two particles at the same vertex. They require measurement of polarizations of two particles at opposite vertices, and are given by the joint decay correlation function $\rho^{\tau_2 \tau'_2}_{\tau_1 \tau'_1}$ with $\tau_1 - \tau'_1$ and $\tau_2 - \tau'_2$ being both odd integers.

All the preceding statements have been depicted in Fig. 2. An arrow on a single particle line means that the polarization of the corresponding particle must be measured. An arrow on a line within a multi-particle packet means that the emission direction of the corresponding particle has been observed, in particular that one can assert whether this particle has been emitted above or below the scattering plane. Figure 2a represents cases where observables are given in terms of incoherent sums over both exchanged naturalities of the type $\sum (T^+T^{+*} + T^-T^{-*})$. Figure 2b represents examples for which separation into natural and unnatural parity exchange is possible. Figure 2c shows in which cases interference contributions are observable. In any of these figures one can of course interchange particles belonging to the same vertex.

3. UNPOLARIZED INITIAL STATE

3.1 Symmetry properties of observables in different decay frames of reference

Equation (2.3), relating density matrix elements to multipolar parameters, can be inverted to give

$$\rho_{mm'}(\varphi_i) \pm (-)^{m'-m} \rho_{-m'-m}(\varphi_i) = \sum_L \frac{2L+1}{\sqrt{2S+1}} \begin{pmatrix} m' & L & S \\ S & M & m \end{pmatrix} t^L_M(\varphi_i) \left(1 \pm (-)^L\right) \quad (3.1)$$

Thus observables available from measurement of the even part of the polarization (corresponding in most of the realistic cases to parity-conserving decays) are given by

$$E_{mm'}(\varphi_i) = \frac{1}{2}\left[\rho_{mm'}(\varphi_i) + (-)^{m-m'}\rho_{-m'-m}(\varphi_i)\right] \tag{3.2}$$

In parity non-conserving decays, one can measure supplementary quantities which we define as:

$$O_{mm'}(\varphi_i) = \frac{1}{2}\left[\rho_{mm'}(\varphi_i) - (-)^{m-m'}\rho_{-m'-m}(\varphi_i)\right] \tag{3.3}$$

From their definitions, and the hermiticity property of the density matrix, the even and odd L polarization observables E and O enjoy the following properties

i)

$$E_{mm'} = E^{*}_{m'm} \tag{3.4}$$
$$O_{mm'} = O^{*}_{m'm}$$

ii)

$$E_{mm'} = (-)^{m-m'} E_{-m'-m} \tag{3.5}$$
$$O_{mm'} = -(-)^{m-m'} O_{-m'-m}$$

iii) Diagonal elements E_{mm} and O_{mm} are real, and all other elements are a priori complex, which means that both their real and imaginary parts can be measured. However, from Eqs. (3.5), $E_{m-m} \equiv 0$ for baryonic resonances and $O_{m-m} \equiv 0$ for bosonic ones.

iv) Trace condition:

$$\sum_m E_{mm} = 1 \tag{3.6}$$

The preceding definitions and properties are independent of the choice of the quantization axis. However, the study of consequences of parity conservation in the production process on multipolar parameters and observables requires us to distinguish between helicity or transversity quantization.

a) In helicity systems, a reflection in the reaction plane yields [11)]

$$t^L_M(\varphi_i) = (-)^L\, t^{L*}_M(-\varphi_i)$$
$$\rho_{\lambda\lambda'}(\varphi_i) = (-)^{\lambda-\lambda'}\rho_{-\lambda-\lambda'}(-\varphi_i) \tag{3.7}$$

and, consequently,

$$E_{\lambda\lambda'}(\varphi_i) = E^*_{\lambda\lambda'}(-\varphi_i)$$
$$O_{\lambda\lambda'}(\varphi_i) = -\,O^*_{\lambda\lambda'}(-\varphi_i) \tag{3.8}$$

The well-known result that in two-body reactions only real parts of density matrix elements can be obtained from parity-conserving decays, and that other observables given by parity non-conserving decays are related to imaginary parts of density matrix elements is thus readily recovered, because in that case only the symmetric parts in φ_i of E and O survive.

b) In transversity systems, parity conservation in the production process now requires

$$\rho_{\tau\tau'}(\varphi_i) = (-)^{\tau-\tau'}\rho_{\tau\tau'}(-\varphi_i) \tag{3.9}$$

giving for the density matrix of Class 1 reactions its characteristic "chequer board" pattern of transversity frames.

Using the C.G. coefficient property

$$\begin{pmatrix} \tau' & L & s \\ s & M & \tau \end{pmatrix} = (-)^{\tau'-\tau}\begin{pmatrix} \tau & L & s \\ s & -M & \tau' \end{pmatrix}$$

the hermiticity property $t^{L*}_M(\varphi_i) = (-)^M t^L_{-M}(\varphi_i)$, and relation (3.1), it is easy to translate (3.9) on multipolar parameters to obtain:

$$t^L_m(\varphi_i) = (-)^M\, t^L_M(-\varphi_i) \tag{3.10}$$

Therefore, in transversity frames, multipolar parameters are symmetric or antisymmetric with respect to azimuthal angles following the parity of M. Consequently, remembering that $M = \tau' - \tau$:

$$\begin{aligned} E_{\tau\tau'}(\varphi_i) &= (-)^{\tau-\tau'} E_{\tau\tau'}(-\varphi_i) \\ O_{\tau\tau'}(\varphi_i) &= (-)^{\tau-\tau'} O_{\tau\tau'}(-\varphi_i) \end{aligned} \tag{3.11}$$

Thus both real and imaginary parts of even and odd polarization observables are symmetric (or antisymmetric) with respect to azimuthal angles when the transversity difference $\tau' - \tau$ is even (or odd). Moreover, $E_{\tau\tau'}$ and $O_{\tau\tau'}$ vanish whenever $\tau' - \tau$ is odd for two-body or inclusive processes.

3.2 Separation into natural and unnatural parity exchange contribution

As pointed out in Refs. 1), 8), 11), 12) and in Section 2, when the initial particle a has spin zero, it is possible to separate the contributions of opposite naturalities in the $a + \bar{1} \rightarrow \bar{b} + 2 + \ldots$ channel.

a) For transversity quantization the exchanged naturality associated with a given density matrix element $\rho_{\tau_1\tau_1'}$ ($\tau_1 - \tau_1'$ even) is simply given by

$$\sigma = \eta_1 \eta_a\, e^{i\pi(\sigma+\tau_1)} \tag{3.12}$$

b) For helicity quantization the situation is a bit more involved. We first define

$$\rho^{\pm}_{\lambda_1\lambda_1'} = \frac{1}{N} \sum_{\{\lambda_j\},\lambda_b} M^{\pm}_{\lambda_1\{\lambda_j\}0\lambda_b}\, M^{\pm *}_{\lambda_1'\{\lambda_j\}0\lambda_b} \tag{3.13}$$

i) For meson exchange the preceding quantity can be expressed in terms of observables as follows:

$$\mathrm{Re}\ \rho^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Re}\left[E_{\lambda_1\lambda'_1} \pm \varepsilon' E_{\lambda_1 -\lambda'_1} \right] \tag{3.14}$$

$$\mathrm{Im}\ \rho^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Im}\left[O_{\lambda_1\lambda'_1} \pm \varepsilon' O_{\lambda_1 -\lambda'_1} \right] \tag{3.15}$$

with

$$\varepsilon' = \eta_1 \eta_a \ (-)^{s_1 - \lambda'_1} \tag{3.16}$$

One should notice in the preceding expressions that both the odd and even parts of the polarization (i.e., for parity-conserving and parity-non-conserving decays) can be separated into natural and unnatural contributions.

For two-body or inclusive reactions (Class 1) relations (3.14) and (3.15) reduce to

$$\rho^{\pm}_{\lambda_1\lambda'_1} = \rho_{\lambda_1\lambda'_1} \pm \varepsilon' \rho_{\lambda_1 -\lambda'_1} \tag{3.17}$$

ii) For baryon exchange in the crossed channel equivalent results are obtained using the following expressions

$$\mathrm{Re}\ \rho^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Re}\ E_{\lambda_1\lambda'_1} \pm \varepsilon' \ \mathrm{Im}\ O_{\lambda_1 -\lambda'_1} \tag{3.18}$$

$$\mathrm{Im}\ \rho^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Im}\ O_{\lambda_1\lambda'_1} \mp \varepsilon' \ \mathrm{Re}\ E_{\lambda_1 -\lambda'_1} \tag{3.19}$$

In that case separation requires measurement of both even and odd parts of the polarization of particle 1. Finally for two-body or inclusive reactions (Class 1), these relations simplify to

$$\rho^{\pm}_{\lambda_1\lambda'_1} = \rho_{\lambda_1\lambda'_1} \mp i\varepsilon' \rho_{\lambda_1 -\lambda'_1} \tag{3.20}$$

3.3 Interference contributions

We now turn to interference contribution between opposite naturalities. We recall that such an experimental investigation is not possible for two-body reactions (Class 1), without measuring the polarization of a particle other than particle 1 (which is not the object of this section). As already stated in Section 2 interference terms are given by observable quantities which are antisymmetric with respect to reflection in the scattering plane, i.e., $\mathrm{Im}\, E_{\lambda_1 \lambda_1'}$ and $\mathrm{Re}\, O_{\lambda_1 \lambda_1'}$ for helicity quantization and $E_{\tau_1 \tau_1'}$ or $O_{\tau_1 \tau_1'}$ with $\tau_1 - \tau_1'$ odd for transversity quantization. Such quantities are measurable only in Class 2 reactions.

In terms of helicity amplitudes we explicitly obtain

$$\left.\begin{matrix} \mathrm{Im}\; E_{\lambda_1\lambda_1'} \\ \\ \mathrm{Re}\; O_{\lambda_1\lambda_1'} \end{matrix}\right\} = \begin{matrix} \mathrm{Im} \\ \\ \mathrm{Re} \end{matrix} \left\{ \frac{1}{2N} \sum_{\substack{\lambda_a,\lambda_b \\ \{\lambda_j\}}} \left(M^{+}_{\lambda_1\{\lambda_j\}\lambda_a\lambda_b} M^{-*}_{\lambda_1'\{\lambda_j\}\lambda_a\lambda_b} + M^{-}_{\lambda_1\{\lambda_j\}\lambda_a\lambda_b} M^{+*}_{\lambda_1'\{\lambda_j\}\lambda_a\lambda_b} \right) \right\} \tag{3.21}$$

When the initial particle a has spin zero we get supplementary relations. Defining first:

$$\rho^{I}_{\lambda_1\lambda_1'} = \frac{1}{N} \sum_{\lambda_b,\{\lambda_j\}} \left(M^{+}_{\lambda_1\{\lambda_j\}0\lambda_b} M^{-*}_{\lambda_1'\{\lambda_j\}\lambda_b} \right) \tag{3.22}$$

we obtain for meson exchange

$$\mathrm{Re}\; \rho^{I}_{\lambda_1\lambda_1'} = \mathrm{Re}\left[O_{\lambda_1\lambda_1'} - \varepsilon' O_{\lambda_1,-\lambda_1'} \right] \tag{3.23}$$

$$\mathrm{Im}\; \rho^{I}_{\lambda_1\lambda_1'} = \mathrm{Im}\left[E_{\lambda_1\lambda_1'} - \varepsilon' E_{\lambda_1,-\lambda_1'} \right] \tag{3.24}$$

and for baryon exchange

$$\mathrm{Re}\; \rho^{I}_{\lambda_1\lambda_1'} = \mathrm{Re}\; O_{\lambda_1\lambda_1'} + \varepsilon'\, \mathrm{Im}\; E_{\lambda_1,-\lambda_1'} \tag{3.25}$$

$$\mathrm{Im}\; \rho^{I}_{\lambda_1\lambda_1'} = \mathrm{Im}\; E_{\lambda_1\lambda_1'} + \varepsilon'\, \mathrm{Re}\; O_{\lambda_1,-\lambda_1'} \tag{3.26}$$

3.4 Applications

We now turn to specific applications of the general expressions given above. We recall that in this section we are concerned with experiments in which only the polarization of one single particle in the final state is measured, while the initial state is unpolarized. We successively consider polarization measurement for a particle with spin 1, 2, $\frac{1}{2}$ or $\frac{3}{2}$.

3.4.1 Measurement of the polarization of a spin 1 particle

We first study vector or axial mesons produced in reaction $a + b \to$ (particle 1 with spin 1) $+ 2 + \ldots$ The polarization is in general expressed in terms of density matrix elements which can be computed from the decay of particle 1. We give in the Appendix the decay angular distribution for $1 \to 0 + 0$, and $1 \to 0 + 0 + 0$, and the relations between observable quantities, density matrix elements and multipolar parameters.

a) Measurable quantities

All independent measurable quantities and their properties both in the helicity and transversity systems are given in Table A.1 of the Appendix. Because the initial state is unpolarized only the columns labelled H-B (for helicity quantization) or T-B (for transversity quantization) have to be considered. For the even polarization of the spin 1 meson there are six independent real measurable quantities. This number reduces to four in the two-body or inclusive case.

One observes for instance from Table A.1 that E_{1-1}, which is equal to ρ_{1-1}, is in general a complex quantity, but that it becomes purely real in two-body reactions for helicity quantization.

b) Separation into natural and unnatural parity exchanges

i) $S_a = 0$

When the spin of particle a is zero, then it is always possible to separate the natural from the unnatural parity exchanged contributions in channel $a + \bar{1} \to \bar{b} + \ldots$, which is therefore necessarily a bosonic channel. We give in Table 1 the d.m.e. or the combinations of d.m.e. which perform explicitly this separation for parity-conserving decay measurements.

In this table σ is the naturality of the exchanged state, and one should be aware that elements on the same line are not necessarily equal.

The two other independent elements, $\mathrm{Im}\, E_{1-1}$ and $\mathrm{Im}\, E_{10}$ for helicity quantization, $\mathrm{Re}\, E_{10}$ and $\mathrm{Im}\, E_{10}$ for transversity quantization, which do not appear in the table, give a measure of interferences between natural and unnatural parity exchanges. They vanish of course in the two-body case (in agreement with statement 1 of Section 2), because, as can be seen in Table A.1, they are antisymmetric with respect to reflection in the scattering plane. They can be measured in multiparticle reactions (Class 2) from an asymmetry measurement relative to particle 2, for instance from $E_{10}^{\uparrow} - E_{10}^{\downarrow}$. The relations between observable quantities and helicity amplitudes are given by Eqs. (3.13), (3.21) and (3.22).

ii) $S_a \neq 0$

When the spin of the initial particle is non-zero, the observable quantities are related either to incoherent sums over both exchanged naturalities, or to interference contributions, according to whether they are symmetric or antisymmetric with respect to reflection in the scattering plane as can be read off from Table A.1. The separation between natural and unnatural parity contribution can be achieved only when the initial particle a is polarized, and this will be studied in the following sections.

3.4.2 Measurement of the polarization of a spin 2 particle

We consider production of a spin 2 particle in a process $a + b \rightarrow$ (particle 1 with spin 2) $+ 2 + \ldots$ Most of what has been said for spin 1 polarization in the preceding paragraph can be readily extended to spin 2 polarization. We give in the Appendix the decay angular distribution for $2 \rightarrow 0 + 0$ and the relations between observable quantities, d.m.e. and multipolar parameters. Also two classes of reactions should be considered.

a) Measurable quantities

All independent measurable quantities and their properties are given in Table A.2 of the Appendix. For unpolarized initial state only columns labelled HB (for helicity quantization) or TB (for transversity quantization) have to be considered, and there are now 15 independent real quantities to describe the even part of the polarization. This number reduces to nine in two-body or Class 1 reactions.

b) Separation into natural and unnatural parity exchanges

i) $S_a = 0$

We give in Table 2 the nine density matrix elements or combinations of density matrix elements which, for parity conserving decay, allow a separation into natural ($\sigma = +1$) or unnatural ($\sigma = -1$) parity exchange in the $a + \bar{1} \rightarrow \bar{b} + 2$ ... channel, when particle a is spinless. The six other quantities which do not appear in the table (i.e., E^T_{21}, E^T_{10} and E^T_{2-1} for transversity quantization, $\mathrm{Im}\, E_{\lambda\lambda'}$ for helicity quantization), give a measure of interferences between opposite naturalities. The relation between the observable quantities and helicity amplitudes are given by Eqs. (3.13), (3.21) and (3.22).

ii) $S_a \neq 0$

When particle a is not spinless, the observable quantities are related either to incoherent sums over both exchanged naturalities, or to interference contributions, according whether they are symmetric or antisymmetric with respect to reflection in the scattering plane as can be read off from Table A.2.

3.4.3 Measurement of the polarization of a spin $\frac{1}{2}$ particle

The situation concerning the measurement of a final state particle with spin $\frac{1}{2}$ in a production reaction $a + b \rightarrow$ (particle 1 with spin $\frac{1}{2}$) $+ 2 + ..$ is slightly different from what we have considered previously for bosons with spin 1 or 2. In fact polarization measurements require either rescattering measurement (for nucleons) or weak decay measurement (for hyperons). Therefore in that case both the odd and even polarizations may be experimentally available.

a) Measurable quantities

All independent measurable quantities and their properties are given in Table A.3 of the Appendix. For unpolarized initial state (columns HB or TB) and two-body or inclusive type reactions, one recovers the well-known fact that the polarization is always normal to the scattering plane (P_y for helicity quantization, P_z for transversity quantization). But for Class 2 reactions a non-zero polarization in the scattering plane [18] (along P_x and P_z for instance for helicity quantization) is now obtained.

b) Separation into natural and unnatural parity exchanges

The unpolarized cross-section or the cross-section with the polarization of the final state spin $\frac{1}{2}$ particle normal to the scattering plane is expressed in terms of incoherent sums over both exchanged naturalities, while the asymmetries with polarization component in the scattering plane depend upon interferences between opposite naturalities.

i) $S_a = 0$

When particle a is spinless, and therefore channel $a + \bar{1} \to b + 2 + ..$ is a baryon exchange channel, the separation between natural and unnatural parity exchanges in this channel is readily obtained by considering the following combinations

$$d\sigma^{\pm} = d\sigma\,(1 \pm \eta_a P_t) \tag{3.27}$$

where η_a is the intrinsic parity of particle a, P_t the usual polarization parameter measured along the normal to the scattering plane, and $d\sigma$ the differential cross-section.

Such an analysis allows one to separate contributions of parity doublet partners and has been studied in Ref. 12).

ii) $S_a \neq 0$

When the initial particle a is not spinless the separation into natural and unnatural parity exchange requires an experiment with polarized initial particle a according to statement 3 of Section 2. This situation will be investigated in the following sections.

3.4.4 Measurement of the polarization of a spin $\frac{3}{2}$ particle

We finally consider measurement of the polarization of a spin $\frac{3}{2}$ particle produced in reaction $a + b \to$ (particle 1 with spin $\frac{3}{2}$) $+ 2 + ..$ From a practical point of view there are essentially two types of such processes:

Δ production with parity-conserving decay into nucleon + pion, from which only the even part of the polarization can be deduced, and Y^* production with a two-step decay including a weak decay of Λ or Σ, from which one can deduce the whole density matrix.

a) Measurable quantities

All independent measurable quantities and their properties are given in Table A.4 of the Appendix. For unpolarized initial states only columns labelled HB (for helicity quantization) or TB (for transversity quantization) should be considered.

b) Separation into natural and unnatural parity exchange

i) $S_a = 0$

We give in Table 3 the eight real observable quantities which allow a separation of opposite naturality contributions. In contrast with the meson polarization measurement case, one should notice that this separation requires measurement of both the even and odd part of the polarization (i.e., E and O quantities). This means, for instance, that one cannot deduce the opposite naturality contributions from measurement of the decay angular distribution of a Δ, which is a parity-conserving decay. On the other hand, this separation can be achieved for $Y^*_{\frac{3}{2}}$, by observing the two-step decay $Y^* \to Y\pi$ and $Y \to N\pi$.

For multiparticle reactions, there are eight more observable quantities (see Table A.4 of the Appendix) which can be measured in Class 2 reactions and are related to interference contributions. The relation between observable quantities and helicity amplitudes are given by Eqs. (3.13), (3.21), (3.25) and (3.26).

ii) $S_a \neq 0$

When the spin of particle a is non-zero, it is not possible to separate the exchanged naturalities, unless particle a is polarized. This case is studied in the next sections. However, it is possible to compute interference terms, even for a parity-conserving decay of the spin $\frac{3}{2}$ particle, in Class 2 reactions as for instance $pp \to \Delta + 2 + 3$. These interference terms are given by $\mathrm{Im}\, E_{31}$ and $\mathrm{Im}\, E_{3-1}$ in helicity quantization, related to helicity amplitudes by Eq. (3.21) and by E_{31} in transversity quantization.

Other combinations giving interference contributions from parity-non-conserving decays can be easily computed from Table A.4 and Eq. (3.21).

4. POLARIZED PROTON TARGET (OR BEAM)

This section and the following one are devoted to reactions induced by polarized protons or photons (assuming that only one initial particle is polarized) with measurement of a single polarization in the final state.

We study first processes with a polarized proton target, or polarized proton beam, and give the specific properties of observable quantities. These properties depend on the space orientation of the polarization vector. When this vector is perpendicular to the production plane, the properties are similar to those described in the unpolarized case (see Section 3.1). They are referred to, following a now standard terminology [14] as Bohr-symmetry properties (B). On the other hand, when the polarization vector lies in the production plane, quite different properties follow from parity conservation in the production process. We shall call them Bohr-antisymmetry properties (BA). Then we discuss the relevant experimental observables which allow one to separate crossed channel exchanges (both mesonic and baryonic) into components of definite naturality, or to detect opposite naturality interferences when the observed final state particle is produced at the same vertex as the polarized proton. The section ends with some considerations on the case where the final state particle and the initial proton are not associated with the same vertex.

4.1 General properties of observables

4.1.1 Initial density matrix and decay distribution

The density matrix describing a polarized initial proton is given by a linear combination of the 2×2 unit matrix and the three Pauli matrices σ_k

$$\rho_i(\vec{P}) = \frac{1}{2}(1 + \vec{\sigma}.\vec{P}) \tag{4.1}$$

where $\vec{P}$ is the polarization vector. The final state is described by a final density matrix:

$$\rho_f(\vec{P}) = F(1 + \vec{\sigma}.\vec{P})\, F^{+}/\mathrm{Tr.}\,(FF^{+}) \tag{4.2}$$

and we expand the density matrix of a final state particle (say particle 1) on the basis $(1, \sigma_k)$ exhibiting the dependence on the polarization vector $\vec{P}$:

$$\rho_{mm'} = \rho^{0}_{mm'} + \sum_k P_k\, \rho^{k}_{mm'} \tag{4.3}$$

where the 0 superscript refers to an unpolarized initial state and

$$\rho^{k}_{mm'} = F_m\, \sigma_k\, F^{+}_{m'} / \mathrm{Tr.}\,(FF^{+}) \tag{4.4}$$

corresponds to an initial proton with polarization along the x, y or z axis for $k = x, y, z$, respectively.

In general, measurement of $\rho_{mm'}$ is achieved by analyzing the decay angular distribution W of particle 1. Since a decay angular distribution is linear in ρ, the representation (4.3) may be also used to expand W:

$$W(\Omega) = W^{0}(\Omega) + \sum_k P_k\, W^{k}(\Omega) \tag{4.5}$$

where $W^{i}(\Omega)$ is a function of ρ^{i} only $(i = 0, x, y, z)$ and $\Omega = (\theta, \varphi)$ refers to the decay products of particle 1 and therefore depends obviously upon the choice of the quantization frame of reference. We recall that helicity and transversity axis are related within our conventions [16] by $(x^H, y^H, z^H) = (x^T, z^T, -y^T)$.

When the initial proton target polarization is parallel to the beam, the observed decay distribution is

$$W^{\parallel}(e_{\parallel}) = \left[W^{0}(\Omega) + \alpha\, e_{\parallel}\, W^{(z^H, -y^T)}(\Omega) \right] \tag{4.6}$$

where α is the degree of polarization of the target and $e_{\parallel} = 1$ (-1) when the polarization is in the same (opposite) direction as the beam. The two upper indices on the right-hand side of Eq. (4.6) refer to a helicity or transversity system.

Consequently two successive measurements, one with $e_{/\!/} = 1$ and the other with a reversed polarization $e_{/\!/} = -1$, give $W^o(\Omega)$ and $W^{z^H}(\Omega)$ [or $W^o(\Omega)$ and $W^{-y^T}(\Omega)$, respectively] in the helicity (or transversity) frame:

$$W^o(\Omega) = \frac{1}{2}\left[W^{/\!/}(1) + W^{/\!/}(-1)\right] \tag{4.7}$$

$$W^{(z^H,-y^T)}(\Omega) = \frac{1}{2\alpha}\left[W^{/\!/}(1) - W^{/\!/}(-1)\right] \tag{4.8}$$

Consider now a proton with polarization perpendicular to the beam, and let us denote by ϕ the angle between the scattering plane and the polarization vector. Then the polarized cross-section is given by

$$W^{\perp}(\phi) = \left(W^o(\Omega) + \alpha\cos\phi\, W^{(x^H,x^T)}(\Omega) + \alpha\sin\phi\, W^{(y^H,z^T)}(\Omega)\right) \tag{4.9}$$

which allows a straightforward determination of the two other distributions:

$$W^{(x^H,x^T)}(\Omega) = \frac{1}{\alpha\pi}\int_0^{2\pi} W^{\perp}(\phi)\cos\phi\, d\phi \tag{4.10}$$

$$W^{(y^H,z^T)}(\Omega) = \frac{1}{\alpha\pi}\int_0^{2\pi} W^{\perp}(\phi)\sin\phi\, d\phi \tag{4.11}$$

and moreover another determination of $W^o(\Omega)$:

$$W^o(\Omega) = \frac{1}{2\pi}\int_0^{2\pi} W^{\perp}(\phi)\, d\phi \tag{4.12}$$

Explicit expressions for the decay angular distributions of spin 1, 2, $\frac{1}{2}$ and $\frac{3}{2}$ are given in the Appendix.

4.1.2 Observable density matrix elements

As in the unpolarized case, it is straightforward to show that available quantities from the measurement of the angular distribution of a parity-conserving decay are defined by:

$$E^i_{mm'}(\varphi_j) = \frac{1}{2}\left[\rho^i_{mm'}(\varphi_j) + (-)^{m-m'}\rho^i_{-m'-m}(\varphi_j)\right] \tag{4.13}$$

and that, in parity-non-conserving decays, supplementary quantities are moreover observables:

$$O^{i}_{mm'}(\varphi_j) = \frac{1}{2}\left[\rho^{i}_{mm'}(\varphi_j) - (-)^{m-m'}\rho^{i}_{-m'-m}(\varphi_j)\right] \tag{4.14}$$

In the two preceding definitions the upper indices refer to the initial polarization. For completeness, we quote some properties of the observables E and O, which are independent of the quantization frame of reference and of the target polarization:

i) hermiticity

$$\begin{aligned} E^{i}_{mm'} &= E^{i\,*}_{m'm} \\ O^{i}_{mm'} &= O^{i\,*}_{m'm} \end{aligned} \tag{4.15a}$$

ii)

$$\begin{aligned} E^{i}_{mm'} &= (-)^{m'-m} E^{i}_{-m'-m} \\ O^{i}_{mm'} &= -(-)^{m'-m} O^{i}_{-m'-m} \end{aligned} \tag{4.15b}$$

thus for half-integer (or integer) spin

$$E^{i}_{m-m} \equiv 0 \quad \left(\text{or } O^{i}_{m-m} \equiv 0\right) \tag{4.15c}$$

iii)

$$\sum_m E^{0}_{mm} = 1 \qquad \text{(trace condition)} \tag{4.15d}$$

4.1.3 Bohr symmetry and Bohr antisymmetry properties

Since for a nucleon, the reflection operator **Y** in the production plane is given by [8)]

$$\begin{aligned} Y^{H} &= -i\,\sigma_y \qquad \text{(in the helicity system)} \\ Y^{T} &= -i\,\sigma_z \qquad \text{(in the transversity system)} \end{aligned} \tag{4.16}$$

We deduce that

$$\begin{aligned} Y^{H}\sigma_k (Y^{H})^{-1} &= \begin{cases} \sigma_k\,, & k = 0,y \\ -\sigma_k\,, & k = x,z \end{cases} \quad \text{for helicity quantization} \\ Y^{T}\sigma_k (Y^{T})^{-1} &= \begin{cases} \sigma_k\,, & k = 0,z \\ -\sigma_k\,, & k = x,y \end{cases} \quad \text{for transversity quantization} \end{aligned} \tag{4.17}$$

Thus the reflection properties of density matrix elements defined by Eq. (4.4), which follow from parity conservation in the production reaction, depend on the polarization direction k of the proton target, the difference being just up to a sign.

i) For positive sign in expressions (4.17), then, as in the unpolarized case, Eqs. (3.7) and (3.9) remain valid, and the matrix elements will be called Bohr symmetric (B). This is the case for $\rho^{y}_{\lambda\lambda'}$ and $\rho^{z}_{\tau\tau'}$. Explicitly

$$\rho^{i}_{\lambda\lambda'}(\varphi_j) = (-)^{\lambda-\lambda'} \rho^{i}_{-\lambda-\lambda'}(-\varphi_j) \quad \text{for} \quad i = 0, y \tag{4.18}$$

in the helicity basis

$$\rho^{i}_{\tau\tau'}(\varphi_j) = (-)^{\tau-\tau'} \rho^{i}_{\tau\tau'}(-\varphi_j) \quad \text{for} \quad i = 0, z \tag{4.19}$$

in the transversity basis.

ii) When an over-all minus sign is introduced by expressions (4.17), the matrix element will be called Bohr antisymmetric (BA). Parity conservation now yields:

$$\rho^{i}_{\lambda\lambda'}(\varphi_j) = -(-)^{\lambda-\lambda'} \rho^{i}_{-\lambda-\lambda'}(-\varphi_j) \quad \text{for} \quad i = x, z \tag{4.20}$$

$$\rho^{i}_{\tau\tau'}(\varphi_j) = -(-)^{\tau-\tau'} \rho^{i}_{\tau\tau'}(-\varphi_j) \quad \text{for} \quad i = x, y \tag{4.21}$$

In other words, when the proton polarization vector lies in the production plane, the density matrix elements describing the polarization of a final state particle obey Bohr antisymmetric reflection properties. Conversely, for an unpolarized initial state or a polarized proton perpendicularly to the production plane, the density matrix elements follow the Bohr symmetric reflection properties.

4.1.4 Symmetry properties through reflection for observables

From the preceding discussion it is very easy to deduce the symmetry properties, through reflection in the scattering plane, of multipolar parameters and observables. For B symmetric polarization these properties are of course just the same as those described in Section 3.1 for the unpolarized case.

i) Helicity system

$$t^{L\,(B)}_{M}(\varphi_j) = (-)^{L}\, t^{L\,(B)*}_{M}(-\varphi_j)$$
$$t^{L\,(BA)}_{M}(\varphi_j) = -(-)^{L}\, t^{L\,(BA)*}_{M}(-\varphi_j) \tag{4.22}$$

and consequently

$$E^{B}_{\lambda\lambda'}(\varphi_j) = E^{B*}_{\lambda\lambda'}(-\varphi_j) \;;\; O^{B}_{\lambda\lambda'}(\varphi_j) = -O^{B*}_{\lambda\lambda'}(-\varphi_j)$$
$$E^{BA}_{\lambda\lambda'}(\varphi_j) = -E^{BA*}_{\lambda\lambda'}(-\varphi_j) \;;\; O^{BA}_{\lambda\lambda'}(\varphi_j) = O^{BA*}_{\lambda\lambda'}(-\varphi_j) \tag{4.23}$$

Thus from a parity-conserving decay measurement, in two-body or inclusive reactions induced by a Bohr symmetric initial polarization, one can obtain real parts of density matrix elements, whereas imaginary parts can be deduced from measurement with B antisymmetric initial polarization.

ii) Transversity system

Parity conservation in the production process requires now

$$t^{L\,(B)}_{M}(\varphi_j) = (-)^{M}\, t^{L\,(B)}_{M}(-\varphi_j)$$
$$t^{L\,(BA)}_{M}(\varphi_j) = -(-)^{M}\, t^{L\,(BA)}_{M}(-\varphi_j) \tag{4.24}$$

Multipolar parameters are symmetric or antisymmetric with respect to reflection, depending on the parity of M and on the initial polarization state. From $M = \tau' - \tau$, we deduce

$$E^{B}_{\tau\tau'}(\varphi_j) = (-)^{\tau-\tau'} E^{B}_{\tau\tau'}(-\varphi_j) \;;\; O^{B}_{\tau\tau'}(\varphi_j) = (-)^{\tau-\tau'} O^{B}_{\tau\tau'}(-\varphi_j)$$

(4.25)

$$E^{BA}_{\tau\tau'}(\varphi_j) = -(-)^{\tau-\tau'} E^{BA}_{\tau\tau'}(-\varphi_j) \;;\; O^{BA}_{\tau\tau'}(\varphi_j) = -(-)^{\tau-\tau'} O^{BA}_{\tau\tau'}(-\varphi_j)$$

Thus even and odd L polarizations are symmetric (antisymmetric) with respect to reflection, when the transversity difference $\tau' - \tau$ is even (odd) for a B symmetric initial polarization, or when $\tau' - \tau$ is odd (even) for a B antisymmetric one.

All preceding properties are summarized in Tables 4 and 5. In Table 4, we give the properties of multipolar parameters T_M^{L1}, with respect to reflection in the production plane. In Table 5, we give the symmetry properties of the observable quantities E and O.

4.2 Study of reactions with meson exchange between the polarized proton and a produced baryon

In this Section, we dwell on the measurement of the polarization of a final baryon (particle 1), produced off a polarized proton target (particle a), assuming a small momentum transfer between the two baryons. The crossed channel $a+\bar{1}\to\bar{b}+2+\ldots$ is therefore a meson exchange channel (e.g., $p^{\uparrow}+K\to Y^{*}+X$). We shall derive the linear combinations of the observable quantities $E^{i}_{mm'}$ and $O^{i}_{mm'}$ (the superscript i refers to the direction of the initial proton polarization), which isolate a well-defined naturality in the crossed mesonic channel, or give interference contributions between opposite naturalities. We shall first use transversity quantization which is the most convenient way to make these combinations explicit, then, for the sake of completeness, we shall give similar expressions for helicity quantization. Throughout this section we shall use, in any quantization frame, the compact notation

$$E^{i+\alpha j}_{mm'} = E^{i}_{mm'} + \alpha\, E^{j}_{mm'}$$

$$O^{i+\alpha j}_{mm'} = O^{i}_{mm'} + \alpha\, O^{j}_{mm'} \qquad (4.26)$$

where $\alpha = \pm 1,\ \pm i$.

a) Transversity quantization

The preceding observable quantities are related to transversity amplitudes as follows:

$$E^{0\pm\gamma}_{\tau\tau'} = \frac{1}{2}\sum_{\tau_b,\{\tau_j\}}\Big[T_{\tau\{\tau_j\}\pm 1/2\,\tau_b}\,T^{*}_{\tau'\{\tau_j\}\pm 1/2\,\tau_b} + (-)^{\tau-\tau'}\,T_{-\tau'\{\tau_j\}\pm 1/2\,\tau_b}\,\times$$
$$\times\, T^{*}_{-\tau\{\tau_j\}\pm 1/2\,\tau_b}\Big] \qquad (4.27a)$$

$$O^{0\pm z}_{\tau\tau'} = \frac{1}{2}\sum_{\tau_b,\{\tau_j\}}\left[T_{\tau\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{\tau'\{\tau_j\}\pm\frac{1}{2}\tau_b} - (-)^{\tau-\tau'} T_{-\tau'\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{-\tau\{\tau_j\}\pm\frac{1}{2}\tau_b}\right] \quad (4.27b)$$

$$E^{x\pm iy}_{\tau\tau'} = \frac{1}{2}\sum_{\tau_b,\{\tau_j\}}\left[T_{\tau\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{\tau'\{\tau_j\}\mp\frac{1}{2}\tau_b} + (-)^{\tau-\tau'} T_{-\tau'\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{-\tau\{\tau_j\}\mp\frac{1}{2}\tau_b}\right] \quad (4.28a)$$

$$O^{x\pm iy}_{\tau\tau'} = \frac{1}{2}\sum_{\tau_b,\{\tau_j\}}\left[T_{\tau\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{\tau'\{\tau_j\}\mp\frac{1}{2}\tau_b} - (-)^{\tau-\tau'} T_{-\tau'\{\tau_j\}\pm\frac{1}{2}\tau_b} T^*_{-\tau\{\tau_j\}\mp\frac{1}{2}\tau_b}\right] \quad (4.28b)$$

Since, as shown in Section 2.3.2, transversity amplitudes $T_{\tau\{\tau_j\}\pm\frac{1}{2}\tau_b}$ are naturality-conserving amplitudes with definite naturality

$$\sigma = \eta_1 (-)^{\tau\mp\frac{1}{2}} \quad (4.29)$$

it is clear from Eqs. (4.27) that, for the Bohr symmetric components $(0,z)$ of the initial polarization, the separation between natural and unnatural parity exchanges can be obtained by isolating observable quantities of even transversity difference $\tau-\tau'$. Elements with odd $\tau-\tau'$ give an evaluation of interference terms between opposite naturalities. For Bohr antisymmetric polarization components (x,y) the situation is reversed: interferences are associated with $\tau-\tau'$ even.

These properties are summarized in Table 6. The first column gives the combinations of measurable quantities which should be considered in order to obtain specific information on the exchanged states (quoted in the last column). One should notice that for Bohr symmetric components (polarization normal to the production plane) both the odd and even polarizations (parity non-conserving decay) of the final baryon are required. The third column refers to symmetry properties (with respect to reflection in the scattering plane) which have been already recorded in Table 5.

b) Helicity quantization

We now turn to helicity quantization. We shall use the simplified notation $M^{\pm}_{\lambda}$ to denote the naturality conserving amplitudes $M^{\pm}_{\lambda \{\lambda_j\} \frac{1}{2} \lambda_b}(\varphi_i)$ defined by Eq. (2.9), and define the following quantities $D^{\pm}_{\lambda \lambda'}$ (or $D^{I}_{\lambda \lambda'}$) related to natural or unnatural parity (or interference) contributions:

$$D^{\pm}_{\lambda\lambda'} = \frac{1}{2N} \sum_{\{\lambda_j\},\lambda_b} \left[M^{\pm}_{\lambda} M^{\pm *}_{\lambda'} + (-)^{\lambda-\lambda'} M^{\pm}_{-\lambda} M^{\pm *}_{-\lambda'} \right]$$
$$D^{I}_{\lambda\lambda'} = \frac{1}{2N} \sum_{\{\lambda_j\},\lambda_b} \left[M^{+}_{\lambda} M^{-*}_{\lambda'} - (-)^{\lambda-\lambda'} M^{+}_{-\lambda} M^{-*}_{-\lambda'} \right] \tag{4.30}$$

These D functions can be experimentally measured when the initial proton has Bohr symmetric polarization (0,y) components *).
Explicitly one obtains:

$$\mathrm{Re}\, D^{\pm}_{\lambda\lambda'} = \mathrm{Re}\, E^{0}_{\lambda\lambda'} \mp \varepsilon(\lambda', 1/2)\, \mathrm{Im}\, O^{y}_{\lambda-\lambda'} \tag{4.31}$$

$$\mathrm{Im}\, D^{\pm}_{\lambda\lambda'} = \mathrm{Im}\, O^{0}_{\lambda\lambda'} \pm \varepsilon(\lambda', 1/2)\, \mathrm{Re}\, E^{y}_{\lambda-\lambda'} \tag{4.32}$$

$$\mathrm{Re}\, D^{I}_{\lambda\lambda'} = \mathrm{Re}\, O^{0}_{\lambda\lambda'} + \varepsilon(\lambda', 1/2)\, \mathrm{Im}\, E^{y}_{\lambda-\lambda'} \tag{4.33}$$

$$\mathrm{Im}\, D^{I}_{\lambda\lambda'} = \mathrm{Im}\, E^{0}_{\lambda\lambda'} - \varepsilon(\lambda', 1/2)\, \mathrm{Re}\, O^{y}_{\lambda-\lambda'} \tag{4.34}$$

with

$$\varepsilon(\lambda', 1/2) = \eta_1 (-)^{s_1-\lambda'}$$

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*) The y axis is the normal to the production plane because we are referring now to helicity quantization.

Similarly for Bohr-antisymmetric initial polarization of the proton (x, z components) we first define bilinear combinations of naturality-conserving amplitudes:

$$C^{\pm}_{\lambda\lambda'} = \frac{1}{2} \sum_{\lambda_b, \{\lambda_j\}} \left[M^{\pm}_{\lambda} M^{\pm *}_{\lambda'} - (-)^{\lambda-\lambda'} M^{\pm}_{-\lambda} M^{\pm *}_{-\lambda'} \right] \tag{4.35}$$

$$C^{I}_{\lambda\lambda'} = \frac{1}{2} \sum_{\lambda_b, \{\lambda_j\}} \left[M^{+}_{\lambda} M^{-*}_{\lambda'} + (-)^{\lambda-\lambda'} M^{+}_{-\lambda} M^{-*}_{-\lambda'} \right]$$

related to observable quantities by the following expressions:

$$\mathrm{Re}\, C^{\pm}_{\lambda\lambda'} = \mathrm{Re}\, O^{z}_{\lambda\lambda'} \pm \varepsilon(\lambda', 1/2)\, \mathrm{Re}\, O^{x}_{\lambda-\lambda'} \tag{4.36}$$

$$\mathrm{Im}\, C^{\pm}_{\lambda\lambda'} = \mathrm{Im}\, E^{z}_{\lambda\lambda'} \pm \varepsilon(\lambda', 1/2)\, \mathrm{Im}\, E^{x}_{\lambda-\lambda'} \tag{4.37}$$

$$\mathrm{Re}\, C^{I}_{\lambda\lambda'} = \mathrm{Re}\, E^{z}_{\lambda\lambda'} - \varepsilon(\lambda', 1/2)\, \mathrm{Re}\, E^{x}_{\lambda-\lambda'} \tag{4.38}$$

$$\mathrm{Im}\, C^{I}_{\lambda\lambda'} = \mathrm{Im}\, O^{z}_{\lambda\lambda'} - \varepsilon(\lambda', 1/2)\, \mathrm{Im}\, O^{x}_{\lambda-\lambda'} \tag{4.39}$$

The above expressions deserve some comments with respect to specific experimental situations.

i) <u>Target polarization orthogonal to the beam</u>

From this kind of experiment only part of the preceding relations can be computed due to lack of information on $E^{y}_{\tau\tau'}$ and $O^{y}_{\tau\tau'}$ in the transversity system, or on $E^{z}_{\lambda\lambda'}$ and $O^{z}_{\lambda\lambda'}$ in the helicity one. Only the Bohr symmetric components of the initial polarization can be used to separate natural and unnatural parity [the two first lines of Table 6 and relations (4.31), (4.32)].

Anyhow the x component of the polarization can be used to isolate interferences (for $\tau - \tau'$ even) or incoherent sums (for $\tau - \tau'$ odd).

ii) Target polarization along the beam

In this case only $E^y_{\tau\tau'}$ and $O^y_{\tau\tau'}$ or $E^z_{\lambda\lambda'}$ and $O^z_{\lambda\lambda'}$ (and obviously E^o and O^o) can be measured. Therefore, the separation into natural and unnatural parity contributions cannot be achieved with this type of experiment. Interference contributions can be obtained from $E^o_{\tau\tau'}$ or $O^o_{\tau\tau'}$ ($\tau - \tau'$ odd) and $E^y_{\tau\tau'}$ or $O^y_{\tau\tau'}$ ($\tau - \tau'$ even), whereas $E^o_{\tau\tau'}$ and $O^o_{\tau\tau'}$ ($\tau - \tau'$ even) or $E^y_{\tau\tau'}$ and $O^y_{\tau\tau'}$ ($\tau - \tau'$ odd) correspond to incoherent sums.

iii) When the polarization of the final baryon is measured by analyzing its parity-conserving decay angular distribution, then, as already quoted in Section 4.1.2, only the even part E^i of the final density matrix can be obtained in general. Thus, it is not possible to measure independently for instance the first and second lines of Table 6. The same remark applies to Eqs. (4.31)-(4.34) in which case only $\mathrm{Re}\, D^+_{\lambda\lambda'} + \mathrm{Re}\, D^-_{\lambda\lambda'}$ and $\mathrm{Im}\, D^+_{\lambda\lambda'} - \mathrm{Im}\, D^-_{\lambda\lambda'}$ can be computed. Interference contributions can be obtained using E measurement only, through the following relations

$$\mathrm{Re}\, D^I_{\lambda\lambda'} - \mathrm{Re}\, D^I_{\lambda'\lambda} = 2\eta_1 (-)^{s_1-\lambda'}\, \mathrm{Im}\, E^y_{\lambda-\lambda'}$$
$$\mathrm{Im}\, D^I_{\lambda\lambda'} - \mathrm{Im}\, D^I_{\lambda'\lambda} = 2\, \mathrm{Im}\, E^o_{\lambda\lambda'} \qquad (4.40)$$

iv) We recall that $E^i_{m-m} \equiv 0$ for half integer spin particles (see Section 4.1.2) and that diagonal elements are real. Looking back to Eqs. (4.30) and (4.35), we find

$$\mathrm{Re}\, D^{\pm}_{\lambda-\lambda} \equiv 0$$
$$\mathrm{Im}\, C^{\pm}_{\lambda-\lambda} \equiv 0 \qquad (4.41)$$

v) For quasi-two-body or inclusive reactions, all interference terms between opposite naturalities are zero, being antisymmetric by reflection in the scattering plane [according to Table 6 in transversity frame or from formulae (4.33), (4.34), (4.38), (4.39) and properties depicted in Table 5 in the helicity system].

4.3 Study of reactions with baryon exchange between the polarized proton and the produced boson

As in the previous section, we are still interested in measuring the polarization of two particles related to the same vertex (particle a and 1), assuming now that the crossed $a + \bar{1} \rightarrow \bar{b} + 2 + \ldots$ channel is a baryonic channel, i.e., that particle 1 is a boson produced off a polarized target with small momentum transfer (e.g., $p\uparrow + \pi \rightarrow \pi + N$, $p\uparrow + K \rightarrow K^* + N$).

a) Transversity quantization

For transversity quantization the relation between the observable quantities $E^{0\pm z}_{\tau\tau'}$, $O^{0\pm z}_{\tau\tau'}$, $E^{x\pm iy}_{\tau\tau'}$ and $O^{x\pm iy}_{\tau\tau'}$ and transversity amplitudes are again given by Eqs. (4.27) and (4.28). The only difference from the previous case is that the transversity amplitudes $T_{\tau\{\tau_j\}\pm\frac{1}{2}\tau_b}$ are naturality-conserving amplitudes with definite naturality σ now given by

$$\sigma = \pm \eta_1 (-)^{\tau} \tag{4.42}$$

instead of Eq. (4.29).

From Eqs. (4.27), (4.28) and (4.42) we easily deduce those observable quantities or combinations of them, which are related to well-defined exchanged naturality or to interference terms. Our results are presented in Table 7, which should be compared with Table 6 for meson exchange. Despite their apparent similarity important differences can be observed between the two tables. In particular it should be noticed that, in the present case, parity non-conserving decays are not required to separate exchanged naturalities when the initial polarization is normal to the production plane.

b) Helicity quantization

As in Section 4.1.2 the bilinear combinations $C^{\pm,I}_{\lambda\lambda'}$ and $D^{\pm,I}_{\lambda\lambda'}$ of naturality-conserving amplitudes, defined by Eqs. (4.30) and (4.35) can be related to combinations of observable quantities. For Bohr-symmetric initial polarization we find

$$\mathrm{Re}\, D^{\pm}_{\lambda\lambda'} = \mathrm{Re}\, E^{o}_{\lambda\lambda'} \pm \varepsilon(\lambda')\, \mathrm{Re}\, E^{y}_{\lambda-\lambda'} \tag{4.43}$$

$$\mathrm{Im}\, D^{\pm}_{\lambda\lambda'} = \mathrm{Im}\, O^{o}_{\lambda\lambda'} \pm \varepsilon(\lambda')\, \mathrm{Im}\, O^{y}_{\lambda-\lambda'} \tag{4.44}$$

$$\mathrm{Re}\, D^{I}_{\lambda\lambda'} = \mathrm{Re}\, O^{o}_{\lambda\lambda'} - \varepsilon(\lambda')\, \mathrm{Re}\, O^{y}_{\lambda-\lambda'} \tag{4.45}$$

$$\mathrm{Im}\, D^{I}_{\lambda\lambda'} = \mathrm{Im}\, E^{o}_{\lambda\lambda'} + \varepsilon(\lambda')\, \mathrm{Im}\, E^{y}_{\lambda-\lambda'} \tag{4.46}$$

where

$$\varepsilon(\lambda') = \eta_{1}\, (-)^{s_1-\lambda'}$$

For Bohr antisymmetric initial polarization the combinations of observable quantities E and O which allow a separation into naturality components $C^{\pm}$ or interference contributions C^{I} are given by

$$\mathrm{Re}\, C^{\pm}_{\lambda\lambda'} = \mathrm{Re}\, O^{z}_{\lambda\lambda'} \pm \varepsilon(\lambda')\, \mathrm{Im}\, E^{x}_{\lambda-\lambda'} \tag{4.47}$$

$$\mathrm{Im}\, C^{\pm}_{\lambda\lambda'} = \mathrm{Im}\, E^{z}_{\lambda\lambda'} \mp \varepsilon(\lambda')\, \mathrm{Re}\, O^{x}_{\lambda-\lambda'} \tag{4.48}$$

$$\mathrm{Re}\, C^{I}_{\lambda\lambda'} = \mathrm{Re}\, E^{z}_{\lambda\lambda'} - \varepsilon(\lambda')\, \mathrm{Im}\, O^{x}_{\lambda-\lambda'} \tag{4.49}$$

$$\mathrm{Im}\, C^{I}_{\lambda\lambda'} = \mathrm{Im}\, O^{z}_{\lambda\lambda'} + \varepsilon(\lambda')\, \mathrm{Re}\, E^{x}_{\lambda-\lambda'} \tag{4.50}$$

To summarize this discussion on boson production off a polarized proton, let us emphasize that it is possible, in this case, to achieve a separation between opposite naturality contributions from even polarization measurement of a meson reonance produced off a nucleon with polarization orthogonal to the scattering plane. This is in fact the most likely experimental situation, which should be contrasted with the more constraining requirements found in the preceding paragraph for baryon production.

4.4 Polarized target and observed particle 1 associated with different vertices

Finally, we consider the case where the final state particle 1 is produced at a different vertex from that of the polarized proton b, as for instance in forward production of ρ in $\pi\ p\uparrow \to \rho N$ (see Fig. 3).

The corresponding transversity amplitudes $T_{\tau_1\,\{\tau_j\}\,\tau_a \pm\frac{1}{2}}$ are naturality-conserving amplitudes, with σ, the naturality of the $a+\bar{1} \to \bar{b}+2+\dots$ channel, given by

$$\sigma = \eta_1\,(-)^{\nu+\tau_1-\tau_a}$$

Looking back to Eqs. (4.27) and (4.28) giving the relation between the transversity amplitudes and the observable quantities, and replacing $T_{\tau\{\tau_i\}\pm\frac{1}{2}\tau_b}$ by $T_{\tau\{\tau_i\}\,\tau_a\pm\frac{1}{2}}$ in order to recover the kinematical configuration depicted in Fig. 3, one can easily be convinced that the observable quantities $E^{o\,\pm z}_{\tau\tau'}$, $O^{o\,\pm z}_{\tau\tau'}$, $E^{x\,\pm iy}_{\tau\tau'}$ and $O^{x\,\pm iy}_{\tau\tau'}$ can be expressed in terms of an incoherent sum over both exchanged naturalities when $\tau-\tau'$ is an even integer, or of interference contributions when $\tau-\tau'$ is an odd integer.

For two-body reactions, according to Table 5, the only non-vanishing quantities with $\tau-\tau'$ odd are those related to Bohr-antisymmetric target polarization, i.e., $E^x_{\tau\tau'}$, $O^x_{\tau\tau'}$, $E^y_{\tau\tau'}$ and $O^y_{\tau\tau'}$.

We formulate now these results in terms of helicity d.m.e. restricting ourselves to Class 1 reactions. From a phenomenological point of view it is useful to know what observables are needed to obtain real or imaginary parts of interference. Thus, for BA initial polarization $i = x,\ z$, we express the related d.m.e. in terms of naturality-conserving amplitudes:

$$\rho^{x}_{\lambda\lambda'} = \frac{1}{2N} \sum_{\{\mu\}\lambda_a} \left[M^{+}_{\lambda\{\mu\}\lambda_a 1/2} \; M^{-\,*}_{\lambda'\{\mu\}\lambda_a -1/2} + \right. \tag{4.51a}$$
$$\left. + \; M^{-}_{\lambda\{\mu\}\lambda_a 1/2} \; M^{+\,*}_{\lambda'\{\mu\}\lambda_a -1/2} \right]$$

and

$$\rho^{z}_{\lambda\lambda'} = \frac{1}{2N} \sum_{\{\mu\}\lambda_a} \left[M^{+}_{\lambda\{\mu\}\lambda_a 1/2} \; M^{-\,*}_{\lambda'\{\mu\}\lambda_a 1/2} + \right. \tag{4.51b}$$
$$\left. + \; M^{-}_{\lambda\{\mu\}\lambda_a 1/2} \; M^{+\,*}_{\lambda'\{\mu\}\lambda_a 1/2} \right]$$

where the $M^{\pm}$ amplitudes are defined in Eq. (2.9) for boson or baryon exchanges. For quasi-two-body or inclusive reactions (where now the unobserved packet is assumed to be produced at the same vertex as the proton) only $\mathrm{Im}\, E^{i}_{\lambda\lambda'}$ and $\mathrm{Re}\, O^{i}_{\lambda\lambda'}$ survive for $i = x, z$ (see Table 5). Imaginary parts of interferences are then obtained through even polarization measurements, whereas odd polarization leads to real parts.

All the preceding results are in agreement with Statement 5 of Section 3.4. Interferences between natural and unnatural parity exchanges can be deduced, in two-body reactions, from polarization measurement of two particles associated with different vertices. For a polarized proton target we have shown that it is necessary, in addition, to have the proton polarization in the scattering plane.

4.5 Applications

We now present some specific applications of the general results obtained in the preceding sections for particle production on a polarized proton target (or with a polarized proton beam), $p\uparrow + b \to 1 + 2 + \ldots + n$. We assume that the polarization of particle 1 in the final state can be measured, and we shall consider that particle 1 may have spin 0, $\frac{1}{2}$, 1, $\frac{3}{2}$ or 2.

4.5.1 $p\uparrow + b \to$ (particle 1 with spin zero) + 2 + ...

In this simple first example one is obviously measuring only the polarized cross-section. We consider separately the two cases where the polarized proton is associated with particle 1 at the same or at the opposite vertex.

i) When the polarized proton and particle 1 are associated with the same vertex (i.e., small momentum transfer between both particles and baryon exchange mechanisms), as for example in backward πN elastic scattering on polarized target, backward pseudoscalar meson photoproduction $p\uparrow \gamma \to \pi N$, $p\uparrow \bar{p} \to \pi^+ \pi^-$ etc., one can separate the exchanges of opposite naturalities. This result has been emphasized in Ref. 12) in particular with respect to the parity doublet problem. From Table 7 we deduce ($\tau = \tau' = 0$, and therefore $\mathcal{O} = 0$) that the separation between opposite naturalities can be achieved by combining the unpolarized cross-section $E^0_{00}\ d\sigma^0/dt = d\sigma/dt$ and the polarization parameter $P = E^z_{00}$ corresponding to the target polarization normal to the scattering plane. The final result, in agreement with Eq. (3.27) is

$$\frac{d\sigma^{\pm}}{dt} = (1 \pm \eta_1 P)\frac{d\sigma}{dt} \tag{4.52}$$

The two other Bohr antisymmetric quantities $E^x_{00}\ d\sigma/dt$ and $E^y_{00}\ d\sigma/dt$ for transversity quantization, corresponding to polarized cross-sections with target polarization in the scattering plane, permit an evaluation of interferences between opposite naturality exchanges. These quantities, being antisymmetric with respect to reflection in the scattering plane (see Table 7), vanish for two-body or inclusive reactions (Class 1) and can be measured in Class 2 reactions from a measurement of asymmetry with respect to the scattering plane. Their relations to helicity amplitudes are given by Eqs. (4.49) and (4.50).

ii) When the polarized proton and particle 1 are associated with different vertices (i.e., small momentum transfer between particles a and 1), then, as stated in Section 4.4, all measurable quantities are given in terms of incoherent sums over both naturalities. It is not possible to measure interference contributions. These interferences are trivially zero if particle a is spinless (well-defined naturality in the crossed channel), and if $S_a \neq 0$ they can be computed when both a and b are polarized from Bohr antisymmetric components of the two initial polarizations as will be shown in Section 6.

4.5.2 $p\uparrow + b \rightarrow$ (particle 1 with spin $\frac{1}{2}$) $+ 2 + \cdots$

a) Measurable quantities

We are now investigating the case where the polarization of a final spin $\frac{1}{2}$ particle is measured. The general properties of observable quantities are given in Table A.3 of the Appendix. These properties clearly depend upon the initial proton polarization, and in agreement with the definitions of Section 4.1 we denote by the superscript i for $E^{i}_{mm'}$, $O^{i}_{mm'}$, or $\rho^{i}_{mm'}$, the polarization orientation of the initial proton $(i = x, y, z$ or 0 for unpolarized initial proton).

b) Separation into natural and unnatural parity exchanges

i) Meson exchange

When the two polarized spin $\frac{1}{2}$ particles are related to the same vertex (i.e., small momentum transfer between the two particles, and meson exchange), then according to the discussion of Section 4.2 and to the results of Table 6, it is possible to separate the contributions of given exchanged naturality by considering suitable combinations of observable quantities. For the sake of clarity and completeness we give in Table 8 the explicit combinations which allow such a separation for a final spin $\frac{1}{2}$ particle.

One should pay particular attention to the second line of Table 8. In fact (for transversity quantization) $O^{0}_{\frac{1}{2}\frac{1}{2}}$ is nothing but the final polarization $P^{f}_{\perp}$ along the normal to the scattering plane for unpolarized initial state, and $E^{z}_{\frac{1}{2}\frac{1}{2}}$ is the up-down asymmetry for the polarized initial proton along the normal to the scattering plane which we denote by

$$\frac{d\sigma^{\perp}}{dt} \Big/ \frac{d\sigma}{dt} = P^{i}_{\perp}$$

Therefore the second line in Table 8 can be rewritten as follows

$$O^{0}_{1/2\,1/2} \pm E^{z}_{1/2\,1/2} = P^{f}_{\perp} \pm P^{i}_{\perp} \qquad (4.53)$$

Using Eqs. (4.30) and (4.38) the preceding quantity is related to helicity amplitudes by

$$P_{\perp}^{f} \pm P_{\perp}^{i} = \frac{1}{N} \,\mathrm{Im} \sum_{\{\lambda_j\}\lambda_b} M^{\pm}_{\frac{1}{2}\{\lambda_j\}\frac{1}{2}\lambda_b} M^{\pm *}_{-\frac{1}{2}\{\lambda_j\}\frac{1}{2}\lambda_b} \qquad (4.54)$$

and therefore gives an evaluation of the imaginary part of products of amplitudes corresponding to a given exchanged naturality.

The interesting point about Eq. (4.53) is that computing this expression does not require experiment with correlation measurements between the two initial and final polarizations. Instead, one can use the results of two separated experiments, one with cross-section measurement on a polarized target, and another one with final polarization measurement on an unpolarized target, and for example combine electronic experiments with bubble chamber experiments.

As a consequence of Eq. (4.53) we recover for $(0^- + \frac{1}{2}^+ \rightarrow 0^- + \frac{1}{2}^+)$ reactions, due to the fact that only natural parity can be exchanged, the well-known relation:

$$P_{\perp}^{f} = P_{\perp}^{i} \qquad (4.55)$$

i.e., measurement of the final polarization is equivalent to measurement of the asymmetry cross-section on a polarized target, as for instance in $\pi N \rightarrow K \Lambda$. Equality (4.55) is of course trivially satisfied when the initial and final states are identical. On the other hand, this equality is not satisfied for more general reactions, such as for instance $\pi N \rightarrow \Lambda$ + anything, $\gamma N \rightarrow K \Lambda$. Joint experimental data from an experiment with a polarized target and from another one with Λ polarization measurement on an unpolarized target will then yield interesting information on the relative phases of helicity amplitudes with a given naturality.

ii) When the two polarized spin $\frac{1}{2}$ particles are related to opposite vertices (i.e., small momentum transfer between particle b and 1), naturality splitting is only possible in particular cases. However, as was underlined in Section 4.4, naturality interferences may be isolated. As an illustrative example, consider the inclusive Λ production (Class 1) on a polarized target: $\pi p\uparrow \rightarrow \Lambda X$ in the pion fragmentation region. Owing to the pseudoscalar nature of the incident particle, hyperon polarization gives the relative amount of naturality exchanges for each Bohr symmetric spin component of the proton target (unpolarized and transverse polarization perpendicular to the production plane). In addition, Bohr-antisymmetric polarizations of the proton lead to interferences evaluation [see Eqs. (4.51a) and (4.51b)].

4.5.3 $p\uparrow + b \to$ (particle 1 with spin 1) + 2 + ...

a) Measurable quantities

All spin 1 observables are given in Table A.1 of the Appendix. Their symmetry properties by space reflection depend on the initial polarization component of the target; for instance, $E^i_{\tau\tau'}$ enjoys opposite symmetry properties according to whether the superscript i refers to a Bohr symmetric or to a Bohr anti-symmetric component of the proton polarization. Only the even part $E^i_{mm'}$ of the density matrix can be measured without ambiguity as briefly discussed in the Appendix. Thus our discussion will be restricted to this part of the density matrix.

b) Separation of opposite naturality exchanges

i) Baryon exchange

We present in Table 9 both in helicity and transversity quantizations those combinations of observables which isolate natural from unnatural parity exchanges, in the baryonic channel $p\uparrow + \bar{1} \to \bar{b} + \dots$.

Obviously an experiment such as $\pi\, p\uparrow \to NV$ is very difficult to achieve since correlation measurements between the vector meson and the target, in the backward direction, are required (small statistics). However, we must emphasize that some combination of particular quantities depicted in Table 9 may be evaluated more precisely and more easily than each of them. This result is readily established by combining lines 1 and 4 of this table and using the normalization condition

$$E^i_{00} + 2\,E^i_{11} = \frac{d\sigma^i}{dt}\Big/\frac{d\sigma}{dt}$$

we obtain in helicity quantization [cf., Eq. (4.43)]:

$$\mathrm{Re}\left(D^{\pm}_{00} - 2\,D^{\pm}_{1-1}\right) = \frac{d\sigma^0}{dt}\left(E^0_{00} - 2\,\mathrm{Re}\,E^0_{1-1}\right) \mp \eta_1 \frac{d\sigma^y}{dt} \qquad (4.56)$$

On the right-hand side of Eq. (4.56), the first term is nothing but the final density matrix elements with unpolarized initial state, whereas the second term is the up-down asymmetry cross-section on polarized target.

ii) We turn now to the case of a spin 1 particle produced in the forward direction (boson exchange), the polarized target and the decaying particle are related to opposite vertices (i.e., small momentum transfer between b and 1). It is then possible to measure interferences between naturality contributions in a Class 1 reaction. As shown in Section 4.4 only BA target polarized states have to be considered and the even non-vanishing density matrix elements are (see Table 5)

$$\mathrm{Im}\ E^{i}_{\lambda\lambda'} \qquad i = x, z$$

Thus only $\mathrm{Im}\,\rho^{x,z}_{1-1}$ and $\mathrm{Im}\,\rho^{x,z}_{10}$ depend on opposite naturality interferences, whereas all the others can be expressed as incoherent sums in Class 1 reactions.

4.5.4 $p\uparrow + b \to$ (particle 1 with spin $\frac{3}{2}$) $+ 2 + \ldots$

a) Measurable quantities

The set of observables related to polarization measurement of a spin $\frac{3}{2}$ particle is given together with detailed properties in Table A.4 of the Appendix. In general the measurement procedure consists in analyzing the strong decay process $\frac{3}{2} \to \frac{1}{2} + 0$, from which the even part only $E^{i}_{mm'}$, of the angular distribution may be extracted (e.g., $\pi\, p\uparrow \to \Delta + \ldots;\ \Delta \to p\,\pi$). But, if the polarization of the final decay produced baryon is analyzed, the odd quantities $O^{i}_{mm'}$, are also accessible, as for instance in $\pi\, p\uparrow \to Y^{*}_{+} \ldots,\ \ Y^{*} \to \Lambda\pi,\ \ \Lambda \to p\,\pi$.

b) Separation of natural and unnatural contributions

i) Boson exchanges

Combinations of density matrix elements which distinguish between natural and unnatural parity contributions exchanged in the channel $p\uparrow + \bar{1} \to \bar{b} + \ldots$ are given in Table 10.

It is apparent that the evaluation of a well-defined naturality contribution requires at least either even and odd polarization measurements of the final baryon when the initial proton polarization is normal to the production plane, or even polarization only when the proton target is polarized in the production plane (normal and parallel to the beam). Thus for inclusive Δ production (in which $E^{i}_{mm'}$ only are available) a polarized target with non-zero component along the beam is required and isolates the following $\sigma = \pm \eta_1$ quantities (helicity quantization):

$$\mathrm{Im}\ E^{y}_{31} \mp \mathrm{Im}\ E^{x}_{3-1}$$
$$\mathrm{Im}\ E^{y}_{3-1} \pm \mathrm{Im}\ E^{x}_{31}$$

Inclusive Y^* production off a polarized target may lead, in principle,, to much more information. In that case, the only quantity which does not require polarization correlation measurements can be obtained from lines 7 and 8 (helicity quantization) of Table 10. From Eq. (4.32) we obtain

$$\mathrm{Im}\left(D^{\pm}_{1-1} - D^{\pm}_{3-3}\right) = \frac{d\sigma^{0}}{dt}\,\mathrm{Im}\left(O^{0}_{1-1} - O^{0}_{3-3}\right) \pm \eta_1 \frac{1}{2} \frac{d\sigma^{y}}{dt} \tag{4.57}$$

ii) We present now some brief comments on the reaction $\pi p\uparrow \rightarrow \Delta \pi$ in the backward direction, in order to illustrate the case of spin $\frac{3}{2}$ production on a polarized target at opposite vertices. A Bohr symmetric initial polarization disentangles natural from unnatural baryonic exchanges in the following way (see preceding discussion, Section 4.5):

$$\frac{d\sigma^{\pm}}{dt} = \frac{d\sigma^{0}}{dt} \mp \frac{d\sigma^{y(z)}}{dt} \tag{4.58}$$

in the helicity (or transversity) system.

Due to the fact that only natural parity can be exchanged in the $\pi\pi \to N\bar{\Delta}$ channel, we deduce from Eq. (4.57) the exact relation

$$\frac{d\sigma^{y}}{dt} = 2\,\frac{d\sigma^{0}}{dt}\left(\mathrm{Im}\,O^{0}_{1-1} - \mathrm{Im}\,O^{0}_{3-3}\right) \tag{4.59}$$

which, on the other hand, can be obtained straightforwardly from parity relations on observables [8]. Substituting Eq. (4.59) into (4.58) we obtain

$$\frac{d\sigma^{\pm}}{dt} = \frac{d\sigma^{0}}{dt}\left(1 \mp 2\,\mathrm{Im}\,O^{0}_{1-1} \pm 2\,\mathrm{Im}\,O^{0}_{3-3}\right) \tag{4.60}$$

in which only unpolarized initial state quantities appear. This result is actually contained in Table 3.

Bohr antisymmetric polarizations of the target or of the Δ isobar are not expected to provide a separation of naturality contributions, since N and Δ are both associated with a spin zero particle. However, interference terms can be obtained from the following even polarization measurements:

$$\mathrm{Im}\,\rho^{x,z}_{31} \quad \text{and} \quad \mathrm{Im}\,\rho^{x,z}_{3-1} \qquad \text{(helicity quantization)}$$

These quantities are not independent, and by a straightforward calculation one finds:

$$\rho^{z}_{31} = -\rho^{x}_{3-1}$$

$$\rho^{z}_{3-1} = \rho^{x}_{31}$$

5. POLARIZED PHOTON BEAMS

In this section we deal with processes induced by polarized photon beams. We shall, of course, work henceforth in the helicity system. Following the same study scheme as in the preceding section, we recall first some general features of polarized photon states, then consider separately the two general possibilities of meson or baryon exchange, and finally turn to some specific examples of photoproduction reaction of low spin particles.

5.1 The photon density matrix

In view of its two-helicity states the photon polarization is described by an initial density matrix similar to that of the proton:

$$\rho_\gamma = \frac{1}{2}\left(\mathbb{1} + \vec{P}\cdot\vec{\sigma} \right) \tag{5.1}$$

However, the vector $\vec{P}$ measuring the polarization degree must not be confused with the electric polarization vector $\vec{\varepsilon}$ of the photon. In fact, this latter is defined by $\vec{\varepsilon} = (\cos\Psi,\ \sin\Psi,\ 0)$ for linearly-polarized photons, where Ψ is the angle between $\vec{\varepsilon}$ and the production plane (see Fig. 4). A helicity basis is implicitly assumed with z axis along the photon momentum $\vec{P}_\gamma$ and y axis along $(\vec{P}_\gamma \times \vec{P}_1)$.

For linearly polarized photons, the three-vector $\vec{P}$ is given by

$$\vec{P} = |\vec{P}|\,(-\cos 2\Psi,\ -\sin\Psi,\ 0) \tag{5.2}$$

and the corresponding initial photon density matrix by

$$\rho_\gamma^{lin} = \frac{1}{2}\begin{pmatrix} 1 & -|\vec{P}|\,e^{-2i\Psi} \\ -|\vec{P}|\,e^{2i\Psi} & 1 \end{pmatrix} \tag{5.3}$$

For circularly polarized photons, with helicity ± 1, one has obviously $\vec{P} = |\vec{P}|\ (0,\ 0,\ \pm 1)$, and consequently

$$\rho_{\gamma}^{circ.} = \frac{1}{2}\begin{pmatrix} 1+|\vec{P}| & 0 \\ 0 & 1-|\vec{P}| \end{pmatrix} \tag{5.4}$$

We consider now the photoproduction reaction $\gamma b \rightarrow 1+2+\ldots$ with measurement of the polarization of particle 1 from its decay angular distribution $W(\theta,\varphi)$. With the same notation that was introduced in the polarized proton target case, we define the decay angular distribution $W^i(\theta,\varphi)$, where i refers to the orientation of the vector polarization $\vec{P}$ of the incident photon. For linearly-polarized photons we therefore get

$$W^L(\psi) = \left[W^0 - |\vec{P}_L|\left(\cos 2\psi\, W^x + \sin 2\psi\, W^y\right)\right] \tag{5.5}$$

with

$$\int d\Omega\, W^0(\Omega) = 1 \tag{5.6}$$

One should keep in mind that the superscript i in W^i refers to the orientation of the vector polarization $\vec{P}$ of the photon (not $\vec{\varepsilon}$). Therefore, the parallel ($\psi=0$) and perpendicular ($\psi=\pi/2$) distributions are defined by

$$W^{\perp} = W^0 + W^x \tag{5.7}$$

$$W^{\parallel} = W^0 - W^x \tag{5.8}$$

Similarly, for circularly polarized photons with helicity ± 1:

$$W^{\pm} = W^0 \pm |\vec{P}_c|\, W^{\gamma} \tag{5.9}$$

The general properties of the observable quantities which can be now computed from W^i are the same as those already given in the polarized proton case, and are summarized in the Appendix. The only difference is that the reflection operator Y is now given by $Y = -\sigma_x$ [to be compared with Eq. (4.16)]. Therefore, the Bohr symmetric part of the final state density matrix corresponds now to $i = 0$, x and the Bohr antisymmetric part to $i = y$, z.

5.2 Meson exchange between the polarized proton and the produced meson 1

We discuss in this section the situation where particle 1 is a meson produced at the same vertex as the photon (i.e., with small momentum transfer between these two particles). The cross channel is therefore a meson exchange channel (e.g., forward ρ production $\gamma\uparrow N \to \rho X$, etc.).

In general, measurement of the Bohr symmetric part of the decay distribution (W^o, W^x), allows one to compute exchanged naturality contributions $D^{\pm}$ and interference terms D^I, related to observable quantities as follows:

$$\mathrm{Re}\ D^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Re}\left(E^{o}_{\lambda_1\lambda'_1} \pm \varepsilon(\lambda'_1)\ E^{x}_{\lambda_1\,-\lambda'_1} \right) \tag{5.10}$$

$$\mathrm{Im}\ D^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Im}\left(O^{o}_{\lambda_1\lambda'_1} \pm \varepsilon(\lambda'_1)\ O^{x}_{\lambda_1\,-\lambda'_1} \right) \tag{5.11}$$

$$\mathrm{Re}\ D^{I}_{\lambda_1\lambda'_1} = \mathrm{Re}\left(O^{o}_{\lambda_1\lambda'_1} - \varepsilon(\lambda'_1)\ O^{x}_{\lambda_1\,-\lambda'_1} \right) \tag{5.12}$$

$$\mathrm{Im}\ D^{I}_{\lambda_1\lambda'_1} = \mathrm{Im}\left(E^{o}_{\lambda_1\lambda'_1} - \varepsilon(\lambda'_1)\ E^{x}_{\lambda_1\,-\lambda'_1} \right) \tag{5.13}$$

where the D's have already been formally defined in Eqs. (4.30) but with naturality-conserving helicity amplitudes now given by [see Eq. (2.5)]:

$$M^{\pm}_{\lambda_1\{\lambda_j\}1\,\lambda_b} = \frac{1}{\sqrt{2}}\left(M_{\lambda_1\{\lambda_j\}1\,\lambda_b} \pm \varepsilon(\lambda_1)\ M_{-\lambda_1\{\lambda_j\}-1\,\lambda_b} \right) \tag{5.14}$$

and

$$\varepsilon(\lambda_1) = -\eta_1 (-)^{s_1 - \lambda_1} \tag{5.15}$$

For Bohr antisymmetric initial polarization of the photon (y, z components), we obtain similar expressions:

$$\mathrm{Re}\, C^{\pm}_{\lambda_1 \lambda'_1} = \mathrm{Re}\, O^{\gamma}_{\lambda_1 \lambda'_1} \mp \varepsilon(\lambda'_1)\ \mathrm{Im}\, E^{y}_{\lambda_1 -\lambda'_1} \tag{5.16}$$

$$\mathrm{Im}\, C^{\pm}_{\lambda_1 \lambda'_1} = \mathrm{Im}\, E^{\gamma}_{\lambda_1 \lambda'_1} \pm \varepsilon(\lambda'_1)\ \mathrm{Re}\, O^{y}_{\lambda_1 -\lambda'_1} \tag{5.17}$$

$$\mathrm{Re}\, C^{I}_{\lambda_1 \lambda'_1} = \mathrm{Re}\, E^{\gamma}_{\lambda_1 \lambda'_1} + \varepsilon(\lambda'_1)\ \mathrm{Im}\, O^{y}_{\lambda_1 -\lambda'_1} \tag{5.18}$$

$$\mathrm{Im}\, C^{I}_{\lambda_1 \lambda'_1} = \mathrm{Im}\, O^{\gamma}_{\lambda_1 \lambda'_1} - \varepsilon(\lambda'_1)\ \mathrm{Re}\, E^{y}_{\lambda_1 -\lambda'_1} \tag{5.19}$$

the C's being defined again by Eqs. (4.35).

Let us briefly discuss some relevant properties of the preceding relations.

For parity-conserving decays, as it is usually the case for mesons, only the B symmetric part of the initial polarization can be used to separate the exchanged naturalities. Interference contributions between different naturalities can be obtained for Class 2 reactions for Bohr-anti-symmetric initial polarization from $\mathrm{Re}\, E^{y,z}$ and for B symmetric polarization from a measurement of $\mathrm{Im}\, E^{o,x}$.

i) For linearly-polarized photons (like those of the SLAC laser beam), and parity-conserving decay of particle 1, one can measure $E^{o,x,y}_{\lambda_1 \lambda'_1}$. In that case it is possible using relations (5.10) and (5.13) to deduce well-defined naturality exchange contributions and interference terms. The y component of the initial polarization can also be used to compute interference terms by rewriting (5.18) in the following way

$$\mathrm{Im}\,(C^{I}_{\lambda_1\lambda'_1} + C^{I}_{\lambda'_1\lambda_1}) = -2\,\varepsilon(\lambda'_1)\ \mathrm{Re}\,E^{y}_{\lambda_1,-\lambda'_1} \tag{5.20}$$

ii) For circularly polarized photons and parity-conserving decay of particle 1, it is not possible to separate the contributions of exchanged naturalities, but one can still obtain interference terms from Eq. (5.18) as:

$$\mathrm{Re}(C^{I}_{\lambda_1\lambda'_1} + C^{I}_{\lambda'_1\lambda_1}) = 2\ \mathrm{Re}\ E^{\gamma}_{\lambda_1\lambda'_1} \tag{5.21}$$

For two-body or inclusive reactions (Class 1), Eqs. (5.12), (5.13) and (5.18), (5.19) vanish identically, as can be checked from the tables of the Appendix, in agreement with the statements of Section 2 that interferences can never be deduced in Class 1 reactions from the measurement of the polarization of two particles related to the same vertex.

5.3 Baryon exchange between the polarized photon and the produced baryon 1

We now consider a baryonic resonance produced via baryonic exchanges, as for example $\gamma\!\uparrow N \to \Delta X$ or $\gamma\!\uparrow N \to Y^*X$ in the backward direction. Measurement of the decay angular distributions of a resonance produced by Bohr symmetric initial polarized photon provides an evaluation of the following quantities

$$\mathrm{Re}\ D^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Re}\ E^{o}_{\lambda_1\lambda'_1} \pm \varepsilon(\lambda'_1)\ \mathrm{Im}\ O^{x}_{\lambda_1-\lambda'_1} \tag{5.22}$$

$$\mathrm{Im}\ D^{\pm}_{\lambda_1\lambda'_1} = \mathrm{Im}\ O^{o}_{\lambda_1\lambda'_1} \mp \varepsilon(\lambda'_1)\ \mathrm{Re}\ E^{x}_{\lambda_1-\lambda'_1} \tag{5.23}$$

$$\mathrm{Re}\ D^{I}_{\lambda_1\lambda'_1} = \mathrm{Re}\ O^{o}_{\lambda_1\lambda'_1} - \varepsilon(\lambda'_1)\ \mathrm{Im}\ E^{x}_{\lambda_1-\lambda'_1} \tag{5.24}$$

$$\mathrm{Im}\ D^{I}_{\lambda_1\lambda'_1} = \mathrm{Im}\ E^{o}_{\lambda_1\lambda'_1} + \varepsilon(\lambda'_1)\ \mathrm{Re}\ O^{x}_{\lambda_1-\lambda'_1} \tag{5.25}$$

where, as previously defined by Eq. (4.30), the $D^{\pm}$ and D^{I} quantities refer to contributions of definite naturality or to interference terms, respectively, and the naturality-conserving amplitudes are defined by

$$M^{\pm}_{\lambda_1\{\lambda_j\}-1\,\lambda_b} = \frac{1}{\sqrt{2}}\left(M_{\lambda_1\{\lambda_j\}-1\,\lambda_b} \pm i\,\varepsilon(\lambda_1)\, M_{-\lambda_1\{\lambda_j\}-1\,\lambda_b} \right) \tag{5.26}$$

with

$$\varepsilon(\lambda_1) = -\eta_1 (-)^{s_1-\lambda_1} \tag{5.27}$$

Similarly, for Bohr antisymmetric initial polarizations we obtain

$$\mathrm{Re}\, C^{\pm}_{\lambda_1\lambda_1'} = \mathrm{Re}\left(O^{z}_{\lambda_1\lambda_1'} \pm \varepsilon(\lambda_1')\, O^{y}_{\lambda_1-\lambda_1'} \right) \tag{5.28}$$

$$\mathrm{Im}\, C^{\pm}_{\lambda_1\lambda_1'} = \mathrm{Im}\left(E^{z}_{\lambda_1\lambda_1'} \pm \varepsilon(\lambda_1')\, E^{y}_{\lambda_1-\lambda_1'} \right) \tag{5.29}$$

$$\mathrm{Re}\, C^{I}_{\lambda_1\lambda_1'} = \mathrm{Re}\left(E^{z}_{\lambda_1\lambda_1'} - \varepsilon(\lambda_1')\, E^{y}_{\lambda_1-\lambda_1'} \right) \tag{5.30}$$

$$\mathrm{Im}\, C^{I}_{\lambda_1\lambda_1'} = \mathrm{Im}\left(O^{z}_{\lambda_1\lambda_1'} - \varepsilon(\lambda_1')\, O^{y}_{\lambda_1-\lambda_1'} \right) \tag{5.31}$$

the C's being defined by Eq. (4.35).

The main features of the preceding expressions can be summarized in the following remarks.

i) Measurements with only Bohr-symmetric initial polarization do not allow a separation of different naturality contributions, unless parity non-conserving quantities are measured in the decay. But, as we stressed earlier in the analogous situation of Bohr antisymmetric polarizations with meson exchange, one can isolate interference contributions. Corresponding expressions are now:

$$\mathrm{Im}\,(D^{I}_{\lambda_1\lambda'_1} - D^{I}_{\lambda'_1\lambda_1}) = 2\,\mathrm{Im}\,E^{0}_{\lambda_1\lambda'_1} \tag{5.32a}$$

$$\mathrm{Re}\,(D^{I}_{\lambda_1\lambda'_1} - D^{I}_{\lambda'_1\lambda_1}) = -2\,\varepsilon(\lambda'_1)\,\mathrm{Im}\,E^{x}_{\lambda_1,-\lambda'_1} \tag{5.32b}$$

ii) Interferences, being always antisymmetric with respect to reflection in the production plane, are never available in two-body or inclusive reactions (see statement 5, Section 2).

iii) For Class 2 reactions use of circularly-polarized photons alone gives information on interference terms only.

5.4 Particle produced at opposite vertex

In the former sections particle 1 was always assumed to be produced at the same vertex as the photon. We study now the case where this outgoing particle is emitted at a vertex opposite to the photon one (see Fig. 5). If the other polarizations involved in the reaction are not observed, as we assume now, it is clear that separation of exchanged naturalities is not possible, unless one is considering a quasi-two-body reaction with a spin zero particle at the same vertex as the photon (e.g., $\gamma\uparrow N \to \pi\Delta$ in the forward direction).

We now show that for this type of process, naturality interferences may be obtained from Bohr antisymmetric initial polarizations (circularly or linearly-polarized photons) without any information required on the other produced particles than particle 1. Limiting our discussion to Class 1 reactions it is easy to show that final density matrix elements will be given by expressions of the type

$$\rho \simeq \Sigma\, M^{+} M^{-*}$$

if the initial polarized state is described by a matrix which is a linear combination of the σ_y and σ_z Pauli matrices.

This corresponds to initial photon polarization of Bohr-antisymmetric type, and therefore to experiments with circularly (giving the z component) or linearly-polarized photons (giving the y component). Thus observables $E^z_{\lambda_1 \lambda_1'}$, $E^y_{\lambda_1 \lambda_1'}$ and $O^z_{\lambda_1 \lambda_1'}$, $O^y_{\lambda_1 \lambda_1'}$ are built up with naturality interferences. In contrast with the situations studied previously, where the resonance, particle 1, is associated with the same vertex as the photon, the crucial point is that now the quantities

$$\mathrm{Im}\ E^{z,y}_{\lambda_1 \lambda_1'} \equiv \mathrm{Im}\ \rho^{z,y}_{\lambda_1 \lambda_1'}$$

$$\mathrm{Re}\ O^{z,y}_{\lambda_1 \lambda_1'} \equiv \mathrm{Re}\ \rho^{z,y}_{\lambda_1 \lambda_1'}$$

are non-vanishing (cf., Tables 4 and 5) and remain measurable for Class 1 reactions. In other words, Bohr antisymmetric initial polarization of the photon and observation of the spin of a particle at the other vertex allow measurement of naturality interferences even in Class 1 reactions.

This is illustrated by the following formulae:

$$\rho^{y}_{\lambda_1 \lambda_1'} = -\frac{i}{2N} \sum_{\mu \lambda_b} \left[M^{+}_{\mu \lambda_1 1 \lambda_b} M^{-*}_{\mu \lambda_1' -1 \lambda_b} + M^{-}_{\mu \lambda_1 1 \lambda_b} M^{+*}_{\mu \lambda_1' -1 \lambda_b} \right] \tag{5.33a}$$

$$\rho^{z}_{\lambda_1 \lambda_1'} = \frac{1}{2N} \sum_{\mu \lambda_b} \left[M^{+}_{\mu \lambda_1 1 \lambda_b} M^{-*}_{\mu \lambda_1' 1 \lambda_b} + M^{-}_{\mu \lambda_1 1 \lambda_b} M^{+*}_{\mu \lambda_1' 1 \lambda_b} \right] \tag{5.33b}$$

In contrast to an experiment with a polarized proton target (see Section 4.4), real <u>and</u> imaginary parts of interferences are available now from even <u>or</u> odd polarization measurements. However, linearly and circularly polarized photons are both required.

Let us consider, for example, the reaction $\gamma\!\uparrow N \to \pi\Delta$. In the forward direction the pseudoscalar character of the π allows the disentanglement of contributions of opposite naturalities from B polarized photons. But, in addition, the Δ density matrix elements give information on interferences for initial BA polarized photons.

Turning now to the more complicated reaction $\gamma\uparrow N \to K^*Y^*$ and without entering into fastidious details, we remark that

i) In the forward direction the K^* decay measurement allows separation of exchanged naturalities from B polarized photons, whereas the Y^* density matrix elements give interferences for BA polarized photons.

ii) In the backward direction, the Y^* density matrix elements measure now different naturality components, and K^* density matrix elements (with BA photon polarization) give information on interference contributions between the various baryonic exchanged states.

5.5 Applications

5.5.1 Pseudoscalar meson photoproduction: $\gamma\uparrow + b \to (\text{spin zero}) + 2 + \ldots$

The most interesting and widely used result for forward photo-produced mesons, is the extension of a theorem obtained long ago by Stichel [3]) for $\gamma N \to \pi N$, which states that natural or unnatural parity contributions are just given respectively by the perpendicular or parallel cross-sections. Such a result is contained in Eq. (5.10) and (5.8):

$$\frac{d\sigma^+}{dt} = (E^0_{00} + E^x_{00}) \frac{d\sigma^0}{dt} \tag{5.34}$$
$$= \frac{d\sigma^\perp}{dt}$$

$$\frac{d\sigma^-}{dt} = (E^0_{00} - E^x_{00}) \frac{d\sigma^0}{dt} \tag{5.35}$$
$$= \frac{d\sigma^{/\!/}}{dt}$$

(For a photoproduced scalar meson, such as $\gamma p \to \sigma + \ldots$, the + and - signs in the preceding relations should be exchanged.)

For Class 2 reactions, both the longitudinal component of the polarization

$$E^\gamma_{00} \equiv \frac{d\sigma^\gamma}{dt} \Big/ \frac{d\sigma^0}{dt} \tag{5.36}$$

and the perpendicular one:

$$E_{\infty}^{y} \equiv \frac{d\sigma^{y}}{dt} \Big/ \frac{d\sigma^{0}}{dt} \tag{5.37}$$

are non-vanishing. Thus experiments with linearly polarized photons provide in this case a measure of the imaginary parts of opposite naturality interferences, whereas circularly polarized beams allow one to obtain real parts of these interferences. More precisely formulae (5.18), (5.19) can be rewritten as

$$\frac{d\sigma^{\gamma}}{dt} = \frac{1}{\mathcal{N}} \operatorname{Re}\left(M^{+} \; M^{-*} \right) \tag{5.38}$$

$$\frac{d\sigma^{y}}{dt} = -\frac{1}{\mathcal{N}} \operatorname{Im}\left(M^{+} \; M^{-*} \right) \tag{5.39}$$

For Class 1 reactions these quantities vanish and no interference terms can be obtained without additional information on another produced particle. This possibility was emphasized in the last section. To illustrate it, let us consider in some detail the two photoproduction reactions $\gamma N \to K \Lambda$ and $\gamma N \to \pi \Delta$, at high energy and near the forward direction.

Whereas Stichel's theorem gives the natural and unnatural parity components of the cross-sections from linearly polarized photons without any spin measurement in the final states, the observation of the produced baryon disintegration allows one to determine a certain amount of naturality interferences from linearly or circularly polarized photons.

Applying our former results, Eqs. (5.33a), (5.33b) to the above mentioned reactions and, using a shorthand notation where the photon helicity is fixed to one, we obtain the following:

i) <u>For $\gamma N \to K \Lambda$</u>

For linearly polarized photons:

$$\rho^{y}_{\frac{1}{2}\frac{1}{2}} = - \operatorname{Im} \Sigma \left(M^{+}_{\frac{1}{2}} \; M^{-*}_{\frac{1}{2}} \right) \tag{5.40}$$

$$\rho^{\gamma}_{1/2\,-1/2} = -\,\mathrm{Im}\,\Sigma\,(\,M^{+}_{1/2}\;\;M^{-*}_{-1/2})$$ (5.41)

and for circularly polarized photons:

$$\rho^{\gamma}_{1/2\,1/2} = \mathrm{Re}\,\Sigma\,(\,M^{+}_{1/2}\;\;M^{-*}_{1/2}\,)$$ (5.42)

$$\rho^{\gamma}_{1/2\,-1/2} = \mathrm{Re}\,\Sigma\,(\,M^{+}_{-1/2}\;\;M^{-*}_{-1/2}\,)$$ (5.43)

($\rho_{\frac{1}{2}\frac{1}{2}}$ and $\rho_{\frac{1}{2}-\frac{1}{2}}$ are respectively proportional to the z and x components of the Λ polarization, Table A.3.)

ii) <u>For $\gamma N \to \pi\Delta$</u>

One can measure only the even parts of the polarization. From Table A.4 one immediately deduces that the only non-vanishing measurable density matrix elements, for Bohr-antisymmetric initial polarization are Im ρ_{31} and Im ρ_{3-1}, given by

$$\mathrm{Im}\,\rho^{\gamma}_{3/2\,\pm1/2} = \frac{1}{2}\,\mathrm{Re}\,\Sigma\left(M^{+}_{3/2}\,M^{-*}_{\pm1/2} - M^{-}_{3/2}\,M^{+*}_{\pm1/2}\,\right)$$ (5.44)

for linearly polarized photons, and by

$$\mathrm{Im}\,\rho^{\gamma}_{3/2\,\pm1/2} = \frac{1}{2}\,\mathrm{Im}\,\Sigma\left(M^{+}_{3/2}\,M^{-*}_{\pm1/2} + M^{-}_{3/2}\,M^{+*}_{\pm1/2}\,\right)$$ (5.45)

for circularly polarized photons.

5.5.2 Photoproduction of spin $\frac{1}{2}$ baryon: $\gamma\uparrow + b \rightarrow (\text{spin } \frac{1}{2}) + 2 + \ldots$

We now turn to a discussion of polarized photoproduction reactions of a spin $\frac{1}{2}$ baryon (with intrinsic parity η). For small momentum transfer between the produced baryon and the photon beam, the interesting combinations of observables quantities can be found in Section 5.5. Those which, in the present case, allow a separation between the two exchanged baryonic naturalities are given in Table 11. It should be noticed that all relevant quantities involve measurement of the polarization of the final baryon. Thus separation into the natural and unnatural parity contributions is only possible when density matrix elements, of the final baryon, are measured in a parity-violating decay or obviously in a rescattering experiment because then one can obtain $\mathrm{Im}\,\rho^{B}_{\frac{1}{2}-\frac{1}{2}}$ and $\mathrm{Re}\,\rho^{BA}_{\frac{1}{2}-\frac{1}{2}}$ (see Tables 11 and A.3). Therefore, processes like $\gamma\uparrow N \rightarrow \Lambda(\Sigma) + \ldots$ are particularly valuable for separating naturality contributions in the backward direction. In fact from the definitions (4.30) as well as from the observable properties (4.15) it is easily seen that formula (5.29) gives a non-zero result only for half-integer spins greater than $\frac{1}{2}$. The situation is analogous to the $a + p\uparrow \rightarrow \frac{1}{2} + \ldots$ case.

A particularly relevant quantity is obtained from the second line of Table 11. Using the normalization condition, $E^{x}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{2}\,(d\sigma^{x}/dt)/(d\sigma^{0}/dt)$, it is easy to show that

$$4\,\mathrm{Im}\,D^{\pm}_{1/2\,-1/2} = 2\,P_y^{\Lambda(\Sigma)}\frac{d\sigma^{0}}{dt} \mp \eta\left(\frac{d\sigma^{\perp}}{dt} - \frac{d\sigma^{\parallel}}{dt}\right)$$

Observe that this quantity can be measured from two independent experiments, one measurement of the decay of the final baryon produced off an unpolarized beam and one measurement of cross-section with linearly polarized photons. Thus valuable information on the relative phases of the same naturality conserving amplitudes can be obtained without spin correlation measurements between the produced baryon and the photon beam.

Moreover, we must emphasize that information on interferences, for Class 2 reactions, can be obtained from experiments with linearly polarized photon beams and measurement only of the even part of the final polarization. In that case relations (5.32a), (5.32b) are trivially zero, but from (5.30) we deduce

$$E^{y}_{1/2\,1/2} = \frac{1}{2N}\,\eta\,\mathrm{Re}\left(M^{+}_{1/2}\,M^{-\,*}_{-1/2} - M^{+}_{-1/2}\,M^{-\,*}_{1/2}\right) \tag{5.46}$$

Circularly polarized photon beams provide other information on these interference contributions. Using Eq. (5.30) again we obtain

$$\mathrm{Re}\,E^{z}_{1/2\,1/2} = \frac{1}{2N}\sum\left(M^{+}_{1/2}\,M^{-\,*}_{1/2} + M^{+}_{-1/2}\,M^{-\,*}_{-1/2}\right) \tag{5.47}$$

5.5.3 Photoproduction of spin 1 meson: $\gamma\uparrow + b \rightarrow (\text{spin } 1) + 2 + \dots$

The particular, but important, case of vector meson photoproduction of nucleons, $\gamma N \rightarrow VN$, has been studied in detail by Schilling, Seyboth and Wolf [5]. Our notation is quite similar to that used in this paper (except that their parallel and perpendicular cross-sections are defined with respect to the normal to the scattering plane) and all their results concerning the separation of measurable quantities into naturality components are contained in our relations (5.10), (5.11), (5.16) and (5.17). More generally we give in Table 12 the explicit observable combinations isolating a given naturality $\sigma = \mp\eta_1$ (η_1 being the intrinsic parity of the spin 1 final particle) exchanged in the cross-channel $\gamma\uparrow + \bar{1} \rightarrow \bar{b} + \dots$ Only those quantities which are above the dashed line in Table 12 can be measured in a parity-conserving decay of the spin 1 particle. Explicit expressions in terms of naturality-conserving amplitudes can be easily obtained in the s channel reference frame from Eqs. (4.30) and (4.35).

As discussed in Sections 5.2 and 5.4 interference contributions between mesonic exchanged states can be obtained for forward production in Class 2 reactions from parity-conserving decay and linearly polarized photons, and between baryonic exchanged states for backward production.

5.5.4 Photoproduction of spin $\frac{3}{2}$ baryon: $\gamma\uparrow + b \rightarrow (\text{spin } \frac{3}{2}) + \dots$

From Eqs. (5.22) to (5.31) one can easily deduce the 16 combinations of observable quantities which allow a separation between natural and unnatural parity exchanges between the polarized photon and the spin $\frac{3}{2}$ particle. They all involve measurement of odd polarizations except the two following ones:

$$\mathrm{Im}\, E^{\gamma}_{31} \pm \eta_1\, \mathrm{Im}\, E^{\gamma}_{3-1}$$

$$\mathrm{Im}\, E^{\gamma}_{3-1} \mp \eta_1\, \mathrm{Im}\, E^{\gamma}_{31} \qquad (5.48)$$

(η_1 = parity of the spin $\frac{3}{2}$ particle)

which in any case require circularly and linearly polarized photons. This means that very little can be said in a model-independent way for backward Δ photoproduction. On the other hand, there are eight combinations of observable quantities obtained with linearly-polarized photons which give a separation of naturality contributions when the odd part of the spin $\frac{3}{2}$ particle polarization can be measured. As in the previous cases it is, however, possible to obtain, via the trace condition:

$$2\left(E^{i}_{11} + E^{i}_{33} \right) = \frac{d\sigma^{i}}{dt} \Big/ \frac{d\sigma}{dt}$$

and Eq. (5.23) a naturality separation which does not require spin correlation measurement:

$$\mathrm{Im}\left(D^{\pm}_{1-1} - D^{\pm}_{3-3}\right) = \frac{d\sigma^{0}}{dt}\,\mathrm{Im}\left(O^{0}_{1-1} - O^{0}_{3-3}\right) \mp \eta_1\, \frac{1}{4}\left(\frac{d\sigma^{\perp}}{dt} - \frac{d\sigma^{\parallel}}{dt}\right)$$

In that case one needs two different experiments measuring the polarization of one particle only (odd polarization of the baryon produced off an unpolarized photon beam, and cross-sections with a linearly polarized initial photon beam), thus avoiding the difficulties of spin correlation measurement.

Finally interferences can be obtained from linearly polarized photons and parity-conserving decay, for Class 2 reactions, by measuring $\mathrm{Im}\, E_{31}$ and $\mathrm{Im}\, E_{3-1}$.

6. POLARIZED BEAM AND TARGET REACTIONS: $a\uparrow + b\uparrow \rightarrow 1 + \ldots$

We consider in this section polarization measurement of a final state particle when both the initial beam and target are polarized. We shall not pursue a detailed study of this case and shall restrict ourselves to a straightforward extension of the results obtained in the preceding sections. The main change in our formulation, when a second initial particle is polarized, occurs in new symmetry properties of observable quantities. Let us define new density matrix elements $\rho^{(i,j)}_{mm'}$, where the two upper indices i and j refer, respectively, to the polarization direction of the beam and the target. As before the two lower indices characterize the spin state of the final particle (particle 1).

Similarly, we define observable $E^{(i,j)}_{mm'}$ and $O^{(i,j)}_{mm'}$ quantities, which enjoy exactly the properties given by Eqs. (3.4) and (3.5) of the E's and O's. Furthermore, their symmetry properties with respect to reflection in the scattering plane are given by Eqs. (4.23) to (4.25) when the second initial particle polarization is Bohr symmetric (i.e., $E^{(B,B)}$, $E^{(BA,B)}$ transform as E^{B}, E^{BA} respectively) and an over-all minus sign should be introduced when the second particle polarization is Bohr antisymmetric (i.e., $E^{(B,BA)}$, $E^{(BA,BA)}$ transform as E^{BA}, E^{B}). From this simple rule it is quite easy to deduce the symmetry properties displayed in Table 5 of these new observables. For instance $\mathrm{Im}\, E^{(B,B)}$ is antisymmetric with respect to reflection in the scattering plane and vanishes in Class 1 reactions. $\mathrm{Im}\, E^{(B,BA)}$ is symmetric and therefore can be measured in two-body or inclusive reactions. More generally all formal observable quantities which were antisymmetric, and therefore vanished by parity conservation in Class 1 reactions when the second initial particle was unpolarized, become now measurable and non-vanishing when the polarization of the second particle is BA. One readily understands what new information can be deduced from these triple correlations, in particular with respect to the measurement of interference contributions. On the other hand, separation into naturality components being a vertex property, all combinations of observables given in this paper which separate the natural and unnatural parity exchanges are independent of the polarization state of one particle at the other vertex, and in this respect are unchanged. Of course, they are related to new subsets of naturality-conserving amplitudes.

Let us consider as an illustration the reaction

$$p\uparrow + p\uparrow \rightarrow 1 + \cdots$$

assuming that particle 1 is a baryon produced with small momentum transfer to the proton beam. We give in Table 13 the properties of observable quantities in that case, as well as their dependence on exchanged naturalities. By comparing this table with Table 6, one observes for instance that the combinations $E^{o}_{\tau\tau'} \pm O^{z}_{\tau\tau'}$ always separate naturality contributions for $\tau - \tau'$ even and interferences for $\tau - \tau'$ odd. However, the symmetry properties with respect to reflection depend on the polarization character of the target and for two-body scattering (for Class 1) only the symmetric quantities survive which allow one, in any case, to compute both interference contributions using $(E^{(0,BA)}_{\tau\tau'} \pm O^{(z,BA)}_{\tau\tau'})$ and contributions of given naturality using $(E^{(0,B)}_{\tau\tau'} \pm O^{(z,B)}_{\tau\tau'})$.

These results can be applied as well to the reaction $p\bar{p} \to \Lambda\bar{\Lambda}$ [19] with now unpolarized initial state but measurement of the Λ and $\bar{\Lambda}$ polarization. The final density matrix can be expanded as follows:

$$\rho = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + \vec{P}_1 \vec{\sigma}^1 \otimes \mathbb{1} + \vec{P}_2 \mathbb{1} \otimes \vec{\sigma}^2 + \sum_{ij} C_{ij}\, \sigma_i^1 \otimes \sigma_j^2\right) \quad (6.1)$$

Polarizations of final state Λ's are given by

$$P_{1i} = \frac{1}{4}\,\mathrm{Tr}\left(\rho\, \sigma_i^1 \otimes \mathbb{1}\right)$$
$$P_{2j} = \frac{1}{4}\,\mathrm{Tr}\left(\rho\, \mathbb{1} \otimes \sigma_j^2\right) \quad (6.2)$$

and polarization correlations by

$$C_{ij} = \frac{1}{4}\,\mathrm{Tr}\left(\rho\, \sigma_i^1 \otimes \sigma_j^2\right) \quad (6.3)$$

The reaction $p\bar{p} \to \Lambda\bar{\Lambda}$ being a Class 1 reaction only symmetric quantities with respect to reflection in the scattering plane are non-vanishing, which means that they are of the (B,B) or (BA,BA) type. In transversity quantization, it is easy to see that the only symmetric quantities are

$$P_{1z},\ P_{2z},\ C_{zz},\ C_{xx},\ C_{yy},\ C_{yx}$$

The density matrix ρ takes the following structure:

$$\rho = \frac{1}{4}\begin{pmatrix} 1+P_{1z}+P_{2z}+C_{zz} & 0 & 0 & C_{xx}-C_{yy}-iC_{xy}-iC_{yx} \\ 0 & 1+P_{1z}-P_{2z}-C_{zz} & C_{xx}+C_{yy}+iC_{xy}-iC_{yx} & 0 \\ 0 & C_{xx}+C_{yy}-iC_{xy}+iC_{yx} & 1-P_{1z}+P_{2z}-C_{zz} & 0 \\ C_{xx}-C_{yy}+iC_{xy}+iC_{yx} & 0 & 0 & 1-P_{1z}-P_{2z}+C_{zz} \end{pmatrix}$$

which simplifies by charge conjugation invariance $(P_{1z} = P_{2z},\ C_{xy} = C_{yx})$.

Finally using Table 13 it is easy to be convinced that all elements of the principal diagonal can be expressed in terms of incoherent sums over both exchanged naturalities, though all elements of the other diagonal measure interference contributions only.

As a second example consider forward meson photoproduction on a polarized target with a linearly polarized proton beam [20]:

$$\gamma\uparrow + p\uparrow \longrightarrow M + \ldots$$

The relevant expressions in that case are given by Eqs. (5.9) to (5.12) and (5.14) to (5.18). Despite the fact that $\mathrm{Im}\, E_{\lambda\lambda'}$ vanishes for two-body reactions, the interference term:

$$\mathrm{Im}\, D^{I}_{\lambda\lambda'} = \mathrm{Im}\left(E^{(0,x)}_{\lambda\lambda'} - \varepsilon(\lambda')\, E^{(x,x)}_{\lambda\lambda'} \right) \tag{6.4}$$

is measurable in two-body reactions due to the Bohr antisymmetric character of the proton polarization along the x direction which makes Eq. (6.4) finally symmetric with respect to reflection in the scattering plane.

As a final comment, let us remark that when no polarization is measured in the final state, there are four different types of polarized cross-sections which can be measured: $\sigma^{(B,B)}$, $\sigma^{(BA,BA)}$, $\sigma^{(B,BA)}$ and $\sigma^{(BA,B)}$. It can be shown that $\sigma^{(B,B)}$ can be expressed in terms of incoherent sums over both naturalities, through $\sigma^{(BA,BA)}$ is given by interference terms only. Of course $\sigma^{(B,BA)}$ and $\sigma^{(BA,B)}$ vanish for Class 1 reactions.

7. G PARITY RELATIONS

Constraints due to G parity conservation at a vertex with two identical particles have already been investigated in the general case by a number of authors [1),8)]. For completeness we just very briefly give some general results and some specific applications.

i) G parity symmetry implies for s channel helicity amplitudes the following relation:

$$M_{\lambda_1\{\lambda_i\}\lambda_a\lambda_b} \simeq g\,\eta\,(-)^{I}\,(-)^{\lambda_a-\lambda_1}\,M_{\lambda_a\{\lambda_i\}\lambda_1\lambda_b} \tag{7.1}$$

if particles a and 1 are of the same species or belong to the same baryonic SU(3) multiplet. In that case the relation is only true in the SU(3) limit, that is $m_a - m_1 \to 0$. We recall that equality (7.1) is not exact because crossing introduces an unavoidable approximation at the order 1/s.

ii) In the transversity system, from relation (2.11) and the property

$$D^{s}(R^{*})^{\lambda}{}_{\tau} = (-)^{\lambda-\tau}\,D^{s}(R)^{\lambda}{}_{\tau} \tag{7.2}$$

one can derive for transversity amplitudes, relations strictly analogous to (7.1):

$$T_{\tau_1\{\tau_i\}\tau_a\tau_b} \simeq g\,\eta\,(-)^{I}\,(-)^{\tau_a-\tau_1}\,T_{\tau_a\{\tau_i\}\tau_1\tau_b} \tag{7.3}$$

If particles a and 1 belong to the $\frac{1}{2}^{+}$ baryon octet, combining Eq. (7.3) with (2.15) we obtain the well-known result that only three types of meson trajectories can be exchanged at a particle-antiparticle vertex, following the values of the naturality $\sigma = \eta\,(-)^{J}$ and the natural charge parity: $\mathcal{C} = C(-)^{J}$

i) Natural parity σ and natural charge parity $\mathcal{C} = +1$ $[C = \eta = (-)^J]$ contribute only to amplitudes $T_{\tau\,\{\tau_i\}\,\tau\,\tau_b}$ with $\tau = \pm\frac{1}{2}$.

ii) Unnatural parity and natural charge parity $\mathcal{C} = 1$ $[C = -\eta = (-)^J]$ correspond to the following combinations:

$$T_{\tau\{\tau_i\}-\tau\,\tau_b} - T_{-\tau\{\tau_i\}\tau\,\tau_b}$$

whereas unnatural parity, unnatural charge parity $\mathcal{C} = -1$ $[C = \eta = -(-)^J]$ dominate the other combination:

$$T_{\tau\{\tau_i\}-\tau\,\tau_b} + T_{-\tau\{\tau_i\}\tau\,\tau_b}$$

Thus further interesting dynamical information may be collected in reactions where G parity symmetry applies or is believed to approximatively work from the SU(3) symmetry classification. Again it is simpler to work in the transversity frame.

In order to understand what type of analysis may be performed in such cases we examine now three examples: $\pi N\uparrow \to VN$ (with a polarized target and measurement of the V decay), $\pi N \to K^*\uparrow \Lambda\uparrow$ (or $KN \to V\uparrow\Lambda\uparrow$, observation of the decay angular correlations of the particles produced by unpolarized initial particles) and $\pi N\uparrow \to \Lambda X$ (inclusive reactions on polarized targets with a measure of the non-conserving parity decay in the final state).

a) $\underline{\pi N\uparrow \to VN}$

This process is described by a set of six independent transversity amplitudes (T_{0++}, T_{0--}, $T_{\pm1+-}$, $T_{\pm1-+}$). The V decay conserving parity allows the measurement of ten independent real quantities when the target proton is polarized perpendicularly and along the beam (cf., Table A.1); they are easily calculated from Eqs. (4.32) to (4.35) for instance:

$$E_{00}^{0\pm z} = |T_{0\pm\pm}|^2$$

$$E_{11}^{0\pm z} = \frac{1}{2}\left(|T_{1\mp\pm}|^2 + |T_{-1\mp\pm}|^2\right)$$

$$E_{1-1}^{0\pm z} = T_{1\mp\pm}\, T_{-1\mp\pm}^{*}$$

$$E_{10}^{x\pm iy} = \frac{1}{2}\left(T_{1\mp\pm}\, T_{0\mp\mp}^{*} - T_{0\pm\pm}\, T_{-1\pm\mp}^{*}\right)$$

Their properties with respect to naturality and charge naturality are presented in Table 14.

b) $\pi N \to K^{*\uparrow} \Lambda^{\uparrow}$

The observation of the decay correlations allows the determination of ten out of the twelve independent parameters necessary to describe these reactions. These parameters have been analyzed by Abramovich et al. [21] and by Field et al. [22]. Our purpose is not to discuss their detailed amplitude analysis, but we want only to recall in this example what type of information may be easily deduced from correlation experiments.

In transversity frame, the twelve measurable elements are the four quantities $\rho_{\pm\pm}^{00}$, $(\rho_{\pm\pm}^{11} + \rho_{\pm\pm}^{-1-1})$, the complex elements $\rho_{\pm\pm}^{-11}$ and the complex combinations $(\rho_{\pm\mp}^{-10} - \rho_{\mp\pm}^{10*})$. We see from Eq. (2.15) that $\rho_{\pm\pm}^{00}$ measures the contribution of the natural parity component exchanged in the crossed channel $\pi \bar{K}^* \to \bar{N} \Lambda$, whereas $(\rho_{\pm\pm}^{11} + \rho_{\pm\pm}^{-1-1})$ and $\rho_{\pm\pm}^{-11}$ are the unnatural components and that $(\rho_{\pm\mp}^{-10} - \rho_{\mp\pm}^{10*})$ gives interferences of these components.

Considering now properties with respect to charge parity exchanges, it is easy to conclude from relation (7.3) that the linear combinations $\rho_{++}^{11} + \rho_{++}^{-1-1} - \rho_{--}^{11} - \rho_{--}^{-1-1}$ and $\rho_{++}^{-11} - \rho_{--}^{-11}$ measure charge parity interferences, whereas $\rho_{++}^{11} + \rho_{++}^{-1-1} + \rho_{--}^{11} + \rho_{--}^{-1-1} \equiv \rho^{11} + \rho^{-1-1}$ and $\rho_{++}^{-11} + \rho_{--}^{-11} \equiv \rho^{-11}$ incoherent sums.

c) $\pi N^{\uparrow} \to \Lambda X$

The generalization of the Wolfenstein parameters [23], to inclusive reactions of the type spin $\frac{1}{2}$ + unpolarized $\to$ spin $\frac{1}{2}$ + anything, have been discussed by Doncel and Mendez [24]. Their seven parameters which constitute, with the differential cross-section, the whole experimental information on these reactions, are related to our density matrix elements (cf., Table A.3) in the transversity system by:

$$
\begin{aligned}
R &= 2\,\mathrm{Re}\, O^{x+iy}_{1/2\,-1/2} \\
A &= 2\,\mathrm{Im}\, O^{x+iy}_{1/2\,-1/2} \\
P &= O^{0}_{1/2\,1/2} + E^{\gamma}_{1/2\,1/2} \\
R' &= 2\,\mathrm{Re}\, O^{x-iy}_{1/2\,-1/2} \\
A' &= -2\,\mathrm{Im}\, O^{x-iy}_{1/2\,-1/2} \\
P' &= O^{0}_{1/2\,1/2} - E^{\gamma}_{1/2\,1/2} \\
D &= 2\, O^{\gamma}_{1/2\,1/2}
\end{aligned}
\qquad (7.4)
$$

In the crossed channel $\bar{\Lambda} N^{\uparrow} \to \pi X$, it is straightforward to deduce from Table 8 that R, A, P, receive contributions from natural parity exchanges only, R', A', P' from unnatural parity exchanges and D is an incoherent sum upon both naturalities. Moreover, to leading order in 1/s and in the SU(3) limit ($m_N \sim m_\Lambda$), it is easy to demonstrate from Eq. (7.3) that R' and D are incoherent sums of charge parity, whereas A' and P' are interferences of natural and unnatural charge parity contributions. Finally, R, A, P correspond both to natural parity exchanges and to natural charge parity contributions. These results are summed up in Table 15.

It is well known that positivity of the transition matrix implies severe restrictions on observables. The positivity domain of these parameters is explicitly discussed in Ref. 24). For convenience we translate their results on our parameters

$$
\left| O^{0}_{1/2\,1/2} + E^{\gamma}_{1/2\,1/2} \right| - 1 \leq 2\, O^{\gamma}_{1/2\,1/2} \leq 1 - \left| O^{0}_{1/2\,1/2} + E^{\gamma}_{1/2\,1/2} \right|
$$

$$
\left| O^{0}_{1/2\,1/2} + E^{\gamma}_{1/2\,1/2} \right|^2 + 4 \left| O^{x+iy}_{1/2\,-1/2} \right|^2 \leq \frac{1}{4} \left(1 + 2\, O^{\gamma}_{1/2\,1/2}\right)^2
$$

$$
\left| O^{0}_{1/2\,1/2} - E^{\gamma}_{1/2\,1/2} \right|^2 + 4 \left| O^{x-iy}_{1/2\,-1/2} \right|^2 \leq \frac{1}{4} \left(1 - 2\, O^{\gamma}_{1/2\,1/2}\right)^2
$$

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APPENDIX

PROPERTIES OF OBSERVABLE QUANTITIES IN POLARIZATION MEASUREMENTS OF SPIN 1, 2, $\frac{1}{2}$ AND $\frac{3}{2}$ PARTICLES

For the sake of completeness and clarity we give in this Appendix a resumé of some general properties of observable quantities in polarization measurements of spin 1, 2, $\frac{1}{2}$ and $\frac{3}{2}$ particles. For each spin case these properties are collected in a Table (Tables A.1, A.2, A.3, and A.4, respectively). The first column in these tables gives a list of all independent measurable quantities $E_{mm'}$ for even polarization and $O_{mm'}$ for odd polarization. The second and third columns give, respectively, the relation between the E's and O's and the density matrix elements $\rho_{mm'}$ or the multipolar parameters t^L_M according to definitions (3.2), (3.3) and relation (3.1).

Other non-independent quantities can be easily obtained from properties (3.4) and (3.5). Our conventions and definitions are those of Doncel, Michel and Minnaert [14] and actually part of the properties displayed in the following tables overlaps with results which can be found in that work which we strongly recommend to the reader.

The remaining columns of our tables give the reality properties of observables, real, pure imaginary, complex or null, and their symmetry character with respect to reflection in the scattering plane, S for symmetric, AS for antisymmetric. When two symbols are enclosed in brackets, the first one refers to the real part and the second to the imaginary part.

We have given separately the properties in the most general case and in the special two-body or inclusive case (Class 1), this former case being deduced from the previous one by retaining only the symmetric elements with respect to reflection. Furthermore one must also consider separately the two possible choices of quantization system, helicity (H) or transversity (T), and finally when the initial state is polarized (or more generally when another particle polarization is measured) one should determine whether the polarization is Bohr symmetric (B) or Bohr antisymmetric (BA). For unpolarized initial state (or more generally for single polarization measurement) only columns labelled HB or TB have to be considered.

TABLE A.1

Observables	Density matrix elements	Multipolar parameters	General case				Two-body or inclusive reactions Class 1			
			H-B	H-BA	T-B	T-BA	H-B	H-BA	T-B	T-BA
E_{00}	ρ_{00}	$\frac{1}{3}(t_o^o - 2\sqrt{\frac{5}{2}}\, t_o^2)$	real S	real AS	real S	real AS	real	0	real	0
E_{11}	$\frac{1}{2}(\rho_{11} + \rho_{-1-1})$	$\frac{1}{3}(t_o^o + \sqrt{\frac{5}{2}}\, t_o^2)$	real S	real AS	real S	real AS	real	0	real	0
E_{1-1}	ρ_{1-1}	$\sqrt{\frac{5}{3}}\, t_2^{2*}$	complex (S,AS)	complex (AS,S)	complex (S,S)	complex (AS,AS)	real	imag.	complex	0
E_{10}	$\frac{1}{2}(\rho_{10} - \rho_{0-1})$	$-\sqrt{\frac{5}{6}}\, t_1^{2*}$	complex (S,AS)	complex (AS,S)	complex (AS,AS)	complex (S, S)	real	imag.	0	complex
O_{00}	0									
O_{11}	$\frac{1}{2}(\rho_{11} - \rho_{-1-1})$		real AS	real S	real S	real AS	0	real	real	0
O_{1-1}	0									
O_{10}	$\frac{1}{2}(\rho_{10} + \rho_{0-1})$		complex (AS, S)	complex (S, AS)	complex (AS,AS)	complex (S,S)	imag.	real	0	complex

Measurement of the polarization of a spin 1 particle

TABLE A.1 (Continued)

Remarks:

i) Trace condition: $t_0^0 = 1$; $E_{00} + 2E_{11} = 1$ (unpolarized initial state)

ii) Number of independent observable real quantities (trace condition not included)

	General case	B symmetric two-body	B antisymmetric two-body
Even part of the polarization	6	4	2
Even and odd measurement	9	5	4

iii) For parity-conserving decay: $1 \to 0 + 0$, the decay angular distribution is given by:

$$W_1(\theta,\varphi) = \frac{3}{4\pi}\left\{E_{00}\cos^2\theta + E_{11}\sin^2\theta - \sin^2\theta \,\mathrm{Re}\left(E_{1-1}e^{2i\varphi}\right) - \sqrt{2}\sin 2\theta \,\mathrm{Re}\left(E_{10}e^{i\varphi}\right)\right\}$$

This formula applies also to the decay $1^- \to 0^- + 0^- + 0^-$, where now (θ, φ) are the polar angles of the normal of the decay plane.

iv) For the decay $1^+ \to 0^- + 0^- + 0^-$, the distribution is [17),25)]

$$W_{1^+}(\theta,\varphi) = \frac{3}{8\pi}\Big\{(1+\cos^2\theta)E_{11} + \sin^2\theta E_{00} + \sin^2\theta \,\mathrm{Re}\left(E_{1-1}e^{2i\varphi}\right) + \sqrt{2}\sin 2\theta \,\mathrm{Re}\left(E_{10}e^{i\varphi}\right) + 2\lambda\left[\cos\theta\, O_{11} + \sqrt{2}\sin\theta \,\mathrm{Re}\left(O_{10}e^{i\varphi}\right)\right]\Big\}$$

where the real parameter λ depends on the dynamics of the decay and vanishes if two of the mesons are identical.

v) $1^+ \to 1^- + 0^-$

We recall that both S and D waves are involved in this decay process. Thus it is not possible to obtain the even part of observables without information on the relative strength of these two waves. Moreover the odd part is never accessible for strong decay. However, in a two-step decay [26)] allowing the measurement of the 1^- polarization as, for instance $A_1 \to \underbrace{\rho\pi}_{\hookrightarrow 2\pi}$, $B \to \underbrace{\omega\pi}_{\hookrightarrow 3\pi}$ or $Q \to \underbrace{K^*\pi}_{\hookrightarrow K\pi}$, the even observables are determined without ambiguity whereas the odd ones may be in principle evaluated up to a sign unless the 1^+ has a pure pole behaviour.

TABLE A.2

Observables	Density matrix elements	Multipolar parameters	General case				Two-body or inclusive reactions Class 1			
			H-B	H-BA	T-B	T-BA	H-B	H-BA	T-B	T-BA
E_{00}	ρ_{00}	$\frac{1}{5}t_0^0-\sqrt{\frac{2}{7}}\,t_0^2+\frac{18}{5}\sqrt{\frac{1}{14}}\,t_0^4$	real S	real AS	real S	real AS	real	0	real	0
E_{11}	$\frac{1}{2}(\rho_{11}+\rho_{-1-1})$	$\frac{1}{5}t_0^0-\sqrt{\frac{1}{14}}\,t_0^2-\frac{3}{5}\sqrt{\frac{8}{7}}\,t_0^4$	real S	real AS	real S	real AS	real	0	real	0
E_{22}	$\frac{1}{2}(\rho_{22}+\rho_{-2-2})$	$\frac{1}{5}t_0^0+\sqrt{\frac{2}{7}}\,t_0^2+\frac{3}{5}\sqrt{\frac{7}{14}}\,t_0^4$	real S	real AS	real S	real AS	real	0	real	0
E_{21}	$\frac{1}{2}(\rho_{21}-\rho_{-1-2})$	$-\sqrt{\frac{3}{7}}\,t_1^{2*}-\sqrt{\frac{9}{70}}\,t_1^{4*}$	complex (S,AS)	complex (AS,S)	complex (AS,AS)	complex (S,S)	real	imag.	0	complex
E_{10}	$\frac{1}{2}(\rho_{10}-\rho_{0-1})$	$-\sqrt{\frac{1}{14}}\,t_1^{2*}+\sqrt{\frac{27}{35}}\,t_1^{4*}$	complex (S,AS)	complex (AS,S)	complex (AS,AS)	complex (S,S)	real	imag.	0	complex
E_{20}	$\frac{1}{2}(\rho_{20}+\rho_{0-2})$	$\sqrt{\frac{2}{7}}\,t_2^{2*}+\frac{3}{5}\sqrt{\frac{15}{4}}\,t_2^{4*}$	complex (S,AS)	complex (AS,S)	complex (S,S)	complex (AS,AS)	real	imag.	complex	0
E_{1-1}	ρ_{1-1}	$\sqrt{\frac{3}{7}}\,t_2^{2*}-\frac{6}{5}\sqrt{\frac{5}{7}}\,t_2^{4*}$	complex (S,AS)	complex (AS,S)	complex (S,S)	complex (AS,AS)	real	imag.	complex	0
E_{2-1}	$\frac{1}{2}(\rho_{2-1}-\rho_{1-2})$	$-\sqrt{\frac{9}{10}}\,t_3^{4*}$	complex (S,AS)	complex (AS,S)	complex (AS,AS)	complex (S,S)	real	imag.	0	complex
E_{2-2}	ρ_{2-2}	$9\sqrt{\frac{1}{5}}\,t_4^{4*}$	complex (S,AS)	complex (AS,S)	complex (S,S)	complex (AS,AS)	real	imag.	complex	0
O_{11}	$\frac{1}{2}(\rho_{11}-\rho_{-1-1})$		real AS	real S	real S	real AS	0	real	real	0
O_{22}	$\frac{1}{2}(\rho_{22}-\rho_{-2-2})$		real AS	real S	real S	real AS	0	real	real	0
O_{21}	$\frac{1}{2}(\rho_{21}+\rho_{-1-2})$		complex (AS,S)	complex (S,AS)	complex (AS,AS)	complex (S,S)	imag.	real	0	complex
O_{10}	$\frac{1}{2}(\rho_{10}+\rho_{0-1})$		complex (AS,S)	complex (S,AS)	complex (AS,AS)	complex (S,S)	imag.	real	0	complex
O_{20}	$\frac{1}{2}(\rho_{20}-\rho_{0-2})$		complex (AS,S)	complex (S,AS)	complex (S,S)	complex (AS,AS)	imag.	real	complex	0
O_{2-1}	$\frac{1}{2}(\rho_{2-1}+\rho_{1-2})$		complex (AS,S)	complex (S,AS)	complex (AS,AS)	complex (S,S)	imag.	real	0	complex

Measurement of the polarization of a spin 2 particle

TABLE A.2 (continued)

i) Trace condition $t_0^0 = 1$; $E_{00} + 2E_{11} + 2E_{22} = 1$ (unpolarized initial state)

ii) $O_{00} \equiv 0$; $O_{1-1} \equiv 0$; $O_{2-2} \equiv 0$

iii) Number of independent observable real quantities (trace condition not included)

	General case	B symmetric two-body (Class 1)	B antisymmetric two-body (Class 1)
Even observable quantities	15	9	6
Even and odd measurement	25	13	12

iv) For parity-conserving decay $2 \rightarrow 0 + 0$, the decay angular distribution is given by:

$$W_2(\theta,\varphi) = \frac{5}{8\pi}\Big\{\frac{1}{2}(3\cos^2\theta - 1)^2 E_{00} + 6\sin^2\theta\cos^2\theta\, E_{11} + \frac{3}{2}\sin^4\theta\, E_{22} + \mathrm{Re}\Big[-6\sin^3\theta\cos\theta\, e^{i\varphi} E_{21} -$$

$$-2\sqrt{6}\sin\theta\cos\theta(3\cos^2\theta - 1)\, e^{i\varphi} E_{10} + \sqrt{6}\sin^2\theta(3\cos^2\theta - 1)\, e^{2i\varphi} E_{20} - 6\sin^2\theta\cos^2\theta\, e^{2i\varphi} E_{1-1} +$$

$$+ 6\sin^3\theta\cos\theta\, e^{3i\varphi} E_{2-1} + \frac{3}{2}\sin^4\theta\, e^{4i\varphi} E_{2-2}\Big]\Big\}$$

v) For parity-conserving decay $2^+ \rightarrow 1^- + 0^-$ [see Ref. 25].

TABLE A.3

Observables	Density matrix elements	Multi-polar para-meters	General case				Two-body or inclusive reactions (Class 1)			
			H-B	H-BA	T-B	T-BA	H-B	H-BA	T-B	T-BA
$E_{\frac{1}{2}\frac{1}{2}}$	$\frac{1}{2}(\rho_{\frac{1}{2}\frac{1}{2}} + \rho_{-\frac{1}{2}-\frac{1}{2}})$	$\frac{1}{2}t_o^o$	real S	real AS	real S	real AS	real	0	real	0
$O_{\frac{1}{2}\frac{1}{2}} = p_z/2$	$\frac{1}{2}(\rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2}-\frac{1}{2}})$	$\frac{\sqrt{3}}{2}\ t_o^1$	real AS	real S	real S	real AS	0	real	real	0
$O_{\frac{1}{2}-\frac{1}{2}} = \frac{p_x + ip_y}{2}$	$\rho_{\frac{1}{2}-\frac{1}{2}}$	$\frac{\sqrt{6}}{2}\ t_{-1}^1$	complex (AS, S)	complex (S, AS)	complex (AS,AS)	complex (S, S)	imag.	real	0	complex

Measurement of the polarization of a spin $\frac{1}{2}$ particle

Remarks:

i) Trace condition: $t_o^o = 1$; $E_{\frac{1}{2}\frac{1}{2}} = \frac{1}{2}$ (for unpolarized initial state)

ii) $E_{\frac{1}{2}-\frac{1}{2}} \equiv 0$

iii) Number of independent observable real quantities (trace condition not included)

	General case	B symmetric Two-body (Class 1)	B antisymmetric (Class 1)
Parity-conserving decay	1	1	0
Parity non-conserving decay	4	2	2

iv) For parity-non-conserving decay $\frac{1}{2} \to \frac{1}{2} + 0$, the decay angular distribution is given by:

$$W_{1/2}(\theta,\varphi) = \frac{1}{2\pi}\left\{E_{1/2\,1/2} + \alpha\left(\cos\theta\, O_{1/2\,1/2} + \cos\varphi\sin\theta\,\mathrm{Re}\,O_{1/2\,-1/2} - \sin\varphi\sin\theta\,\mathrm{Im}\,O_{1/2\,-1/2}\right\}\right.$$

where α is the asymmetry parameter, relating the interference of S and P partial waves of the decay.

TABLE A.4

Observables	Density matrix elements	Multipolar parameters	General case				Two-body or inclusive reactions (Class 1)			
			H-B	H-BA	T-B	T-BA	H-B	H-BA	T-B	T-BA
$E_{11} = E_{-1-1}$	$\frac{1}{2}(\rho_{11} + \rho_{-1-1})$	$\frac{1}{4}(t_o^0 - \sqrt{5}\, t_o^2)$	real S	real AS	real S	real AS	real	0	real	0
$E_{33} = E_{-3-3}$	$\frac{1}{2}(\rho_{33} + \rho_{-3-3})$	$\frac{1}{4}(t_o^0 + \sqrt{5}\, t_o^2)$	real S	real AS	real S	real AS	real	0	real	0
$E_{31} = -E_{-1-3} = E^*_{13}$	$\frac{1}{2}(\rho_{31} - \rho_{-1-3})$	$-\sqrt{\frac{5}{8}}\, t_1^{2*}$	complex (S, AS)	complex (AS, S)	complex (AS,AS)	complex (S, S)	real	imag.	0	complex
$E_{3-1} = E_{1-3} = E^*_{-13}$	$\frac{1}{2}(\rho_{3-1} + \rho_{1-3})$	$\sqrt{\frac{5}{8}}\, t_2^{2*}$	complex (S, AS)	complex (AS, S)	complex (S, S)	complex (AS,AS)	real	imag.	complex	0
$O_{11} = O_{-1-1}$	$\frac{1}{2}(\rho_{11} - \rho_{-1-1})$	$\frac{3}{4}(\sqrt{\frac{1}{15}}\, t_o^1 - \sqrt{\frac{7}{5}}\, t_o^3)$	real AS	real S	real S	real AS	0	real	real	0
$O_{33} = -O_{-3-3}$	$\frac{1}{2}(\rho_{33} - \rho_{-3-3})$	$\frac{3}{4}(\sqrt{\frac{3}{5}}\, t_o^1 + \sqrt{\frac{7}{45}}\, t_o^3)$	real AS	real S	real S	real AS	0	real	real	0
$O_{31} = O_{-1-3} = O^*_{13}$	$\frac{1}{2}(\rho_{31} + \rho_{-1-3})$	$-\frac{3}{4}(\sqrt{\frac{2}{5}}\, t_1^{1*} + \sqrt{\frac{28}{45}}\, t_1^{3*})$	complex (AS, S)	complex (S, AS)	complex (AS,AS)	complex (S, S)	imag.	real	0	complex
$O_{3-1} = -O_{1-3} = O^*_{-13}$	$\frac{1}{2}(\rho_{3-1} - \rho_{1-3})$	$\sqrt{\frac{7}{8}}\, t_2^{3*}$	complex (AS, S)	complex (S, AS)	complex (S, S)	complex (AS,A)	imag.	real	complex	0
$O_{1-1} = O^*_{-11}$	ρ_{1-1}	$-\sqrt{\frac{3}{10}}(t_1^{1*} - \sqrt{\frac{7}{2}}\, t_1^{3*})$	complex (AS, S)	complex (S, AS)	complex (AS,AS)	complex (S, S)	imag.	real	0	complex
$O_{3-3} = O^*_{-33}$	ρ_{3-3}	$-\sqrt{\frac{7}{4}}\, t_3^{3*}$	complex (AS,S)	complex (S,AS)	complex (AS,AS)	complex (S, S)	imag.	real	0	complex

Measurement of the polarization of a spin $\frac{3}{2}$ particle (for simplicity we use in this table the notation $E_{2m2m'}$, $O_{2m2m'}$).

TABLE A.4 (continued)

Remarks:

i) Trace condition $t_0^0 \equiv 1$; $E_{11}+E_{33}=\frac{1}{2}$ (for unpolarized initial state)

ii) $E_{1-1} \equiv E_{3-3} \equiv 0$

iii) Number of independent observable real quantities (trace condition not included)

	General case	B symmetric two-body	B antisymmetric two-body
Even part of the polarization	6	4	2
Even and odd measurement	16	8	8

iv) For parity non-conserving decay: $\frac{3}{2} \to \frac{1}{2} + 0$, the decay angular distribution is given by:

$$W_{3/2}(\theta,\varphi) = \frac{3}{4\pi}\Big\{\left(\frac{1}{3}+\cos^2\theta\right)E_{11} + \sin^2\theta\, E_{33} - \frac{2}{\sqrt{3}}\sin 2\theta\,\mathrm{Re}\left(E_{31}e^{i\varphi}\right) - \frac{2}{\sqrt{3}}\mathrm{Re}\left(E_{3-1}e^{2i\varphi}\right) +$$

$$+\alpha\Big[\left(\frac{4}{3}\cos\theta - \frac{3}{2}\sin\theta\sin 2\theta\right)O_{11} - \frac{4}{\sqrt{3}}\sin\theta\left(1-\frac{3}{2}\sin^2\theta\right)\mathrm{Re}\left(O_{31}e^{i\varphi}\right) + \frac{\sin\theta\sin 2\theta}{2}O_{33} -$$

$$-\sqrt{3}\sin\theta\sin 2\theta\,\mathrm{Re}\left(O_{3-1}e^{2i\varphi}\right) + \frac{8}{3}\sin\theta\left(1-\frac{9}{8}\sin^2\theta\right)\mathrm{Re}\left(O_{1-1}e^{i\varphi}\right) - \sin^3\theta\,\mathrm{Re}\left(O_{3-3}e^{3i\varphi}\right)\Big]\Big\}$$

In fact when parity is conserved in this decay [e.g., $\Sigma(\frac{3}{2}^+) \to \Lambda\pi$] one of the two P and D partial waves has to vanish. In this case, α is zero, because it is related to the interference of these two waves.

v) For cascade decays (in which the final $\frac{1}{2}$ polarization is measured), see Ref. 26).

TABLE 1

Exchanged naturality	Transversity quantization		Helicity quantization	
	Class 2	Class 1	Class 2	Class 1
$\sigma = \eta_a \eta_1$	E^T_{oo}	ρ^T_{oo}	$E^H_{11} + \mathrm{Re}\, E^H_{1-1}$	$\rho^H_{11} + \rho^H_{1-1}$
$\sigma = -\eta_a \eta_1$	E^T_{11} $\mathrm{Re}\, E^T_{1-1}$ $\mathrm{Im}\, E^T_{1-1}$	$\rho^T_{11} + \rho^T_{-1-1}$ $\mathrm{Re}\, \rho^T_{1-1}$ $\mathrm{Im}\, \rho^T_{1-1}$	E^H_{oo} $E^H_{11} - \mathrm{Re}\, E^H_{1-1}$ $\mathrm{Re}\, E^H_{10}$	ρ^H_{oo} $\rho^H_{11} - \rho^H_{1-1}$ $\mathrm{Re}\, \rho^H_{10}$

Experimental quantities which allow a separation of natural $(\sigma = +1)$ and unnatural parity $(\sigma = -1)$ in the $a + \bar{1} \to \bar{b} + 2 + \ldots$ channel, when particle a is spinless and particle 1 has spin 1.

TABLE 2

	Transversity quantization		Helicity quantization	
Exchanged naturality	Class 2	Class 1	Class 2	Class 1
$\sigma = \eta_a \eta_1$	E^T_{oo} E^T_{22} Re E^T_{20} , Im E^T_{20} Re E^T_{2-2} , Im E^T_{2-2}	ρ^T_{oo} $\rho^T_{22} + \rho^T_{-2-2}$ Re ρ^T_{20}, Im ρ^T_{20} Re ρ^T_{2-2}, Im ρ^T_{2-2}	E^H_{oo} Re E^H_{10} E^H_{11} - Re E^H_{1-1} Re E^H_{20} Re E^H_{21} - Re E^H_{2-1} E^H_{22} + Re E^H_{2-2}	ρ^H_{oo} Re ρ^H_{10} $\rho^H_{11} - \rho^H_{1-1}$ Re ρ^H_{20} Re $(\rho^H_{21} - \rho^H_{2-1})$ ρ^H_{22} + Re ρ^H_{2-2}
$\sigma = -\eta_a \eta_1$	E^T_{11} Re E^T_{1-1} , Im E^T_{1-1}	$\rho^T_{11} + \rho^T_{-1-1}$ Re ρ^T_{1-1}, Im ρ^T_{1-1}	E^H_{11} + Re E^H_{1-1} Re E^H_{21} + Re E^H_{2-1} E^H_{22} - Re E^H_{2-2}	ρ^H_{11} + Re ρ^H_{1-1} Re $(\rho^H_{21} + \rho^H_{2-1})$ ρ^H_{22} - Re ρ^H_{2-2}

Experimental quantities separating natural ($\sigma = +1$) and unnatural parity ($\sigma = -1$) contributions in the $a + \bar{1} \to \bar{b} + 2 + \ldots$ channel, when particle a is spinless and particle 1 has spin 2.

TABLE 3

Exchanged naturality	Transversity quantization		Helicity quantization	
	Class 2	Class 1	Class 2	Class 1
$\sigma = \eta_1 \eta_a$	$E^T_{33} + O^T_{33}$ $E^T_{3-1} + O^T_{3-1}$ $E^T_{11} - O^T_{11}$	ρ^T_{33} ρ^T_{3-1} ρ^T_{-1-1}	$\mathrm{Re}\, E^H_{33} + \mathrm{Im}\, E^H_{3-3}$ $\mathrm{Re}\, E^H_{11} - \mathrm{Im}\, O^H_{1-1}$ $\mathrm{Re}\, E^H_{31} - \mathrm{Im}\, O^H_{3-1}$. $\mathrm{Im}\, O^H_{31} + \mathrm{Re}\, E^H_{3-1}$	$\rho^H_{33} + \mathrm{Im}\, \rho^H_{3-3}$ $\rho^H_{11} - \mathrm{Im}\, \rho^H_{1-1}$ $\rho^H_{31} + i\, \rho^H_{3-1}$
$\sigma = -\eta_1 \eta_a$	$E^T_{33} - O^T_{33}$ $E^T_{3-1} - O_{3-1}$ $E^T_{11} + O^T_{11}$	ρ^T_{-3-3} ρ^T_{1-3} ρ^T_{11}	$\mathrm{Re}\, E^H_{33} - \mathrm{Im}\, O^H_{3-3}$ $\mathrm{Re}\, E^H_{11} + \mathrm{Im}\, O^H_{1-1}$ $\mathrm{Re}\, E^H_{31} + \mathrm{Im}\, O^H_{3-1}$ $\mathrm{Im}\, O^H_{31} - \mathrm{Re}\, E^H_{3-1}$	$\rho^H_{33} - \mathrm{Im}\, \rho^H_{3-3}$ $\rho^H_{11} + \mathrm{Im}\, \rho^H_{1-1}$ $\rho^H_{31} - i\, \rho^H_{3-1}$

Experimental quantities which allow a separation of natural ($\sigma = +1$) and unnatural parity ($\sigma = -1$) in the $a + \bar{1} \rightarrow \bar{b} + 2 + \ldots$ channel, when particle a is spinless and particle 1 has spin $\frac{3}{2}$.

TABLE 4

	Helicity		Transversity	
	B	BA	B	BA
General case	$\mathrm{Re}\, t^L_M(\varphi_i) = (-)^L\, \mathrm{Re}\, t^L_M(-\varphi_i)$ $\mathrm{Im}\, t^L_M(\varphi_i) = (-)^{L+1}\, \mathrm{Im}\, t^L_M(-\varphi_i)$	$\mathrm{Re}\, t^L_M(\varphi_i) = (-)^{L+1}\, \mathrm{Re}\, t^L_M(-\varphi_i)$ $\mathrm{Im}\, t^L_M(\varphi_i) = (-)^L\, \mathrm{Im}\, t^L_M(-\varphi_i)$	$t^L_M(\varphi_i) = (-)^M t^L_M(-\varphi_i)$	$t^L_M(\varphi_i) = (-)^{M+1} t^L_M(-\varphi_i)$
Two-body and inclusives	$\mathrm{Re}\, t^L_M \equiv 0$, L odd $\mathrm{Im}\, t^L_M \equiv 0$, L even	$\mathrm{Re}\, t^L_M \equiv 0$, L even $\mathrm{Im}\, t^L_M \equiv 0$, L odd	$t^L_M \equiv 0$, M odd	$t^L_M \equiv 0$, M even

Symmetry properties with respect to reflection in the scattering plane of multipolar parameters.

TABLE 5

		Helicity		Transversity	
		B	BA	B	BA
General case	Symmetric quantities	Re E , Im 0	Im E , Re 0	E and 0 $\tau - \tau'$ even	E and 0 $\tau - \tau'$ odd
General case	Anti-symmetric quantities	Im E , Re 0	Re E , Im 0	E and 0 $\tau - \tau'$ odd	E and 0 $\tau - \tau'$ even
Two - body and inclusives		Im E = Re 0 ≡ 0	Re E = Im 0 ≡ 0	E = 0 ≡ 0 for $\tau - \tau'$ odd	E = 0 ≡ 0 for $\tau - \tau'$ even

Symmetry properties with respect to reflection in the scattering plane of observable quantities.

TABLE 6

Observable quantities	$\tau - \tau'$	Symmetry properties by reflection in the scattering plane	Dependence on exchanged naturalities
$E^{0}_{\tau\tau'}$, $\pm O^{z}_{\tau\tau'}$	Even	Symmetric	$\sigma = \pm \eta_1 (-)^{\tau - \frac{1}{2}}$
	Odd	Antisymmetric	Interference
$O^{0}_{\tau\tau'}$ $\pm E^{z}_{\tau\tau'}$	Even	Symmetric	$\sigma = \pm \eta_1 (-)^{\tau - \frac{1}{2}}$
	Odd	Antisymmetric	Interference
$E^{x\pm iy}_{\tau\tau'}$; $O^{x\pm iy}_{\tau\tau'}$	Even	Antisymmetric	Interference
	Odd	Symmetric	$\sigma = \pm \eta_1 (-)^{\tau - \frac{1}{2}}$

Properties of the observable quantities related to the polarization measurement of a final baryon produced off a polarized proton target (transversity quantization).

TABLE 7

Observable quantities	$\tau - \tau'$	Symmetry properties by reflection in the scattering plane	Dependence on exchanged naturalities
$E^{o\pm z}_{\tau\tau'}$; $O^{o\pm z}_{\tau\tau'}$	Even	Symmetric	$\sigma = \pm\eta_1(-)^{\tau}$
	Odd	Antisymmetric	Interferences
$E^{x}_{\tau\tau'} \pm iO^{y}_{\tau\tau'}$	Even	Antisymmetric	Interferences
	Odd	Symmetric	$\sigma = \pm\eta_1(-)^{\tau}$
$O^{x}_{\tau\tau'} \pm iE^{y}_{\tau\tau'}$	Even	Antisymmetric	Interferences
	Odd	Symmetric	$\sigma = \pm\eta_1(-)^{\tau}$

Properties of the observable quantities related to the polarization measurement of a final boson produced off a polarized proton target (transversity quantization) for baryon exchange reactions.

TABLE 8

	Transversity quantization	Helicity quantization
Bohr symmetric initial polarization	$\frac{1}{2} \pm O^{z}_{\frac{1}{2}\frac{1}{2}}$ $O^{o}_{\frac{1}{2}\frac{1}{2}} \pm E^{z}_{\frac{1}{2}\frac{1}{2}}$	$\frac{1}{2} \pm \mathrm{Im}\, O^{y}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Im}\, O^{o}_{\frac{1}{2}-\frac{1}{2}} \pm E^{y}_{\frac{1}{2}\frac{1}{2}}$
Bohr antisymmetric initial polarization	$O^{x}_{\frac{1}{2}-\frac{1}{2}} \pm i O^{y}_{\frac{1}{2}-\frac{1}{2}}$	$O^{z}_{\frac{1}{2}\frac{1}{2}} \mp \mathrm{Re}\, O^{x}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Re}\, O^{z}_{\frac{1}{2}-\frac{1}{2}} \pm O^{x}_{\frac{1}{2}\frac{1}{2}}$

Combination of observable quantities which allow a separation of natural $(\sigma = 1)$ and unnatural $(\sigma = -1)$ parity exchange contributions for a spin $\frac{1}{2}$ particle produced off a polarized proton target.

TABLE 9

	Transversity quantization	Helicity quantization
Bohr symmetric initial polarization	$E^{o}_{11} \mp \eta E^{z}_{11}$	$\mathrm{Re}\, E^{o}_{1-1} \pm \eta E^{y}_{11}$
	$E^{o}_{oo} \pm \eta E^{z}_{oo}$	$E^{o}_{11} \pm \eta\, \mathrm{Re}\, E^{y}_{1-1}$
	$\mathrm{Re}\, E^{o}_{1-1} \mp \eta\, \mathrm{Re}\, E^{z}_{1-1}$	$\mathrm{Re}\, E^{o}_{1o} \mp \eta\, \mathrm{Re}\, E^{y}_{10}$
	$\mathrm{Im}\, E^{o}_{1-1} \mp \eta\, \mathrm{Im}\, E^{z}_{1-1}$	$E^{o}_{oo} \mp \eta E^{y}_{oo}$

Combinations of even observable quantities which allow a separation of natural (upper sign) and unnatural parity exchange (lower sign) contributions between a polarized proton and a final spin 1 meson with parity η .

TABLE 10

Proton polarization	Transversity quantization	Helicity quantization
B	$E^{o}_{33} \mp O^{z}_{33}$ $E^{o}_{3-1} \mp O^{z}_{3-1}$ $E^{o}_{11} \pm O^{z}_{11}$ $O^{o}_{33} \mp E^{z}_{33}$ $O^{o}_{3-1} \mp E^{z}_{3-1}$ $O^{o}_{11} \mp E^{z}_{11}$	$E^{o}_{33} \mp \mathrm{Im}\, O^{y}_{3-3}$ $\mathrm{Re}\, E^{o}_{31} \pm \mathrm{Im}\, O^{y}_{3-1}$ $\mathrm{Re}\, E^{o}_{3-1} \mp \mathrm{Im}\, O^{y}_{31}$ $E^{o}_{11} \pm \mathrm{Im}\, O^{y}_{1-1}$ $\mathrm{Im}\, O^{o}_{31} \mp \mathrm{Re}\, E^{y}_{3-1}$ $\mathrm{Im}\, O^{o}_{3-1} \pm \mathrm{Re}\, E^{y}_{31}$ $\mathrm{Im}\, O^{o}_{1-1} \pm E^{y}_{11}$ $\mathrm{Im}\, O^{o}_{3-3} \mp E^{y}_{33}$
BA	$O^{x}_{31} \mp iO^{y}_{31}$ $O^{x}_{1-1} \pm iO^{y}_{1-1}$ $O^{x}_{3-3} \mp iO^{y}_{3-3}$	$O^{z}_{11} \mp \mathrm{Re}\, O^{x}_{1-1}$ $\mathrm{Re}\, O^{z}_{1-1} \pm O^{x}_{11}$ $O^{z}_{33} \pm \mathrm{Re}\, O^{x}_{3-3}$ $\mathrm{Re}\, O^{z}_{31} \mp \mathrm{Re}\, O^{x}_{3-1}$ $\mathrm{Re}\, O^{z}_{3-1} \pm \mathrm{Re}\, O^{x}_{31}$ $\mathrm{Re}\, O^{z}_{3-3} \mp O^{x}_{33}$
	$E^{x}_{31} \mp iE^{y}_{31}$	$\mathrm{Im}\, E^{z}_{31} \mp \mathrm{Im}\, E^{x}_{3-1}$ $\mathrm{Im}\, E^{z}_{3-1} \pm \mathrm{Im}\, E^{x}_{31}$

Combinations of observable quantities allowing a separation of naturality $\sigma = \pm \eta_1$ exchanged between a polarized proton and a spin $\frac{1}{2}$ baryon of parity η_1 (quantities under the dashed line need only an even polarization measurement).

TABLE 11

Photon polarization	Class 2 reactions	Class 1 reactions
B	$E^{o}_{\frac{1}{2}\frac{1}{2}} \mp \eta \, \mathrm{Im}\, O^{x}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Im}\, O^{o}_{\frac{1}{2}-\frac{1}{2}} \mp \eta \, E^{x}_{\frac{1}{2}\frac{1}{2}}$	$\rho^{o}_{\frac{1}{2}\frac{1}{2}} \mp \eta \, \mathrm{Im}\, \rho^{x}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Im}\, \rho^{o}_{\frac{1}{2}-\frac{1}{2}} \mp \eta \, \rho^{x}_{\frac{1}{2}\frac{1}{2}}$
BA	$O^{z}_{\frac{1}{2}\frac{1}{2}} \mp \eta \, \mathrm{Re}\, O^{y}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Re}\, O^{z}_{\frac{1}{2}-\frac{1}{2}} \pm \eta \, O^{y}_{\frac{1}{2}\frac{1}{2}}$	$\rho^{z}_{\frac{1}{2}\frac{1}{2}} \mp \eta \, \mathrm{Re}\, \rho^{y}_{\frac{1}{2}-\frac{1}{2}}$ $\mathrm{Re}\, \rho^{z}_{\frac{1}{2}-\frac{1}{2}} \pm \eta \, \rho^{y}_{\frac{1}{2}\frac{1}{2}}$

Combinations of observable quantities which allow a separation of natural (upper sign) and unnatural parity exchange (lower sign) contributions between a polarized photon and a final spin $\frac{1}{2}$ baryon with parity η .

TABLE 12

Photon polarization	Observables
B	$E^{o}_{11} \pm \mathrm{Re}\, E^{x}_{1-1}$
	$\mathrm{Re}\, E^{o}_{10} \mp \mathrm{Re}\, E^{x}_{10}$
	$\mathrm{Re}\, E^{o}_{1-1} \pm E^{x}_{11}$
	$E^{o}_{oo} \mp E^{x}_{oo}$
	$\mathrm{Im}\,(O^{o}_{10} \mp O^{x}_{10})$
BA	$O^{z}_{11} \mp \mathrm{Im}\, E^{y}_{1-1}$
	$\mathrm{Re}\, O^{z}_{10} \pm \mathrm{Im}\, E^{y}_{10}$
	$\mathrm{Im}\, E^{z}_{1-1} \pm O^{y}_{11}$
	$\mathrm{Im}\, E^{z}_{10} \mp \mathrm{Re}\, O^{y}_{10}$

Combinations of observable quantities which allow a separation of naturality $\sigma = \mp \eta_1$ exchanged between a polarized photon and a spin 1 meson with parity η_1, in $\gamma\uparrow b \rightarrow (\text{spin } 1) + \ldots$

TABLE 13

Observable quantities	$\tau-\tau'$ parity	Symmetry properties by reflection in scatter. plane	Dependence on exchanged naturalities
$E^{(0,B)}_{\tau\tau'} \pm O^{(z,B)}_{\tau\tau'}$	Even	S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$
	Odd	A S	Interferences
$E^{(0,BA)}_{\tau\tau'} \pm O^{(z,BA)}_{\tau\tau'}$	Even	A S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$
	Odd	S	Interferences
$O^{(0,B)}_{\tau\tau'} \pm E^{(z,B)}_{\tau\tau'}$	Even	S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$
	Odd	A S	Interferences
$O^{(0,AB)}_{\tau\tau'} \pm E^{(z,AB)}_{\tau\tau'}$	Even	A S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$
	Odd	S	Interferences
$E^{(x\pm iy,B)}_{\tau\tau'}$; $O^{(x\pm iy,B)}_{\tau\tau'}$	Even	A S	Interferences
	Odd	S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$
$E^{(x\pm iy,BA)}_{\tau\tau'}$; $O^{(x\pm iy,BA)}_{\tau\tau'}$	Even	S	Interferences
	Odd	A S	$\sigma = \pm \eta_1 (-)^{\tau-\frac{1}{2}}$

Properties of the observable quantities related to the polarization measurement of a final baryon produced from polarized proton beam and target in transversity frame (compare with Table 6).

TABLE 14

Observables	Naturality properties	Natural charge parity
$E_{oo}^{o\pm z}$	Natural	Natural
$E_{1o}^{\pm iy}$	Interferences	Interferences + Incoherent Sum
E_{1-1}^{o}	Unnatural	Incoherent Sum
E_{1-1}^{3}	Unnatural	Interferences
E_{11}^{o}	Unnatural	Incoherent Sum
E_{11}^{3}	Unnatural	Interferences

Relations between observable quantities and exchanged naturality or natural charge parity in $\pi + N\uparrow \rightarrow V\uparrow + N$ (V = vector meson).

TABLE 15

Parameters	R	A	P	R'	A'	P'	D
Naturality	+	+	+	–	–	–	Incoherent Sum
Natural charge parity	+	+	+	Incoherent Sum	Interference	Interference	Incoherent Sum

Relations between observable quantities and exchanged naturality or natural charge parity in $\pi + N\uparrow \rightarrow \Lambda\uparrow + X$.

REFERENCES

1) G. Cohen-Tannoudji, A. Morel and Ph. Salin, Nuovo Cimento 55A, 412 (1968); J.P. Ader, M. Capdeville, G. Cohen-Tannoudji and Ph. Salin, Nuovo Cimento 56A, 952 (1968).

2) K. Gottfried and J.D. Jackson, Nuovo Cimento 33, 309 (1964).

3) P. Stichel, Z.Physik 180, 170 (1964).

4) F. Cooper, Phys.Rev. 170, 1602 (1968).

5) K. Schilling, P. Seyboth and G. Wolf, Nuclear Phys. B15, 397 (1970).

6) R. Thews, Phys.Rev. 175, 1749 (1968).

7) G. Ringland and R. Thews, Phys.Rev. 170, 1569 (1968).

8) R. Saenger and W. Schmidt, Ann.Phys. 54, 307 (1969).

9) W. Meggs and L. Van Hove, Nuclear Phys. B26, 245 (1971).

10) J.D. Kimel and E. Reya, Phys.Rev. D8, 1519 (1973).

11) J.P. Ader, C. Meyers and Ph. Salin, Nuclear Phys. B47, 397 (1972).

12) J.P. Ader, C. Meyers and Ph. Salin, Nuclear Phys. B58, 621 (1973).

13) R.L. Sekulin, Nuclear Phys. B56, 227 (1973).

14) M.G. Doncel, L. Michel and P. Minnaert, Nuclear Phys. B38, 477 (1972), and Matrices densité de polarisation, Lectures at the Summer School of Gif-sur-Yvette (1970) (Ed: R. Salmeron, Laboratoire de Physique, Ecole Polytechnique, Paris).

15) G. Cohen-Tannoudji, A. Morel and H. Navelet, Ann.Phys. 46, 239 (1968).

16) A. Kotanski, Acta Phys.Pol. 29, 699 (1966); 30, 629 (1966).

17) See for instance J.D. Jackson, in High Energy Physics, Les Houches (1965) (Eds: C. de Witt and M. Jacob) (Gordon and Breach, New York, 1965), p. 351 and Ref. 14).

18) I.T. Drummond and G.L. Kane, Nuclear Phys. B45, 157 (1972).

19) B. Sadoulet, Phénoménologie des réactions d'échange d'hypercharge, Thesis (1971) Paris-Sud University and Nuclear Phys. B53, 135 (1973); G. Plaut, Nuclear Phys. B35, 221 (1971).

FIGURE CAPTION

Figure 1 :

Kinematics of the production process $ab \rightarrow 1 + \ldots + n$ in the over-all c.m. system.

Figure 2 :

Classification of various types of experiments. See text.

Figure 3 :

Schematic representation of reaction $a + b\uparrow \rightarrow 1\uparrow + 2 + \ldots + n$.

Figure 4 :

Photoproduction reactions: definition of the scattering plane and electric polarization vector.

Figure 5 :

Schematic representation of reaction $\gamma\uparrow + b \rightarrow 1\uparrow + 2 + \ldots + n$.

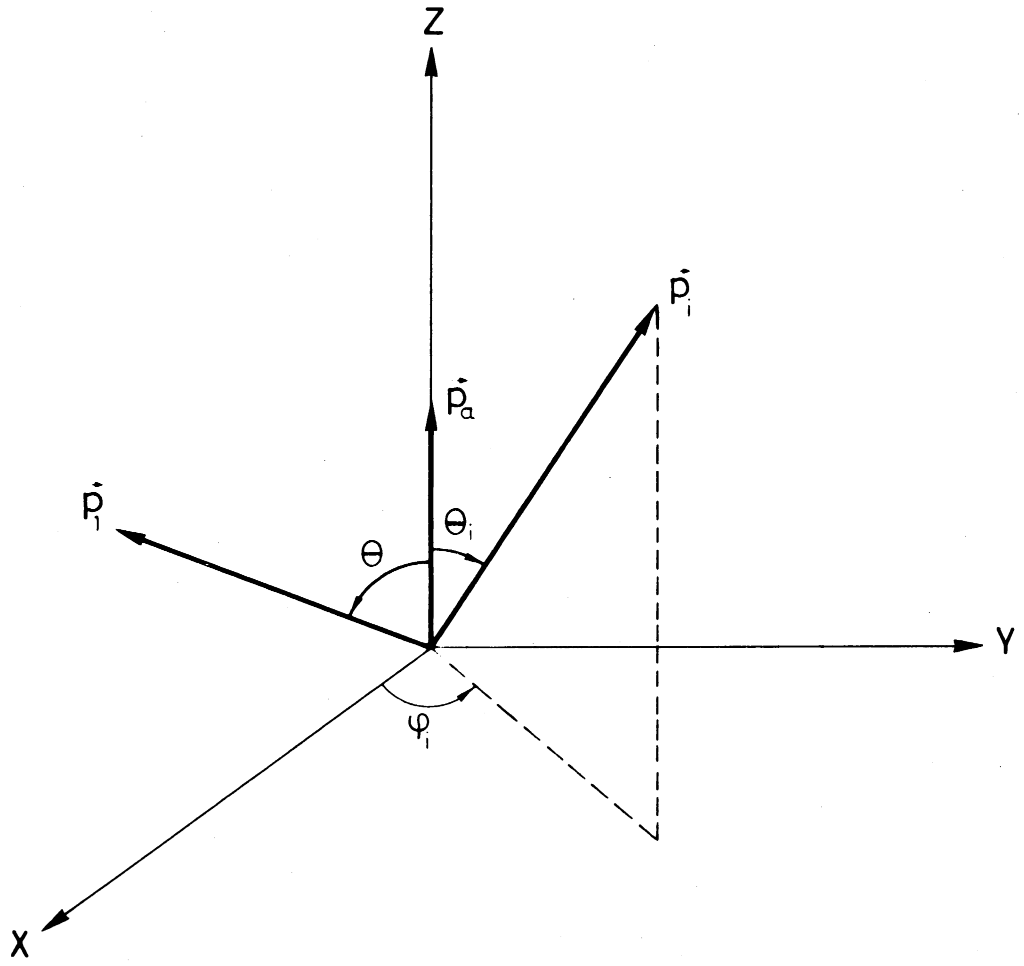
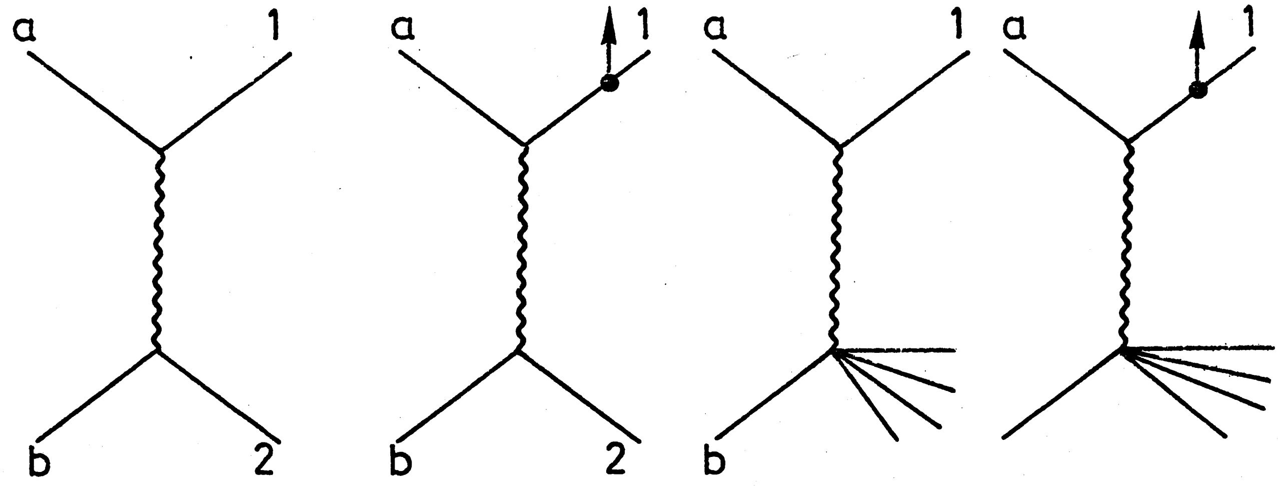
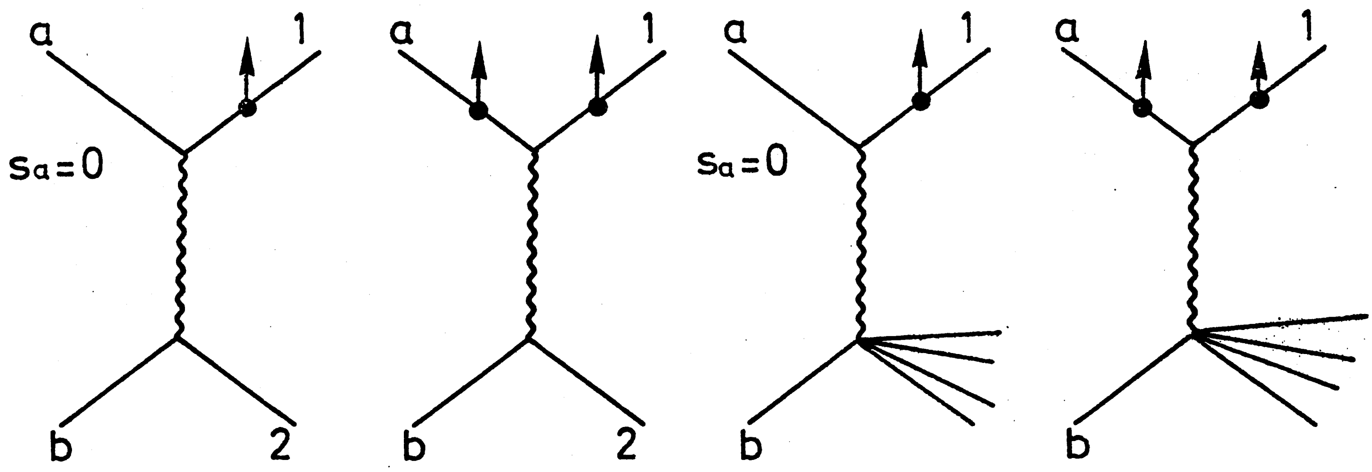


FIG.1



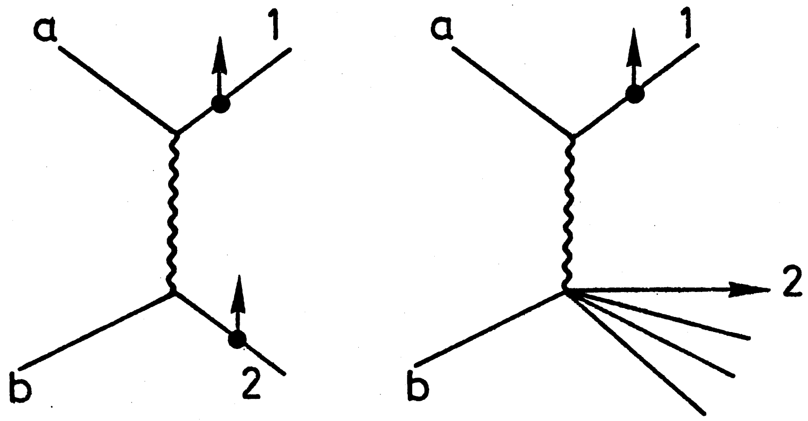
Incoherent Sums

FIG. 2.a



Naturality Separation

FIG. 2.b



Interferences

FIG. 2.c