PROC. 19th INT. CONF. HIGH ENERGY PHYSICS
TOKYO, 1978

C 10 Strong and Weak CP-Violation in Gauge Theories: A Review

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Present status of gauge theories of weak CP-violation is reviewed. We also discuss the constraints imposed on the models of weak CP-violation if the strong CP-non-invariant effect induced by the Quantum Chromodynamic model of strong interactions is to be either absent or natually suppressed to an acceptable order of magnitude. Several possible suggestions to understand the small value of weak η_{+-} are also mentioned.

§1. Introduction

The phenomenon of CP-non-invariance in elementary particle interactions was discovered by Christenson, Cronin, Fitch and Turlay in 1964 in the $K\rightarrow 2\pi$ decays. Fourteen years have passed since then and yet, observed CP-violation remain confined only to the K-meson system. The parameters η_{+-} and η_{00} describing the ratio of $K_L\rightarrow 2\pi$ to $K_S\rightarrow 2\pi$ decay amplitudes have been measured quite accurately: the result is[2]

$$|\eta_{+-}/\eta_{00}|=(1.05\pm 0.046)$$

and

$$\phi_{+-}=(46\pm 1.7)^\circ \text{ and } \phi_{00}=(43\pm 19)^\circ \tag{1}$$

The most recent values for CP-violating parameters in $K_S\rightarrow 3\pi$ decays are[2]

$$|\eta_{+-0}|<\cdot 2 \text{ and } |\eta_{000}|^2<\cdot 38.$$

Another very interesting result relevant for understanding the nature of CP-violation is the upper limit on the electric dipole moment (edm) of neutron obtained by Ramsey *et al.*[3]

$$d_n^e=(\cdot 4\pm 1.5)\times 10^{-24}\text{ ecm} \tag{2}$$

Phenomenologically, there exist two very straightforward ways (*i.e.*, without making additional dynamical assumptions) to understand the equality of η_{+-} and η_{00}:

(a) *Superweak model*[4] The CP-violating nonleptonic Hamiltonian in this case obeys $\Delta S=2$ selection rule and has a strength of $\sim G_F\times 10^{-9}$ (whence the name Superweak). As a result, the on-shell part of the CP-violating amplitude (usually called ε') vanishes and CP-violation arises only through the mixing of K_1^0 and K_2^0 (usually called ε) and eq. (1) obtains. The prediction for the neutron edm in this model relies on additional assumptions and is estimated to be $\lesssim 10^{-29}$ e.c.m. CP-violation in this model will remain confined only to $K^0-\bar{K}^0$ system.

(b) *Isoconjugate models*[5] This class of models is milliweak in character; however, if the P- and CP-violating parts of the Hamiltonian ($H^{(-)}_{wk,P.V.}$) obey the following isospin property, (+ and − stand for CP-even or CP-odd parts),

$$[I_3,\ H^{(-)}_{wk,P.V.}]=iaH^{(+)}_{wk,P.V.} \tag{3}$$

the relation $\eta_{+-}=\eta_{00}$ follows automatically even though $H^{(\pm)}_{wk,P.V.}$ have both the $\Delta I=1/2$ as well as 3/2 parts in them. In these models, the edm of neutron will in general be higher ($\lesssim 10^{-24}$ ecm) than the superweak model. Measurement of $K_S\rightarrow 3\pi$ decay amplitudes will distinguish these models from the superweak model.

After these phenomenological remarks, we proceed to discuss the problem of understanding CP-violation in gauge theories. The nontriviality of this task is due to the constraints of local gauge invariance and renormalizability of gauge models. To generate CP-violation in gauge theories, it is sufficient to introduce a genuine phase in the fermion mixing matrix. The word "genuine" is relevant here since the fermion fields we are dealing with are complex and their phases are arbitrary. Thus, we must make sure that the phase we introduce to get CP-violation does not disappear on redefinition of the phase of the fermion fields. There are two ways to generate a complex mass matrix for quark-fields out of the gauge invariant, renormalizable Yukawa couplings:

(i) *Hard CP-Violation.* If the CP-violating Lagrangian $L_{wk}^{(-)}$ has canonical dimension four, this model is said to have hard CP-violation. This happens when either the Yukawa or the φ^4-coupling or both are made complex.

(ii) *Soft CP-Violation.* If $L_{wk}^{(-)}$ has canonical dimension $d(L_{wk}^{(-)})\leq 3$, the resulting CP-violation is soft since in higher orders, it introduces finite CP-violation into matrix elements of operators O with $d(\mathrm{O})\geq 4$. In this case, CP-violation may be spontaneous in origin in which case, it will disappear at very high energies.

The rest of the talk will deal with the various way to understand this phenomenon in gauge theories and is divided with four parts:

(i) CP-violation in current-gauge boson interactions;

(ii) CP-violation through Higgs-boson exchange graphs;

(iii) QCD-constraints on CP-violation models;

(iv) Possible ways to understand the magnitude of η_{+-}.

§2. CP-Violation in Current-Gauge Boson Interactions

It is by now well-known that, for pure left-handed $SU(2)_L\times U(1)$ models with $2N$ flavors (N of which have $Q=+2/3$ and other N have $Q=-1/3$), the number of phases $\#_{CP}$ which cannot be removed by redefinition of fermion field phases is

$$\#_{CP}=(N-1)(N-2)/2 \tag{4}$$

Therefore, for pure left-handed models with four quarks, there is no CP-violation through currents. The standard WSGIM model,[7] which has only one Higgs multiplet, cannot accout for CP-violation and must, therefore, be modified. Two ways were suggested to remedy the situation:

(a) *Use right-handed currents* It was suggested by the author[8] that right-handed weak currents ought to be used to generate CP-violating interactions. Many such models were subsequently written down.[5,9,10] We will present in some detail the model of ref. 5, which uses the left-right symmetric gauge group $SU(2)_L\times SU(2)_R\times U(1)$ to generate CP-violation with four quarks and has several interesting features. With an arbitrary fermion mass matrix, in this case, one has two genuine phases (δ_L and δ_R). The quark assignment is purely left-right symmetric:

Left-doublets:

$$\begin{pmatrix} u_L \\ d_L\cos\theta_L+s_L\sin\theta_L e^{i\delta_L} \end{pmatrix}$$
$$\begin{pmatrix} c_L \\ -d_L\sin\theta_L e^{-i\delta_L}+s_L\cos\theta_L \end{pmatrix} \tag{5}$$

Right-doublets: replace L by R in eq. (5).

The resulting CP-violating Hamiltonian is milliweak in character and satisfies the iso-conjugate relation of eq. (3) and therefore leads to $\eta_{+-}=\eta_{00}$ ignoring Higgs interactions. Furthermore, in this model, the magnitude of CP-violation η_{+-} gets related to the P-violation parameter *i.e.*,

$$\eta_{+-}\simeq\sin(\delta_L-\delta_R)\cdot(m_{W_L^+}/m_{W_R^+})^2 \tag{6}$$

The model predicts d_n^e in the range of 10^{-24} to 10^{-29} ecm. This relation between CP and P-violation has been extended to the six quark case in an interesting paper by Hagiwara *et al.*[10] The most important feature of this model is the direct link between CP and P-violation both of which owe their existence to the mechanism of spontaneous symmetry breaking. It is worth stressing the alternative of spontaneous P-violation is a viable and philosophically distinct mechanism that needs to be tested. The SLAC experiments with polarized electrons may have ruled out one chain of symmetry breaking of the model; but real test of the model will not come until experiments are done to look for the existence of right-handed charged currents as suggest, for example, by K. Winter. The model of ref. 9 also uses right-handed currents to generate a gauge model with superweak CP-violation.

(b) *Six quark pure left-handed model*[11] It is clear from the formula in eq. (4) that for the case of six quarks ($N=3$), there is one non-trivial CP-phase for pure-left-handed models. This was first noted by Kobayashi and Maskawa,[11] who wrote down the following left-handed charged weak current:

$$J_{\mu L}^{-}=(\bar{u},\bar{c},\bar{t})\gamma_\mu(1+\gamma_5)\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\varrho} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{7}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ and θ_1, θ_2 and θ_3 are the generalized Cabibbo angles. The model has the generalized GIM mechanism; so the neutral currents conserve all flavor naturally. The model has only one Higgs doublet. Therefore, CP-violation is "hard" in this model. The model has been analyzed in detail subsequently.[12] Higher order induced flavor-changing effects constrain $\sin^2\theta_2 < .25$ and $\theta_3^2 < .06$ and θ_1 is, of course, the usual Cabibbo angle. Universality of weak interaction is not natural in this model. Coming to CP-violation, the model is milliweak in character, but it is usually argued (I believe, not entirely convincingly!!) that the on-shell part of the CP-violating amplitude is suppressed due to an OZI rule type mechanism. As a result, for $K^0-\bar{K}^0$ system, the model becomes effectively superweak and one predicts

$$\eta_{+-} \sim s_2 s_3 \sin\delta. \tag{8}$$

Correct magnitude of η_{+-} obtains if $\theta_2 \approx \theta_3 \approx \delta \approx 1/10$. The d_n^e in this model is predicted to be $\sim 10^{-29}$ e.c.m. It must however be mentioned that, the model will lose its attractiveness if more than six quark flavors are discovered. But even as it is, the model will have arbitrary strong CP-violation, as we will see subsequently.

§3. CP-Violation through Higgs–Bosons Interactions

It has been suggested by Lee,[13] Weinberg[14] and Sikivie[15] that contrary to the conventional thinking, CP-violation may arise from the interaction of the Higgs bosons. Although not an overwhelmingly attractive idea, it finds its most natural place in pure left-handed models with four quarks, where current gauge boson interactions are necessarily CP-conserving. The model usually requires two or three Higgs doublets, depending on whether natural flavor diagonality of neutral currents is not or is imposed on the model.[15a] The model of ref. 14, which employs at least three Higgs multiplets (9 physical Higgs mesons), is milliweak in character and predicts a value for $d_n^e \approx 10^{-25}$ e.c.m. and will become untenable should the upper limit on d_n^e go down an order of magnitude. Also, the model predicts $|\eta_{+-}/\eta_{00}| \approx 1.04$. So, a better measurement of $K_L \to 2\pi$ decay amplitudes will also be a crucial test of the model. The model of ref. 15 on the other hand is superweak and requires very heavy Higgs bosons. As we see subsequently, QCD constraints on four quark models often pick this alternative for CP-violation.

§4. QCD-Constraints on CP-Violation Models

An analysis of the vacuum structure in the Quantum Chromodynamic Model of strong interaction has established the existence of a strong P and T-noninvariant term in the effective Lagrangian of the form:

$$L_{st}^{(-)} \simeq c[\mathrm{e}^{i\theta}\,\mathrm{Det}\,\bar{q}_L q_R + \mathrm{e}^{-i\theta}\,\mathrm{Det}\,\bar{q}_R q_L] \tag{9}$$

where the determinant is taken over the flavor space. The present upper limits on d_n^e, however, constrain $\theta \lesssim 10^{-9}$ to 10^{-10}, an awfully small number. The problem then is either to eliminate θ altogether from the theory by requiring some continuous symmetry or make it "naturally" of the acceptable order of magnitude. The first possibility advocated by Peccei and Quinn[16] requires that the Higgs sector of the flavor-dynamics Lagrangian must have an axial U(1) symmetry. It was subsequently observed by Wilczek[17] and Weinberg,[17] that since in any realistic model of weak interactions, the Higgs particles acquire non-zero vacuum-expectation values, this U(1) symmetry is spontaneously broken leading to light mass pseudo-Goldstone particles (mass coming from the instanton effects, see eq. (9)), called the axion. It must be stressed here, that accomodating the axion in a realistic theory of CP-violation makes the models further complicated: for example, in the pure left-handed four quark model, absence of strong CP-violation via the Peccei–Quinn mechanism requires at least four Higgs doublets. Similarly, an axion-ated pure left-handed six quark model too requires at least four Higgs doublets, thus considerably reducing their appeal. In any case, naive estimates yield an axion mass ~ 100 keV and axion-hadron coupling $\approx 10^{-3}$ or axion-electron coupling $\approx 10^{-6}$. This is apparently inconsistent[18] by several orders of magnitude with the present data from reactor neutrino experiment of Reines *et al.* and also the beam dump experiments at CERN. It is therefore not premature to start constructing theories without axion but with $\theta \lesssim 10^{-9} \sim 10^{-10}$ "naturally."

The construction of theories with natural suppression of strong P and T-non-invariance proceeds by the following steps:

(a) First, impose a discrete symmetry such as P or CP on the Lagrangian prior to spontaneous breakdown of gauge symmetry so that $\theta=0$ vacuum in eq. (9) is chosen.

(b) Subsequent to spontaneous breakdown of the gauge symmetry, the fermion mass matrix will in general be complex to account for weak CP-violation. If $M^{\pm}$ are the mass matrices for the up $(+)$ and down$(-)$ quark sectors respectively, they are diagonalized by:

$$U_L^{(\pm)-1} M^{(\pm)} U_R^{(\pm)} = M_{\mathrm{diag.}}^{(\pm)} \qquad (10)$$

when eq. (9) is reexpressed in terms of physical quark fields, that will in general induce a $\bar{\theta}$. Therefore, $M^{(\pm)}$ must be so chosen that no such $\bar{\theta}$ is induced subsequent to spontaneous breakdown. The requirement on $M^{(\pm)}$ is:

$$\det M^{(+)}. \quad \det M^{(-)} = \mathrm{Real} \qquad (11)$$

Once this is done, $\theta=0$ "naturally" at the tree level.

(c) In such theories since the original discrete symmetry is spontaneously broken, θ will be induced in higher orders. To estimate the effective θ induced effective $M_{\mathrm{eff}}^{(\pm)}$ up to the corresponding loop must be calculated and the condition in equation (11) must be checked. The amount of deviation from eq. (11) gives the magnitude of θ induced (*i.e.*, $\theta_{\mathrm{ind.}} \approx$ phase in eq. (11)). This technique has been employed in the ref. 19, 20 and 21. Ref. 20 employs the left-right symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)$ whereas in ref. 21, the standard $SU(2)_L \times U(1)$ group is used. In the first two references, P-invariance is invoked to set $\theta=0$ prior to spontaneous breakdown whereas in ref. 21, softly broken CP-invariance is invoked. Vanishing of θ is achieved naturally up to the one loop level in ref. 19 and 20 whereas in ref. 21, non-vanishing θ arises at the one-loop level. Its prediction is therefore a bit large. For the four quark case, all three cases lead to a superweak model of CP-violation. The model of ref. 20 provides an interesting generalization to the six quark case, where one obtaines a left-right symmetric generalization of the K–M model. By ingenious choice of discrete symmetries and Higgs multiplets, it is assured in refs. 19 and 20 that, the fermion mass matrices satisfy the condition

$$M = M^{\dagger} \qquad (12)$$

As a result, the charged weak currents satisfy manifest left-right symmetry. In this case, we check that, at the one-loop level, hermiticity of the mass matrices is maintained; As a result, θ remains zero up to one loop level. Thus, the lowest order in which θ is induced, is at the two-loop level and is therefore estimated to be $\lesssim 10^{-9}$ to 10^{-11}.

It is worth pointing out at this stage that the Kobayashi-Maskawa model as well as the model of ref. 5 would lead to an arbitrary value of θ without further ado.

§5. Possible Ways to Understand the Magnitude of η_{+-}: an Excursion into Exotica

The observed magnitude of $\eta_{+-} = \alpha/\pi$. Time and again, this has led to speculation that CP-violation may have something to do with higher order electromagnetic and weak interactions. In the context of gauge models also, this speculation has been advanced[22] and pursued.[23] One necessary condition for this to happen[23] is that, the Higgs potential must have a larger accidental symmetry than the gauge group. No realistic model has been presented.

Another alternative is to relate the magnitude of the CP-phase to other approximately known parameters in the theory such as quark masses and mixing angles etc. There have been two interesting attempts in this direction[24,25] within the four quark framework. These directions of research is worth pursuing and appears to hold a lot cf promise.

References and Footnotes

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