RESONANT FISSION WITH DAMPING

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ABSTRACT

In the decay of the compound nucleus by fission some effects of the doubly-humped fission barrier are discussed. The fission penetrability is calculated in an "optical model of fission" in which the real potential characteristic of each saddle-point channel is assumed to be doubly-humped. The coupling of the vibrational states in the intermediate well to more complicated states is described by an imaginary potential. Thus, the damping of the fission channel states is made in the same way as in the conventional optical model for particle scattering. The problem of subsequent distribution of the dissipated flux is discussed. The theory is illustrated by the ^{239}Pu(d,pf)reaction.

1. INTRODUCTION

In (d, pf) experiments[1], in the A=230-250 region, resonances have been observed in the fission probability as a function of the excitation energy of the fissioning nucleus. Both the resonances and the fission isomers[2] have been interpreted as quasi-bound vibrational states in the second minimum of the doubly humped fission potential barrier[3]. These types of collective strongly deformed states are usually referred to as class II states. Besides the vibrational states, one has observed more densely distributed class II states[4]. Clearly, these states are more complicated than the vibrational states, and they are usually interpreted as compound states of class II.

For the analysis of data for energies near the isomeric state, the fission penetrability has been calculated[1,8] in the channel theory[5] with a doubly humped real potential barrier. At energies above the fission barrier, the possibility of strong damping of the class II vibrational states has been taken into account by a two-step decay model[3,9]. As seen in ref.1 the calculated resonances (fig.1) in the analysis of (d,pf) are more narrow and more peaked than those observed in the experiment. Some experiments[6] have shown fine structure of typical class II character in the fission resonance in ^{239}Pu(d,pf). These observations and the requirement to connect the two rather different models for low and high energy analyses has led to the construction of a model which includes both the features of the doubly-humped real potential barrier and of the damping[7].

In principle, the damping could be introduced by coupling between several fission channels. Since, however, we know very little about the channel states and less about the possible coupling between them, it is difficult to obtain a set of coupled equations. Therefore, we suggest a simpler model in which the damping of the fission

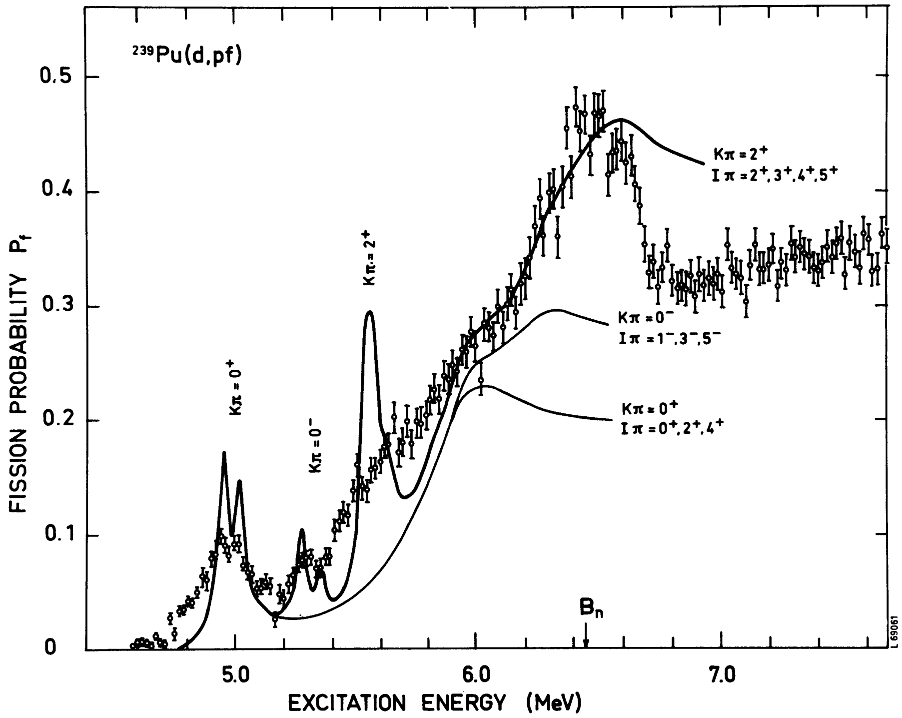

Fig.1. Fission probability for ^{239}Pu(d,pf). The figure is taken from ref. [1].The experimental points are the fission probability $P_f(\exp) = d\sigma(d,pf)/d\sigma(d,p)$.The theoretical fission probability is calculated with the branching ratio (2) and no damping in class II. For the details of the calculation, see ref. [1].

channels is described by a complex fission barrier. We shall briefly formulate this "optical model of fission" and apply it to some (d,pf) data.

2. OPTICAL MODEL OF FISSION

We consider reactions in which compound states λ of spin J and parity π are populated and we assume that these states are of normal deformation (class I states). For simplicity, we shall assume that only γ-decay is possible besides fission. The branching ratio for decay of the state λ by fission is then

$$\mathcal{B}(\lambda J\pi) = \frac{\Gamma_f^{\lambda J\pi}}{\Gamma_f^{\lambda J\pi} + \Gamma_\gamma^{\lambda J\pi}} \tag{1}$$

Its average over many levels λ can be written

$$\langle \mathcal{B}(J\pi)\rangle = \frac{T_f^{J\pi}}{T_f^{J\pi} + T_\gamma^{J\pi}} F(J\pi) \tag{2}$$

where the fission transmission-coefficient is a sum over partial coefficients for the saddle-point channels K [5].

$$T_f^{J\pi} = \sum_K T_K^{J\pi} \tag{3}$$

and $T_\gamma^{J\pi}$ is a corresponding γ-ray coefficient. The transmission coefficient for a channel c is related to the partial width and the compound level distance $D^{J\pi}$ by

$$T_c^{J\pi} = 2\pi \frac{\langle \Gamma_c^{J\pi}\rangle}{D^{J\pi}} \tag{4}$$

$F(J\pi)$ is a fluctuation factor of the order of unity.

In the compound nuclear theory, one usually calculates the transmission coefficient from an optical model for channel c for which

$$T_c = 1 - |\eta_c|^2 \tag{5}$$

where η_c is the scattering amplitude in channel c. This is because T_c is that part of the incident flux which goes to compound nucleus formation. In our case of fission, we want to adopt the same philosophy and thus ask: how much flux is absorbed into the class I region in an analogous scattering problem for a fission channel ? The Schrödinger equation for the collective fission motion in channel $J\pi K$ is

$$\left(-\frac{\hbar^2}{2B}\frac{d^2}{d\beta^2} + V_{J\pi K}(\beta) + \varepsilon_{J\pi K} - E \right)\Psi_{J\pi K}(\beta, E) = 0 \tag{6}$$

where $V_{J\pi K}(\beta)$ is the complex barrier potential (fig. 2) which is usually assumed to be independent of $J\pi$ and K. We assume that the mass parameter B is independent of the deformation coordinate β. The energy $\varepsilon_{J\pi K} = \varepsilon_K + \Delta_K J(J+1)$ is the intrinsic energy of the channel state. The quantity Δ_K is the rotational energy constant for the saddle-point state. For the doubly humped barrier potential one may choose an absorptive imaginary part which is located in regions I and II. This is because it is expected that the damping occurs locally with two essentially different strengths in the two regions. The imaginary potential is therefore put to zero at point A(fig.2). One now considers scattering of fission waves in the channel. The boundary condition at large values of β is

$$\Psi_K(\beta_{max}, E) = \exp(-ik'\beta_{max}) - \eta_K \exp(ik'\beta_{max}) \qquad (7)$$

where k' is the wave number at the distance $\beta_{max} \geq \beta_{scission}$. It is found by calculation that β_{max} can be chosen rather arbitrarily.

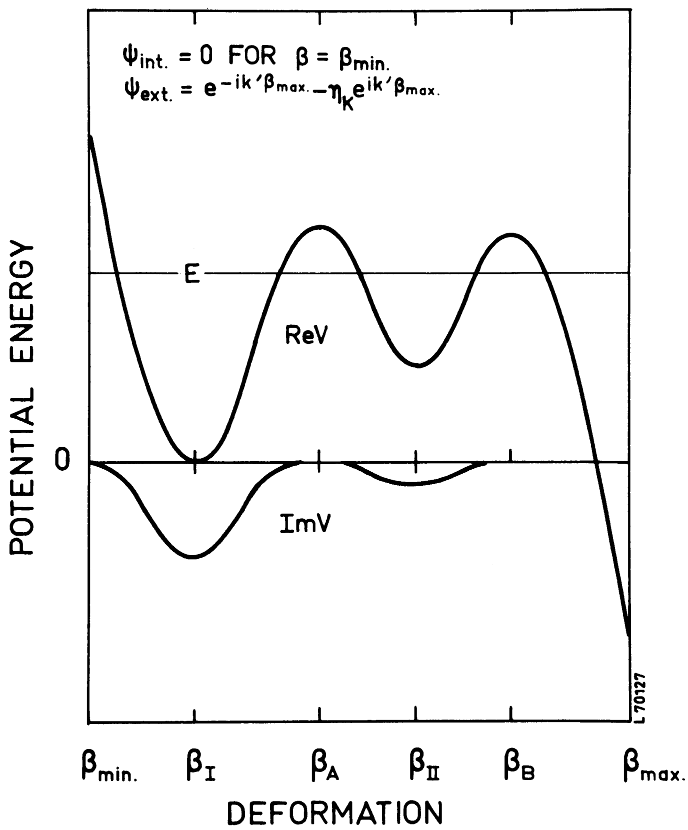

Fig. 2. Complex fission barrier used in the discussion of eq.(6).

For some minimum value β_{min}, one has

$$\Psi_K(\beta_{min}, E) = 0 \tag{8}$$

(For the sake of brevity, we have dropped the $J\pi$ quantum numbers). We now calculate some relevant fluxes for the wave function Ψ_K . The ingoing flux (positive towards the left) is defined

$$S_K = -\frac{\hbar}{2Bi}\left(\Psi_K^* \frac{\partial}{\partial\beta}\Psi_K - \Psi_K \frac{\partial}{\partial\beta}\Psi_K^*\right) \tag{9}$$

One finds that, at the top of the second barrier (point B), the flux is

$$S_K^B = 1 - |\eta_K|^2 \tag{10}$$

At the point A, the flux is $S_K^A < S_K^B$. Thus, the flux which is absorbed in region II is

$$\mathcal{F}_K = S_K^B - S_K^A \tag{11}$$

It is an essential feature of the model that this flux is again distributed into region I and the continuum. At this point, it is worth noting that the absorbed flux in class II may have lost the K quantum number due to mixing and thus only "remembers" $I\pi E$ and the fact that it is of class II. We assume that the re-distribution can be made by some distribution probabilities p for the barriers A and B. (There could be other decay modes of the class II compound states but we neglect them and refer to ref. [7].

In this way, we obtain the total flux transmitted into region I, or the transmission coefficient, which can now be used in formula (2):

for no K-mixing and one channel

$$T_K = S_K^A + \mathcal{T}_K p_K^A \qquad (12)$$

for full K-mixing and all channels

$$\sum_K T_K = \sum_K S_K^A + p^A \sum_K \mathcal{T}_K \qquad (13)$$

The redistribution probabilities could, for example, be related to the Hill-wheeler penetrabilities[12] for single barriers P so that

$$p_K^A = \frac{P_K^A}{P_K^B + P_K^A} \qquad (14a)$$

$$p^A = \frac{\sum_K P_K^A}{\sum_K (P_K^B + P_K^A)} \qquad (14b)$$

Usually one deals with very strong absorption in region I. In this case, the use of an imaginary potential gives some extra oscillatory effects in the wave function. This can be avoided by using, instead of eq.[8], a purely ingoing wave and no absorptive potential in region I. In this way, one has instead, for $\beta = \beta_{min}$, where β_{min} now corresponds to some value close to β_I

$$\Psi(\beta_{min}, E) = \mathcal{C}_K e^{-ik\beta_{min}} \qquad (15)$$

Eq.(15) was used in refs. [1,7,14].

3. RESONANCES

The transmission coefficient for γ-decay, T_γ, is usually a rather small number, for example of the order of 10^{-3} to 10^{-2}, while T_K varies strongly from much smaller values at low energy to the order of unity at higher energy and in the neighbourhood of resonances. We shall investigate how the average branching ratio

$$b = \langle \mathcal{B} \rangle = \frac{\sum_K T_K}{\sum_K T_K + T_\gamma} \tag{16}$$

is varying by the presence of a resonance located at $E = E_o$. (The fluctuation factor $F(J\pi)$ has been put equal to 1 as an approximation).

In a one-level approximation, one can write [11,10,7]

$$S_K^B = \frac{\Gamma_K^B(\Gamma_K^A + \Gamma_K^S)}{(E - E_o)^2 + \frac{1}{4}\Gamma_K^2} \tag{17}$$

$$S_K^A = \frac{\Gamma_K^B\,\Gamma_K^A}{(E - E_o)^2 + \frac{1}{4}\Gamma_K^2} \tag{18}$$

where

$$\Gamma_K = \Gamma_K^A + \Gamma_K^B + \Gamma_K^S \tag{19}$$

The spreading with Γ_K^S is a positive number which is roughly proportional to the strength W_{II} of the imaginary potential in region II. For simplicity, we stick to one K-value only (no K-mixing) and therefore omit the K quantum number. The resonance widths Γ^A and Γ^B can be approximated by

$$\Gamma = \frac{D_{II}^{vib}}{2\pi} P \tag{20}$$

where P is the single-barrier penetration factor and D_{II}^{vib} the vibrational energy distance in region II. If one uses the re-distribution factor (14a), one obtains the branching ratio

$$b = \frac{\Gamma^A\Gamma^B\Gamma/((\Gamma^A + \Gamma^B)\,T_\gamma)}{(E - E_o)^2 + \frac{1}{4}\gamma^2} \tag{21}$$

with the width

$$\gamma = 2\sqrt{\Gamma^A\Gamma^B\Gamma/((\Gamma^A+\Gamma^B)T_\gamma) + \Gamma^2/4} \tag{22}$$

and the integrated area

$$a = \int_{-\infty}^{\infty} b\, dE = \frac{\pi}{\sqrt{\frac{\Gamma^A+\Gamma^B}{\Gamma^A\Gamma^B}T_\gamma\left(\frac{1}{\Gamma} + \frac{\Gamma^A+\Gamma^B}{\Gamma^A\Gamma^B}T_\gamma\right)}} \tag{23}$$

It is seen that both γ and a increase as $\Gamma^S = \Gamma-(\Gamma^A+\Gamma^B)$ increases, or as T_γ decreases. This effect can be rather large, as seen in the next section.

4. COMPARISON WITH EXPERIMENT

In fig. 4, the experimental $^{239}Pu(d,pf)$ fission probability

$$P_f^{(exp)} = \frac{d\sigma(d,pf)}{d\sigma(d,p)} \tag{24}$$

is compared with the theoretical fission probability

$$P_f^{(th)} = \frac{\sum'_{J\pi}\,[d\sigma_{d,p}(J\pi\, E\, \theta_p)]_{av} \langle \mathcal{B}(J\pi)\rangle}{\sum_{J\pi}\,[d\sigma_{d,p}(J\pi\, E\, \theta_p)]_{av}} \tag{25}$$

where $\langle\mathcal{B}(J\pi)\rangle$ is taken from eq.(2). The (d,p) part of the cross section has been calculated as discussed in ref.[13]. The values of the various parameters used in the calculation of $\langle\mathcal{B}\rangle$ are given in table 1. In the calculation, the in-going wave boundary condition method ref.[15] has been used.

Table 1

Some parameters in the calculation

Real potential parameters

Energies	E_A	E_{II}	E_B	$\hbar\omega_A$	$\hbar\omega_{II}$	$\hbar\omega_B$
in MeV	6.05	2.35	5.55	1.00	1.18	0.70

Channel energies

$K\pi$	0+	0-	2+	1-
ε_K(MeV)	0	0.35	0.70	0.90

The potential has been constructed by joining smoothly three vertical parabolas defining the barriers A and B and the well in region II. They can be described by the oscillatory frequences $\hbar\omega_A$, $\hbar\omega_{II}$ and $\hbar\omega_B$ and by the extremum energies E_A, E_{II} and E_B. The values for E_B and $\hbar\omega_B$ are chosen so that they give the observed lifetime of the isomeric state, which is 4 ns. The γ-decay width has been fixed to 0.03 eV and the level distances D(Jπ) are determined from ref. [15]. The rotational constant was Δ_K = 0.005 MeV.

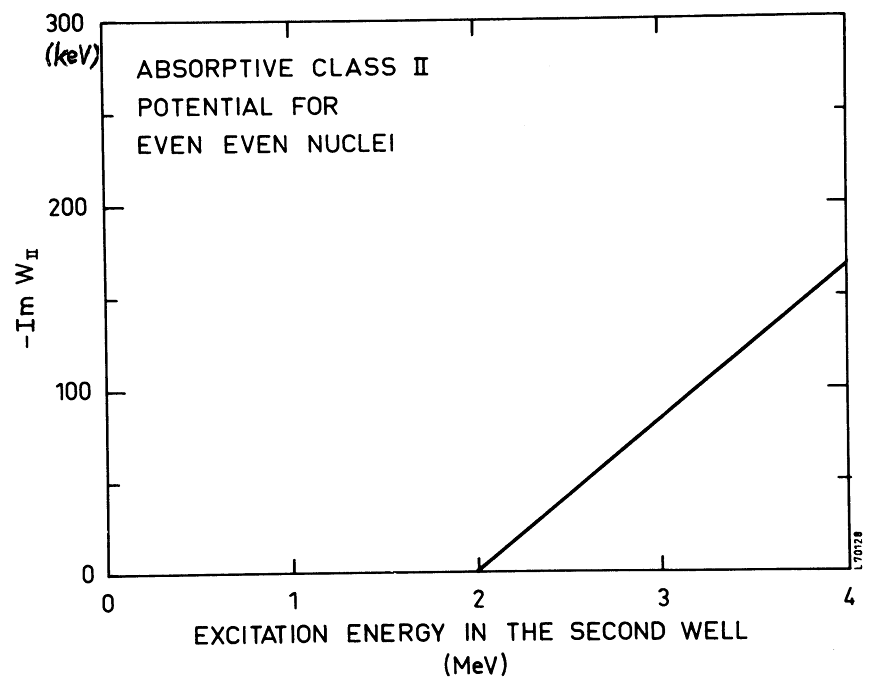

Fig. 3 The imaginary maximum potential strength W_{II} (keV) in the second well as a function of the excitation energy.

In this paper, the absorption potential in region II is of special interest. We expect that the damping of the vibrational states increases with the excitation energy of the fissioning nucleus. In order to describe this, we have assumed a linear dependence of W_{II} on the energy in the energy interval of interest (fig.3). To reproduce the width of the big resonance at 5.0 MeV, we need $W_{II} \approx 70$ keV. Part of the observed width of this and the other resonances is interpreted as due to the rotational energy. For odd nuclei which show less resonance structure, the necessary W_{II} is considerably higher[15]. The effect of the damping and re-distribution on the fission probability is clearly shown in fig.5. The curve without damping has

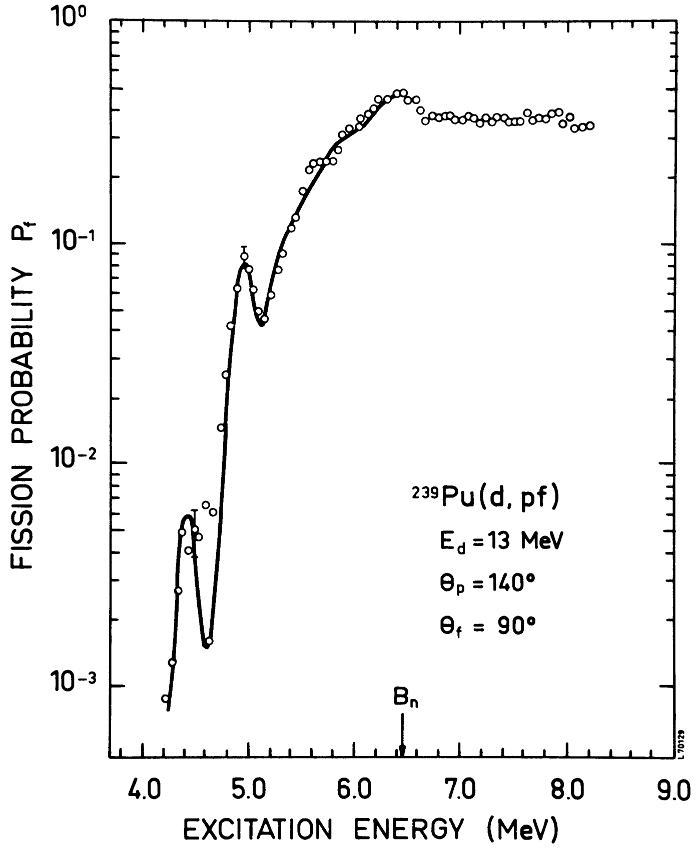

Fig. 4 Fission probability for ^{239}Pu (d,pf). The same data as shown in fig. 1. The theoretical fission probability is calculated with the branching ratio (2) with damping in class II. The parameters used in the calculation are different from those of fig. 1 and described in detail in the text, table 1 and ref.[14].

rather narrow resonances . They are considerably enhanced and broadened by the damping, as shown in the full drawn curve.

In conclusion, one may say that the model with damping and re-destribution gives rise to considerable modification of the calculated fission probability as compared with calculations with no damping. In this way, it has been possible to obtain an agreement with the measured fission probability and to give a consistent set of barrier parameters for a range of nuclei [14]. However, the large number of parameters in the calculations give rise to some uncertainty in the obtained parameters.

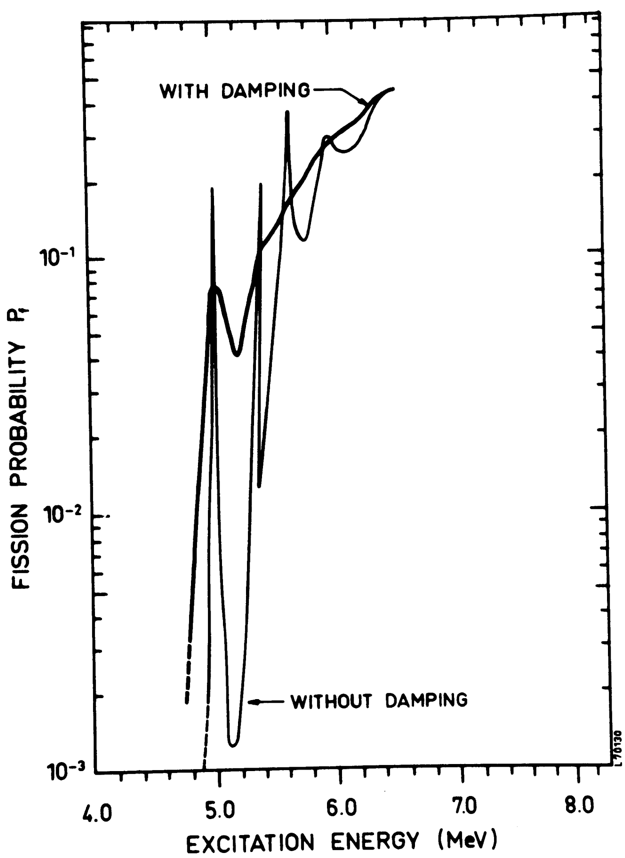

Fig. 5. Calculated fission probability with and without damping. The full drawn curve is identical to that of fig. 4. The thin curve is calculated from a set of parameters identical to that of fig. 4 with the exception that the absorption in the second well is now zero, and that the rotational constant Δ_K is zero for simplicity.

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