The Problem of Radiative Depolarization in the "Siberian Snake"

R. Schwitters

As pointed out by Derbenev and Kondratenko[1] and by the LEP Study Group[2], a "Siberian Snake" may be a convenient method for providing longitudinally polarized beams at LEP. We wish to show that at the highest LEP energies synchrotron radiation with spin-flip[3,4] may depolarize the beams.

As is well known[3,4], the transition probability per unit time for emission of synchrotron radiation with spin-flip is given by:

$$\omega = \frac{5\sqrt{3}}{16} \frac{e^2 \hbar}{m^2 c^5} \gamma^5 |\vec{\dot{\beta}}|^3 \left[1 - \frac{2}{9} \left(\hat{\xi} \cdot \hat{\dot{\beta}} \right)^2 + \frac{8\sqrt{3}}{15} \hat{\xi} \cdot \left(\hat{\beta} \times \hat{\dot{\beta}} \right) \right] \tag{1}$$

where $\hat{\xi}$ is the initial spin direction in the electron rest frame, $\vec{\dot{\beta}}$ is the laboratory acceleration and $\hat{\beta}$, $\hat{\dot{\beta}}$ are unit vectors along the velocity, acceleration directions respectively. γ is the Lorentz factor of the particle.

It can be shown[5] that the spin motion of particles moving on the equilibrium orbit of a storage ring, when viewed at some particular azimuthal position s along the orbit, is simply precession about a fixed direction $\hat{n}(s)$ with a constant phase advance per orbital resolution. In the absence of depolarization phenomena, a beam of particles initially polarized along $\hat{n}(s)$ will remain polarized in this direction. The direction $\hat{n}(s)$ and the spin phase advance per revolution ϕ depend on the lattice of the storage ring. For example, in a conventional storage ring with horizontal bending magnets and no solenoids, $\hat{n}$ points in the vertical direction (direction of the guide magnetic field) and ϕ is given by

$$\phi = 2\pi\nu \qquad \nu \equiv \gamma\left(\frac{g-2}{2}\right) \tag{2}$$

$$= \frac{E_{beam}}{0.44063 \; GeV}$$

In the case of the Siberian Snake, $\hat{n}$ depends on azimutal position, but is perpendicular to the guide field in the normal arcs of the machine and is parallel to the beam direction in the interaction region opposite the snake magnets. In the snake magnets, $\hat{n}$ depends on the exact choice of magnet configuration. In all cases, the phase advance ϕ is equal to π.

Equation (1) shows that the spin states of particles moving in a storage ring change through emission of synchrotron radiation. Neglecting depolarization and spin-orbit coupling effects, the various quantities entering into Eq. (1) are periodic or nearly periodic functions of azimuthal position and the evolution of polarization along the direction $\hat{n}$ can be calculated by averaging Eq.(1) around one orbital resolution. This leads to the following time dependence of the beam polarization along $\hat{n}$:

$$P(t) = P_i \, e^{-t/T} + P_\infty \left(1 - e^{-t/T}\right)$$

$$\text{where} \quad \frac{1}{T} = \frac{5\sqrt{3}}{8} \frac{e^2 \hbar}{m^2 c^5} \gamma^5 \frac{1}{2\pi R} \int_0^{2\pi R} ds \, |\vec{\beta}|^3 \left[1 - \frac{2}{9}(\hat{n}\cdot\hat{\beta})^2\right]$$

$$P_\infty = \frac{8\sqrt{3}}{15} \frac{\displaystyle\int_0^{2\pi R} ds \, |\vec{\beta}|^3 \, \hat{n}\cdot(\hat{\beta}\times\hat{\dot{\beta}})}{\displaystyle\int_0^{2\pi R} ds \, |\vec{\beta}|^3 \left[1 - \frac{2}{9}(\hat{n}\cdot\hat{\beta})^2\right]} \tag{3}$$

P_i is the initial beam polarization and R is the mean radius of the storage ring.

For a conventional storage ring, Eq. (3) gives the familiar result

$$T_0 = \left[\frac{5\sqrt{3}}{8} \frac{e^2 \hbar \gamma^5}{m^2 c^2 \rho^2 R}\right]^{-1} \simeq \frac{98.7 \, \left[\rho(meters)\right]^2 \left[R(meters)\right]}{\left[E(GeV)\right]^5} \; seconds \tag{4}$$

$$P_\infty = \frac{8\sqrt{3}}{15} \simeq 92.4\%$$

where ρ is the bending radius of the machine.

In the simplest case of the Siberian Snake, where the special spin rotation is made with a solenoid, Eq. (3) gives:

$$T = \frac{T_0}{\left[1 - \frac{1}{9}\left(1 - \frac{\sin 2\pi\nu}{2\pi\nu}\right)\right]} \simeq \frac{9}{8} T_0 \tag{5}$$

and $P_\infty = 0$

Therefore, the longitudinal beam polarization will decay exponentially with a time constant nearly equal to the build-up time for transverse polarization in a conventional storage ring. In LEP, T_0 is given by:

$$T_{LEP} \simeq 19 \left(\frac{70 \text{ GeV}}{E} \right)^5 \text{minutes} \tag{6}$$

Other configurations of snake magnets such as the bending magnet schemes suggested in the LEP Design Study will have slightly different time constants and may even leave the beams partially polarized. These have to be studied in more detail.

In this analysis, other depolarization mechanisms which are present in conventional storage rings have been neglected. These are expected to be small because of the inherent stability of the Siberian Snake topology. It has been suggested[1,6] that spin-orbit coupling terms may actually cause a build-up of longitudinal polarization in a Siberian Snake Storage Ring. It is difficult to know whether this effect is practical in a machine like LEP, but it should be studied carefully, as it may provide the best source of polarized beams.

In conclusion, while the Siberian Snake is an attractive idea for providing longitudinally polarized beams at LEP, it appears likely that for energies above 60 or 70 GeV, synchrotron radiation with spin-flip may seriously reduce the average beam polarization.

References

1) Ya.S. Derbenev and A.M. Kondratenko, Novosibirsk Preprint 76-84 (1976).

2) CERN/ISR-LEP/78-17, p. 162 (1978).

3) A.A. Sokolov and I.M. Ternov, Soviet Phys. Dokl. 8, 1203 (1964).

4) J.D. Jackson, Rev. Mod. Phys. 48, 417 (1976).

5) Ya. S. Derbenev and A.M. Kondratenko, Soviet Phys. JETP 35, 230 (1972).

6) Ya. S. Derbenev and A.M. Kondratenko, Soviet Phys. JETP 37, 968 (1973).