WEAK INTERACTIONS WITH MUON BEAMS

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ABSTRACT

The feasibility of studying weak interactions with muon beams at the CERN SPS has been investigated. The measurements of cross-section asymmetries induced by the interference of the one photon and Z^0 propagators were found to be the best suited for the SPS energy domain. The utilization of the large luminosity BCDMS spectrometer could provide $\sin^2\theta_W$ measurements with an error of ± 0.01 and an accurate determination of the muon couplings to the Z^0.

The study of weak interactions can be done profitably with charged leptons at very high energies where the weak cross-sections become considerable. In particular high energy electron proton collision achievable with colliding beam machines such as in the HERA project could produce new heavy leptons coupled to new weak currents and could determine the mass of the W or infer the existence of several Ws through the damping of the charged current cross-section with Q^2. At the CERN SPS the weak force is still dwarfed by the electromagnetic one and the study of its effects remains difficult. Nevertheless it is instructing to see what can be measured with the high quality muon beams available at the SPS along three main topics: exotics, weak charged currents and weak neutral currents.

1. EXOTICS

1.1 Neutrino Counting

In the accepted theory of lepton-hadron interactions the leptons and quarks are arranged into left-handed doublets and right-handed singlets.

The discovery of the τ lepton in 1975¹⁾, which hinted to the existence of a new lepton doublet, was followed in 1977²⁾ by the discovery of a new quark, the bottom, reinforcing our prejudice that to each lepton doublet there corresponds a quark doublet. The number of doublets, or generations, is not fixed by the theory although the existence of three generations of quarks and leptons has been used³⁾ to explain in a natural way CP violation.

Experimentally it might be very difficult to produce new generations of charged leptons or of quarks if they are very massive. Neutrinos can also be used to determine the number of generations. Astrophysical considerations⁴⁾ limit the number of neutrino types, N_ν, to 4 or to some very large number $N_\nu > 10^3$. Phenomenological arguments⁵⁾ set limits on N_ν that vary from 10 to 137 according to the assumptions made for the masses of the charged leptons in the new generations.

The most direct determination of N_ν can be done with the construction of the LEP accelerator through the process$^{6)}$

$$e^+ e^- \to \gamma + Z^0 \tag{1.1}$$

which permits to measure the branching ratio of the Z^0 to $\nu\bar{\nu}$ which is proportional to the number of neutrino generations.

In fixed target experiments N_ν can in principle be measured through the production of $\nu\bar{\nu}$ pairs in the Coulomb field of heavy nuclei$^{7)}$:

$$\mu + A \to \mu + \nu\bar{\nu} + A \tag{1.2}$$

This reaction is mediated by the Feynman diagrams shown in fig. 1. The first two graphs represent the production of $\nu_i\bar{\nu}_i$ pairs via the Z^0. The third graph contributes only to $\nu_\mu\bar{\nu}_\mu$ production via the charged W boson and the contribution of the last graph is negligible since it involves two W exchanges.

A detailed calculation of the cross-sections and of the distributions of relevant kinematical variables has been performed by Barger et al.$^{8)}$. The most striking feature is that the scattered μ is relatively soft, $\langle P_\mu \rangle = 23$ GeV with a beam energy of 280 GeV, contrary to the case of vector meson production, see fig. 2, where the primary muon retains most of the beam energy.

This characteristic can be used to eliminate the only source of background which is given by the production of vector mesons (ρ, ϕ, ψ, T,..) and decaying into $\nu\bar{\nu}$,

$$\mu + A \to \mu + \underset{\hookrightarrow \nu\bar{\nu}}{V^0} + A \tag{1.3}$$

Unfortunately the cross-section for reaction (1.2) is extremely small $\sim 10^{-41}$ cm^2 and cannot be measured with the present generations of synchrotrons unless N_ν is very large $\sim 10^2$. Furthermore, the cross-section grows with the center of mass energy (S) only as S ℓn S, fig. 3, so that this process will not be measurable even at the Tevatron.

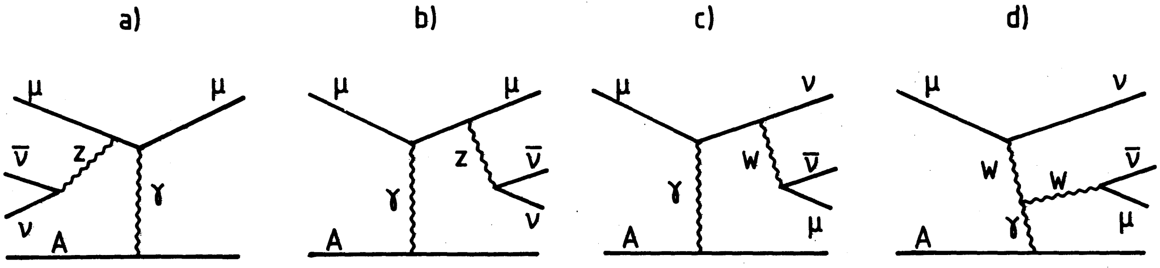

Fig. 1: Feynman diagrams for neutrino pair production by muons.

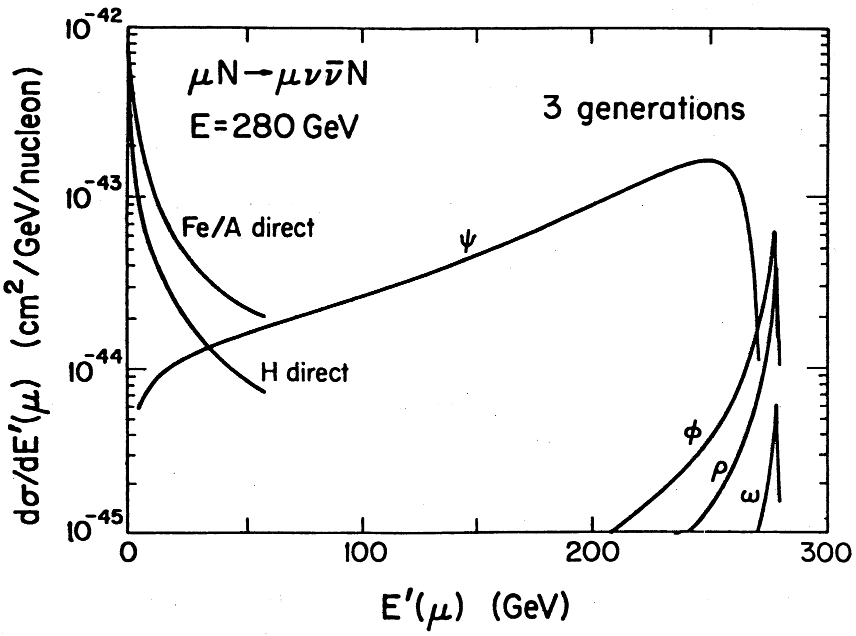

Fig. 2: Final muon energy (E') dependence of neutrino pair production with N_ν = 3 and E = 280 GeV. The contributions from vector mesons decays are also shown (from ref. 8).

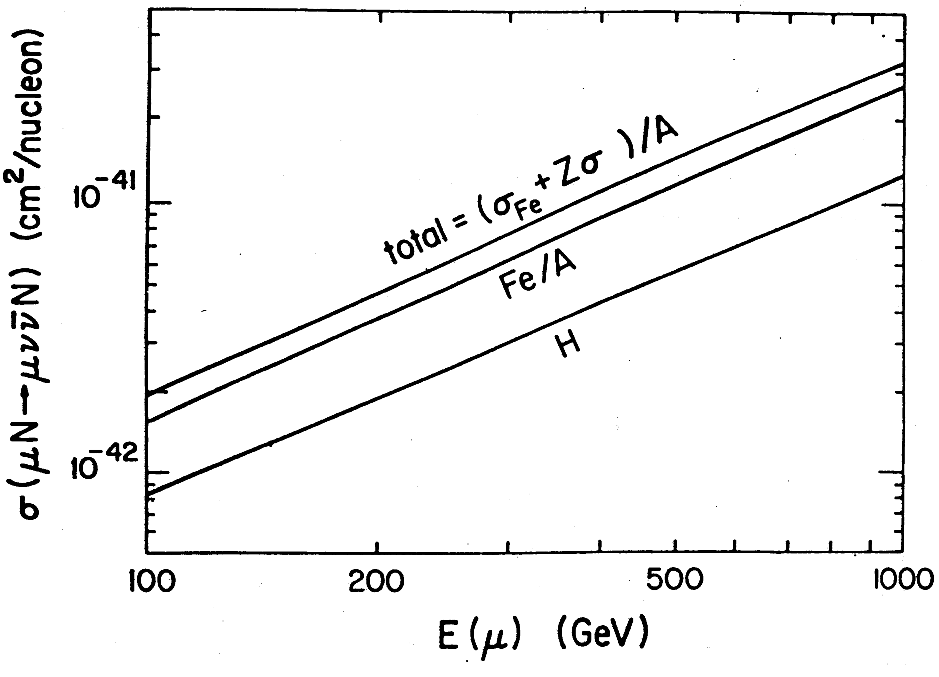

Fig. 3: Total cross-sections per nucleon for neutrino pair production by muons with N_ν = 3 versus incident muon energy E (from ref. 8).

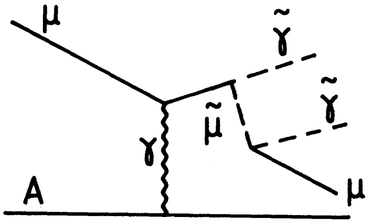

Fig. 4: Dominant Feynman diagram for photino pair production by muons.

1.2 Photino Pair Production

Super symmetric theories (SUSY) predict[9)] a large number of new particles in particular for each fundamental fermion there should be two scalar particles (i.e. for a muon there should be two spin zero particles the super symmetric muons or Smuons ($\tilde{\mu}$) to each fundamental vector particle should correspond a spin 1/2 object (i.e. for the photon there should be a SUSY photon called photino $\tilde{\gamma}$). Little is known about these particles and it is therefore important to set limits on their production although fixed target experiments with present accelerators are not likely to yield any positive result.

In occasion of this workshop J. Ellis and D. Nanopoulos[10)] have pointed out that photinos can be pair produced in the Coulomb field of the nucleus in the reaction

$$\mu + Z \rightarrow \mu + \tilde{\gamma}\tilde{\gamma} + Z \qquad (1.4)$$

which proceeds primarily via the diagram of fig. 4. This process is analogous to the pair production of neutrinos, fig. 1c, with the W propagator being replaced by a $\tilde{\mu}$ and the neutrinos by the photinos.

The cross-section $\sigma(\tilde{\gamma}\tilde{\gamma})$ can be expressed in terms of that for reaction (1.2) ($\sigma(\nu\bar{\nu})$) as follows[10)]:

$$\frac{\sigma(\tilde{\gamma}\,\tilde{\gamma})}{\sigma(\nu\,\bar{\nu})} \simeq \frac{8\sin^4\theta_W}{F_\nu}\left[\frac{m_W}{m_{\tilde{\mu}}}\right]^4$$

with

$$F_\nu = N_\nu\,[4\,\sin^4\theta_W - 2\,\sin^2\theta_W + 1/2] + 4\,\sin^2\theta_W$$

where N_ν is the number of neutrino generations and θ_W is the Weinberg angle. Using the present limit on the mass of the $\tilde{\mu}$ ($m_{\tilde{\mu}}$) of 15 GeV, the world average value for $\sin^2\theta_W$ of 0.23 and three generations of leptons we get

$$\sigma(\tilde{\gamma}\,\tilde{\gamma}) \simeq 185\;\sigma\,(\nu\,\bar{\nu}) \simeq 1.85\;10^{-39}\;\mathrm{cm}^2$$

in the hypothesis of light photinos, $m_{\tilde{\gamma}} \simeq 150$ MeV.

This cross-section albeit small is measurable with a massive uranium target calorimeter as it has been proposed[11)] to study the muon production of $B\bar{B}$ and the Υ. Conversely the absence of a signal can be used to set a lower limit on $m_{\tilde{\mu}}$ within the assumption for $m_{\tilde{\gamma}}$.

The kinematics of $\tilde{\gamma}\,\tilde{\gamma}$ production are the same as those for $\nu\,\bar{\nu}$ production, in particular the interactions have very little hadronic energy deposition and are characterized by a large missing energy since the photinos are expected to escape detection.

The detector need not be particularly sophisticated, the main requirement being a massive calorimeter with good segmentation both lateral and longitudinal to detect the scattered μ and to withstand intense muon beams ($\sim 10^7$ μ/sec). A magnetized calorimeter would be optimal since it would provide the best acceptance for measuring the sign and energy of the scattered muon.

The detector[11)] shown in fig. 5 which consists of a rearrangement of the EMC spectrometer can nevertheless give some useful information for the study of reaction (1.4). Its salient features are an uranium calorimeter with a fiducial mass of 7324 g/cm^2, an open air spectrometer and a muon identifier. A typical run of 100 days assuming an incident beam of 2 10^7 muons/spill with a 2 seconds spill length and 5000 effective spills per day would provide an integrated luminosity of 4.6 10^{40} cm^{-2}.

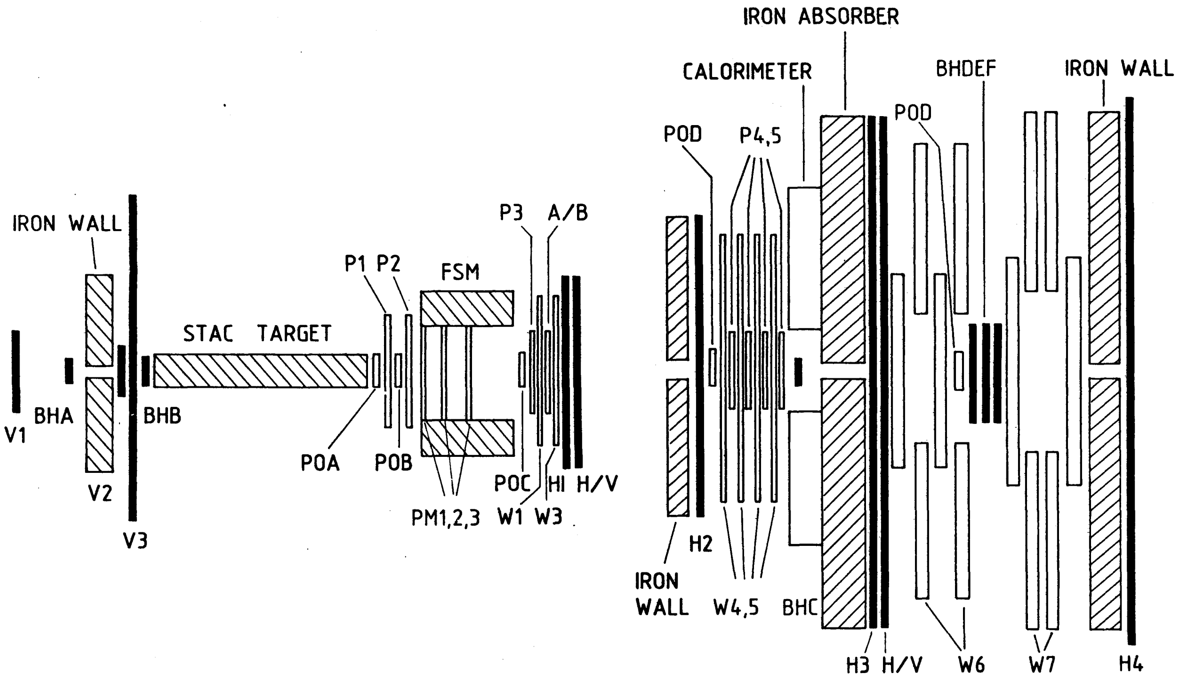

Fig. 5 : Schematic layout of a high luminosity spectrometer with a uranium target calorimeter (from ref. 11).

With a beam energy of 300 GeV this luminosity would yield the following events in function of $m_{\tilde{\mu}}$

$N(\tilde{\gamma}\,\tilde{\gamma})$	$m_{\tilde{\mu}}$ (GeV)
102	17
53	20
22	25
10	30

Since the average scattered muon energy is about 30 GeV it is safe to assume a 50% detection efficiency on the above numbers.

In conclusion the detector proposed$^{11)}$ for the study of multimuon production could be used to increase the limit on the Smuon mass up to 25, 30 GeV.

2. WEAK CHARGED CURRENT INTERACTIONS OF MUONS

In neutrino beams the helicity of the beam particles is fixed to be -1 (left-handed) for neutrinos and +1 (right-handed) for antineutrinos. For muon beams instead the helicity of the beam particles can be adjusted by varying the ratio of the muon momentum P_μ with respect to the parent pion momentum $P_\pi^{12)}$. Muons with $P_\mu \simeq P_\pi$ correspond to pion decays with the μ being emitted along the direction of motion of the pion in the pion rest system (forward decays). The helicity of the muon is then +1 for μ^- (μ^-_R) and -1 for μ^+ (μ^+_L). Muons with $P_\mu \simeq (m_\mu/m_\pi)^2\, P_\pi \simeq 0.57\, P_\pi$ correspond to backward decays and the muons have natural helicity, that is -1 for μ^- (μ^-_L) and +1 for μ^+ (μ^+_R).

Thus with muon beams the following processes can be studied :

$$\mu^-_L + N \rightarrow \nu + X \qquad (2.1)$$

$$\mu^+_R + N \rightarrow \bar{\nu} + X \qquad (2.2)$$

$$\mu^-_R + N \rightarrow \nu_R + X \qquad (2.3)$$

$$\mu^+_L + N \rightarrow \bar{\nu}_L + X \qquad (2.4)$$

The first two reactions are completely equivalent to the standard V-A charged current weak interactions:

$$\nu + N \rightarrow \mu^-_L + X \qquad (2.5)$$

$$\bar{\nu} + N \rightarrow \mu^+_R + X \qquad (2.6)$$

while the other two reactions which cannot be studied with neutrinos are strictly forbidden in pure V-A theories. Therefore they can be used to set stringent limits on the existence of V+A currents as was pointed out by K. Winter$^{13)}$ long time ago.

The interest in righ-handed currents (V+A) has been recently revived by the realization that the present V-A (left-handed) character of the weak interaction may be simply the low energy limit of a more complex reality which embodies in a symmetric way both left-handed and right-handed currents[14]. In models such as the $SU(2)_L \times SU(2)_R \times U(1)$ the electroweak interaction is generated by a set of gauge Bosons $\vec{W}_L$, $\vec{W}_R$ and B according to the Lagrangian.

$$\mathcal{L} = g_L \vec{J}_L \cdot \vec{W}_L + g_R \vec{J}_R \cdot \vec{W}_R + g_y J^y B \tag{2.7}$$

The left right symmetry is then given by the equality of g_L and g_R. The apparent left-handed aspect of the low energy world is attributed to the spontaneous symmetry breaking process which makes the W_R bosons much heavier than the W_L bosons.

The mass eigenstates W_L and W_R are linear combinations of the unmixed chiral eigenstates $W_L^{(0)}$ and $W_R^{(0)}$

$$W_L = W_L^{(0)} \cos\zeta + W_R^{(0)} \sin\zeta \tag{2.8}$$

$$W_R = -W_L^{(0)} \sin\zeta + W_R^{(0)} \cos\zeta \tag{2.9}$$

with

$$m_R >> m_L.$$

Experimentally we are interested in determining two quantities: the mixing angle ζ and the mass ratio $\delta = (m_L/m_R)^2$. The pure V-A world being obtained by $\delta \rightarrow 0$ and $\zeta \rightarrow 0$.

The experimental situation on the limits of δ and ζ and on the possibility of improving such limits has been discussed in detail by G. Vesztergombi[15] at this workshop. Here we briefly recall the main points of that study.

Neutrino experiments are sensitive to the right-handed currents only through the mixing angle given the inherent left-handness of ν beams. They can therefore set limits only on a combination of the mass ratio and the mixing angle. This is usually achieved[16] by comparing the y distributions (hadronic energy/incoming energy) for neutrino and antineutrinos interactions. For large values of the Bjorken x variable (i.e. valence quark region), the presence of a right-handed current term will manifest itself as an excess of events at large y.

With muon beams the mass ratio can be measured directly

$$\frac{\sigma(\mu^-_R N)}{\sigma(\mu^+_L N)} \simeq \left(\frac{m_L^2}{m_R^2}\right)^2 \tag{2.10}$$

as long as the right-handed neutrino is not too heavy (few GeV). An upper limit of 1% for the cross-section ratios would give a lower limit of about 250 GeV for the mass of the right-handed W. At present the best limits have been obtained from the beta decay of the muon$^{17)}$

$$m_R > 220 \text{ GeV}$$
$$|\zeta| < 0.06$$

which is valid only for very light ν_R, and from the K_L-K_S mass difference$^{18)}$

$$m_R > 1.6 \text{ TeV}$$

independently of the mass of the ν_R but with considerable theoretical uncertainties. It is worth recalling that an ep machine of the type proposed in HERA is also expected to set limits on m_R of the order of 500 GeV. Hence a measurement of the cross-section ratio would contribute significantly to the present understanding.

The experimental difficulties in performing this measurement at the SPS are overwhelming. The weak interaction cross-section is still about six orders of magnitude lower than the electromagnetic cross-section and the most severe background is given by the decay of the primary μ inside the hadronic shower produced in low Q^2 interactions.

To overcome this background the target calorimeter must be dense in order to provide a large mass with relative short decay path and must have enough granularity in the beam spot to detect the decay of the primary μ after the shower has been absorbed. The uranium calorimeter of ref. 11) (fig. 5) could be optimized to meet these requirements. Furthermore the presence of the open air spectrometer would be a valuable assets since it could remove the off-momentum muons from the beam spot so that they could be vetoed in a low intensity environment.

As a practical example of background calculation the case of a 200 GeV μ beam with zero polarization impinging on the uranium calorimeter has been considered . The measurement is restricted to $0.2 < y < 0.8$ and a decay length of one meter was chosen for the scattered muon. The muons decaying in this length are considered undetected and the electron energy produced in the decay is added to the hadronic energy. The muons surviving this cut are considered identifiable since they are out of the hadronic shower. Under these assumptions a signal to background ratio of ∿ 10% for μ^+ and 10-20% for μ^- was obtained depending on the y of the interaction.

From the analysis of the transition curves of the hadronic showers it will be possible to identify the decay electrons and to eliminate part of the background. The degree of rejection clearly depends on the degree of segmentation of the calorimeter.

Experimentally the following cross-section can be measured

$$\sigma = \frac{1-\lambda}{2}\,\sigma_L + \frac{1+\lambda}{2}\,\sigma_R + \sigma_B \qquad (2.11)$$

where λ is the polarization of the muon beam, σ_L, σ_R are the cross-sections for left and right-handed currents and σ_B is the normalized background. To a good approximation the background is independent of the beam polarization and measuring (2.11) with different λ (for instance with a 200 GeV μ^- beam usable polarizations can be obtained from $\lambda \simeq + 0.95$ to $\lambda \simeq 0$) it is possible to separate the three contributions.

The proposed uranium calorimeter could be used for a measurement of σ_L but it is doubtful that with such a detector the background could be controlled with the accuracy needed to set a competitive upper limit on σ_R.

3. WEAK NEUTRAL CURRENT INTERACTIONS WITH MUON BEAMS

In the deep inelastic scattering of charged leptons the weak neutral current interaction introduces a dependence of the cross-section on the charge and polarization of the beam. Given the relative strengths of the weak and electromagnetic interaction the only possible way to observe the weak neutral current with charged leptons is through the interference of the photon and the Z^0 propagator, fig. 6. The magnitude and properties of this effect have been studied in detail by many authors$^{19)}$. It is amusing to recall that experiments with muon beams were suggested as a method to search for weak neutral currents prior to their discovery.

The interference cross section is given by

$$d\sigma^{\mp}_{\gamma Z} \equiv \frac{d\sigma^{\mp}_{\gamma Z}}{dQ^2 d\nu} = \frac{G_F}{\sqrt{2}}\;\frac{\alpha}{Q^2\nu}\left\{\left[1+(1-y)^2\right]G_2(x)\;(-v_\mu \pm \lambda a_\mu)\right.$$
$$\left. + \left[1-(1-y)^2\right] \times G_3(x)\;(\pm a_\mu - \lambda V_\mu)\right\} \qquad (3.1)$$

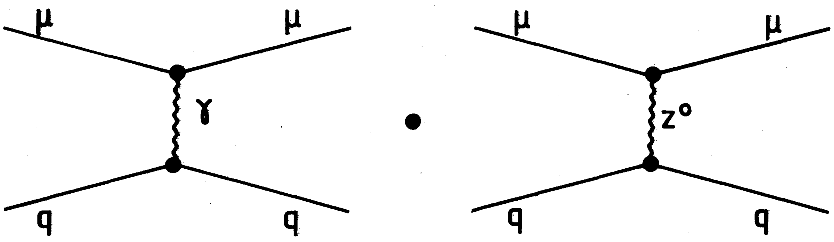

Fig. 6: First order contribution to the electroweak asymmetries.

where v_μ and a_μ represent the vector and axial coupling of the μ to the Z^0 and λ is the helicity of the beam. $G_2(x)$ and $x\, G_3(x)$ are "interference structure functions" analogous to the well known nucleon structure functions $F_2(x)$ and $xF_3(x)$ but containing the coupling of the quarks both to the γ and the Z^0:

$$G_2(x) = 2x\ \Sigma_i v_i Q_i\ (q(x) + \bar{q}(x)) \tag{3.2}$$

$$xG_3(x) = -2x\ \Sigma_i a_i Q_i\ (q(x) - \bar{q}(x)) \tag{3.3}$$

where Q_i, v_i and a_i are respectively the charge and the vector and axial coupling of the quark of type i to the Z^0.

We recall that the deep inelastic cross-section is given in first order by

$$\begin{aligned} d\sigma_0 = \frac{d\sigma_\gamma}{dQ^2 d\nu} &= \frac{2\pi\alpha^2}{Q^4\nu} \left[\frac{y^2}{1+R} + 2\ (1-y) \right] F_2(x,Q^2) \\ &= \frac{2\pi\alpha^2}{Q^4\nu} \left[1+(1-y)^2 \right] F_2\ (x,Q^2) \end{aligned} \tag{3.4}$$

since $R = \sigma_L/\sigma_T$ is known to be very small[20)]. The magnitude of the interference term relative to $d\sigma^0$ is

$$\begin{aligned} \frac{d\sigma^{\mp}_{\gamma Z}(\lambda)}{d\sigma^0} &= \frac{G_F}{\sqrt{2}} \frac{Q^2}{2\pi\alpha} \left\{ \frac{G_2(x)}{F_2(x)} (-v_\mu \pm \lambda a_\mu) + \right. \\ &\quad \left. + \left[\frac{1-(1-y)^2}{1+(1+y)^2} \right] \frac{xG_3\ (x)}{F_2(x)} (\pm\, a_\mu - \lambda v_\mu) \right\} \\ &= K\ Q^2 \left[V(-v_\mu \pm \lambda a_\mu) + g(y)\ A(\pm\, a_\mu - \lambda v_\mu) \right] \end{aligned} \tag{3.5}$$

The constant K, equal to 1.79 10^{-4} GeV^{-2}, determines the size of the effect; about 2% at Q^2 of 100 GeV^2. For the case of an isoscalar target and in the valence quark approximation the structure function ratios reduce to:

$$A = \frac{6}{5} \left(a_d - 2a_u \right) \tag{3.6}$$

$$V = \frac{6}{5} \left(2v_u - v_d \right) \tag{3.7}$$

There are two different kinds of cross-section asymmetry that can be measured:

A. Changing the helicity of the beam

$$\begin{aligned} A^{\mp}(\lambda_1,\lambda_2) = \frac{\sigma^{\mp}(\lambda_1) - \sigma^{\mp}(\lambda_2)}{\sigma^{\mp}(\lambda_1) + \sigma^{\mp}\ (\lambda_2)} &= \frac{\sigma^{\mp}_{\gamma Z}\ (\lambda_1) - \sigma^{\mp}_{\gamma Z}(\lambda_2)}{2\sigma_0} \\ &= -K\ Q^2\ \frac{\lambda_1 - \lambda_2}{2} \left[v_\mu A\ g(y) \mp a_\mu V \right] \end{aligned} \tag{3.8}$$

this asymmetry is manifestly parity violating since it involves only vector, axial-vector combinations of the quark and lepton couplings to the Z^0.

B. Changing the beam charge and helicity

$$B(\lambda_1,\lambda_2) = \frac{\sigma^+(\lambda_1) - \sigma^-(\lambda_2)}{\sigma^+(\lambda_1) + \sigma^-(\lambda_2)} = -K\,Q^2 \left[\frac{\lambda_2 + \lambda_1}{2}\, a_\mu V + \left(\frac{\lambda_1 - \lambda_2}{2}\, v_\mu + a_\mu \right) A\, g(y) \right] \tag{3.9}$$

This asymmetry contains both parity violating and parity conserving terms. As we shall see the measurements is most profitably done with right-handed μ^- ($\lambda_2 = |\lambda|$) and left-handed μ^+ ($\lambda_1 = -|\lambda|$) so that

$$B(|\lambda|) = -K\,Q^2\, g(y) \left[a_\mu - |\lambda|\, v_\mu \right] A \tag{3.10}$$

In the Glashow Weinberg Salam model (standard model)with the world averaged value of the mixing angle $\sin^2\theta_W = 0.23$, the vector coupling of the μ is very small and the asymmetry is essentially parity conserving.

Both types of asymmetry, are very small and their magnitude increases linearly with Q^2. An apparatus designed to measure these asymmetries must therefore have a good acceptance over a long target in order to provide the required high luminosity, a good selectivity on Q^2 at the triggering level to overcome the $1/Q^4$ dependence of the cross-section and a high degree of redundancy in the detectors to minimize systematic errors.

The BCDMS spectrometer, utilized in the CERN SPS NA4 experiment, was designed to study inclusive deep inelastic scattering at very large Q^2 and is therefore particularly well suited for the asymmetry measurements. In facts possible scenarios for measuring both types of asymmetry have been considered[21)] in several collaboration meetings dating back to 1977, and a first measurement of the B asymmetry has been recently completed.

In the following we reconsider the feasibility and scientific interest of performing the asymmetry measurements at the CERN SPS in the coming years using this detector.

Before going into the details of the measurements we recall the salient features of the BCDMS spectrometer shown in fig. 7. A more extensive description has been given elsewhere[22)]. Briefly it consists of ten iron toroids magnetized to saturation (2 Tesla), eight target sections each five meters long and 12 cm in diameter located in the first supermodules, eighty planes of Multiwire Proportional Chambers (MWPC) which provide the measurement of the scattered μ and twenty planes of trigger counters with an annular segmentation which permit a Q^2 dependent trigger. In addition a set of four hodoscopes is used both to define the beam through the target and to measure its divergence. Finally the whole spectrometer is shielded from the beam halo by a wall of veto counters. The trigger used for data taking required the coincidence of four consecutive trigger planes (11 meter long tracks) with the beam $\cdot\ \overline{\text{halo}}$ pulse.

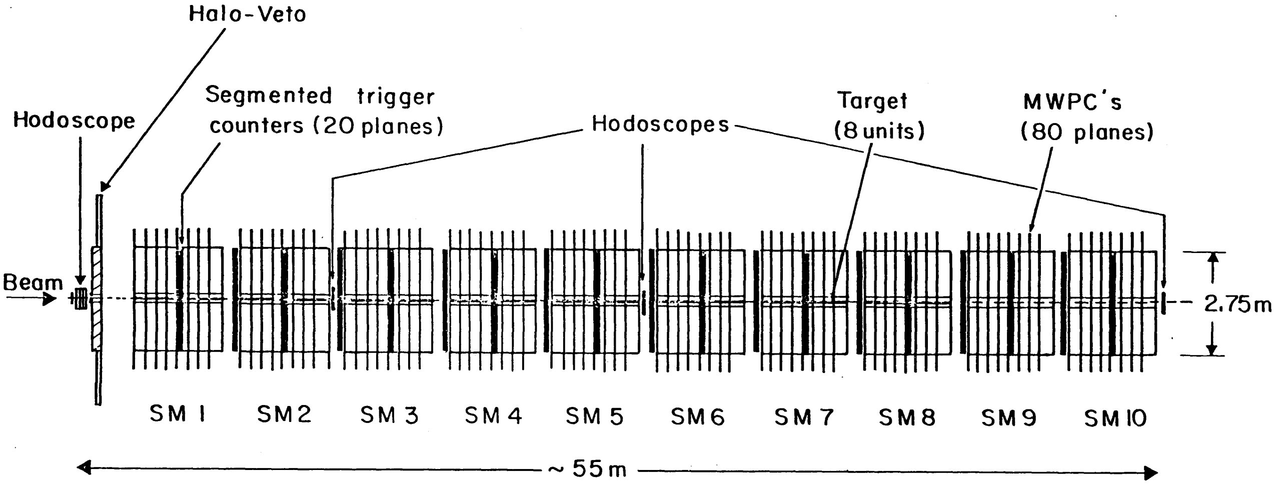

Fig. 7: Schematic layout of the BCDMS spectrometer.

3.1 The B Asymmetry

Experimentally this asymmetry is the least demanding since it can be measured with μ^-_R and μ^+_L that is with muons coming from the forward pion decays which, therefore, provide intense beam at high energies ($E_\mu \sim 200$ GeV). Furthermore, the size of the asymmetry is about four times larger than for the $A^\mp$ asymmetry.

However, higher order electromagnetic processes introduce an asymmetry which tends to compensate the electroweak term. The major contribution comes from the interference between the one photon and two photon exchanges and between lepton and hadron bremsstrahlung. The size of the correction$^{23)}$ is shown in fig. 8 for the measurement of B with 200 GeV muons.

A measurement of the B asymmetry has been performed by the BCDMS collaboration and is described elsewhere$^{24)}$. Here we recall some of the crucial points of the experiment. In this measurement the charge and polarization of the beam are changed by reversing the polarity of all the magnets in the beam line. The field in the spectrometer is also reversed to ensure equal acceptance for the scattered muons.

A reproducibility of the field in the spectrometer with a relative accuracy of $2\ 10^{-4}$ was obtained by keeping the magnet on the same hysteresis loop with a computer control of the excitation current. Similarly the reproducibility of the field in the magnets used to define the beam energy (Beam Momentum Station, BMS) was kept with a precision of $6\ 10^{-4}$. The incident beam intensity was maintained approximately constant at $2\ 10^7$ μ/spill for μ^+ and μ^- in order to minimize systematic errors in the corrections for dead time in the flux counting and in the trigger electronics. This condition severely limited the frequency of polarity switches since the yield of μ^- per proton is about a factor three less than the yield of μ^+ so that a major change in the SPS beam sharing was needed for each polarity switch. Typically the beam polarity was changed three times for each data taking period of about 12 days.

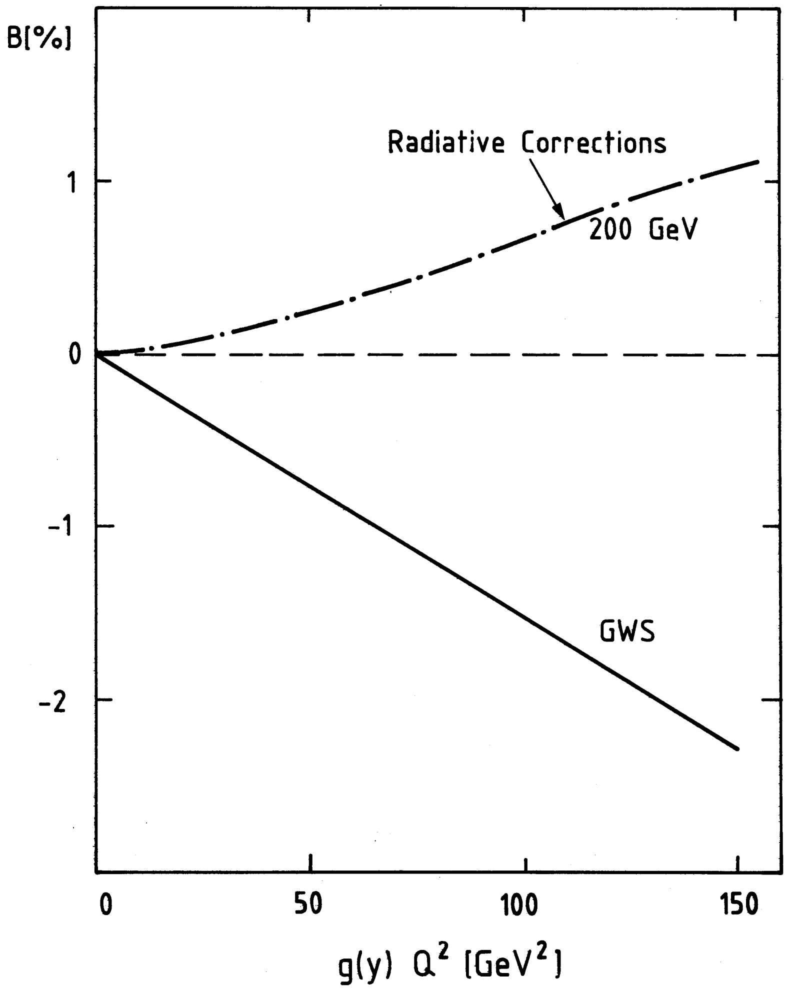

Fig. 8: The B asymmetry from γ-Z^0 interference to first order, calculated for polarization λ = .81 and $\sin^2\theta_W$ = .23 (solid line), and the asymmetry expected from higher order electromagnetic processes at a beam energy of 200 GeV (dash-dotted line).

The data were taken with beam energy of 200 GeV and 120 GeV, the main thrust of the experiment being in the high energy data. The low energy data were taken mostly as a cross-check on systematic errors and radiative corrections.

The asymmetry measurements at both energies are shown in fig. 9 together with a linear fit of the form $B = a + b\, g(y)\, Q^2$. The results of the fit for the slope parameter b:

$$b_{200\ \mathrm{GeV}} = [-\ 1.47 \pm 0.37\ (\mathrm{stat}) \pm 0.2\ (\mathrm{syst})]\ 10^{-4}\ \mathrm{GeV}^{-2}$$
$$b_{120\ \mathrm{GeV}} = [-\ 1.74 \pm 0.75\ (\mathrm{stat}) \pm 0.3\ (\mathrm{syst})]\ 10^{-4}\ \mathrm{GeV}^{-2}$$

are in good agreement with the values of $-\ 1.51\ 10^{-4}$ and $-\ 1.53\ 10^{-4}$ predicted by the standard model for $\sin^2\theta_w = 0.23$ and with $|\lambda| = 0.81$ (200 GeV data) and $|\lambda| = 0.66$ (120 GeV data).

The largest sources of systematic errors are due to the leakage of a halo component in the data which is different for μ^+ and μ^- and to an insufficient accuracy in the reproducibility of the field in the BMS.

A new measurement of the B asymmetry would benefit from several improvements in the BCDMS spectrometer, in particular the construction of a new set of trigger counters, one per supermodule, which improve considerably the redundancy of the trigger. Furthermore, a new fast decision logic based on the pattern of MWPC hits has been implemented with a reduction of about a factor 10 in the raw trigger rate. As a result of these changes the spectrometer can be operated at much higher luminosities either by increasing the target density or the beam intensity. For instance using a 30 m copper target would increase the luminosity of the spectrometer by a factor four and with the SPS running at 450 GeV it would be possible to increase the beam energy to 250 GeV. In these conditions a typical run of 60 days would yield a measurement of the slope parameter b with an accuracy of $\pm\ 0.08\ 10^{-4}\ \mathrm{GeV}^{-2}$ and, benefitting from past experience, the systematic error could be reduced to $\pm\ 0.05\ 10^{-4}\ \mathrm{GeV}^{-2}$.

In the context of the standard model assuming factorization the measured b slope can be used to determine the only free parameter of the model, $\sin^2\theta_w$. The published result of the old measurement is:

$$\sin^2\theta_w = 0.23 \pm 0.07\ (\mathrm{stat}) \pm 0.04\ (\mathrm{syst}),$$

while a new measurement could give an accuracy of

$$\delta\ \sin^2\theta_w = \pm\ 0.016\ (\mathrm{stat}) \pm 0.01\ (\mathrm{syst}).$$

However, the measurement would still be affected by the theoretical uncertainties in the calculations of the radiative corrections which account for a sizeable fraction of the effect itself.

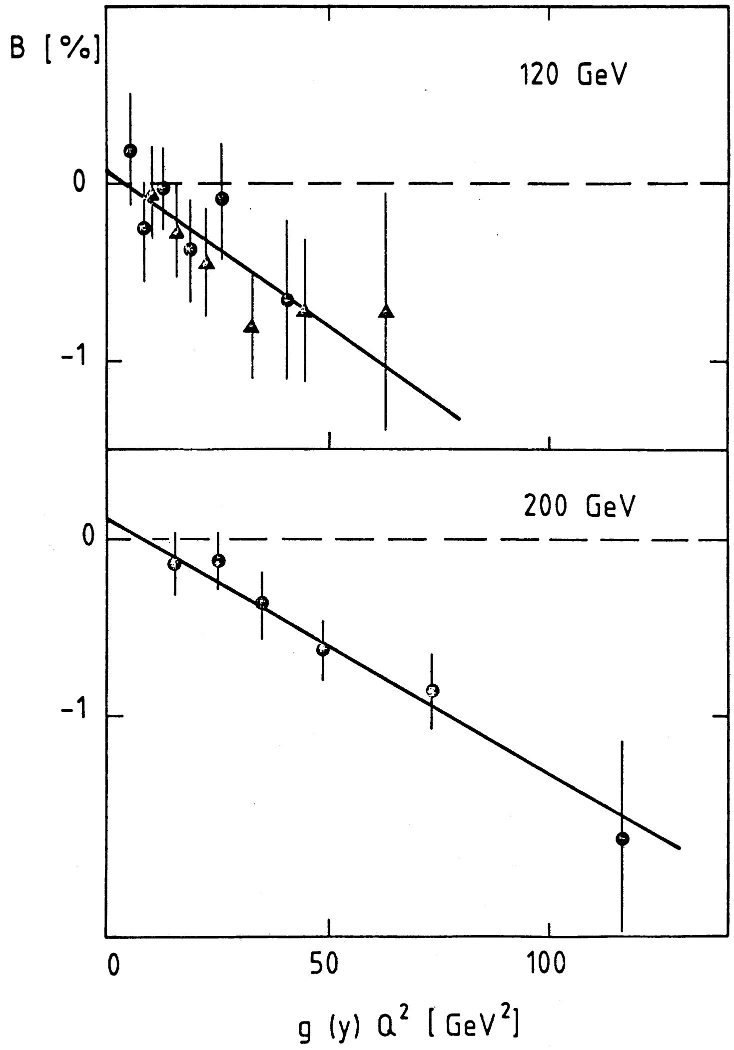

Fig. 9: The measured B asymmetry after radiative corrections at 120 GeV and 200 GeV beam energy vs. $g(y)Q^2 = Q^2 \cdot [1-(1-y)^2]/[1+(1-y)^2]$ for the 120 GeV data, circles represent data with $Q^2 > 15$ GeV^2 and triangles data with $Q^2 > 25$ GeV. Solid lines are straight line fits to the data.

Finally, a high statistics measurement of the B asymmetry could be used to determine the interference structure function $xG_3(x)$ by subtraction of the two cross sections[25]:

$$\sigma^+(-|\lambda|) - \sigma^-(|\lambda|) = -\frac{G_F}{\sqrt{2}}\,\frac{\alpha}{Q^2\nu}\,[\,1-(1-y)^2]\;x\;G_3(x)(a_\mu-\lambda v_\mu)$$

for x bewtween 0.3 and 0.7. This measurement is particularly sensitive to normalization errors between the two data sets requiring a relative accuracy of few parts in 10^{-4} which might not be feasible.

3.2 A asymmetry

This asymmetry was measured for the first time at SLAC[26] using polarized electron beams on a deuterium target and provided the first evidence of parity violation in weak neutral current interactions.

With the conventional hypothesis of factorization and validity of the quark parton model QPM the A asymmetry is given by:

$$A^{\mp} = \pm K\,\frac{\Delta\lambda}{2}\,\frac{6}{5}\,[\,a_\mu(2\,v_u-v_d) \mp v_\mu(a_d-2a_u)\;g(y)]Q^2 \qquad (3.11)$$

where $\Delta\lambda$ is the net helicity change in the two data taking conditions. To illustrate the relative importance of the two terms we introduce the explicit couplings of the standard model:

$$A^{\mp} = \pm K\,\frac{\Delta\lambda}{2}\,\frac{6}{5}\,[\,-\frac{3}{4}+\frac{5}{3}\,\sin^2\theta_w) \pm \frac{3}{2}\,(-\frac{1}{2}+2\,\sin^2\theta_w)\;g(y)]Q^2 \qquad (3.12)$$

With the standard value of $\sin^2\theta_w = 0.23$ both terms are negative with the second term being about 1/4 of the first. In the measurements of A^+ the two contributions subtract and the asymmetry is quite insensitive to the value of $\sin^2\theta_w$. This is illustrated in fig. 10 where the asymmetry at fixed Q^2 is plotted in function of $\sin^2\theta_w$. Hence the measurement of A^+ alone is not recommended although it is experimentally easier given the higher fluxes achievable with μ^+ beams.

On the other hand the measurement of A^- by itself would give a competitive determination of $\sin^2\theta_w$ and would also provide a confirmation of parity violation at Q^2 values roughly two orders of magnitude larger than in the original SLAC experiment.

For a model independent analysis of the weak neutral current couplings it is important to have as many independent determinations as possible of the various couplings. From the dependence of the A asymmetry, on y introduced by the term g(y) it is in principle possible to separate the two axial-vector and vector combinations. In practice the resulting errors are rather large but a much better measurement can be obtained by combining the A^+ and A^- measurements.

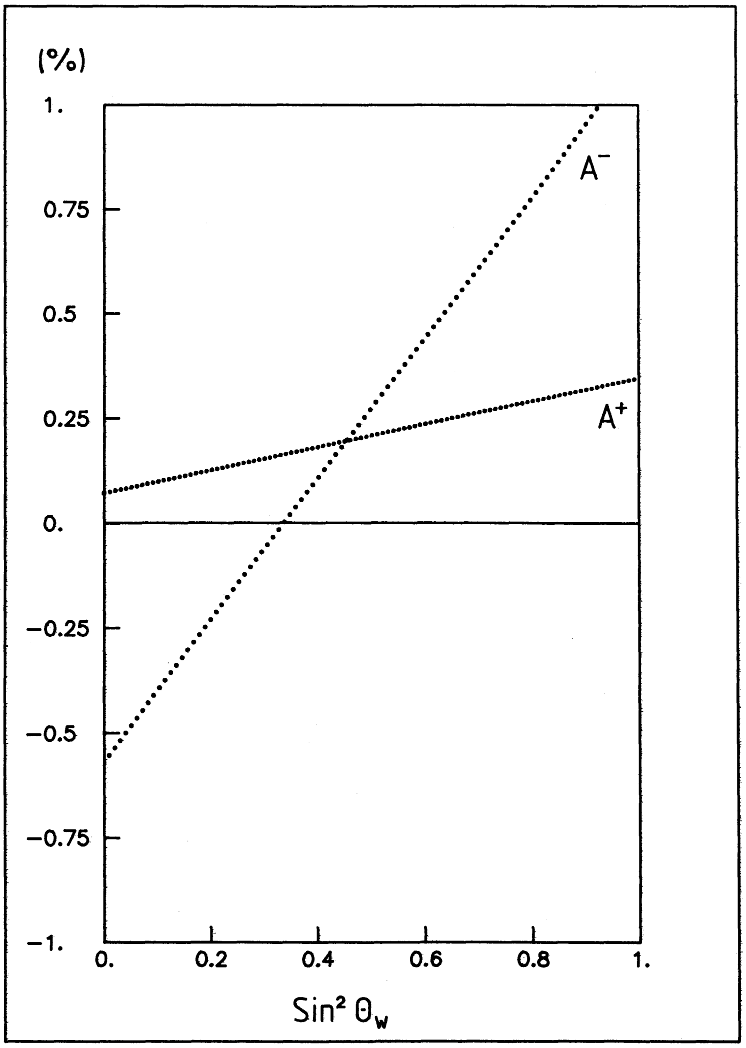

Fig. 10: $A^{\pm}$ asymmetries at Q^2 = 100 GeV^2 and $\Delta\lambda$ = 1.0 versus $\sin^2\theta_W$.

The size of these asymmetries is directly proportional to $\Delta\lambda$ and Q^2 hence the choice of the running conditions depends on two conflicting requirements, obtaining a large statistics of events at large Q^2 (100-200 GeV^2) and retaining a sufficiently large $\Delta\lambda$. The first condition requires intense beams at high energy while changing the helicity of the beam requires increasing values of the parent pion energy relative to the energy of the muon beam, so that the yield of muons per interacting proton quickly decreases with increasing muon energy. The situation is of course worse for the A^- measurement since the yield of energetic π^- is less than for π^+.

With the SPS operating at 450 GeV it is feasible to measure $A^\mp$ with a beam energy of 200 GeV, a $\Delta\lambda$ of 1.0 and an integrated flux of 1.6 10^{12} muons at each beam condition. A Monte-Carlo calculation based on this integrated flux on a 30 meter copper target in the BCDMS spectrometer predicts 8 10^6 reconstructed interactions in the kinematic domain $Q^2 > 30$ GeV^2, $.2 < x < .8$, $.2 < y < 0.85$ where the acceptance of the detector is well behaved.

Assuming the standard model with $\sin^2\theta_w = 0.23$ the Monte-Carlo calculation gives the asymmetries for A^+ and A^- plotted in fig. 11 in function of Q^2. The errors are purely statistical and the points are not randomized. A linear fit to these points gives a Q^2 slope of $(0.33 \pm .11)10^{-4}$ for A^+ and $(-0.46 \pm -0.11)10^{-4}$ for A^- that is a raw signal of 3 to 4 standard deviations.

In the standard model assuming factorization and universality we can determine $\sin^2\theta_w$ from a fit to (3.11) with an accuracy of $\pm .10$ for A^+ and ± 0.01 for A^-. From the combined fit to A^+ and A^- the following axial coupling combinations can be determined in a model independent way

	μ Cu	SLAC ed 26)
$a_\mu (2v_u - v_d) =$	$- 0.37 \pm 0.04$	(-0.45 ± 0.12)
$v_\mu (2a_u - a_d) =$	$- 0.06 \pm 0.05$	(0.23 ± 0.38)

Assuming the quark couplings known these values determine completely the couplings of the muon:

$$a_\mu = - 0.5 \pm 0.05$$
$$v_\mu = - 0.04 \pm 0.04$$

The systematic errors in the case of the $A^\mp$ asymmetry are less demanding than for the B asymmetry since the only change during the data taking is the setting of the pion channel. This however will affect the composition of the halo in the beam and to some extent the beam phase space. Furthermore, the polarization of the beam must be known accurately since the size of $A^\pm$ is directly proportional to $\Delta\lambda$. An accuracy of 10% on $\Delta\lambda$ propagates an error of ± 0.005 for $\sin^2\theta_w$, similarly a systematic uncertainty of 10% introduces again a systematic error of ± 0.005. The

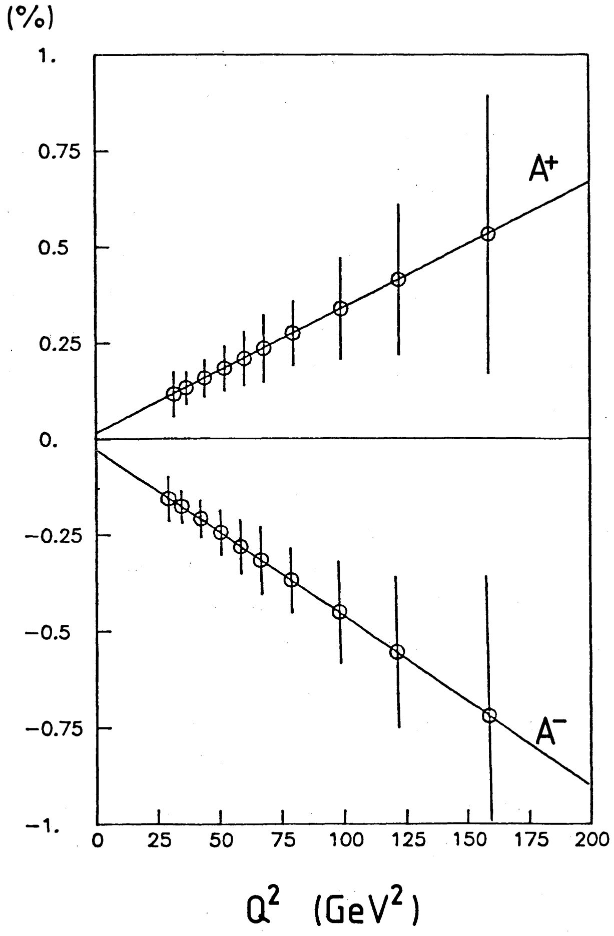

Fig. 11: Monte-Carlo predictions for $A^{\pm}$ measurements with $\sin^2\theta = 0.23$, $E_\mu = 200$ GeV, $\Delta\lambda = 1.0$ and 1.6 10^{12} muons on a a 30 meter long copper target.

polarization of the M2 beam line has already been measured with 200 GeV energy for the measurement of the B asymmetry. In this workshop F.L. Navarria[27)] presented a detailed plan for measuring and monitoring the polarization of the muon beam with an accuracy of ± 0.05 which is sufficient for the proposed accuracy on $\sin^2\theta_W$. Finally since no other part of the detector or of the beam line is changed it is relatively easy to maintain the apparatus stable for the period of the measurement (∿ 100 days).

3.3 Beam Requirements for $A^{\pm}$ measurements

The best estimates for muon yields with the SPS operating at 450 GeV are based on some old measurements of muons yields and pion production yields[28)].

SPS Energy 450 GeV	E_π(GeV)	E_μ(GeV)	$< \lambda >$	Yield μ/proton
	220^+	200	-0.8	$1.6\ 10^{-5}$
	300^+	200	+0.1	$2.8\ 10^{-6}$
	220^-	200	+0.8	$6.1\ 10^{-6}$
	300^-	200	-0.1	$1.\ 10^{-6}$
	212^+	200	-0.9	$8.\ 10^{-6}$
	212^-	200	+0.9	$3.1\ 10^{-6}$

where the yields for $|<\lambda>| = 0.9$ have been estimated from Monte-Carlo calculations of the muon yield versus polarization.

The required integrated muon flux of $1.6\ 10^{12}$ could be obtained with the following beam conditions:

	$< \lambda >$	Proton/Pulse	Days	Integrated Protons
μ^-	0.9	$5.2\ 10^{12}$	20	$5.2\ 10^{17}$
	-0.1	$5.2\ 10^{12}$	65	$17.\ 10^{17}$
μ^+	-0.9	$3.2\ 10^{12}$	15	$2.4\ 10^{17}$
	+0.1	$3.2\ 10^{12}$	35	$5.6\ 10^{17}$

assuming an overall efficiency for data taking of about 70% which was found realistic in the past.

The instantaneous beam intensities are rather low, at most $1.3\ 10^7$ μ/sec for μ^+ with $< \lambda > = -0.9$ and a spill length of 2 sec, and will not pose any problems for beam counting or in the performance of the hardware. It is therefore advantageous to maintain a constant intensity of protons per pulse so that the operation of the SPS will not be affected by the changes in polarization.

With the standard 100 days of operation per year of the SPS for fixed target physics the A^- measurement could be done within one calendar year and the A^+ with somewhat less time. The overall cost in protons is of 2.2 10^{18} for A^- and 0.8 10^{18} for A^+.

3.4 Two Measurements of B

An interesting alternative consists in making two measurements of the B asymmetry at two different beam polarizations λ_1 and λ_2. From the values of the fitted slopes in function of $Q^2 \cdot g(y)$:

$$b_1 = -(a_\mu - \lambda_1 v_\mu)\, K\, A \tag{3.13}$$

$$b_2 = -(a_\mu - \lambda_2 v_\mu)\, K\, A \tag{3.14}$$

is possible to obtain a model independent relation between the vector and axial vector couplings of the muon

$$v_\mu = a_\mu \quad \frac{b_1 - b_2}{b_1\lambda_1 - b_2\lambda_2} \tag{3.15}$$

This relation is independent of the validity of the QPM and is also independent on the parameter ρ which measures the relative strength of the weak neutral current and charge current interaction. In the previous discussion a value of $\rho = 1$ was implicitely assumed in the definition of the constant K. In the standard model (3.15) determines $\sin^2\theta_W$:

$$\sin^2\theta_W = \frac{1}{4} - \frac{1}{4} \quad \frac{b_1 - b_2}{b_1\lambda_2 - b_2\lambda_1} \tag{3.16}$$

Using the same beam conditions as for the $A^\pm$ measurements and utilizing the same integrated μ flux of 1.6 10^{12} μ, the measurements of B at $\lambda_1 = 0.9$ and $\lambda_2 = -0.1$ would yield the following values for the μ couplings:

$a_\mu = -\ .5 \pm .04$
$v_\mu = -\ .0 \pm .06$

and an accuracy on $\sin^2\theta_W$ of ± 0.03 but without any assumption on the value of ρ or on the validity of the QPM. These values are however still sensitive to the theoretical uncertainties involved in the radiative corrections to the data.

Finally it is important to notice that an experiment carried out to measure the B asymmetry at two different beam polarizations cannot in practice give also a measurement of $A^\pm$ since the strategy of data taking is quite different for the two cases in order to minimize the systematic errors.

4. PROSPECTIVES FOR ASYMMETRY MEASUREMENTS

It is difficult to foresee the relevance of a combined A^+, A^- measurement which would give results about two years after completion of data taking considering the huge quantity of data to be processed, $\sim 50\ 10^6$ interactions. This would mean 1987-1988 for final results, just before the foreseen LEP operation. Furthermore, the operation of the $p\bar{p}$ colliders at CERN and at Fermilab should by then have detected the Z^0 and measured its mass with a few percent accuracy.

Nevertheless, accurate determination of $\sin^2\theta_w$ will still be relevant even after the Z^0 has been discovered and its mass measured at LEP since they will permit to check higher order weak contributions and thus prove the renormalizability of the theory. Grand unified theories that include the standard model as a low energy limit also make predictions which are very close to the experimental values for $\sin^2\theta_w$.

A sharpening of the experimental errors is extremely important$^{29)}$ to test these theories. The threshold of interest for new determinations of $\sin^2\theta_w$ seems to be about $\delta\sin^2\theta_w \sim 0.01$ but again it should be remembered that different experiments will have different systematic errors and theoretical uncertainties so that many independent determinations of $\sin^2\theta_w$ will still be valuable.

For instance the asymmetry measurements will provide a $\sin^2\theta_w$ using interactions at very high Q^2 and high x in a well definite kinematic domain of x and Q^2. The experiments with neutrino beams have little control of the x and Q^2 region used for the measurement since inclusive weak neutral current interactions contain only one well measured variable, the hadronic energy. Furthermore, in these experiments the determination of $\sin^2\theta_w$ depends on the measurement of the ratio of neutral to charged current interactions both for neutrinos and antineutrinos:

$$\frac{1}{2} - \sin^2\theta_w = \frac{R - r\bar{R}}{1 - r}$$

$$\text{with } R = \frac{\sigma^{\nu}_{Nc}}{\sigma^{\nu}_{cc}} \ , \quad \bar{R} = \frac{\sigma^{\bar{\nu}}_{Nc}}{\sigma^{\bar{\nu}}_{cc}} \ , \quad r = \frac{\sigma^{\bar{\nu}}_{cc}}{\sigma^{\nu}_{cc}}$$

The most important theoretical uncertainties involve the size and composition of the non strange component of the sea which is sampled differently by neutral currents (flavour conserving) and charged current (flavour changing) interactions, and the effects of scaling violations.

In the asymmetry measurements the contribution of the sea is small because the interactions are at $x > 0.2$ and the error on the correction is negligible. Similarly scaling violations appear in the same way in the numerator and denominator and cancel out from the final asymmetry. Finally the higher order weak contributions affect differently the $\sin^2\theta_w$ determination from neutrino interactions and from the $A^{\pm}$ measurements so that the two independent measurements would be largely complementary.

Finally it has been recently pointed out by Sehgal[30)] that the determination of the axial-vector, vector combinations of the quark muon couplings which are measured with $A^{\pm}$ can put severe constraints on theories like the left right symmetric model which have more than one Z^0.

In summary the asymmetry measurements can still contribute to our understanding of weak neutral currents and to test the standard model in a more stringent way.

CONCLUSIONS

In the context of this workshop we have considered what could be done with muon beams among three major topics: exotics, weak charged current and weak neutral currents. The first two topics require a construction of a massive target calorimeter with fine segmentation that could also be used to study the muon production of the T and $D\bar{D}$, $B\bar{B}$ pairs. With such a detector the limit on the mass of the scalar muon could be increased from 15 GeV to 26 GeV, by studying photino pair production in the field of the nucleus. Weak charged current interactions could also be detected and their cross-section measured. A limit on the existence of right-handed currents as implied by the left right symmetric models is much harder to achieve since the measurement requires the control of the background to a degree which might not be realistic to obtain. For both experiments the Tevatron could provide a serious competition since the factor two increase in energy with respect to CERN will be a very valuable asset and the BFP detector[31)], already scheduled to run, could do a good job.

The weak neutral current interactions can still be studied quite profitably at the CERN SPS with the BCDMS spectrometer. The experience gained in the measurement of the charge asymmetry "B" could be utilized in the measurement of the parity violating asymmetries $A^{\pm}$ thus establishing parity violation in weak neutral current interactions at a Q^2 about two orders of magnitude larger than in the first measurement at SLAC. This measurements using a well tested apparatus would provide a determination of $\sin^2\theta_W$ with an error of ± 0.01 and with theoretical uncertainties which are quite different and in many respects smaller than with the traditional neutrino experiments. Finally the concept of μ-e universality could be tested in a new Q^2 regime and new more accurate constraints would be provided both for the standard model and for larger models which incorporate the standard model as a low energy limit. The measurement of the $A^{\pm}$ requires a good control of the systematics in the beam conditions and in the detector. Clearly the experiment is not first generation and the factor two increase in energy at the Tevatron could not offset the advantage of a well tested beam and a particularly well suited detector.

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