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FURTHER ANALYSIS OF $N^*\rho$ AND $N^*\omega$ QUASI-TWO-BODY REACTIONS BY 5 GeV/c π^+ MESONS ON PROTONS

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INTRODUCTION

An analysis of double resonance production by 5 GeV/c π^+ mesons on protons has been presented at the Heidelberg Conference ([1]). The analysis was based on a sample of approximately 6000 events of the type:

$$\pi^+ p \to \pi^+ p \pi^+ \pi^- \qquad (1)$$

and approximately 6500 events of the type:

$$\pi^+ p \to \pi^+ p \pi^+ \pi^- \pi^o \qquad (2)$$

For both the reactions:

$$\pi^+ p \to N^{*++}_{3/2,3/2} \rho^o \qquad (3)$$

and

$$\pi^+ p \to N^{*++}_{3/2,3/2} \omega^o \qquad (4)$$

we presented results for $d\sigma/d|t|$ and the (single-vertex) density matrix elements as a function of $|t|$.

Comparisons with the absorption model predictions were made. For the $N^*\rho$-channel a satisfactory over-all agreement with the predictions of the one-pion-exchange absorption (OPEA-)model was found. Our $N^*\omega$-results however, differed appreciable from the predictions of the absorption model with ρ-meson exchange.

For both channels we looked for a possible correlation between the polar angles of the vector meson- and the isobar-decay ($\Theta_{\rho(\omega)}$ and Θ_{N^*}) using the method originally employed by G. Goldhaber et al. at 3.65 GeV/c. Ref. ([2]). For the $N^*\rho$-channel the existence of such a correlation was established; its magnitude was found to be in rough agreement with the expectations from the OPEA-model. For the $N^*\omega$-channel no statistically significant correlation of the above type could be detected.

The present paper deals with preliminary results of a further and more detailed analysis of the reactions (3) and (4).

Five topics will be (briefly) discussed:

I. The differential cross-section as a function of $|t - t_{min}|$ ($|t_{min}|$ being the minimum value of $|t|$ allowed by the masses at both vertices).

II. The constraints implied on the single-vertex density matrix-elements by the positivity of spin density matrix (in function of $|t|$).

III. Preliminary results of an attempt to evaluate the effect of background on the spin-density matrix-elements.

IV. The results of an evaluation of the average value of all joint-decay density matrix-elements (including those expressing correlations between the two vertices) and a study of the variation of these elements as a function of $|t - t_{min}|$.

V. A test of the relations between the spin-density matrix elements obtained by the quark-model and the so-called additivity assumption (Bialas & Zalewski).

We refer to the Heidelberg-paper for all notations and conventions used in this paper and not explicitely redefined here ([1]).

I. DIFFERENTIAL CROSS-SECTIONS $d\sigma/d|t - t_{min}|$.

The differential cross-section for reactions (3) and (4) as a function of $|t - t_{min}|$ is shown in figs. 1 and 2 ([4]).

For the $N^*\rho$-channel the "round-off" of the $d\sigma/d|t|$-distribution at small $|t|$-values has completely disappeared and the $d\sigma/d|t - t_{min}|$-curve remains a pure exponential in $|t - t_{min}|$ up to the lowest transfer allowed. For the $N^*\omega$-channel we observe a plateau extending from zero to about 0.2 $(GeV/c)^2$; above 0.2 $(GeV/c)^2$ the $d\sigma/d|t - t_{min}|$-distribution shows a smooth exponential decrease.

In table I we compare the slopes of the $d\sigma/d|t|$- and $d\sigma/d|t - t_{min}|$-distributions for both the $N^*\rho$ and $N^*\omega$-channel.

II. POSITIVITY CONDITIONS ON THE SPIN-DENSITY MATRIX-ELEMENTS.

P. Minnaert has stressed that the (single-vertex) spin-density matrix-elements must satisfy certain conditions due to the fact that this (hermitian) matrix has positive eigenvalues and unit trace ([5]).

For a spin 1 particle, and for any frame of reference which has the quantization axis in the (2-body) production plane (i.e. the Jackson reference-frame), these conditions are:

$$|\rho^{1,-1}| \leqslant \frac{1 - \rho^{00}}{2} \tag{5}$$

$$|\mathrm{Re}\ \rho^{10}| \leqslant \frac{1}{2}\sqrt{\rho^{00}(1 - \rho^{00} - 2\rho^{1,-1})} \tag{6}$$

For a spin 3/2 particle we find:

$$0 \leqslant \rho_{33} \leqslant \frac{1}{2} \tag{7}$$

$$(\mathrm{Re}\ \rho_{3,-1})^2 + (\mathrm{Re}\ \rho_{31})^2 + (\rho_{33} - \tfrac{1}{4})^2 \leqslant \frac{1}{16} \tag{8}$$

Fig. 3 shows a plot for the $N^*\rho$-channel of $\rho^{1,-1}$ and $\mathrm{Re}\ \rho^{10}$ ($\mathrm{Re}\ \rho_{3,-1}$ and $\mathrm{Re}\ \rho_{31}$) in function of ρ^{00} (ρ_{33}). The projection of the positivity domain on the 2 dimensional space of the parameters considered, is shown by dashed lines. Positivity requires the points to lie inside this domain. The points with error-flags correspond to the values obtained for the 3 different regions of momentum transfer considered. (See fig. 4 - Ref. (1)). The black dots correspond to the values of the density matrix elements averaged over a $|t|$-interval from 0-0.3 $(GeV/c)^2$.

Fig. 4 shows a similar plot for the $N^*\omega$-channel. In this case 4 different $|t|$ regions were considered (See fig. 8 - Ref. (1)) and the black dots now correspond to averages over a $|t|$ range from 0-0.6 $(GeV/c)^2$.

The conclusion is that for both the $N^*\rho$ and $N^*\omega$ channel, the density matrix elements at each vertex are in excellent agreement with the positivity requirements.

III. BACKGROUND EFFECTS.

We have tried to obtain a feeling for the importance of the background-effects on the (single-vertex) density-matrix-elements by the following very simple-minded procedure:

a) evaluation of the density-matrix-element expressions for events lying resp. above, below, to the left and to the right of the double resonance mass-region considered (6);

b) averaging of the values obtained under a) (ρ_{BG});

c) estimation of the number of background events in the double resonance region by 2 dimensional interpolation (N_{BG});

d) calculation of a corrected density-element value ρ_c by means of the expression:

$$\rho_c = \frac{\rho \cdot N - \rho_{BG} \cdot N_{BG}}{N - N_{BG}} \tag{9}$$

where N is the total number of events in the double resonance region and ρ the uncorrected density-matrix-element.

The results are shown in table II. No significant effects are observed. In view of the uncertainties connected with the procedure sketched above (especially in the case of $N^*\rho$-production) we prefer to look upon the results of table II as an indication that no dramatic background effects are distorting our data and we will continue to use the uncorrected values in our analysis.

IV. JOINT SPIN-DENSITY MATRIX-ELEMENTS.

The general joint-decay angular distribution for a spin 1 and spin 3/2 resonance in terms of the orthogonal functions of the two polar and two azimuthal (Jackson) angles and the joint-decay matrix-elements involved, was given by Pilkuhn and Svensson (7) to be of the form:

$$W(\Omega_1, \Omega_{3/2}) = 1 + W_1(\Omega_1) + W_2(\Omega_{3/2}) + W_3(\Omega_1, \Omega_{3/2}) \qquad (10)$$

where the index 1 stands for the vector-meson and the index 3/2 for the N^*.

Inspection shows that $1 + W_1(\Omega_1)$ and $1 + W_2(\Omega_{3/2})$ are the expressions used in the separate spin-density matrix-analyses of the corresponding vertices. $W_1(\Omega_1)$ is a linear expression of the density elements labeled 1 to 3 in table III. Similarly $W_2(\Omega_{3/2})$ is a linear expression of the left-hand side quantities labeled 4 to 6 in table III.

Without any physical angular correlation between the two vertices the overall distribution would read:

$$W'(\Omega_1, \Omega_{3/2}) = 1 + W_1(\Omega_1) + W_2(\Omega_{3/2}) + W_1(\Omega_1)\, W_2(\Omega_{3/2}) \qquad (11)$$

This implies that the presence of correlations between the two vertices will manifest itself by the non-vanishing of:

$$W'' = W_3(\Omega_1, \Omega_{3/2}) - W_1(\Omega_1)\, W_2(\Omega_{3/2}) \qquad (12)$$

The expression W" can be written as a linear sum of the quantities labeled 7 to 19 in table III (7).

For the case of $N^*\rho$ we have introduced two additional angular terms in the expansion of Pilkuhn & Svensson in order to parameterize the observed ρ^o-decay asymmetry in terms of an S-wave background interference [8]. The corresponding elements are labeled 20 and 21.

In table III we present our experimental results (derived by means of the so-called method of moments) for the quantities defined in terms of the joint-decay matrix-elements as shown on the L.H.S. The results represent averages over $|t - t_{min}|$ between 0 and 0.2 $(GeV/c)^2$ for $N^*\rho$ and between 0 and 0.6 $(GeV/c)^2$ for $N^*\omega$ (*).

From table III we conclude that there exist three significant correlation terms in the $N^*\rho$-channel, labeled {7}, {12} and {18}, and three in the $N^*\omega$-channel, labeled {12}, {16} and {18} resp. (Significant here means more than 2 standard deviations away from zero). The following is a list of the angular functions which have the above correlation-terms as a co-factor in the Pilkuhn & Svensson-expansion:

$$\begin{aligned}
\{7\} &\rightarrow (1 - 3\cos^2\Theta_{\rho(\omega)})(1 - 3\cos^2\Theta_{N^*}) \\
\{12\} &\rightarrow \sin 2\Theta_{\rho(\omega)} \cdot \sin 2\Theta_{N^*} \cos(\phi_{\rho(\omega)} + \phi_{N^*}) \\
\{16\} &\rightarrow \sin 2\Theta_{\rho(\omega)} \sin^2\Theta_{N^*} \cos(\phi_{\rho(\omega)} + 2\phi_{N^*}) \\
\{18\} &\rightarrow \sin^2\Theta_{\rho(\omega)} \sin^2\Theta_{N^*} \cos(2\phi_{\rho(\omega)} + 2\phi_{N^*})
\end{aligned} \qquad (13)$$

The correlation implied by the non-vanishing of {7} for $N^*\rho$ is nothing but the Θ_ρ, Θ_{N^*} correlation first noted by Goldhaber et al. at 3.65 GeV/c [2] and already reported in our previous paper [1].

From the remaining correlations especially the coefficient {12} in the case of $N^*\rho$, and coefficient {18} in the case of $N^*\omega$ are very significant. The genuineness of these correlations has been checked directly by plotting appropiate projections of the W-distributions.

In figs. 5 and 6 we show the $|t - t_{min}|$ dependence of the most significant correlation terms.

Donohue has pointed out that, for simple kinematical reasons, only the $\Theta_{\rho(\omega)}$ - Θ_{N^*} correlation term and those depending on the

(*) Differences with the results obtained when averaging in terms of |t| are as required negligible. The reader can see this for himself by comparing the first six density elements of table III with those presented in the Heidelberg paper [1] and repeated in table II.

azimuthal angles ϕ_1 and ϕ_2 through the sum $(\phi_1 + \phi_2)$ can have non-vanishing values in the forward direction ([10]). From the Pilkuhn-Svensson expansion ([7]) one can see that only the expressions labeled {7}, {12} and {18} contain terms satisfying this requirement. It is reassuring (and to some extend an internal-consistency check on our data and our analysis procedure) that the expressions {7}, {12} and {18} correspond exactly to correlation coefficients which we found to be significant. The fourth one, i.e. {16}, actually the least significant one, is furthermore compatible with becoming zero in the forward direction (see fig. 6). We also stress that our strongest correlation elements, i.e. $\mathrm{Re}(\rho_{31}^{10} - \rho_{31}^{0,-1})$ in the case of $N^*\rho$, and $N^*\omega$ and $\mathrm{Re}(\rho_{3,-1}^{1,-1})$ in the case of $N^*\omega$, obey, within the error-limits, the forward-direction kinematic limits given by Donohue ([10]):

$$|\mathrm{Re}(\rho_{31}^{10} - \rho_{31}^{0,-1})| \leqslant \sqrt{\tfrac{1}{2}\rho^{00}\rho_{33}} \qquad (14)$$

$$|\mathrm{Re}(\rho_{3,-1}^{1,-1})| \leqslant \sqrt{\tfrac{1}{2}(1 - \rho^{00} - 2\rho_{33})\rho_{33}} \qquad (15)$$

V. BIALAS & ZALEWSKI QUARK-MODEL PREDICTIONS.

From the quark-model and using only the so-called additivity assumption Bialas & Zalewski derived a set of predictions for the joint-decay density matrix-elements in double resonance production shown in table IV ([3]). In table V we have compared our results of section IV with these predictions. For both the $N^*\rho$ and the $N^*\omega$ channel, the agreement is very satisfactory.

The first Bialas-Zalewski prediction is a relation between single-vertex density matrix-elements. From relation (2) to (6) the L.H.S. contains but correlation terms, the R.H.S. but single-vertex elements. It is trivial to note that each time a single-vertex R.H.S. term (or any combination of R.H.S. terms) has a value significantly different from zero (at a certain momentum transfer), the relations (2) to (6) imply that a certain sum of correlation-elements should also differ significantly from zero (at the same momentum transfer). A particularly interesting example, in which the sum of correlation-terms can be reduced to a single one, is obtained by looking, for collinear production (i.e. for $|t - t_{min}| \to 0$), at the relation derived by adding the Bialas-Zalewski relations (3) and (4). From the Donohue-argument mentioned above, we know that in this situation, all but one of the L.H.S. coefficients vanish and that therefore, at $|t - t_{min}| = 0$, we should have the equality:

$$4\sqrt{3}\mathrm{Re}(\rho^{1,-1}_{3,-1}) = 1 + 2(\rho_{33} - \rho_{11}) + 4/\sqrt{3}\mathrm{Re}(\rho_{3,-1}) \qquad (16)$$

In fig. 7 we show the dependence on $|t - t_{min}|$ of both sides of Eq.(16) for the reaction $\pi^+ p \to N^* \omega$. The hatched boundary represents the upper limit imposed on the L.H.S. by the second Donohue Eq.(15). The agreement required by Eq.(16) is indeed present. The significance of the test illustrated in fig. 7 resides in the fact that we are looking at a prediction of the quark-model, which is clearly satisfied, and which requires a correlation-element to be equal to a combination of single-vertex quantities (used as "input"), in a situation where this equality does not seem to be imposed by kinematics.

Table I: Comparison of $d\sigma/d|t|$ and $d\sigma/d|t - t_{min}|$-Slopes.

Reaction	$\pi^+ p \rightarrow N^* \rho$	$\pi^+ p \rightarrow N^* \omega$
$d\sigma/d|t|$-Slopes $(GeV/c)^{-2}$	(11.8 ± 0.5)	(3.4 ± 0.3)
$d\sigma/d|t - t_{min}|$-Slopes $(GeV/c)^{-2}$	(15.5 ± 1.0)	(3.9 ± 0.3)
$|t|$-band $(GeV/c)^2$	$0.05 \rightarrow 0.35$	$0.1 \rightarrow 0.8$
$|t - t_{min}|$-band $(GeV/c)^2$	$0.0 \rightarrow 0.25$	$0.2 \rightarrow 0.6$

Remark: The mass-bands used are:

1.14 - 1.30 GeV for the N^*

.66 - .86 GeV for the ρ

.74 - .82 GeV for the ω

Table II: Density-Matrix-Elements in π^+p-collisions at 5 GeV/c corrected for background as described in section III.

Reaction	$\pi^+p \rightarrow N^*\rho$		$\pi^+p \rightarrow N^*\omega$	
	Before Correct.	After Correct.	Before Correct.	After Correct.
Vector Meson (ρ^o or ω^o)				
ρ^{00}	0.781 ± 0.024	0.791	0.388 ± 0.039	0.404
Re ρ^{10}	-0.087 ± 0.016	-0.090	-0.126 ± 0.021	-0.135
$\rho^{1,-1}$	-0.058 ± 0.015	-0.069	0.077 ± 0.031	0.085
Baryon (N^{*++})				
ρ_{33}	0.127 ± 0.018	0.129	0.210 ± 0.028	0.215
Re ρ_{31}	-0.059 ± 0.018	-0.070	-0.042 ± 0.028	-0.035
Re $\rho_{3,-1}$	-0.016 ± 0.016	-0.019	0.004 ± 0.026	-0.003
$\|t\|$-range $(GeV/c)^2$	$\leqslant$ 0.30		$\leqslant$ 0.60	

Table III: Joint Density Matrix Elements of Resonances in π^+p collisions at 5 GeV/c.

Reaction	$N^*\rho$ $\lvert t-t_{min}\rvert < .2\ GeV^2$	$N^*\omega$ $\lvert t-t_{min}\rvert < .6\ GeV^2$
{1} ρ^{00}	0.79 ± 0.02	0.39 ± 0.04
{2} $\mathrm{Re}(\rho^{10})$	-0.06 ± 0.02	-0.13 ± 0.02
{3} $\rho^{1,-1}$	-0.05 ± 0.02	0.07 ± 0.03
{4} ρ_{33}	0.13 ± 0.02	0.23 ± 0.03
{5} $\mathrm{Re}(\rho_{31})$	-0.05 ± 0.02	-0.05 ± 0.03
{6} $\mathrm{Re}(\rho_{3,-1})$	-0.02 ± 0.02	0.01 ± 0.03
{7} $(\bar{\rho}_{33} - \bar{\rho}_{11}) - 2(\rho^{11} - \rho^{00})(\rho_{33} - \rho_{11})$	0.19 ± 0.08	0.10 ± 0.14
{8} $\mathrm{Re}(\underline{\rho}^{10}) - 2(\rho_{33} - \rho_{11})\mathrm{Re}(\rho^{10})$	0.01 ± 0.04	0.02 ± 0.05
{9} $\underline{\rho}^{1,-1} - 2(\rho_{33} - \rho_{11})\rho^{1,-1}$	-0.00 ± 0.04	-0.05 ± 0.07
{10} $\mathrm{Re}(\bar{\rho}_{31}) - 2(\rho^{11} - \rho^{00})\mathrm{Re}(\rho_{31})$	0.07 ± 0.04	0.04 ± 0.07
{11} $\mathrm{Re}(\bar{\rho}_{3,-1}) - 2(\rho^{11} - \rho^{00})\mathrm{Re}(\rho_{3,-1})$	0.00 ± 0.04	0.07 ± 0.07
{12} $\mathrm{Re}(\rho_{31}^{10} - \rho_{31}^{0,-1}) - 2\mathrm{Re}(\rho^{10})\mathrm{Re}(\rho_{31})$	-0.14 ± 0.03	-0.09 ± 0.04
{13} $\mathrm{Re}(\rho_{31}^{01} - \rho_{31}^{-1,0}) - 2\mathrm{Re}(\rho^{10})\mathrm{Re}(\rho_{31})$	-0.02 ± 0.03	0.03 ± 0.03
{14} $\mathrm{Re}(\rho_{31}^{1,-1}) - \rho^{1,-1}\mathrm{Re}(\rho_{31})$	-0.01 ± 0.01	0.03 ± 0.03
{15} $\mathrm{Re}(\rho_{31}^{-1,1}) - \rho^{1,-1}\mathrm{Re}(\rho_{31})$	-0.01 ± 0.01	0.00 ± 0.03
{16} $\mathrm{Re}(\rho_{3,-1}^{10} - \rho_{3,-1}^{0,-1}) - 2\mathrm{Re}(\rho^{10})\mathrm{Re}(\rho_{3,-1})$	-0.04 ± 0.02	0.10 ± 0.03
{17} $\mathrm{Re}(\rho_{3,-1}^{01} - \rho_{3,-1}^{-1,0}) - 2\mathrm{Re}(\rho^{10})\mathrm{Re}(\rho_{3,-1})$	-0.03 ± 0.02	-0.02 ± 0.03
{18} $\mathrm{Re}(\rho_{3,-1}^{1,-1}) - \rho^{1,-1}\mathrm{Re}(\rho_{3,-1})$	0.03 ± 0.01	0.14 ± 0.02
{19} $\mathrm{Re}(\rho_{3,-1}^{-1,1}) - \rho^{1,-1}\mathrm{Re}(\rho_{3,-1})$	-0.02 ± 0.01	-0.02 ± 0.03
{20} $\rho^{0\varepsilon}$	0.215± 0.02	———
{21} $\rho^{1\varepsilon}$	-0.029± 0.01	———

Table IV.

Bialas & Zalewski Quark-Model Predictions.

(Based on the Additivity-Assumption only)

$$\frac{1}{3}\left(\rho^{11} - \rho^{00}\right) + \rho^{1,-1} = \frac{2}{3}\left(\rho_{33} - \rho_{11}\right) + \frac{4}{\sqrt{3}}\,\mathrm{Re}\,\rho_{3,-1} \qquad (1)$$

$$\sqrt{3}\left(\mathrm{Re}\,\rho_{3,-1}^{1,-1} + \mathrm{Re}\,\rho_{3,-1}^{-1,1}\right) + \frac{1}{2}\,\underline{\rho^{1,-1}} - \sqrt{3}\,\mathrm{Re}\,\rho_{\overline{3,-1}} - \frac{1}{2}\left(\rho_{\overline{33}} - \rho_{\overline{11}}\right) = \frac{1}{2}\left(\rho^{11} - \rho^{00}\right) - \frac{1}{2}\,\rho^{1,-1} \qquad (2)$$

$$\sqrt{3}\left(\mathrm{Re}\,\rho_{3,-1}^{1,-1} + \mathrm{Re}\,\rho_{3,-1}^{-1,1}\right) + \frac{1}{\sqrt{3}}\,\mathrm{Re}\,\rho_{\overline{3,-1}} - \frac{3}{2}\,\underline{\rho^{1,-1}} - \frac{1}{2}\left(\rho_{\overline{33}} - \rho_{\overline{11}}\right) = \left(\rho_{33} - \rho_{11}\right) - \frac{2}{\sqrt{3}}\,\mathrm{Re}\,\rho_{3,-1} \qquad (3)$$

$$3\sqrt{3}\left(\mathrm{Re}\,\rho_{3,-1}^{1,-1} + \mathrm{Re}\,\rho_{3,-1}^{-1,1}\right) + \frac{3}{2}\,\underline{\rho^{1,-1}} + \sqrt{3}\,\mathrm{Re}\,\rho_{\overline{3,-1}} + \frac{1}{2}\left(\rho_{\overline{33}} - \rho_{\overline{11}}\right) = 1 + \left(\rho_{33} - \rho_{11}\right) + 2\sqrt{3}\,\mathrm{Re}\,\rho_{3,-1} \qquad (4)$$

$$-\sqrt{3}\,\mathrm{Re}\,\underline{\rho^{10}} - 3\left[\mathrm{Re}\left(\rho_{3,-1}^{10} - \rho_{3,-1}^{0,-1}\right) + \mathrm{Re}\left(\rho_{3,-1}^{01} - \rho_{3,-1}^{-1,0}\right)\right] = \sqrt{3}\,\mathrm{Re}\,\rho^{10} \qquad (5)$$

$$-\sqrt{2}\,\mathrm{Re}\,\rho_{\overline{31}} - 3\sqrt{2}\left(\mathrm{Re}\,\rho_{31}^{1,-1} + \mathrm{Re}\,\rho_{31}^{-1,1}\right) = 2\sqrt{2}\,\mathrm{Re}\,\rho_{31} \qquad (6)$$

Table V.

Bialas-Zalewski Quark Model Predictions

Experimental Test for 5 GeV/c π^+ on Protons.

$\pi^+ p \to N^* \rho$			
Relation	L.H.S.	R.H.S.	L.H.S.-R.H.S.
(1)	-.28±.02	-.21±.05	-.07±.05
(2)	-.29±.10	-.32±.02	.03±.10
(3)	-.26±.12	-.22±.04	-.04±.12
(4)	.40±.14	.69±.07	-.29±.16
(5)	.11±.12	-.15±.03	.26±.13
(6)	-.17±.11	-.15±.05	-.02±.11

$\pi^+ p \to N^* \omega$			
Relation	L.H.S.	R.H.S.	L.H.S.-R.H.S.
(1)	.04±.04	-.03±.07	.07±.08
(2)	.00±.15	-.08±.04	.08±.15
(3)	.28±.15	-.08±.06	.36±.16
(4)	.70±.20	.95±.11	-.25±.22
(5)	-.32±.17	-.22±.03	-.10±.17
(6)	-.18±.18	-.14±.08	-.04±.20

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(6) The 9 mass-regions included are defined by the following mass-bands (in GeV):

for the ρ^0 : 0.56 → 0.66 and 0.86 → 0.96
for the ω^0 : 0.60 → 0.70 and 0.85 → 0.95
for the N^{*++}: 1.06 → 1.14 and 1.30 → 1.38

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Fig. 1

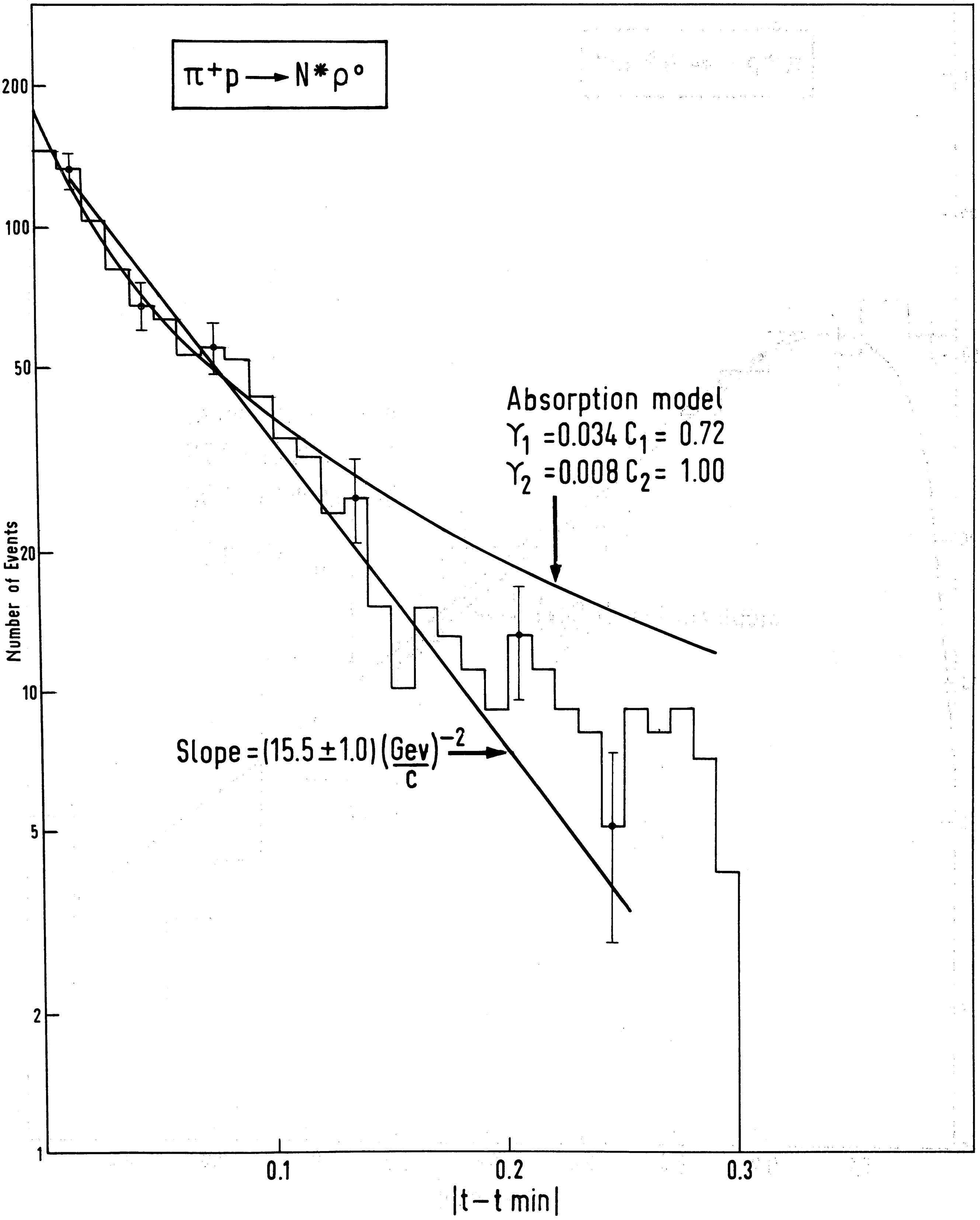

π⁺p → N*ρ°
Absorption model
γ_1 = 0.034 C_1 = 0.72
γ_2 = 0.008 C_2 = 1.00
Slope = (15.5 ± 1.0) (Gev/c)^-2
Number of Events
200
100
50
20
10
5
2
1
0.1
0.2
0.3
|t − t min|

Fig. 2

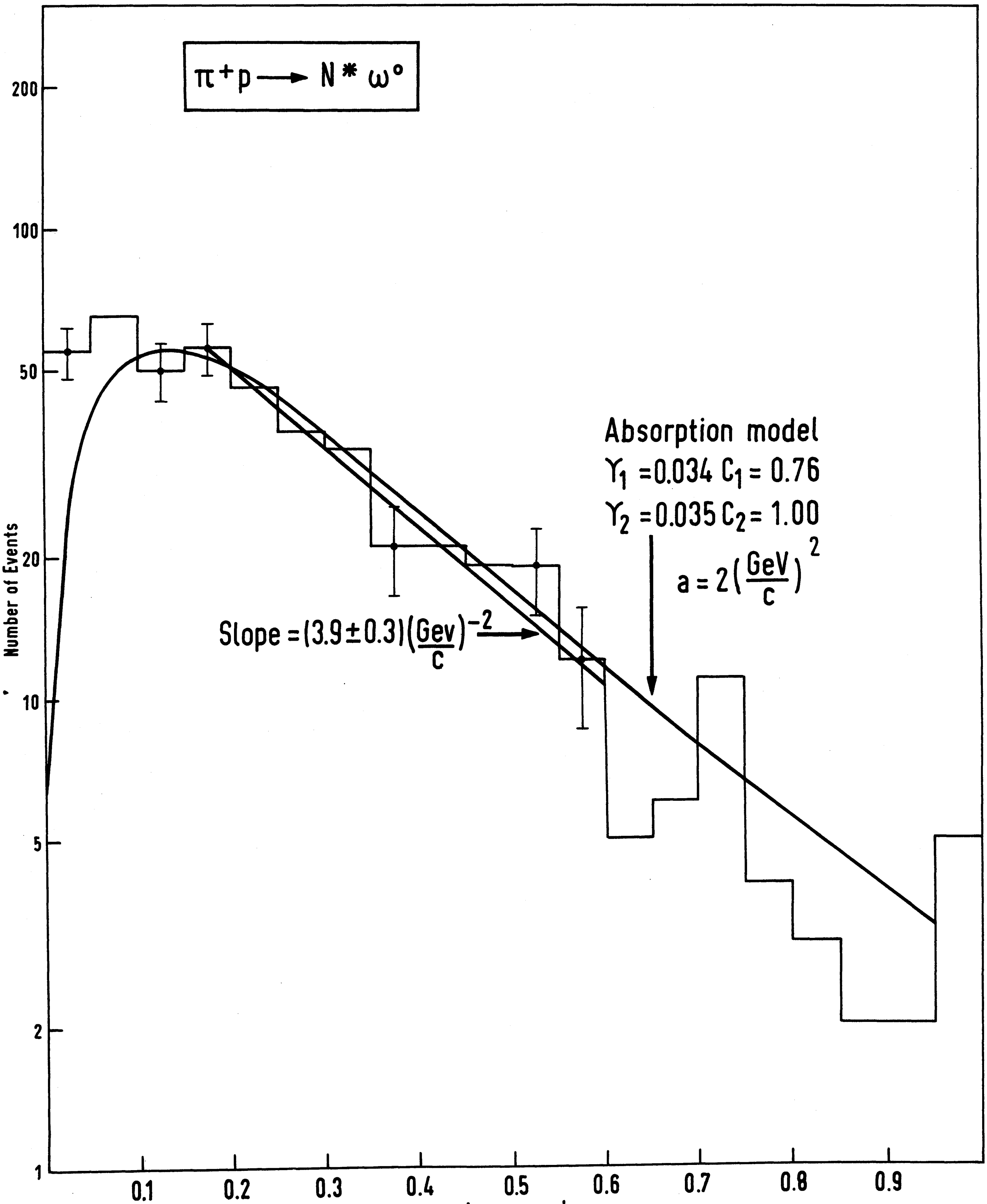

π⁺p ⟶ N* ω°
Absorption model
Υ_1 =0.034 C_1 = 0.76
Υ_2 =0.035 C_2 = 1.00
a = 2 (GeV/c)²
Slope = (3.9 ± 0.3) (Gev/c)⁻²
Number of Events
200
100
50
20
10
5
2
1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
|t−t min|

Fig. 3

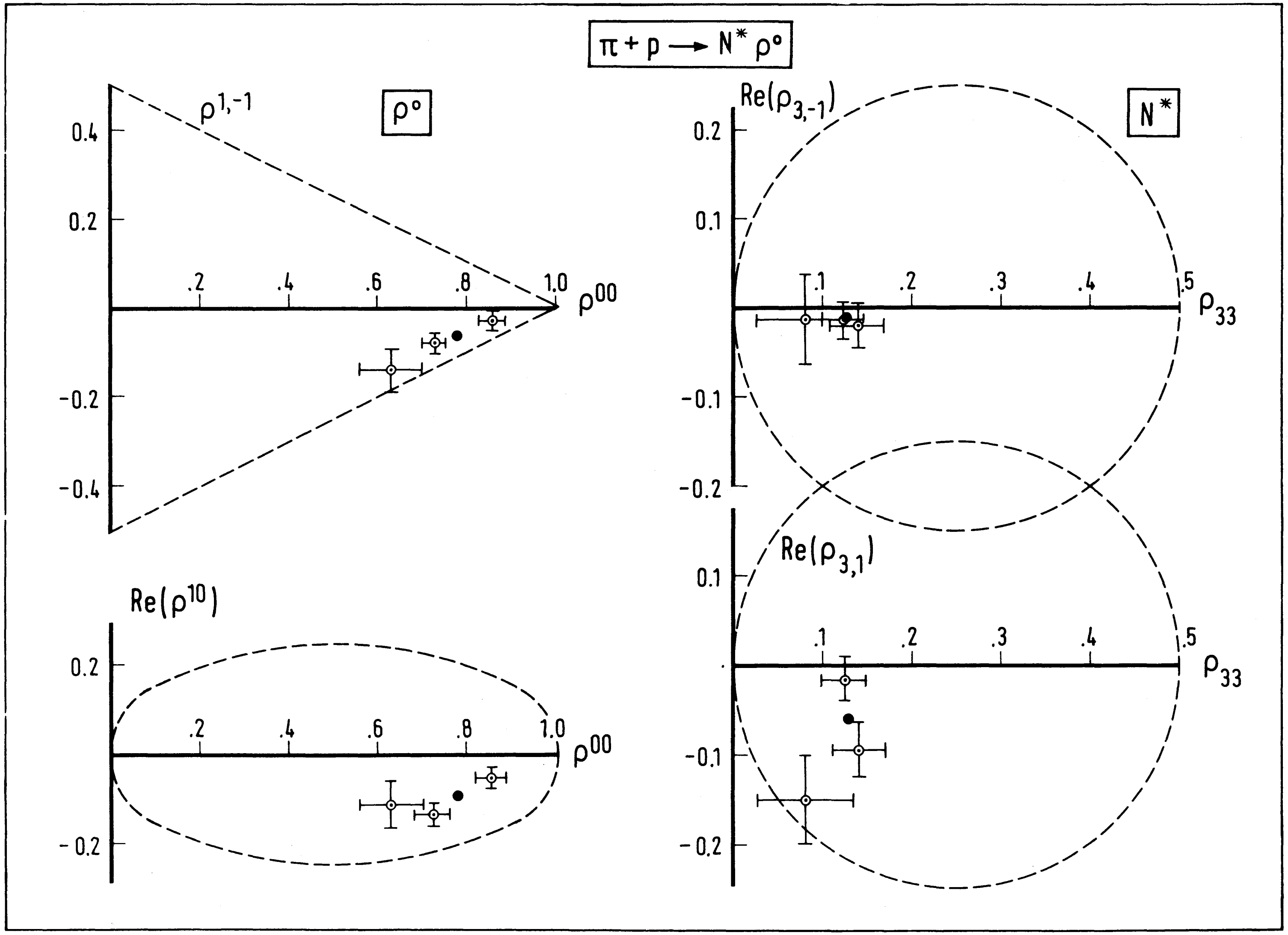

π + p ⟶ N* ρ°
ρ°
$\rho^{1,-1}$
0.4
0.2
-0.2
-0.4
.2
.4
.6
.8
1.0
ρ^{00}
$Re(\rho^{10})$
0.2
- 0.2
.2
.4
.6
.8
1.0
ρ^{00}
N*
$Re(\rho_{3,-1})$
0.2
0.1
-0.1
-0.2
.1
.2
.3
.4
.5
ρ_{33}
$Re(\rho_{3,1})$
0.1
-0.1
-0.2
.1
.2
.3
.4
.5
ρ_{33}

Fig. 4

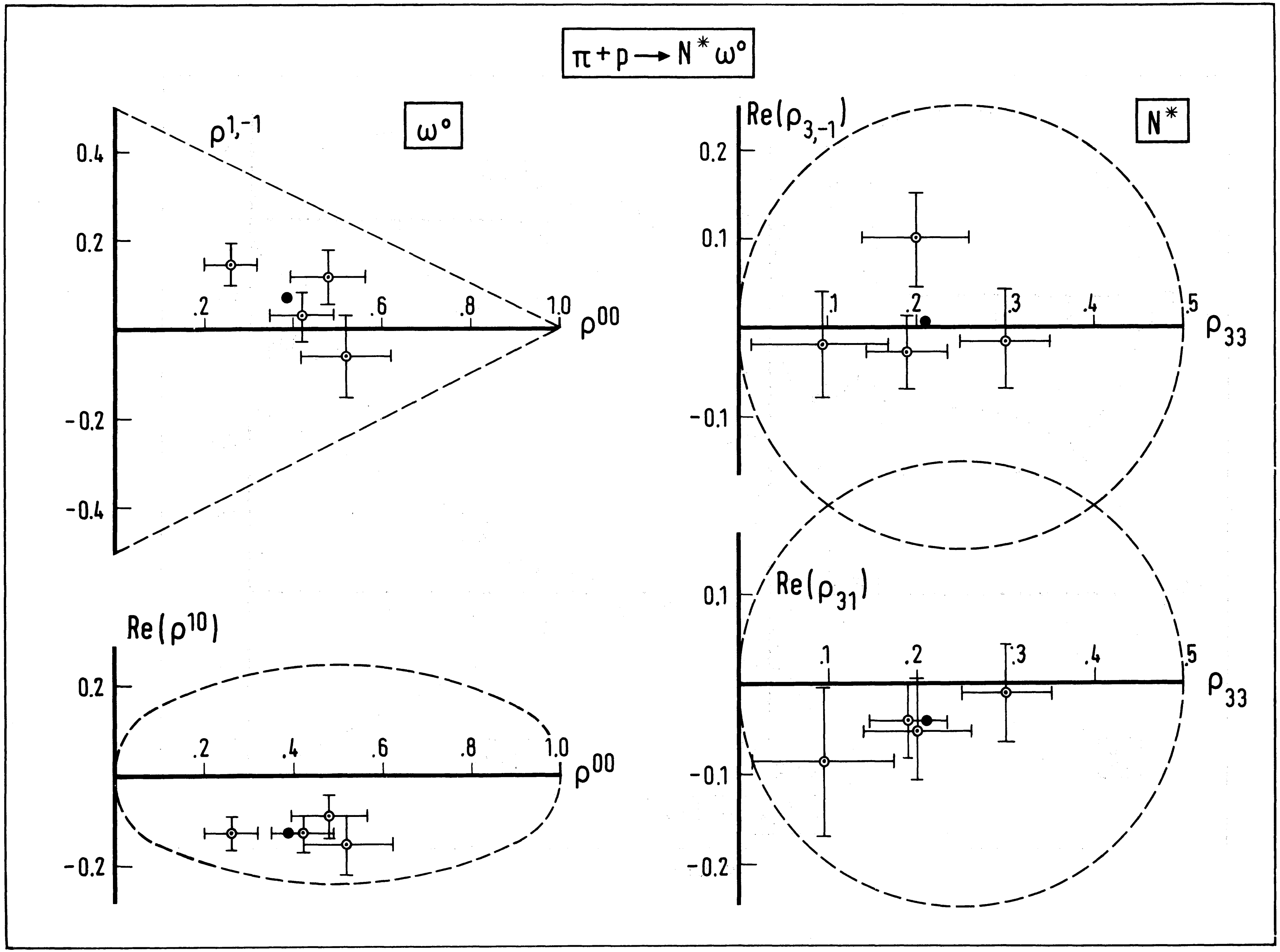
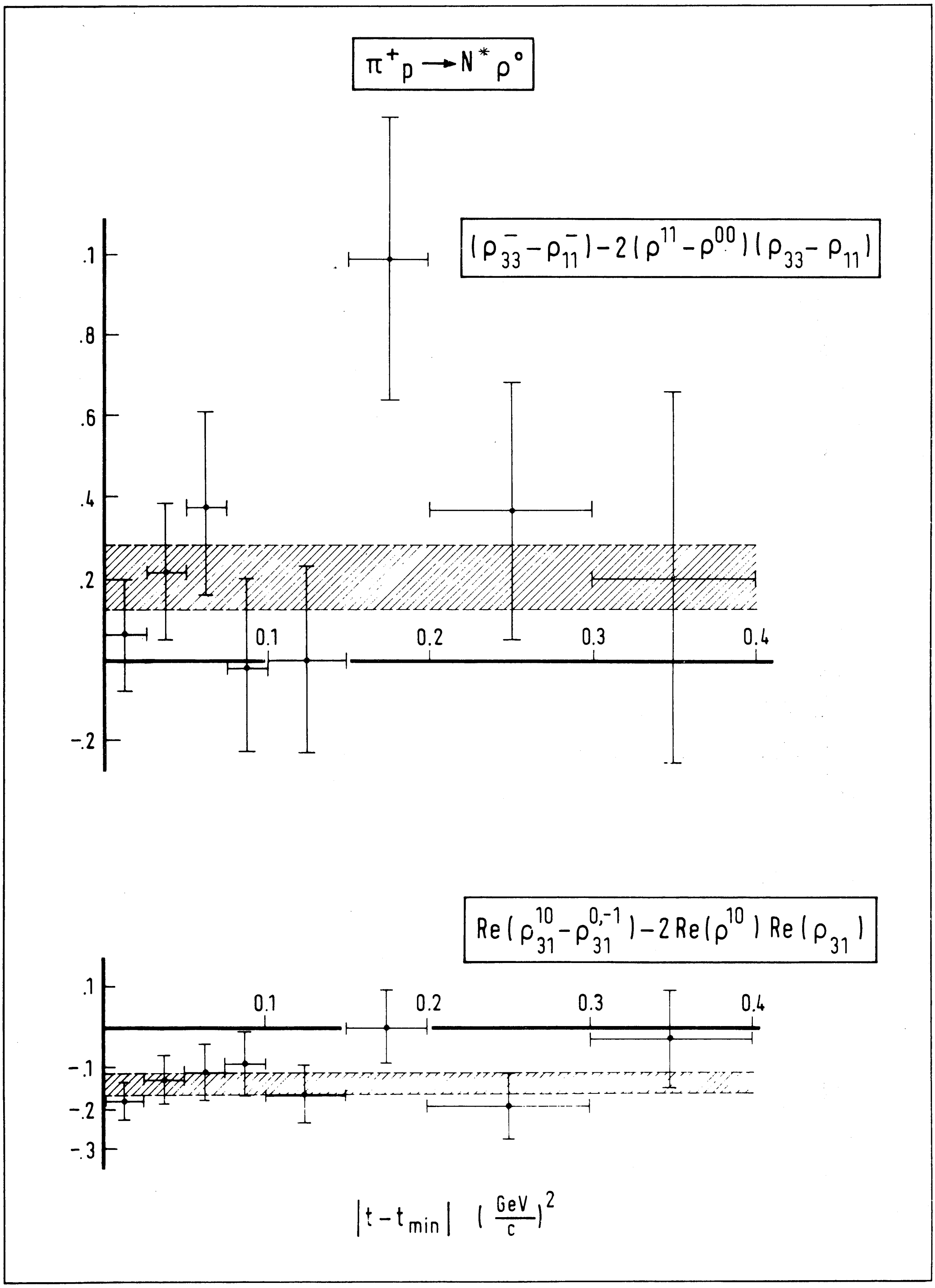

π+p ⟶ N* ω°
ω°
N*
ρ1,-1
ρ00
Re(ρ10)
Re(ρ3,-1)
Re(ρ31)
ρ33
0.4
0.2
-0.2
-0.4
0.1
-0.1
-0.2
.2
.4
.6
.8
1.0
.1
.3
.5

Fig. 5



π⁺p → N* ρ°
(ρ⁻₃₃ − ρ⁻₁₁) − 2(ρ¹¹ − ρ⁰⁰)(ρ₃₃ − ρ₁₁)
.1
.8
.6
.4
.2
-.2
0.1
0.2
0.3
0.4
Re(ρ¹⁰₃₁ − ρ⁰,⁻¹₃₁) − 2 Re(ρ¹⁰) Re(ρ₃₁)
.1
-.1
-.2
-.3
0.1
0.2
0.3
0.4
|t − t_min| (GeV/c)²

Fig. 6

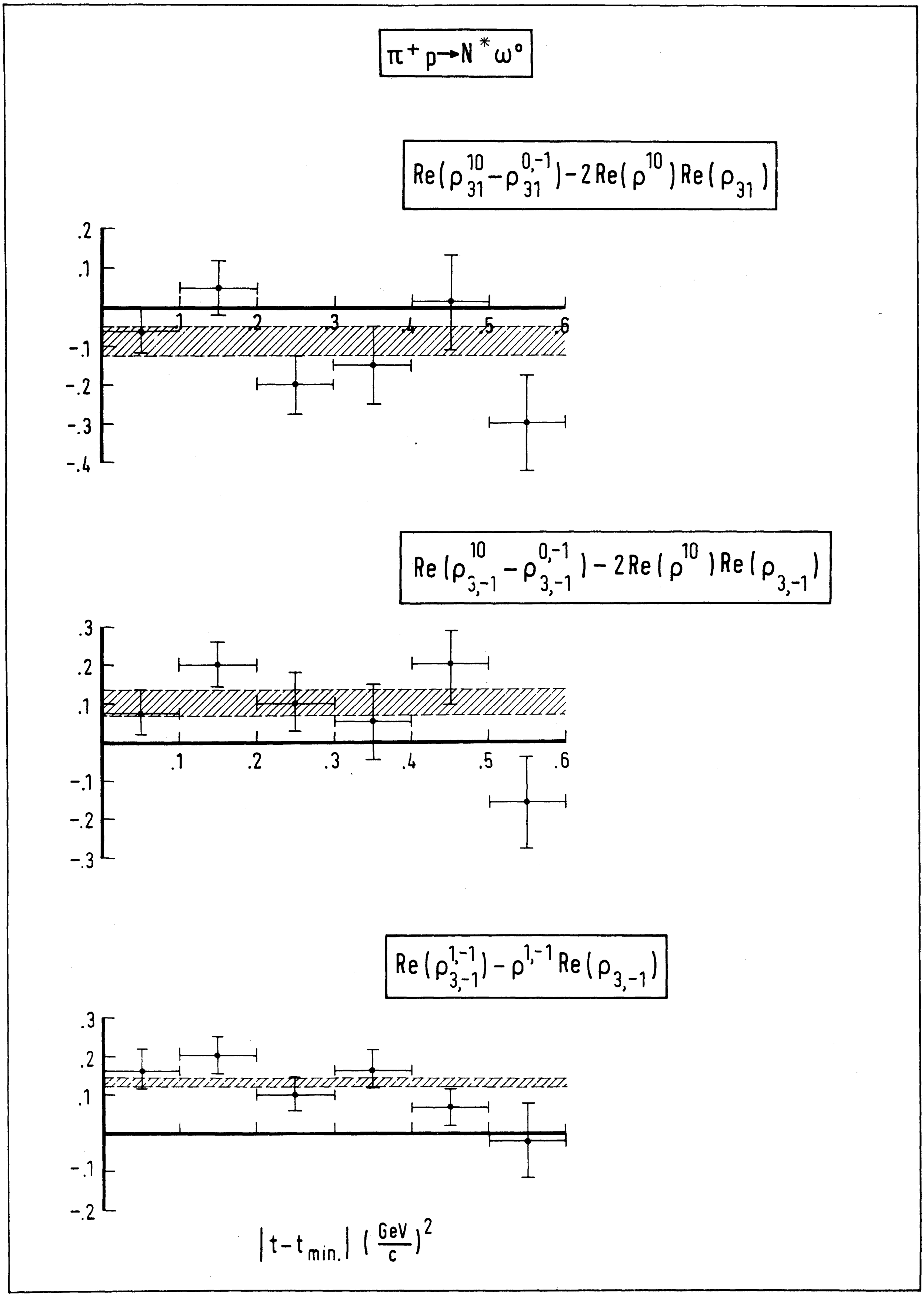
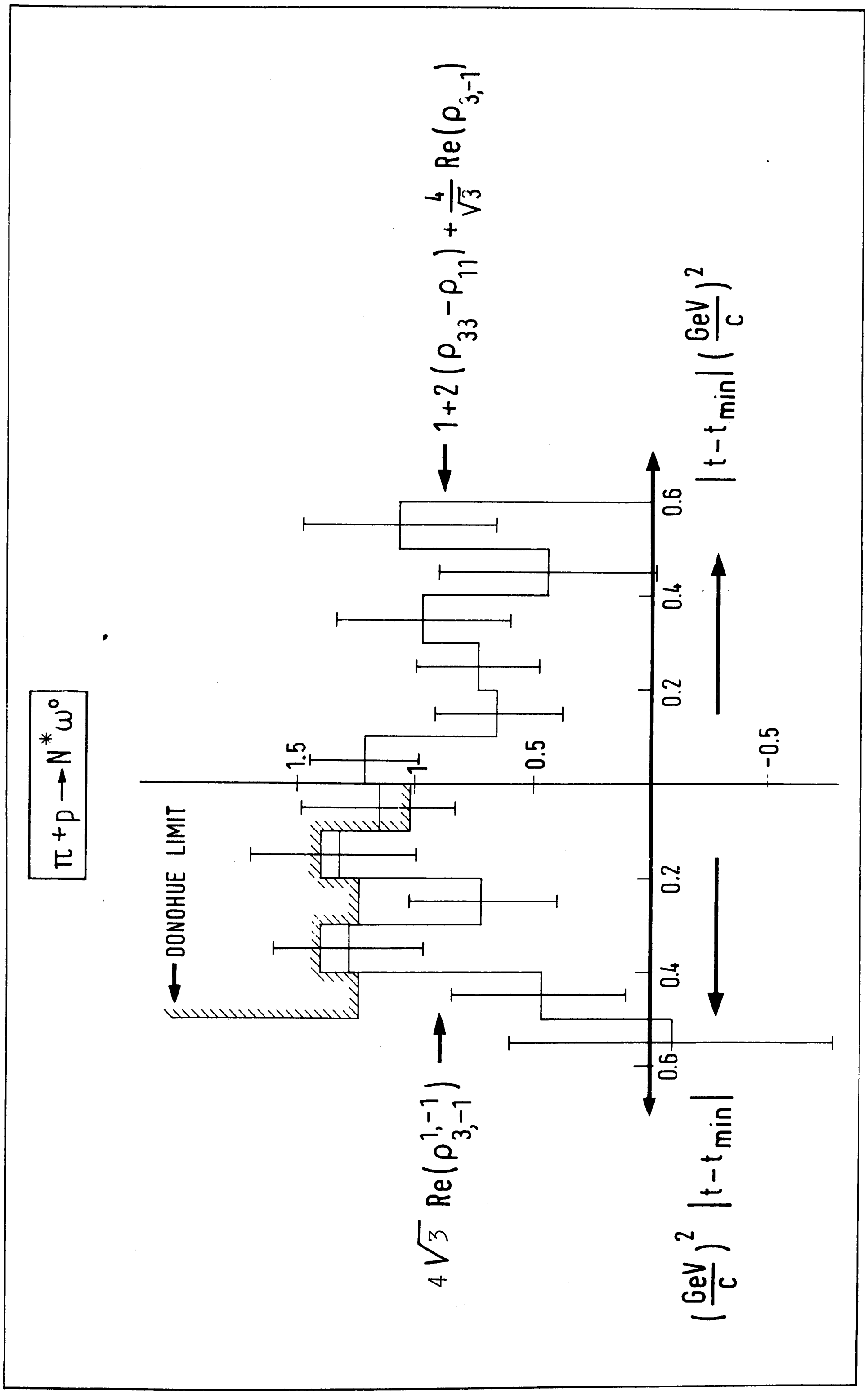

π⁺p→N*ω°
Re(ρ^10_31 − ρ^0,-1_31) − 2Re(ρ^10)Re(ρ_31)
.2
.1
-.1
-.2
-.3
-.4
.1
.2
.3
.4
.5
.6
Re(ρ^10_3,-1 − ρ^0,-1_3,-1) − 2Re(ρ^10)Re(ρ_3,-1)
.3
.2
.1
-.1
-.2
-.3
Re(ρ^1,-1_3,-1) − ρ^1,-1 Re(ρ_3,-1)
.3
.2
.1
-.1
-.2
|t-t_min.| (GeV/c)^2

Fig. 7


π + p → N* ω°
DONOHUE LIMIT
4 √3 Re(ρ^{1,-1}_{3,-1})
(GeV/c)^2 |t − t_min|
1 + 2 (ρ_33 − ρ_11) + 4/√3 Re(ρ_3,-1)
|t − t_min| (GeV/c)^2
1.5
1
0.5
−0.5
0.2
0.4
0.6
0.2
0.4
0.6