REGGE RESIDUA, FORM FACTORS, AND COMPOSITE PARTICLES

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1. INTRODUCTION

1.1 The Van Hove formula

It was observed by Van Hove[1] in 1966 that for small $|t|$, the relation

$$\frac{d\sigma}{dt} \underset{s \to \infty}{\sim} [G_E(t)]^4 \qquad (1)$$

for proton-proton scattering is apparently very well satisfied experimentally. He gave a derivation based on a quark model, wherein both the strong and electromagnetic interaction amplitudes of the quarks were assumed to be slowly varying with t compared to the variation of the structure factors. This picture is quite analogous to the deuteron's properties. We will refer to (1) as the Van Hove formula, to emphasize the small momentum transfer spirit in which it will be regarded in this work.[2]

In Figure 1 we present the right and left sides of relation (1), using data for proton-proton scattering at 20 GeV/c,[3] and the proton magnetic form factor G_M.[4] The agreement is very impressive for a strong interaction dynamics result.

1.2 Regge pole residua and form factors

We wish to explore the significance of a relation such as (1) (generalized to other hadrons, as well as nucleons) in the context of the analysis of singularities in the angular momentum plane of high energy scattering amplitudes. In particular suppose that a Pomeranchuk

Regge pole, with $\alpha(0) = 1$, is responsible for the high energy asymptotic behavior at small $|t|$ of elastic scattering amplitudes. Then if the Van Hove formula is to be satisfied, this pole must have the following two properties:

(a) $\alpha(t)$ remains very close to 1 for the range of t under consideration (i.e., the pole must be fixed, or nearly so);

(b) The pole residue $\beta(t)$ must have factors proportional to the electromagnetic form factors of the hadrons to which the pole couples, e.g., $\beta_{12}(t) \propto G_{E_1}(t)\, G_{E_2}(t)$.

Property (a) is necessary if $d\sigma/dt$ is to become asymptotically a function of t only for the range of t under consideration (e.g., $|t| <$ .50 GeV^2); property (b) then yields the relation (1) since

$$\frac{d\sigma}{dt} \underset{s \to \infty}{\to} s^{2\alpha(t)-2}[\beta(t)]^2 \quad .$$

A natural question to ask is this; do all the <u>other</u> Regge poles (e.g., ρ, ω, A_2, . . .) have residua given in such a way by the electromagnetic form factors? We will consider a model wherein this can be answered; we will also be able to discuss corrections to the Van Hove formula, and whether all corrections die out for large enough energy. We also show in the model that relations such as (1) hold even if there are singularities near $\alpha = 1$ other than poles which dominate the asymptotic quark-quark scattering behavior.

1.3 Outline of model

We consider the known low-mass hadrons to be composed of three quarks (baryons) or quark and antiquark (mesons). To simplify the discussion[5)] we will later take a meson mass around $M_\omega \approx .80$ GeV and baryon mass around $M_\Sigma \approx 1.2$ GeV; this will permit us to introduce quarks of mass M slightly above $\frac{M_\omega}{2}$ (e.g., .42 GeV) and ultimately consider a moderately weak-binding limit.[6)] However, the initial discussion is independent of these details.

The Regge poles, and other singularities in the ℓ plane, will be introduced as singularities in the $Q\bar{Q} \rightarrow Q\bar{Q}$ t channel.

The QQ, $Q\bar{Q}$ scattering amplitudes at high energies and Q form factors are to be input quantities from which we derive the hadron properties using an approach which is essentially the impulse approximation generalized to the complex angular momentum plane. In practice, the $Q\bar{Q}$ Regge pole reduced residua are taken as constants over the t region of interest ($|t| < .50\ \mathrm{GeV}^2$), and the trajectories taken as straight lines passing through known resonances in the J plane. The formulation of the model relies on analyticity and unitarity rather than nonrelativistic ideas.

Assuming the t channel in hadron-hadron scattering is dominated by $Q\bar{Q}$ states, corresponding to the picture of hadrons with a quark structure, a coupled channel N/D formulation for amplitudes in the t-channel at complex angular momentum is constructed, continued when necessary to binding energies for which anomalous thresholds appear. We will find that near each singularity ℓ_s in the ℓ plane where the QQ or $Q\bar{Q}$ scattering amplitude $A_{11}(\ell, t)$ has an infinity, the (hadron)$_j$-(hadron)$_k$ scattering amplitude A_{jk} can be factored into 3 terms:

$$A_{jk}(\ell, t) \cong [D_{1j}(\ell_s, t)]^T A_{11}(\ell, t) D_{1k}(\ell_s, t) \tag{2}$$

for ℓ near ℓ_s, where the D_{1j} contain the structure parameters of hadron j. (The transpose notation refers only to spin indices, when present.) This generalizes the Van Hove formula, which is obtained from singularities with $\ell = 1$, as $D_{1j}(1, t)$ is closely related to the electromagnetic form factor of hadron j, the two coinciding when the quark form factors are constants.

2. TRIPLE FACTORIZATION AT SINGULARITIES

Consider the elastic scattering of hadron j on hadron k, in a system composed of hadrons and quarks. We define a suitably normalized scattering amplitude $A_{jk}(\ell, t)$, whose relation to differential cross-sections will be specified later, to be obtained from an N/D formalism (with proper threshold and analytic properties built in) as:

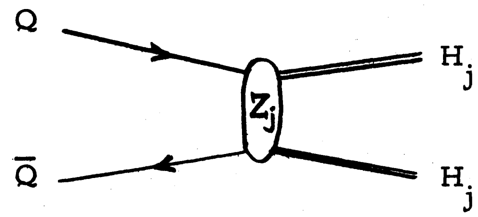
$$\underset{\sim}{A}(\ell, t) = \underset{\sim}{N}(\ell, t)\underset{\sim}{D}^{-1}(\ell, t), \tag{3}$$

where $\underset{\sim}{D}$ has only a right hand (unitarity) cut in t and $\underset{\sim}{N}$ has only left hand or anomalous cuts. Let channel 1 be $Q\overline{Q}$, and let $\rho_j(\ell, t)$ be the phase space for channel j with appropriate threshold factors. We follow to a great extent the approach of Blankenbecler, Cook and Goldberger, [7] who treated Regge poles in a coupled channel formalism including composite particles. The unitarity condition, keeping many coupled two-body channels, imposes the relation

$$\underset{\sim}{D}(\ell, t) = \underset{\sim}{I} - \frac{1}{\pi}\int_{4M^2}^{\infty} \frac{dt'}{t' - t - i\epsilon}\underset{\sim}{\rho}(\ell, t')\underset{\sim}{N}(\ell, t') \tag{4}$$

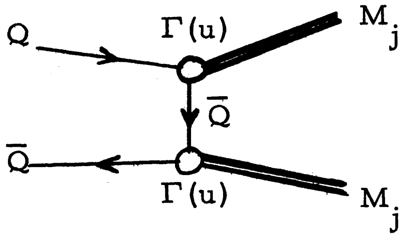
where $\underset{\sim}{\rho}$ is the diagonal matrix of phase space factors; $4M^2$ is the lowest normal threshold. (The lower limit will be extended to the anomalous threshold when the continuation is performed to obtain weak binding.)

The input quantities for this formalism can be taken as $\underset{\sim}{B}(\ell, t)$, the left hand or "dynamical" contributions to the amplitude, representing a generalization of the Born approximation for processes such as $j + \overline{j} \rightarrow k + \overline{k}$ in the t channel. The N function satisfies a linear integral equation whose kernel and inhomogeneous term are determined by $\underset{\sim}{B}$. The hadron structure information is contained in the B_{1j}'s in a way well-known from deuteron theory. [8] These functions arise from the partial wave projection of the following graphs:



These contain the wave function of hadron H_j as a composite of quark Q with state Z_j. In the meson case we take Z to be $\bar{Q}$, whence for mesons

B_{1j} =



where u is the square of the invariant momentum transfer between Q and M_j.

The vertex functions $\Gamma_j(u)$ now are essentially the wave function [of M_j as a bound state of $Q\bar{Q}$] in momentum space divided by a factor representing the bound state pole.[8)]

In accordance with the idea that hadron reactions at high energy are mediated by Q interactions, we postulate $B_{jk} = 0$ unless either j or k = 1 (absence of "direct" hadron-hadron potentials). However, B_{11} is assumed arbitrarily strong. Now we make the following observations:

(A) If the vertex Γ_j refers to bound state wave functions which vanish rapidly enough at small distances, B_{1j} will drop off rapidly with increasing t.

(B) Since the hadron masses are assumed appreciably larger than the quark mass M, $4M^2$ is the lowest normal threshold. [In the weak binding configuration the anomalous threshold will be $t_0 < 4M^2$.] Thus the thresholds $t_k (k > 1)$ which define the support of ρ_k will be high, and presumably in a region of t where B_{1j} is small.

(C) We assume the singularities in B_{11} are at high mass (short range $Q\bar{Q}$ forces).

As a consequence of (A), (B) and (C), we can (for j, $k \neq 1$) approximate N_{1j} by B_{1j}; and further, D_{j1} and $(D_{jk}-1)$ for j, $k \neq 1$ will have magnitudes much less than D_{1j}, so we can ignore the former two classes of terms in computing D^{-1}. We obtain an upper-triangular form for D, with only the top row and diagonal elements nonzero, and all terms but D_{11} on the diagonal are unity. This yields a determinant equal to the determinant of the (1, 1) channel only, i.e., quark-antiquark dynamics determine the pole positions (zeros of det D). The form of D^{-1} is the same as the form of D, with $(D^{-1})_{11} = (D_{11})^{-1}$, and $(D^{-1})_{1j} = -(D_{11})^{-1} D_{1j}$.

The meson-meson scattering amplitude, computed using these expressions, can be written in the form:

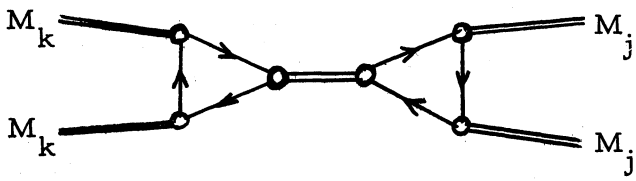
$$A_{jk} = -(N_{1j})^T D_{1k} + (D_{1j})^T A_{11} D_{1k} \tag{5}$$

where the transpose notation refers only to possible spin indices in the channels specified by the explicit subscript notation. [In obtaining this expression it is necessary to use the proof of Bjorken and Nauenberg[9] that a symmetric input B (as we assume) necessarily yields a symmetric.] At any singularity of the quark-antiquark scattering amplitude A_{11}, e.g., poles (or branch points yielding an infinity) in the ℓ plane, the second term dominates yielding equation (2).

For later reference, the amplitude for t channel annihilation of mesons into quark and antiquark is given by:

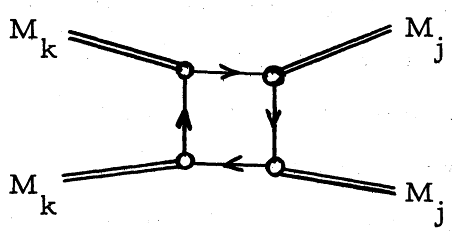
$$A_{1j} = N_{1j} - A_{11} D_{1j} \tag{6}$$

The second term in (5), which gives the contributions for $Q\bar{Q}$ scattering singularities, is generated by diagrams of the following form:



Note that these diagrams are exactly those given by Van Hove[1] in his derivation.

The first term in (5) can be shown to be generated by the box diagrams (since the imaginary part of this term is $N^+ \rho N$, which is the imaginary part of the box):



A similar explanation can be attached to the two terms in (6); the second involves $Q\bar{Q}$ scattering, while the first is just the Born term.

For our particular case involving $Q\bar{Q}$ states with spin 1/2 quarks, it can be shown that the first term in (5) contributes an asymptotic term in the meson-meson s channel scattering amplitude of order s^{-1} compared to the Pomeranchuk singularity, and can always be ignored asymptotically.

The expression (2) now exhibits a form of relativistic impulse approximation, since D_{1j} is calculated directly from the wave functions of the hadrons through Γ_j.

3. FORM FACTORS

Let G_j be the j^{th} hadron form factor (set of $2s_j + 1$ form factors, if hadron j has spin s_j). Then using coupled channel unitarity, writing G as a column matrix with elements G_j,

$$\mathrm{Im}G(t) = A^+(1,t)\rho(1,t)G(t) \tag{7}$$

We assume only $Q\bar{Q}$ states dominate the absorptive parts in the t channel for all processes, as in scattering. In addition, unsubtracted dispersion relations are assumed for the form factors. Using the expressions (6) for the relevant amplitudes in (7), and noting that $\mathrm{Im}D = -\rho N$, we can obtain

$$G_j(t) = [D_{1j}(1,t)]^T G_1(t) \tag{8}$$

exhibiting the factorization of the j^{th} hadron form factor into products of quark form factor [presumably dominated by vector meson poles, where det $D(1, t) = 0$] and structure factor $D_{1j}(1, t)$.

Now (8) together with (2) imply the Van Hove formula if G_1 and the residua of A_{11} are constant; but a correction is obtained to the VH formula in general which depends on the relative t dependence of the strong and electromagnetic properties of the quark. This faithfully reflects the physical assumptions implicit in the simple derivations given originally.[1] The empirical data on high energy scattering shown in Figure 1, together with the form factor data, suggest that this correction must be unimportant in fact.

The relation (8) has another consequence. If the normalization appropriate to charge conservation is chosen, then $G_{1E}(0) = G_j(0) = 1$ (here, E denotes the quark electric form factor).

Taking j to be a meson, the condition $D_{1j}(1, 0) = 1$ implied by these relations can be interpreted as imposing a condition on the magnitude of the wave function normalizations, or Γ functions. It can be shown that in the extreme weak binding limit this condition is identical to that of the zero-range model of Freund and Predazzi,[10] and thus our model contains theirs as a special case.

The condition that $D_{1k}(1, 0)$ is a universal constant for all hadrons guarantees the retention of universality and symmetry properties as in a nonrelativistic bound-state quark model, but (strictly speaking) only for singularities near $\ell = 1$ such as the Pomeranchuk singularity.

Now our theory of Regge residua in general can be stated as follows: Let the vertex function Γ_j (related to wave functions of hadron j) be parametrized with any number of parameters; e.g., a sum of poles (cf. Gourdin et al., Ref. 8), leading to a sum of Legendre functions for B_{1j}. These parameters can be determined (in principle) by accurate enough measurements of the electromagnetic form factor $D_{1j}(1, t)$. Then the expression for B_{1j} exhibits explicitly the dependence on ℓ, from which we obtain $D_{1j}(\ell, t)$ at the position of Regge poles (or cuts) in $Q\bar{Q}$ scattering amplitudes $A_{11}(\ell, t)$. The meson-meson residua then are obtained from equation (2).

We will explore the consequences in detail of a zero-range approximation for Γ which contains only one parameter: the binding energy of the $Q\bar{Q}$ pair constituting the meson.

4. STRUCTURE FACTORS AND CONTINUATIONS

4.1 D functions (structure factors) for meson residua

The reaction $MM \rightarrow Q\bar{Q}$ for spinless mesons and spin 1/2 quarks can be described with two invariant amplitudes, A and B, as in $\pi\pi \rightarrow N\bar{N}$. The latter reaction has been discussed extensively by Frazer and Fulco[11] and by Singh;[12] starting with the normal-threshold (tight binding) case we can carry over their kinematics directly to $MM \rightarrow Q\bar{Q}$. If Λ is the absolute value of the sum of quark and antiquark helicities (Λ = 0 or 1), B^{Λ}_{1j} are the appropriate parital wave projections (with threshold factors extracted), and assuming A = 0 [as we have only a positive parity spin 1/2 pole as in $\pi\pi \rightarrow N\bar{N}$] , from ref. (12) one obtains:

$$\begin{aligned} B^{0}_{1j}(\ell, t) &= (pq)^{-\ell} \int du\ Z\ Q_{\ell}(Z)\ \mathrm{disc}_u B(u,t) \\ B^{1}_{1j}(\ell, t) &= \frac{1}{2}(pq)^{-\ell} \int du[Q_{\ell+1}(Z) - Q_{\ell-1}(Z)]\,\mathrm{disc}_u B(u,t) \end{aligned} \qquad (9)$$

and

$$Z(u) = (u + p^2 + q^2)/2pq\ ;$$

$$p = (t/4 - \mu^2)^{1/2}, \quad q = (t/4 - M^2)^{1/2}\ .$$

Now if the zero-range approximation for the wave function is used (Γ constant), the discontinuity is concentrated only at $u = M^2$ and we obtain

$$\begin{aligned} B^{0}_{1j}(\ell, t) &= \Gamma^2_j (pq)^{-\ell}\ Z_0 Q_{\ell}(Z_0) \\ B^{1}_{1j}(\ell, t) &= \Gamma^2_j (pq)^{-\ell}\ [\ Q_{\ell+1}(Z_0) - Q_{\ell-1}(Z_0)]\ /2 \end{aligned} \qquad (10)$$

where

$$Z_0 = (M^2 + p^2 + q^2)/2pq \quad .$$

In the general case we can parametrize Γ^2 by a sum of poles,[8] and the resulting B's will be sums of terms of the form (10). The continuation in angular momentum (in the normal threshold case) is now simple, since the Q functions are analytic in ℓ and damp out at infinity sufficiently rapidly to insure uniqueness via Carlson's theorem.

For practical applications, instead of using a multi-pole form for Γ which naturally damps out at high t we will use a zero-range approximation as given in (10), and after the D functions are expressed in terms of these B's, the masses continued to the weak binding region and the dominant anomalous threshold contribution exhibited, the t region above the normal threshold will be dropped.[13] The explicit form of the elements of the channel 1 phase space matrix, chosen to take into account the normal threshold behavior of the partial waves in the t channel and the analytic properties at t = 0 of the helicity amplitudes, are:

$$\rho_1^+(\ell, t) = \frac{2[q(t)]^{2\ell+1}}{\sqrt{t}}$$

$$\rho_1^-(\ell, t) = \left(\frac{t}{4M^2}\right) \frac{2[q(t)]^{2\ell+1}}{\sqrt{t}}$$

Finally, the result of the above continuation (keeping only the anomalous region) is:

$$\begin{aligned} D_j^+(\ell, t) &= \Gamma_j^2 \cdot \frac{2}{\pi} \int_{t_0}^{4M^2} \frac{dt'q'}{\sqrt{t'}(t'-t)} Z_0(t') P_\ell[Z_0(t')] (q'/p')^\ell \\ D_j^-(\ell, t) &= \Gamma_j^2 \cdot \frac{1}{\pi} \int_{t_0}^{4M^2} \frac{dt'q'}{\sqrt{t'}(t'-t)} \{P_{\ell+1}[Z_0(t')] - P_{\ell-1}[Z_0(t')]\} \\ &\quad \times (q'/p')^\ell (t'/4M^2) , \end{aligned} \qquad (11)$$

where

$$t_0 = \frac{\mu^2}{M^2}(4M^2 - \mu^2) .$$

[The notation (±) has been introduced instead of $\Lambda = (0, 1)$.]

For $M = .425$ GeV and $\mu = .80$ GeV, $t_0 = .29$ GeV; this gives roughly the same binding energy as Freund and Predazzi[10] obtain to fit the coupling constants. With this set of masses we obtain (assuming a constant quark form factor) a satisfactory fit to the pion form factor $F_\pi(t)$, as shown in Figure 2, taking F_π from the VH formula and high energy πp scattering data. (Also shown in Fig. 2 is the empirical proton form factor G_{Mp}, which drops off somewhat more rapidly, illustrating a probable difference in structure between proton and pion.)

With the same masses, putting $\ell = \alpha_\rho(t) \simeq .5 + 1.0\,t$, we find $D^+(\ell, t)$ from (11) to be comparatively flat for $|t| < .50\ \mathrm{GeV}^2$, and D^- remains small compared to D^+. These D's, for ρ exchange (e.g., in charge exchange reactions), are shown in Figure 3. (The relative normalization between Figs. 2 and 3 is significant, but not the absolute normalization.) If the binding energy is much smaller, one obtains similar behavior for the form factors and the residua. This is illustrated in Figure 4, where the value $M = .41$ GeV is used and the $D^\pm$ calculated both for $\ell = 1$ and $\ell = \alpha_\rho(t)$ (arbitrary absolute normalization).

The qualitative behavior of these functions can be seen from the expressions (11). For very small t_0, the integrands are sharply peaked at the lower limit from the $(t')^{-1/2}$ factor. At the lower limit, however, $Z_0 = 1$ and there is no variation of the P_ℓ factor as ℓ changes. Near $t' = 0$ the factor $(q'/p')^\ell$ becomes $(M/\mu)^\ell$, which changes by only 40% when ℓ varies over the range $0 < \ell < 0.5$ [as $\alpha_\rho(t)$ does for $|t| < .50$], and is significant only for precise symmetry considerations. The D's therefore differ only slightly for $\ell = \alpha_\rho(t)$ compared to $\ell = 1$. At the same time, we find $|D^-| \ll |D^+|$ since the integrand of D^- vanishes at the lower limit. As the binding energy is increased, the contribution of higher t' becomes more important; the ℓ dependence becomes significant, and D^+ does not dominate so much over D^-.

It may be noted that expressions (10), inserted in the D integrals before continuation, are just the lowest order ($Q\bar{Q}$ intermediate states) expressions for the vertex functions of a spin ℓ elementary meson coupling to spinless mesons, with constant vertices in the triangle diagram.

4.2 Effect of quark properties on meson residua

In order to compute the Regge residua in meson-meson scattering, given the D functions, we must assume some properties of the channel 1 ($Q\bar{Q}$) scattering amplitudes A_{11}. If we assume slowly varying reduced residua in channel 1, which have similar spin-amplitude ratios for each pole, we can relate the A_{11} residua to the electromagnetic properties of the quarks, e.g., magnetic moments. In this case we can write, near each pole under consideration,

$$\underset{\sim}{T}_{11}(\ell, t) \equiv \underset{\sim}{\rho}_1^{1/2}\, \underset{\sim}{A}_{11}(\ell, t)\, \underset{\sim}{\rho}_1^{1/2} = \underset{\sim}{\beta}_1(t)/[\ell - \alpha(t)] \qquad (12)$$

where $\alpha(t)$ is the pole trajectory in the ℓ plane, and $\underset{\sim}{\beta}_1$ is essentially constant. (We neglect signature here.)

Since residua factorize in helicity indices, we can express $\underset{\sim}{\beta}_1$ as:

$$\underset{\sim}{\beta}_1 = \begin{pmatrix} \gamma_1^2 & \gamma_1\gamma_2 \\ \gamma_1\gamma_2 & \gamma_2^2 \end{pmatrix}$$

If we examine the unitarity equation (7) for the quark e.m. form factors, near the lowest vector meson pole (assuming it dominates the quark form factor), we find that

$$\frac{G_{1-}(0)}{G_{1+}(0)} = \frac{\gamma_2}{\gamma_1} \quad .$$

This relation connects the magnetic moment to charge ratio of the quark with the residue $\underset{\sim}{\beta}_1$ appearing in hadron residua, through (2) and (12), leaving only an overall normalization factor independent of s or t.

If the quark is pure Dirac particle, i.e. with no anomalous magnetic moment, then $G_-(0)/G_+(0) = 1$. We find then the contribution from terms involving D^- in meson-meson scattering are quantitatively unimportant because $|D^-| << |D^+|$ for moderately weak (~ 50 MeV) binding; e.g., for ρ exchange

$$[\beta_\rho(t)]_{jk} \cong D_j^+[\alpha_\rho(t), t] \times D_k^+[\alpha_\rho(t), t] .$$

In particular, from Figure 3, this means the ($\rho\,\pi\pi$) vertex should be slowly varying with t compared to typical electromagnetic form factors. If we assume a similar situation holds for the nucleon vertex, then we find agreement with phenomenological analysis[14] of $\pi^- p \to \pi^0 n$, wherein constant residua give a good fit for $|t| < .50\ \text{GeV}^2$.

We remark again that the same model leads to a Pomeranchon residue which (on the contrary) drops off sharply like the electromagnetic form factor (cf. Fig. 2), as suggested by the Van Hove formula, (1).

At this point it is appropriate to record the connection between differential s-channel meson-meson cross-sections and A_{22}. If

$$\left(\frac{d\sigma}{d\Omega}\right)_{22} = \frac{1}{s}|F|^2, \quad \text{then}$$

$$F \underset{s \to \infty}{\to} (s/s_0)^\alpha [D^T(\alpha, t)\beta_1 D(\alpha, t)](2\alpha + 1)\left[\frac{1 \pm \exp(-i\pi\alpha)}{\sin\pi\alpha}\right] \tag{13}$$

for each pole, where we have restored the signature factor; s_0 is the scale factor[15] appropriate for $Q\bar{Q}$ scattering, presumably in the neighborhood of $2M^2$.

4.3 Baryon vertices

Since the baryons are assumed conventionally to have a three-quark structure, the formulation of the vertex functions and their relation to wave functions is not such a well-known procedure as with the mesons. Consequently, we cannot rely on previous work and we will only treat the baryons in a simplified way, which is similar to the mesons.

Guided by SU(6) baryon wave function symmetry with all quarks in relative S-waves, we suppose that when a baryon emits a (virtual) quark the remaining part (QQ state)[16] A has 1^+ or 0^+ quantum numbers only, and the ($A\bar{Q}B$) coupling has the simplest possible, i.e., non-derivative form. Then the kinematics for $B\bar{B} \to Q\bar{Q}$ resemble those in $N\bar{N} \to Y\bar{Y}$ with scalar and axial vector exchange with non-derivative coupling. This case can now be treated with the formalism of Chan,[17] who considered exchange of Regge and elementary poles in $N\bar{N} \to Y\bar{Y}$.

In a zero-range approximation for the ($A\bar{Q}B$) vertex, we obtain a pole structure (similar to the meson case) for Chan's invariant amplitudes F_1 and F_4, yielding with normal thresholds

$$[\rho_1 B^J_{13}]_{11} = C_1 Q_J[Z_A(t)]$$

$$[\rho_1 B^J_{13}]_{22} = C_4 Q_J[Z_A(t)]$$

where $Z_A(t) = (M_A^2 + q^2 + n^2)/(2qn)$; C_1 and C_4 are residua;

and $$[\rho_1 B^J_{13}]_{12} = [\rho_1 B^J_{13}]_{21} = 0$$

where (in all the above) channel 3 refers to $N\bar{N}$, $n^2 = t/4 - M_B^2$ where M_B is the baryon mass, and the second set of subscripts refers to helicity indices as defined by GGMW.[18]

Finally, inserting these in the expressions for $[D_{13}]_{jk}$ and analytically continuing in baryon mass (and/or A mass) to describe weak binding, keeping only the anomalous region, we obtain:

$$[D_{13}]_{11} = C_1 D_0(\ell, t) \ ;$$

$$[D_{13}]_{22} = C_4 D_0(\ell, t) \ ;$$

and

$$[D_{13}]_{12} = [D_{13}]_{21} = 0$$

where

$$D_0(\ell, t) = \frac{1}{\pi} \int_{t_A}^{4M^2} \frac{dt' q'}{\sqrt{t'}(t' - t)} \left(\frac{q'}{n'}\right)^{\ell} P_\ell[Z_A(t')]$$

The lower limit t_A is that value of t which makes $Z_A = +1$.

Since both spin terms have the same t-dependence in this approximation we see immediately that both the electric and magnetic form factors of the nucleon have the same t-dependence, in agreement with experiment. Furthermore, the qualitative properties (ℓ and t dependence) of these D_{13} are similar to those of D^{+}_{12} which, as previously discussed, leads to satisfactory agreement with $\pi^{-}p$ charge exchange data, as well as yielding the Van Hove formula for πN scattering.

The residua C_1 and C_4 can, in principle, be fixed from electromagnetic properties of nucleons. However, we see that such a procedure would lead to a ratio of helicity-flip to helicity non-flip which is independent of the trajectory; this is undesirable, since the ratio as determined in $\pi^{-}p \rightarrow \pi^{0}n$ is much larger[19)] than that of the static nucleon properties, i. e., magnetic moment. This means that our simple baryon model is not quite adequate to describe the details of the accepted phenomenological residue fits.

5. CONCLUDING DISCUSSION

The central point of our model has been the generalized impulse approximation formula (2). This equation follows from one-channel dominance for exchange processes and one of the following assumptions: (a) spin 1/2 constituents and asymptotically high energies, or: (b) dominance of singularities above $\ell = 0$ in the ℓ plane (usually relevant at high energies) in the scattering of constituents. Such a formula is already enough to obtain the Van Hove relation (1) when diffraction singularities (near $\ell = 1$) dominate, and constituents' form factors and scattering amplitudes vary weakly with t.

To draw further conclusions concerning moving Regge poles one needs additional dynamical assumptions. In particular, if moderately weak binding is assumed such that the anomalous region ($t_0 < t' < 4M^2$) is most important in the D functions (equivalently, in the form factors), our general formulation allows a connection to be made between form factors and residua, represented by equations (8) and (13).

This assumption is not necessary if one is willing to parametrize with sufficient freedom the input Born terms. Thus, using the general form (9), one can use (2) together with (8) in the normal case, and avoid the assumption of anomalous thresholds (weak binding) altogether. However, in such a case it would be difficult to justify the one-channel approximation.

When we specialize to the weak-binding case we find consistency between: (a) Diffraction (Pomeranchuk) residua falling with t (like electromagnetic form factors) in agreement with asymptotic πp elastic scattering data; (b) Residua for ρ exchange varying slowly with t, as indicated by $\pi^- p$ charge-exchange data; (c) Absolute value of strong-interaction coupling constants, as computed by Freund and Predazzi,[10)] all with a binding energy of around 50 MeV.

Finally, we find that it is not possible to use a simple ZRA and extrapolate from $t = 0$ to the vector meson poles, as one would like to do in pole models. A more detailed model for the Γ's is necessary.

1) L. Van Hove, at 1966 Topical Conference on Two-Body Reactions at High Energy at Stony Brook, L.I., N. Y. (CERN preprint, 1966).

2) This relation was first postulated for large momentum transfer by Wu and Yang, [Phys. Rev. 137, B708 (1965)] and later discussed for all momentum transfer by Chou and Yang at the International Conference on High Energy Physics and Nuclear Structure, Weizmann Institute, Israel, 1967 (State Univ. of N. Y. at Stony Brook preprint, 1967) in terms of a droplet model for high energy hadron reactions. Such a model is not appropriate, however, for a discussion of Regge poles. The essential idea common to both types of models has been stated clearly by Feynman, in a remark at the XIIIth International Conference on High Energy Physics, Session 6 (Univ. of Calif. Press, Berkeley and Los Angeles, 1967).

3) Small angle pp and πp data were taken from K. J. Foley et al., Phys. Rev. Letters 11, 425 (1963); form factor data was obtained from the compilation of Hand, Miller and Wilson, Rev. Mod. Phys. 35, 335 (1963).

4) The Sachs magnetic form factor G_M has been plotted since this is more accurately determined than the electric form factor G_E; quark models suggest the two are proportional, in agreement with experiment to date. In the case of the scattering data we have chosen pp data only, ignoring $p\bar{p}$ results, since the former seem to be approaching an asymptotic behavior more rapidly than the latter, indicating that for $p\bar{p}$ a simple singularity in the ℓ plane is not yet dominant (e.g., at 12 GeV/c).

5) We will find later that with weak binding there is a correlation between meson mass and the width of the forward peak obtained with $\beta(t)$. If the mass is too small (i.e., μ_π) the peak is too narrow. Therefore, we use a meson mass around M_ω.

6) We offer no explanation concerning the experimental nonexistence of light quarks.

7) R. Blankenbecler, L. F. Cook, Jr., and M. L. Goldberger, Phys. Rev. 128, 2440 (1962).

8) F. Gross, Phys. Rev. 134, B405 (1964); ibid. 136, B140 (1964). See also M. Gourdin, M. LeBellac, F. M. Renard and J. Tran Thanh Van, Nuovo Cimento 37, 524 (1965); and for a discussion of basic principles, R. Blankenbecler and Y. Nambu, Nuovo Cimento 18, 595 (1960).

9) Bjorken and Nauenberg, Phys. Rev. 121, 1250 (1961).

10) P. G. O. Freund and E. Predazzi, Physics 3, 81 (1967).

11) W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1603 (1960); ibid., p. 1069.

12) V. Singh, Phys. Rev. 129, 1889 (1963).

13) The expressions from the ZRA, (10), lead to a logarithmic divergence in D if taken literally at large t. A more realistic approximation for the wave functions would agree with the ZRA in the small t region but provide a damping necessary for larger t. If we keep only the anomalous region our procedure would yield the same physics.

14) The relatively slow variation of ρ residua can be noticed in the fits of Phillips and Rarita [Phys. Rev. 139, B1326 (1965)] and early considerations of Logan [Phys. Rev. Letters 14, 414 (1965)]. We ignore the "crossover phenomenon" discussed by Phillips and Rarita, and suggest the charge-exchange fits of Yokosawa [Phys. Rev. 159, 1431 (1967)] as our reference point.

15) For moving poles ($\alpha \neq 1$), the choice of S_0 is important, especially in comparing symmetry predictions. This has been discussed by

James and Watson [Phys. Rev. Letters 18, 179 (1967)] using a simple bound state picture. Changing S_0 in a purely phenomenological approach from S_{01} to S_{02} introduces a multiplicative factor $(S_{01}/S_{02})^{\alpha(t)}$ for each pole, resulting in a different residue. In our model, on the other hand, the residua are obtained unambiguously from the D functions and S_0 is the same for any hadron reaction. We find that in the extreme weak-binding limit, the result of James and Watson appears in our model *via* the threshold factor $(p')^{-\ell}$ in the meson D function integrand which may be approximated by $\mu^{-\ell}$ for sufficiently weak binding. A similar factor occurs with baryon vertices. In our approach we use a degenerate (mean) meson-mass and would not make a correction when comparing π with K data, but such factors become important when comparing MB and BB reactions.

16) We are well aware that there is no QQ bound state, A. We use the single-particle Born term only as a guide.

17) C. H. Chan, Phys. Rev. 133, B431 (1964).

18) M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960); referred to in text as GGMW.

19) G. Hohler, J. Baacke, H. Schlaile, and P. Sonderegger, Physics Letters 20, 79 (1966); B. R. Desai, Phys. Rev. 142, 1255 (1966).

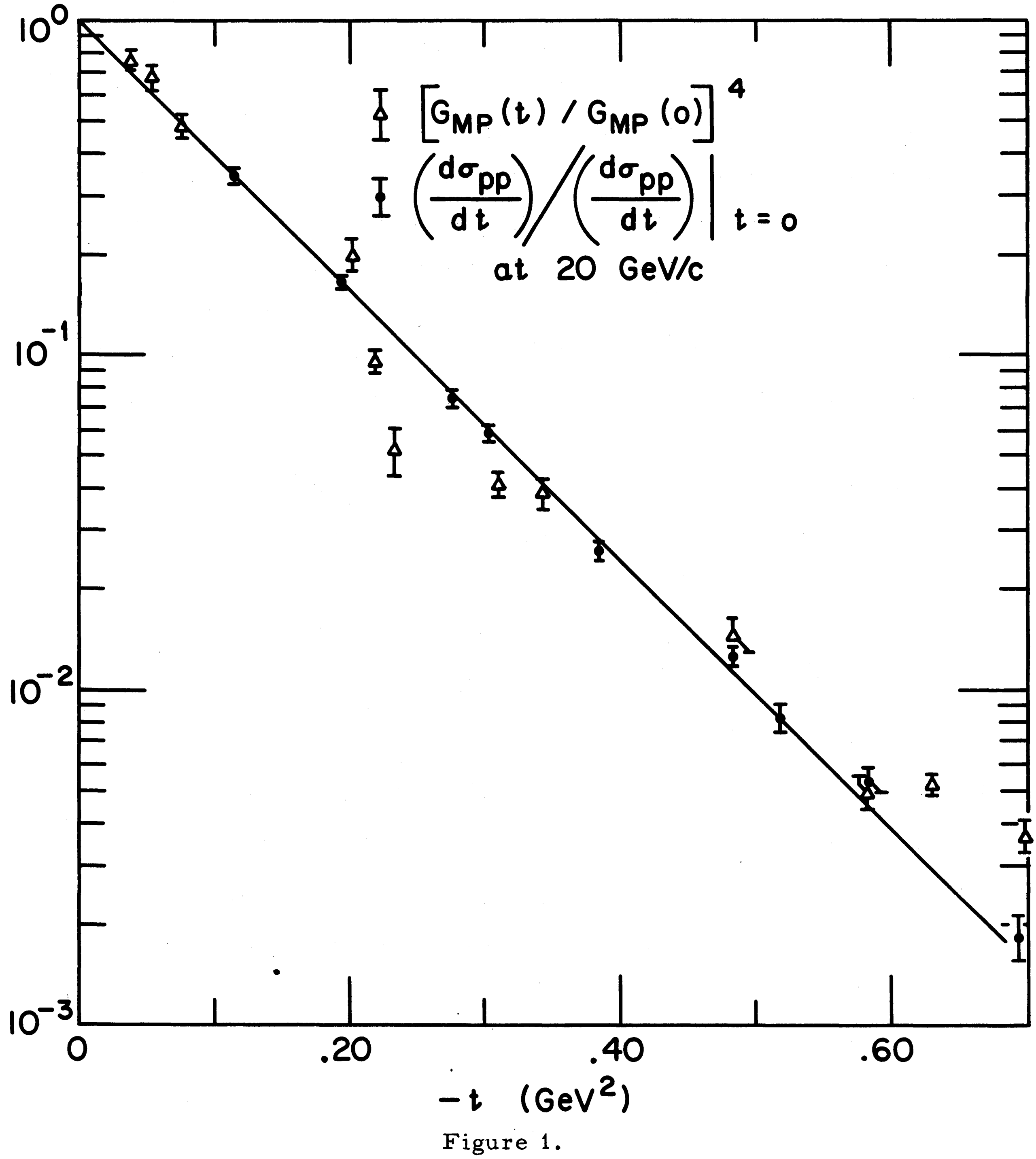


Figure 1.

Comparison of Differential Cross Section for p-p elastic scattering as a function of t at 20 GeV/c with prediction of Van Hove relation (Eq. 1), using magnetic Sachs form factor of proton as representative electromagnetic form factor. Data sources are given in references 2 and 3. The straight line is to guide the eye only.

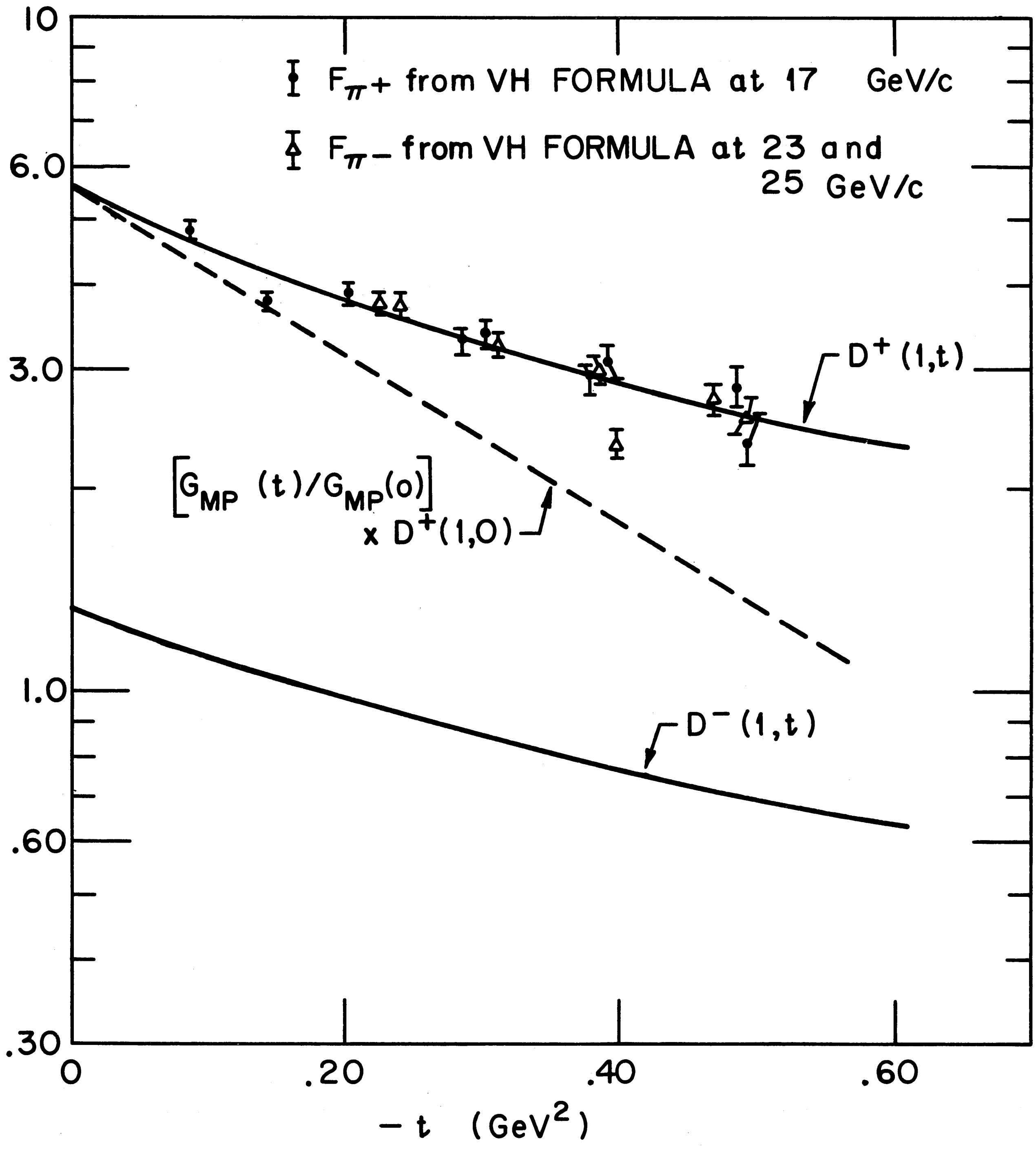


Figure 2.

Results of calculation for structure factors $D^{\pm}$ of mesons, using zero-range approximation with M = 425 MeV and μ = 800 MeV, evaluated at ℓ = 1. For comparison, the pion electromagnetic form factors $F_{\pi+}$ and $F_{\pi-}$ are shown as deduced from small-angle high-energy $\pi^{\pm}p$ scattering using the Van Hove formula. Data sources are given in reference 3. Also shown (dotted line) is proton form factor.

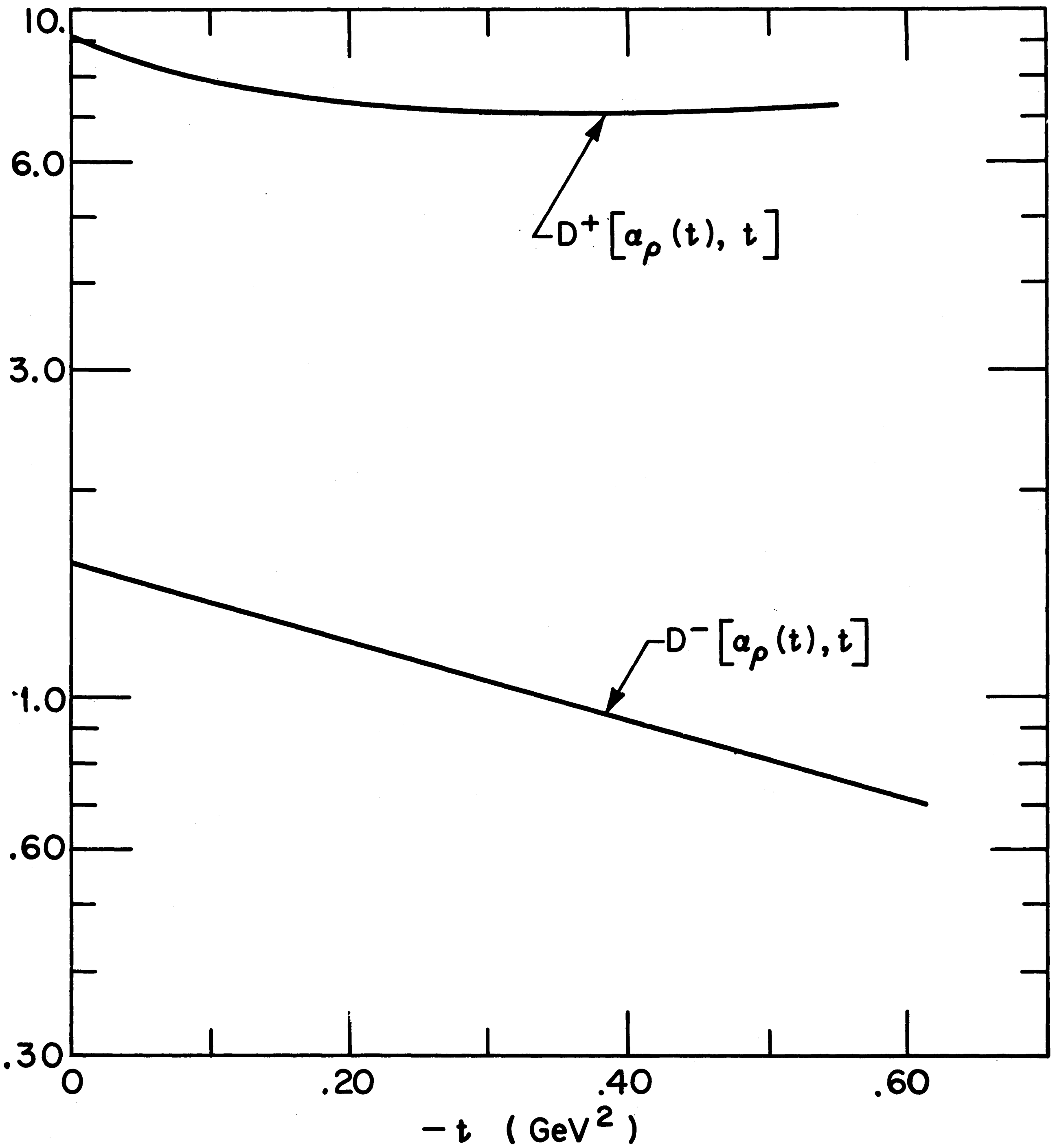


Figure 3.

Calculated residue structure factors $D^{\pm}$ for the ρ trajectory, $a_{\rho}(t) = .5 + 1.0\, t\,(GeV^2)$, with same masses as in Figure 2.

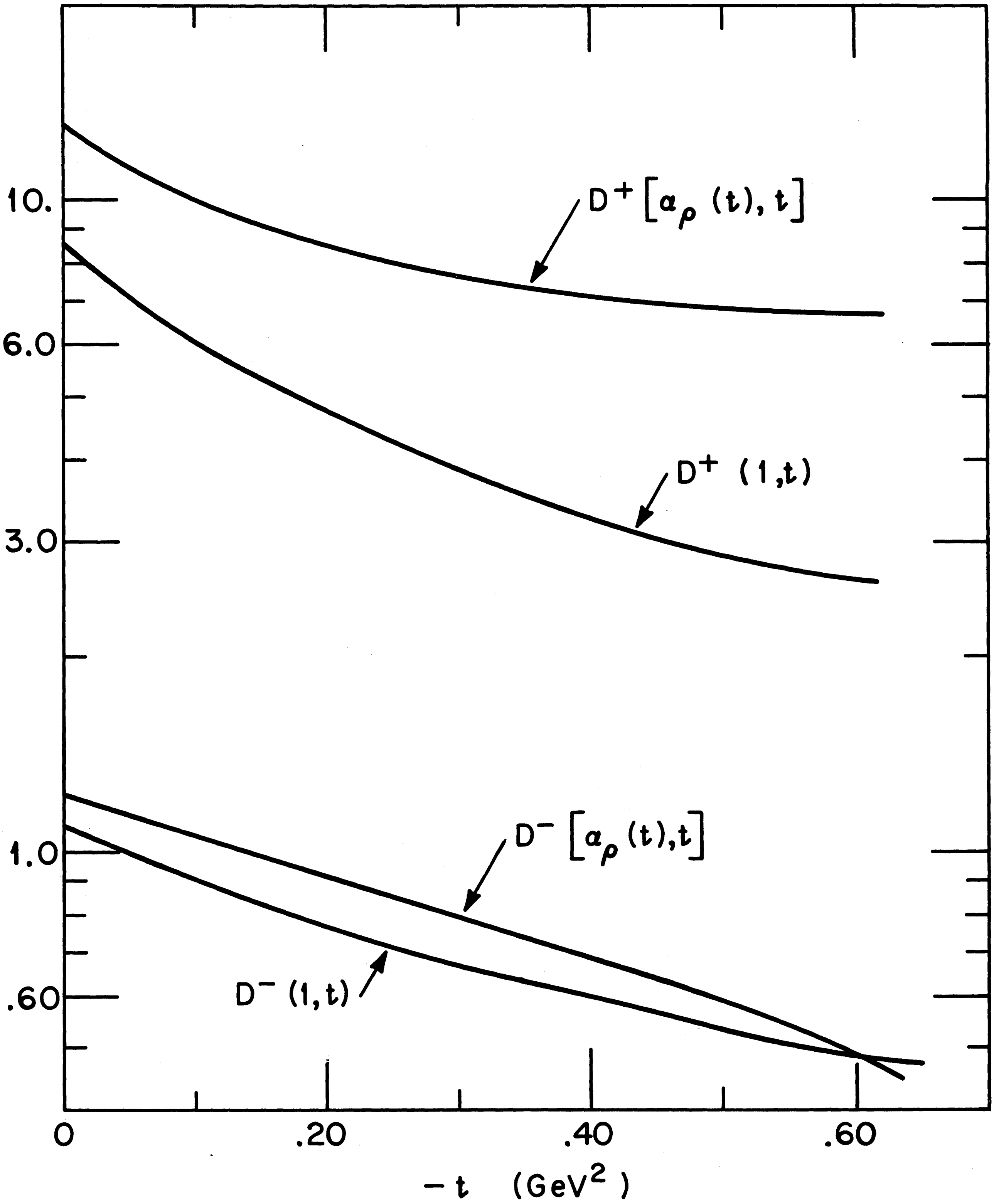


Figure 4.

Structure factors $D^{\pm}$ for M = 410 MeV and μ = 800 MeV, both with ℓ = 1 and $\ell = \alpha_{\rho}(t)$.