ON THE HIGH-ENERGY LIMIT OF SCATTERING AMPLITUDES

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I shall discuss

1) some results on the high-energy behaviour of scattering amplitudes which follow from ordinary dispersion relations;

2) a paper by Gribov and Pomeranchuk which was submitted to this conference and will be included in the proceedings*).

I think it is in agreement with the purpose of this meeting if I try to summarize some arguments which are relevant to the high-energy behaviour of amplitudes, rather than describe the papers I mentioned in great detail.

A high-energy collision depends on at least two important parameters: the total energy or centre of mass momentum K of the incoming particles and the momentum transfer q between particles. It has been stressed repeatedly that most high-energy events involve only low values of $q \approx m_\pi$, corresponding to collisions with large impact parameter $r \sim \frac{1}{q}$. This follows convincingly from the large values of total cross-sections at high energies. If the interaction were limited to impact parameters $r \lesssim \frac{1}{m_N}$, cross-sections should be of order $\sigma \approx \pi \cdot \frac{1}{m_N^2} \approx 1$ mb. Hence, at most a few per cent of all events will be due to distances corresponding to the inner structure of the particles as against their pion cloud. I repeat this only to emphasize that there are two rather different limits of scattering amplitudes to be considered: total energy or $K \to \infty$ for fixed q; or both $K \to \infty$ and $q \to \infty$, i.e. in a two particle reaction $K \to \infty$ at fixed scattering angle. I consider here only the first limit.

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*) Compare Gribov-Pomeranchuk: p. 376

The arguments on the behaviour of amplitudes at very high energies are normally of the form of statements referring to infinite energy. They make sense only if some asymptotic expansion of the amplitudes is possible and we may consider the first term of this expansion to be a reasonable approximation at sufficiently high energies. At what energies can we expect such asymptotic statements to be applicable? We can only make a guess by extrapolating from our present knowledge of particles and interactions: the scale of energies is determined by the particle masses which fix the thresholds of reactions. We may expect that an asymptotic behaviour is reached if the available kinetic energies are large compared to the masses of the heavier particles, i.e. large compared to m_N. From this criterion it appears that for many reactions (enough perhaps for a general survey) a total c.m. energy of the order of 10-20 GeV is sufficient. Such an energy also allows the study of momentum transfers of $\approx$ 10 GeV which by the same criterion ($9 \gg m_N$) might be enough to reach an asymptotic region.

Two reservations are to be made:

1) The scale of energies may be altered very much if small numerical parameters are involved; i.e. the fine-structure constant and the Fermi coupling constant. Quite different energies may therefore be necessary to investigate electromagnetic and in particular weak interactions.

2) Effects of a qualitatively different nature may occur at very high energies. This we certainly cannot find out by extrapolating from present knowledge.

I discuss now in particular the expected asymptotic behaviour of amplitudes referring to elastic scattering of two particles. Very little is known as a rigorous consequence of general principles and all statements make some assumptions on the mathematical form of the involved functions.

Let $T(s,t)$ be an elastic amplitude with c.m. energy $\sqrt{s}$ and momentum transfer $q = \sqrt{-t}$. By the optical theorem the total cross-section is related to the imaginary part of the forward amplitude by

$$\sigma_T(s) = \frac{4\pi^2}{s} \cdot \text{Im } T(s,0)$$

if the amplitude is correspondingly normalized. Therefore, if σ_T approaches a constant as $s \to \infty$,

$$\text{Im } T(s,0) \to s \,.\, \frac{\sigma_T(\infty)}{4\pi^2} \quad .$$

The so-called diffraction picture of elastic scattering assumes then (with some support from partial wave analysis) in its simplest form that the imaginary part has an analogous behaviour for finite momentum transfer ($t < 0$)

$$\text{Im } T(s,t) \to s \,.\, f(t)$$

and that the real part is negligible.

The latter assumption can be studied by analyzing the high-energy properties of ordinary dispersion relations. These essentially determine the high-energy real part if the high-energy imaginary part is given. The result which follows from ordinary dispersion relations can be stated as follows*):

If the imaginary part of an amplitude has an asymptotic expansion in some power of s:

$$\text{Im } T(s,t) = s \,.\, f_1(t) + s^{1-\alpha} \,.\, f_2(t) + \cdots$$

then

$$\text{Re } T(s,t) \to s^{1-\alpha} \,.\, g(t) \quad \text{if} \quad 0 < \alpha < 1$$

and

$$|\text{Re } T(s,t)| \to \leq \ln s \,.\, g(t) \quad \text{if} \quad \alpha < 1 \,.$$

Dispersion relations therefore support the diffraction picture insofar as the leading term $\sim s$ of the imaginary part does not contribute to the real

*) For details compare H. Lehmann: On the high-energy limit of ordinary dispersion relations (to appear in Nuclear Physics).

part. The ratio of real to imaginary part approaches zero more strongly, the better the imaginary part is represented by its leading term. If the imaginary part does not behave $\sim s$ at high energies, the diffraction pattern is less pronounced; e.g. if cross-sections decrease logarithmically, the ratio of real to imaginary part vanishes only logarithmically.

If neither of the two particles is self-conjugate, the given behaviour of the real part is valid only if the difference of particle-particle and antiparticle-particle amplitudes satisfies in addition a sum-rule which is well known in the case of forward scattering. Otherwise the real part would have a term $\sim s$.