"Strong Interaction IVB Models"

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The calculation of the $(K_L^\circ - K_S^\circ)$ mass difference has been shown[1], using plausible approximations, to lead to a small value of the weak interaction cutoff $\Lambda(\simeq 5$ Bev), rather independent of the hadronic strong interactions. The fact that $\Lambda << 300$ Bev (the unitarity limit) implies that the weak interaction saturates at a relatively low energy and a fortiori that the weak interaction never becomes "strong". To state the result another way, it appears that the effective expansion parameter for weak interactions is $G\Lambda^2 \simeq 2 \times 10^{-4}$ (G is the weak coupling constant) so that it is justified to make a sharp separation between the lowest order and higher order weak processes.

While it is agreeable to predict a reasonable high energy behaviour for the weak interaction and to expect much reduced higher order weak decay rates, the origin of such a low value of Λ becomes a problem of great interest. Most speculations concerning this question have been presented within the framework of some form of intermediate vector boson (IVB) model. Ioffe[2] has conjectured that the weak interaction will be cut off by the electromagnetic form factor of the W boson at energies where the electromagnetic interaction of W becomes "strong"; however, this should happen when $\Lambda \simeq \frac{m_W}{\sqrt{\alpha}}$ (m_W is the mass of W, α is the fine structure constant). If we take $\Lambda \simeq 5$ Bev, we find $m_W \simeq 0.5$ Bev, which is substantially smaller than the experimental lower limit. Gell-Mann et al[1] have attempted to achieve the low value of Λ by introducing an octet of scalar bosons, which eliminates the divergence resulting from the W bosons; apart from the large number of new (unobserved) bosons entering this theory, a dichotomy between "self-current" and "non-self-current" weak processes is set up which goes counter to the usual argument of universality.

We have suggested[1] that the low value of Λ can be explained by some form of strong interaction of the IVB (assuming it exists) which would induce a "strong" form factor for W and thereby provide a "natural" cutoff at $\Lambda \simeq m_W$. Moreover, the low value of the weak interaction cutoff is not the only reason for giving serious consideration to the possibility of a strong interaction for the IVB. We also give weight to the argument that the pattern observed in nature, to wit that the more massive particles enjoy strong interactions (e.g. the average mass of hadrons is substantially larger than the average mass of leptons), would be contradicted if the IVB only possessed semi-weak and electromagnetic interactions as in the usual IVB model. A strong interaction of W would provide a simple mechanism[3] for generating its large mass as a "self-energy" effect. For generating this self-mass, the strong interaction of W can have a hadronic origin[3] or result from a self-interaction among the W's themselves[4]. It will turn out that one version of the model involving a strong W interaction with hadrons will involve a Sakata-type triplet of W's (i.e. two neutral W's and one charged) and a strong quadratic interaction of this W triplet with hadrons[3]. On the other hand, the alternative of strong self-interaction of W will arise as a W^3 ("cubic") interaction in a theory which regards CP violation as maximal at the semi-weak level[4]. We propose to examine some of the predictions of these two models in order to encourage experimental tests of the hypothesis of a strongly interacting W.

The idea that the IVB hypothesized as a particle interacting semi-weakly and electromagnetically with leptons and hadrons might also interact strongly with itself or hadrons, dates back to at least 1964. Several authors[5] pointed out that the large mass of W and several other interesting consequences would follow from a sufficiently strong W interaction. This proposal, in our opinion, acquired greater interest when it was combined with some SU(3) arguments concerning the representation to which the W boson should be assigned[6]. One is then led to a strong quadratic interaction of W's with hadrons by the following sequence of arguments. In view of the successful application of unitary symmetry to weak hadron currents, it is reasonable to assign W to a representation of the SU(3) group if the IVB actually exists. The smallest representation for W, which gives a simple explanation of the $\Delta I = 1/2$ selection rule (as well as the "octet rule") in non-leptonic decays, is the triplet representation[7]. If one assigns W to the triplet representation, one is confronted by the dilemma of fractional electric charge Q and hypercharge Y unless one enlarges the SU(3) group to the U(3) group through the introduction of the triality quantum number t. By redefining the electric charge $\bar{Q} = Q + t/3$ and the hypercharge $\bar{Y} = Y + 2t/3$, one may obtain integral values for these quantum numbers if one assigns the value $t = 1$ to the W triplet. The W's then comprise a Sakata-type triplet consisting of an "isotopic doublet" (W^+, W^o) and an "isotopic singlet" $W^{o\prime}$. The next step is to note that in writing down the semi-weak interaction of the W triplet (with $t = 1$) with the hadron current (which belongs to the octet representation with $t = 0$), the triality quantum number is not conserved. If we continue to insist that the total strong interaction Hamiltonian consists of a unitary singlet plus a unitary octet (the symmetry-breaking term) in SU(3), it follows that any strong interaction assumed to exist between the W triplet and hadrons must conserve triality. In this fashion we arrive at the conclusion that a strong quadratic interaction of the W triplet with hadrons can be postulated without altering the semi-weak interaction and, a fortiori, without modifying in any way the stability of the IVB against strong decays. This model is called the strong quadratic IVB model[3,8].

Despite the attractiveness of the triplet version of the IVB model in helping to define the nature of the hadronic strong interaction of the W's, the apparent compatibility with the usual semi-weak interaction (with hadrons and leptons) must be scrutinized more carefully before spelling out any further consequences. The first obstacle which results from a strong quadratic interaction of W's with normal hadrons is the possibility that too large a cross-section will be predicted for the direct production of W in high energy neutrino collisions. It will be recalled that the dominant mechanism customarily considered for W production is through the coherent (or incoherent) electromagnetic reaction: $\nu_\ell + Z \rightarrow W + \ell + Z$.

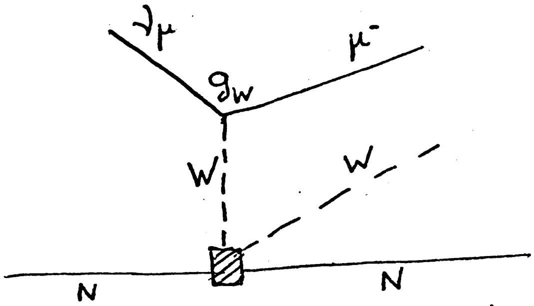

Fig. 1: Production of W in reaction $\nu_\mu + N \rightarrow W^+ + \mu^- + N$ on strong quadratic IVB model

A new mechanism is opened up for W production in neutrino-nucleon collisions by a possible strong quadratic interaction of W's with hadrons as shown in Fig. 1. This new mechanism is necessarily incoherent (in contrast to the electromagnetic mechanism) but the coupling constant is strong (at the $W\bar{W}N\bar{N}$ vertex) and should be the dominant mechanism for W production despite the possibility of coherent electromagnetic production. The W production cross-section which is predicted by the diagram in Fig. 1 will depend, of course, on the strength and structure of the ($W\bar{W}N\bar{N}$) vertex. However, it can be shown that for reasonable choices of the (strong) coupling constant and a variety of structures[9], the predictions of the quadratic strong IVB model will not contradict the measured upper limit to date on the W production cross-section as long as m_W exceeds 2.5 - 3 Bev. This lower limit for m_W is only approximate in view of the crudeness of the theoretical calculations and lack of precise knowledge concerning the experimental neutrino spectrum but the value of interest for the weak interaction cutoff - in the vicinity of 5 Bev - is acceptable. It should be remarked in passing that while in the gauge-dependent version[9] of the strong quadratic IVB model, a self-mass of the IVB is generated in perturbation theory, this is not the case for the guage-invariant version[10].

The strong quadratic IVB model also raises some potential difficulties for certain of the "higher order" weak processes. For example, consider the scattering process $\nu_\mu + N \to \nu_\mu + N$ which is a second-order weak process in the usual IVB model. In the strong quadratic IVB model, the elastic scattering of neutrinos by nucleons can occur in the same (lowest) order of the weak interaction as $\nu_\mu + n \to p + \mu^-$ by means of the diagram shown in Fig. 2.

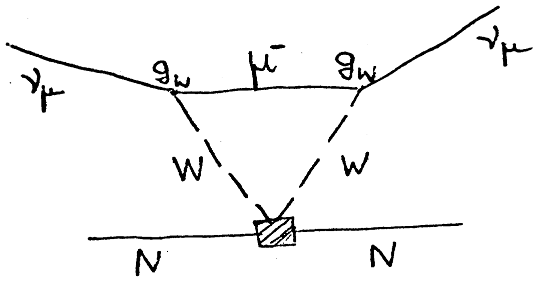

Fig. 2: Elastic neutrino-nucleon scattering on strong quadratic IVB model

However, it can be shown[8] that as long as the total strong interaction Hamiltonian is invariant under the replacement $W_\mu \to W_\mu^+$, which is true for the hypothesized strong quadratic interaction of the W's with hadrons, the cross-section for neutrino-proton scattering will vanish at zero energy and therefore depend on the factor q^2/m_W^2 (q is the four-momentum transfer). For sufficiently large m_W (say greater than 3 Bev), it is therefore easy to reconcile the cross-section predicted by the strong quadratic IVB model for ν_μ - N scattering with the findings of the high energy neutrino experiments thus far performed[11]. One might also be concerned by the compatibility of the strong quadratic IVB model prediction for the decay $K_L^\circ \to \mu^+ + \mu^-$ with the measured upper limit on the branching ratio of 1.6×10^{-6}. One might think that a diagram of the type shown in Fig. 3 would lead to an excessive branching ratio for $K_L^\circ \to \mu\,\bar{\mu}$ but it is evident that the charges and hypercharges of the (Sakata-type) W triplet forbid the diagram of Fig. 3 (since the $K_L^\circ W\bar{W}$ vertex must conserve both charge and hypercharge).

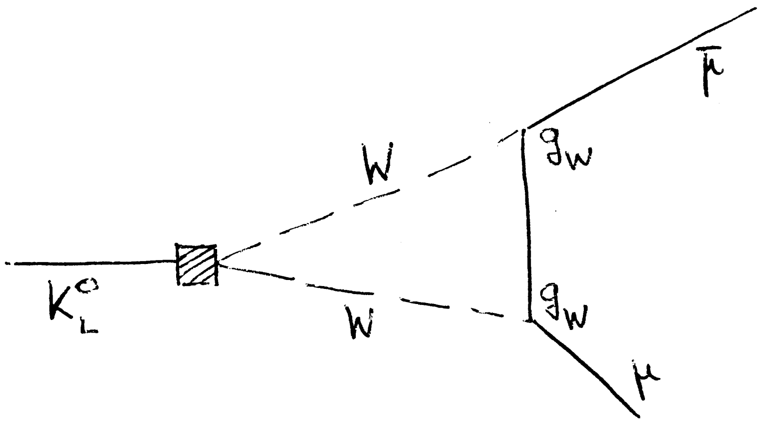

Fig. 3: $K_L^o \to \mu\, \bar{\mu}$ decay on strong quadratic IVB model

Hence, the decay $K_L^o \to \mu\, \bar{\mu}$ is unaffected by the quadratic strong interaction of the IVB. It can similarly be shown that the strong quadratic IVB model does not induce too large an "effective" neutral current interaction - insofar as existing experimental data are concerned - for other possible weak processes which are of higher order in the usual IVB theory.

There is another consequence of the strong quadratic IVB model to which attention should be called, namely whether the discrepancy between the muon decay constant, G_μ, and the vector coupling constant in β decay, G_β must be explained by setting $G_\beta = G_\mu \cos\theta$ (θ is the Cabibbo angle). It turns out that in the strong quadratic IVB model, the discrepancy between G_β and G_μ can be explained as a renormalization affect, i.e. as a deviation from the CVC hypothesis due to the strong quadratic W interaction with the hadrons. That is to say, there are additional diagrams contributing to the decay of the neutron that arise from the postulated strong quadratic interaction of the IVB (of the type shown in Fig. 4)

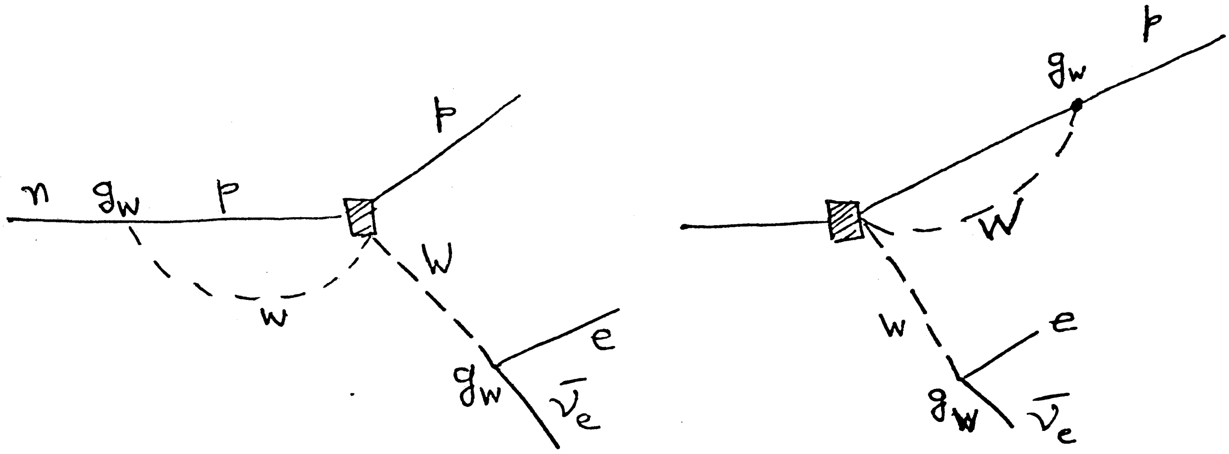

Fig. 4: Additional diagrams contribution to β decay renormalization in strong quadratic IVB model

which may or may not introduce renormalization affects depending on whether the strong quadratic interaction is gauge-dependent or gauge-invariant. Thus, if one writes down the semi-weak semi-leptonic interaction, the matrix element for β decay contains the term $M_\mu \equiv \langle p|W_\mu(0)|n\rangle$. It is then easy to show[3,8] that in the limit of zero four-momentum transfer, this matrix element becomes:

$$M_\mu^V = \frac{g_W}{m_W^2} \langle p \,|J_\mu^V\,(0)|\, n\rangle \tag{1}$$

where the superscript V indicates that we are interested in the vector contribution to the matrix element. In the usual IVB model, J_μ^V becomes the conserved isospin current so that G_β suffers no renormalization. However, in the strong quadratic IVB model, J_μ^V receives an effective contribution from diagrams like those in Fig. 4 and the proof of non-renormalization of G_β does not hold for the gauge-dependent type of interaction. Following along these lines, one can derive the correct sign and approximate order of magnitude[12] of the discrepancy between G_β and G_μ. It should be emphasized, though, that the CVC result is maintained even in the presence of strong quadratic W interactions with hadrons as long as these interactions have derivative coupling (i.e. are gauge-invariant). In that case, the contribution to the matrix element (1) of the diagrams in Fig. 4 and all higher order diagrams arising from the strong quadratic W interaction with hadrons will be proportional to the four-momentum transfer between proton and neutron and therefore, will vanish identically for zero momentum transfer. The conclusion is that the strong quadratic IVB model is capable - when a gauge-dependent strong interaction is employed - to explain the discrepancy between G_β and G_μ without introducing ab initio the Cabibbo factor $\cos\theta$. On the other hand, if one chooses a gauge-invariant strong quadratic W interaction with hadrons, the CVC hypothesis continues to hold and then the Cabibbo formulation is preserved.

The final important point to note concerning the strong quadratic IVB model is its prediction that "strong" W pair production (i.e. with cross-sections of the order 10^{-29} cm^2) should take place in reactions like $\bar{N} + N \rightarrow \bar{W} + W$, $\pi N \rightarrow W \bar{W} N$, etc. Some threshold energies for "strong" W pair production in nucleon-anti-nucleon collisions are: 17, 33 and 53 Bev (in the laboratory system for the anti-nucleon for m_W = 3, 4, 5 Bev respectively). The only existing accelerator on which the search for "strong" W pair production can be carried out is the Serpukhov machine and it is to be hoped that this crucial experiment can be undertaken at the earliest opportunity.

We have discussed the type of strong IVB model involving hadrons and now we wish to examine the type of strong IVB model in which there is solely a strong interaction among the W's. Just as the first type of model led to a triplet of W's and a strong quadratic interaction with hadrons, so the second type of model can be formulated in terms of a W triplet and a "cubic" interaction among the members of the W triplet (hereafter referred to as the "strong cubic" IVB model). However, the most striking difference between the strong quadratic and strong cubic IVB models lies in the fact that the former model has nothing special to say about CP violation in weak interactions whereas the latter model actually has its genesis in a unified treatment of both CP-conserving and CP-violating weak processes.

The starting point of the strong cubic IVB model is the observation[13] that a unified treatment of the CP-conserving and CP-violating processes requires the assignment of CP = -1 to the total semi-weak interaction, at the same time demanding that all first-order effects in g_W are forbidden. Again, the smallest SU(3) representation which is of interest for the W's is the triplet representation, and the theory will be sketched for this case[14]. If we now wish to postulate a strong cubic interaction among the members of the W triplet $[W_\mu^{(a)};\ a = 1, 2, 3]$, we may write down the following Lagrangian:

$$\mathcal{L}_0(x) = -\frac{1}{2}\left[\partial_\nu \overline{W}_\mu^{(a)}(x) - \partial_\mu \overline{W}_\nu^{(a)}(x)\right]\left[\partial_\nu W_\mu^{(a)}(x) - \partial_\mu W_\nu^{(a)}(x)\right] - m_0^2\, \overline{W}_\mu^{(a)}(x)\, W_\mu^{(a)}(x) - i f_0\, \varepsilon_{abc}\left[W_\mu^{(a)}(x)\, W_\nu^{(b)}(x)\, \partial_\mu W_\nu^{(c)}(x) - \overline{W}_\mu^{(a)}(x)\, \overline{W}_\nu^{(b)}(x)\, \partial_\mu \overline{W}_\nu^{(c)}(x)\right] \tag{2}$$

where f_0 is a strong coupling constant. From the structure of the W interaction term, it follows that the total charge of the three members of the W triplet must be zero and a unique assignment is Q = 0, -1, +1 (a = 1, 2, 3) for the three W bosons, if we restrict ourselves to $|Q| \leq 1$. An important property of Eq.(2) is its invariance under the triality transformation[15] given by:

$$W_\mu^{(a)}(x) \rightarrow \lambda W_\mu^{(a)}(x)\,, \quad \overline{W}_\mu^{(a)}(x) \rightarrow \lambda^* \overline{W}_\mu^{(a)}(x) \tag{3}$$

where λ is a constant satisfying the cubic equation $\lambda^3 = 1$. With the charge conjugation operation defined by:

$$C:\ W_\mu^{(a)}(x) \longrightarrow -\overline{W}_\mu^{(a)}(x) \tag{4}$$

the simplest CP = -1 total semi-weak interaction which can be written down is (θ is the Cabibbo angle):

$$H_W = i g_W \Big\{ W_\mu^{(1)} \left[\cos\theta\, \bar{J}_{\mu 3}^{2} + \sin\theta\, \bar{J}_{\mu 3}^{3}\right] - \bar{W}_\mu^{(1)} \left[\cos\theta\, J_{\mu 2}^{3} + \sin\theta\, \bar{J}_{\mu 3}^{3}\right]$$
$$+ W_\mu^{(2)} \left[\cos\theta\, \bar{J}_{\mu 2}^{1} + \bar{l}_\mu\right] - \bar{W}_\mu^{(2)} \left[\cos\theta\, \bar{J}_{\mu 1}^{2} + l_\mu\right]$$
$$+ W_\mu^{(3)} \left[\sin\theta\, \bar{J}_{\mu 1}^{3} + l_\mu\right] - \bar{W}_\mu^{(3)} \left[\sin\theta\, \bar{J}_{\mu 3}^{1} + \bar{l}_\mu\right] \Big\} \quad (5)$$

It is easily seen that Eq.(5) possesses CP = -1 due to the insertion of the coefficient i and this is non-trivial because C and P are now defined by the strong cubic interaction among the W's. It is now possible to show that all first-order effects in g_W are forbidden by the invariance of Eq.(5) under the triality transformation[16]; this forbiddenness extends to any process which is first order in g_W and of arbitrary order in e (electric charge) and therefore excludes the occurrence of an electric dipole moment of the neutron in this order.

The first non-vanishing effects in the strong cubic IVB model occur in second order of g_W^2 since a term like $\langle W_\mu(x)\ \bar{W}_\nu(y)\rangle_o$ is consistent with triality conservation. In this way one obtains the usual CP = +1 leptonic, semi-leptonic and non-leptonic weak interactions [with the $\Delta I = 1/2$ rules actually following from the choices of the semi-weak interaction (5)]. At this order in g_W^2, the strong cubic IVB model is essentially indistinguishable from the usual IVB model. The differences first arise in order g_W^3 where one encounters a term of the type $\langle W_\mu(x).\ W_\nu(y).\ W_\lambda(z)\rangle_o$ which does not vanish because of the cubic self-coupling among the W bosons [and which conserves triality as a result of condition (3)]. Moreover, the weak processes occurring in order g_W^3 possess CP = -1 and are therefore CP-violating when they interfere with terms of order g_W^2. It also follows that the order of magnitude of the CP=violating effects observed in nature ($K_L^o \rightarrow 2\pi$ decay and the charge asymmetry in $K_L^o \rightarrow \pi \ell \nu_\ell$) is correctly reproduced[17] since the semi-weak coupling constant[18] is of order 10^{-3} (which, of course, is not unexpected since this is the basis of the theory). It can be shown that the semi-weak Hamiltonian (5) does not predict any $\Delta Y = 2$ semi-leptonic or non-leptonic CP-violating transitions to order g_W^3. A number of other interesting consequences follow from the strong cubic IVB model in the version defined by Eqs.(2) - (5) but here we limit ourselves to several processes for which the predictions differ in the strong quadratic and strong cubic IVB models.

Two weak processes, already mentioned in connection with the strong quadratic IVB model, are the production of W's in neutrino-nucleon collisions and neutrino-proton scattering. In the strong quadratic IVB model, these two processes could only be reconciled with existing experimental data for a sufficiently large mass of the W (namely $m_W \gtrsim 2.5 - 3$ Bev). In the strong cubic IVB model, the requirements are much less stringent. Thus, the diagrams representing W production and $\nu_\mu - N$ scattering in the strong cubic IVB model are given respectively in Figures 5 and 6 (to be compared with Figures 1 and 2 in the strong quadratic IVB model).

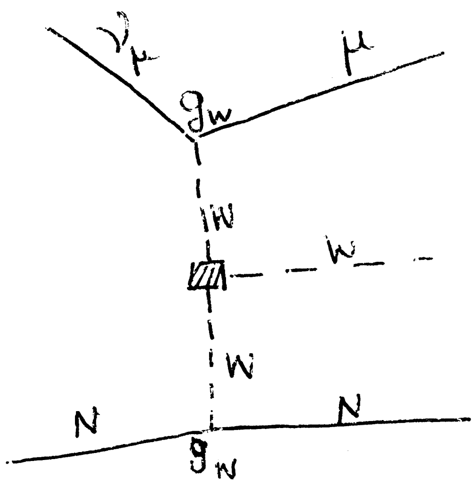

Fig. 5: Production of W in neutrino-nucleon collision on strong cubic IVB model

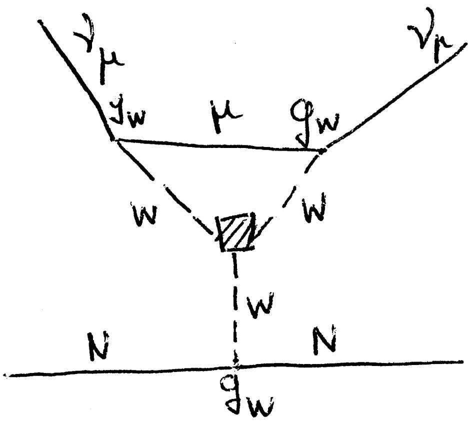

Fig. 6: Elastic neutrino - nucleon scattering on strong cubic IVB model

It is seen that both cross-sections (for W production and ν_μ - N scattering) are reduced in the strong cubic IVB model by a factor roughly of the order $g_W^2 \simeq 10^{-5}$ compared to the cross-sections in the strong quadratic IVB model. More explicitly, the ratio of the cross-section for (single) W production in neutrino-nucleon collisions in the strong cubic IVB model compared to the electromagnetic mechanism in the usual IVB model should be of the order:

$$\sim \quad g_W^2 \quad f_o^2/e^4 \tag{6}$$

Since f_o is of the order of unity (actually larger[19]), one would a priori expect the two cross-sections to be fairly comparable although with different angular distributions (since the photon and IVB propagators are quite different). Actually, the use of Eqs.(2) and (5) yields a vanishing matrix element for the diagram of Fig. 5 in the limit of exact SU(3) symmetry (for the W's) when one assumes the emerging W to decay into a lepton pair[20] (the most favorable experimental situation). One finds for this matrix element (E is the energy of the outgoing W in its rest systen

$$|M|^2 \sim \left| \frac{1}{E - m^{(2)} + i\frac{\Gamma^{(2)}}{2}} - \frac{1}{E - m^{(3)} + i\frac{\Gamma^{(3)}}{2}} \right|^2 \tag{7}$$

where $m^{(a)}$ and $\Gamma^{(a)}$ are the mass and decay width of the $W^{(a)}$ boson; evidently

$|M|^2 = 0$ when $m^{(2)} = m^{(3)}$ and $\Gamma^{(2)} = \Gamma^{(3)}$ [SU(3) limit]. This would be a discouraging result were it not for the fact that one expects departures from exact SU(3) symmetry (among W bosons) and in particular $[m^{(2)} - m^{(3)}] \gg i\frac{\Gamma^{(2)}}{2}$. With this last condition, Eq.(7) reduces to:

$$|M|^2 \sim \frac{4}{(\Gamma^{(2)})^2} \qquad (E \sim m^{(2)}) \tag{8}$$

which is equivalent to the single Breit-Wigner case. One of the most tantalizing possibilities is that CP, or more precisely, time-reversal, violation effects could show up in a gross fashion since the "strong cubic" and electromagnetic contributions to the $\Delta Y = 0$ reaction $\nu_\mu + N \to \mu^- + W^+ + N$ have opposite values of CP in the strong cubic IVB model. These violations of time-reversal may be detected in polarization-correlation measurements.

It should be pointed out that there is no possibility of cancellation, even in the SU(3) limit, when one considers, say, the $|\Delta Y| = 1$ W production reaction: $\nu_\mu + N \to \mu^- + W^+ + \Lambda$; in this case, the matrix element becomes:

$$|M|^2 \sim \left| \frac{1}{E - m^{(3)} + i\,\Gamma^{(3)}/2} \right|^2 \sim \frac{4}{(\Gamma^{(3)})^2} \qquad (E \sim m^{(3)}) \tag{9}$$

which is identical with the $\Delta Y = 0$ matrix element in broken SU(3) (except for a different dependence[21] on the Cabibbo angle). There is, however, no possibility now of detecting CP (or time-reversal) violation effects since the electromagnetic mechanism is inoperative in a $|\Delta Y| = 1$ reaction. Nevertheless, it would be very interesting to search for the $|\Delta Y| = 1$ W production reaction since it could have a substantially larger cross-section in the strong cubic than in the usual IVB model.

Another reaction mentioned above: ν_μ - N scattering - provides a less crucial test of the strong cubic IVB model; while the cross-section for ν_μ - N scattering in the strong cubic IVB model is again much larger (by a factor g_W^{-2}) than in the usual IVB model, we estimate from Fig. 6 that the cross-section will be of the order:

$$\sigma \sim g_W^6 \frac{1}{m_W^2} \sim 10^{-44} cm^2 \quad (10)$$

which will be rather difficult to measure.

Still another process which shows up to order g_W^3 in the strong cubic IVB model - the same order as ν_μ - N scattering - is the decay[22] $K_S^o \to \mu \bar{\mu}$. The diagram for this process is shown in Fig. 7a and is evidently closely related to the diagram (Fig. 6) for ν_μ - N scattering. The roughly estimated branching ratio on the basis of the diagram in Fig. 7a is $10^{-8} - 10^{-6}$, compared to a branching ratio of $10^{-8} - 10^{-7}$ when $K_S^o \to \mu \bar{\mu}$ decays according to Fig. 7b; the branching ratio in the usual IVB or strong

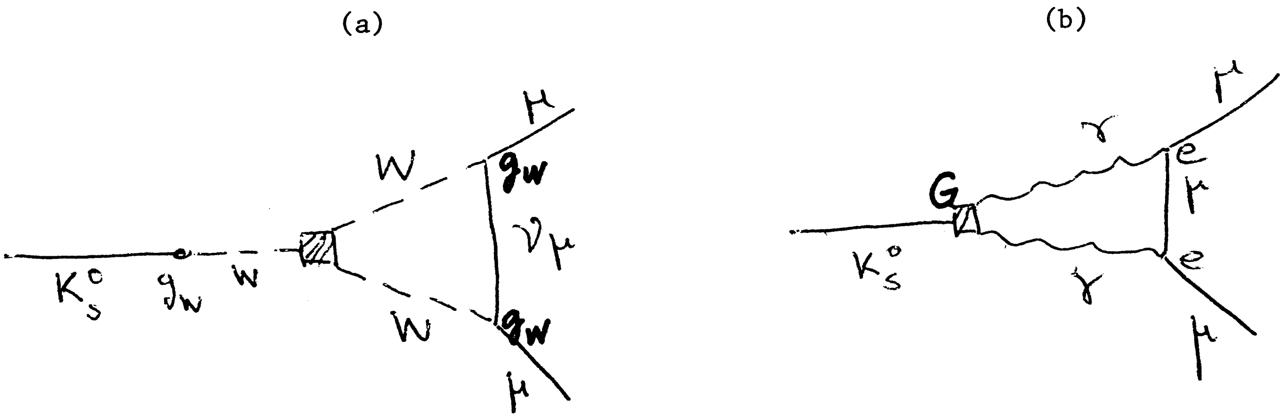

Fig. 7: Decay $K_S^o \to \mu \bar{\mu}$ on (a) strong cubic IVB model; (b) combined first-order weak and fourth-order electromagnetic mechanism.

quadratic IVB model is much smaller. Of particular interest in $K_S^o \to \mu \bar{\mu}$ decay - as in any process wherein the "strong cubic" and electromagnetic contributions to the matrix element differ by one order in g_W - is the possibility of gross CP violation. Thus, in general[23], the final state in $K_S^o \to \mu \bar{\mu}$ decay is an admixture of both ^1S_o (CP violating) and ^3P_o (CP conserving) parts which receive their contributions from Fig. 7a and Fig. 7b respectively. A crude estimate yields (Λ_1 is the electromagnetic cutoff energy):

$$\frac{P_{wave}}{S_{wave}} \sim \frac{8\pi B e^4 \ln(\Lambda_1/m_K)}{g_W \sin\theta\, f_K f_0 (\Lambda/m_W)^2} \tag{11}$$

where $\frac{Ge^3B}{\sqrt{2}}$ stands for the $K_S^o \to 2\gamma$ coupling (Fig. 7b). In view of our lack of knowledge of the $K_S^o \to \mu\bar{\mu}$ coupling, it is not clear whether the ratio (11) is of order unity and hence whether gross CP violation should actually be present. Fortunately, if we allow for SU(3) breaking (as we did earlier for the $\Delta Y = 0$ W production reaction), then the more experimentally favorable $K_L^o \to \mu\bar{\mu}$ decay can also occur; we estimate a branching ratio $\sim 10^{-7}$ [for $(m^{(2)} - m^{(3)})/m^{(2)} \simeq 1/10$] and expect gross CP violation as a result of interference with the electromagnetic mechanism (the diagrams are similar to those for $K_S^o \to \mu\bar{\mu}$ in Fig. 7). Similarly, one may expect to have gross CP violation for the decays $K^+ \to \pi^+ \mu\bar{\mu}$ and $\Sigma^+ \to p e \bar{e}$ since the electromagnetic contribution to the matrix element is of the order e^2 (rather than e^4 as in $K_S^o \to \mu\bar{\mu}$) and SU_3 symmetry need not be broken.

From the brief discussion which we have given of the strong cubic IVB model, it is clear that existing experimental data are completely consistent with this theory and, indeed, tests of this model will require considerable effort. On the other hand, an experimental test of the strong quadratic IVB model should be forthcoming as soon as the threshold for strong W pair production is attained. From the point of view of the generation of its large mass (assuming that the IVB exists) and the possible connection of the "strong" form-factor with the weak interaction cutoff, the strong quadratic and strong cubic IVB models seem to be equally likely candidates. On the other hand, it should be emphasized that the strong cubic IVB model has the very attractive feature that it incorporates automatically a theory of CP violation which the strong quadratic IVB model does not. Moreover, the charges which must be chosen for the W triplet (0, -, +) in the strong cubic IVB model mirror the charges of the leptons (ν, e^-, μ^+) (with $\nu \equiv \nu_e$, $\bar{\nu}_\mu$) when one assigns opposite values of a single lepton number L to the (ν_e, e^-) and (ν_μ, μ^-) pairs to explain the high energy neutrino experiments[24]. This coincidence of charges between the triplet of L = +1 four-component lepton fields (ν, e^-, μ^+) and the "strong cubic" W triplet, if it were ever confirmed, might have relevance for the future theory of composite particles.

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7. Cf. also B. d'Espagnat, Phys. Letters 7, 204 (1963).
8. C. G. Callan, Jr., Phys. Rev. Letters 20, 809 (1968).
9. We may classify the strong quadratic interactions into two categories: (1) those which are invariant under the gauge transformation $W_\mu(x) \to W_\mu(x) - \frac{\partial\Lambda(x)}{\partial x_\mu}$ – [$\Lambda(x)$ arbitrary] – called the gauge-invariant interactions or (2) those which are not invariant under this gauge transformation – called the gauge-dependent interactions. The gauge-invariant and gauge-dependent versions of the strong quadratic IVB model differ in certain important respects (see text).
10. There is, of course, always the possibility that non-perturbative calculations might yield a non-vanishing self-mass [cf. J. Schwinger, Phys. Rev. 125, 396 (1962)] for the gauge-invariant version of the strong quadratic IVB model.
11. So far we only know that the cross-section for ν_μ - N elastic scattering does not exceed about 15% of the cross-section for the reaction $\nu_\mu + n \to p + \mu^-$ or $\bar{\nu}_\mu + p \to n + \mu^+$ [cf. E. C. M. Young "High Energy Neutrino Interactions", CERN report (1967)].

12. By matching the strong coupling constant of the W's and hadrons to the self-generated mass m_W.

13. S. Okubo, Annals of Phys. (N.Y.) 49, 219 (1968), Nuovo Cimento 57A, 794 (1968); cf. also ref. 4 and subsequent Univ. of Rochester preprints.

14. One of us (S.O.) has elaborated the strong cubic IVB model for CP violation chiefly for an octet (or nonet) of intermediate vector bosons where it is also possible to describe the W's by Yang - Mills fields. Since the strong quadratic IVB model is based on a W triplet, it seems more illuminating for present purposes to treat the consequences of the strong cubic IVB model in terms of a W triplet (but with different charges - see text).

15. This triality transformation is of the "multiplicative" SU(3) type and should not be confused with the "additive" U(3) type of triality transformation which underlies the strong quadratic IVB model (cf. ref. 6).

16. The typical term which appears is $\langle W_\mu(x)\rangle_o$ [the expectation value is understood with respect to the vacuum state of the W bosons with the Lagrangian $\mathcal{L}_o$ defined by Eq.(2)] and this must vanish by triality conservation.

17. More precisely, one finds $\dfrac{M(K_L^o \to 2\pi)}{M(K_S^o \to 2\pi)} \simeq 10^{-3}\, f_o \left(\dfrac{\Lambda}{m_W}\right)^4 g_W \sim 10^{-3}$ if one sets $\Lambda \sim 2m_W$ and uses the value of f_o derived from the self-mass (see ref. 20).

18. From the relation $\dfrac{G}{\sqrt{2}} = \dfrac{g_W}{m_W^2}$, it follows that $\dfrac{g_W^2}{4\pi} \sim \dfrac{10^{-5}}{4\pi\sqrt{2}} \left(\dfrac{m_W}{m_N}\right)^2 \sim 10^{-5}$ for $m_W \sim 5m_N$.

19. If we use the cubic interaction in Eq.(5) to generate the self-mass of W, we estimate $f_o\, \Lambda^2/m_W^2 \simeq 2\pi$ (Λ is the cutoff).

20. The matrix element does not vanish for the reaction $\nu_\mu + N \to \mu^- + W^+ + N$ when the decay products of W are not lepton pairs (see Fig. 5); however, W production will be more difficult to detect if, say, $W \to 2\pi$.

21. With the choice (5) for H_W, Eq. (9) would have the factor $\cos^2\theta$ while Eq.(8) would have a $\sin^2\theta$ dependence; the reverse dependence is also consistent with the chief assumptions of the theory.

22. It can easily be shown that in the exact SU_3 limit for the W's, $K_L^{\circ} \to \mu \bar{\mu}$ is forbidden.

23. Cf. A. Pais and S. Treiman, preprint (1968).

24. The assignment of L = +1 to (ν_e, e^-) and ($\bar{\nu}_\mu$,μ^+) can equally well explain $\nu_e \neq \nu_\mu$ as the introduction of a second lepton (muon) number [cf. Chap. 3 of "Theory of Weak Interactions" by R. E. Marshak, Riazuddin and C.P. Ryan, John Wiley (1969)].