INTERFERENCE BETWEEN K_L AND K_S AMPLITUDES IN THE $\pi^+\pi^-$ DECAY MODE

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We report here final results from an experiment[1] in which we measured the decay rate of $K^0 \rightarrow \pi^+\pi^-$ behind a regenerator.

A beam of long-lived kaons passed through material (carbon), and short-lived kaons were regenerated. The state behind the regenerator is described by

$$|\psi(\tau)> = (1 + |\rho|^2)^{-1/2} \left[e^{-iM_L\tau} |K_L> + \rho \; e^{-iM_S\tau} |K_S> \right] \qquad (1)$$

where ρ is the amplitude of K_S at the downstream end of a regenerator of length L or $\ell = L/\Lambda_S$ in units of the K_S mean decay length, τ time in the kaon rest system, M_L and M_S are the complex eigenvalues, $M_{L,S} = M_{L,S} + i\Gamma_{L,S}/2$.

$$\rho = \frac{i\pi N}{K} \; \frac{(f - \bar{f})\Lambda_S\Gamma_S}{\Delta M} \left(1 - e^{i\Delta M \ell}\right). \qquad (2)$$

N is the number of scattering centers per unit volume, K is the kaon wave number, f and $\bar{f}$ are the forward scattering amplitudes of K^0 and $\bar{K}^0$, respectively, Γ_S is the total decay width of short-lived kaons and $\Delta M = M_L - M_S$ is the difference of the complex eigenvalues.

The data have been analysed under the hypothesis that the state $|K_L>$ has an amplitude for decay into $\pi^+\pi^-$. We expect then a rate per unit time of

$$\left.\begin{array}{l} \Gamma_{+-}(\tau) = \Gamma^S_{+-}(1 + |\rho|^2)^{-1/2} \left(|\rho|^2 \; e^{-\Gamma_S\tau} + |\eta_{+-}|^2 \; e^{-\Gamma_L\tau} \right. \\ \quad |\rho| \; |\eta_{+-}| \; e^{-1/2(\Gamma_S+\Gamma_L)\tau} \cos[\varphi + \tau \cdot \mathrm{Re}(M_L - M_S)]. \end{array}\right\} \qquad (3)$$

φ is the relative phase between the amplitude ρ and η_{+-},
$\varphi = \varphi_\rho - \varphi_\eta$,

$$\Gamma^S_{+-} = \Gamma(K^o_S \to \pi^+\pi^-),\ \eta_{+-} = \frac{<\pi^+\pi^-|K_L>}{<\pi^+\pi^-|K_S>}$$

Comparison of the experimentally observed rate and the theoretical expectation confirms the validity of the assumptions used in deriving eq.(3) : the state $|K_L>$ has an amplitude for decay into $\pi^+\pi^-$, identity of final states from K^o_L and K^o_S decays and the superposition principle. Best values of $|\eta_{+-}|$, $Re(M_L - M_S) = \Delta m$, φ, $R = |\rho|/|\eta_{+-}|$ and Γ_S are determined by a fit.

The experimental arrangement has been described before[1]. $K^o \to \pi^+\pi^-$ decays have been selected by an accurate determination of the vector momenta of charged secondaries, $\vec{p}_+$ and $\vec{p}_-$, and by identification of electrons from K_{e3} decays in a gas Čerenkov counter and of muons from $K_{\mu 3}$ decays by transmission through 620 gr/cm^2 of iron. $K^o \to \pi^+\pi^-\pi^o$ decays are outside the acceptance of the detector. Events which do not trigger the Čerenkov counter or the muon counters are candidates for $K^o \to \pi^+\pi^-$. This hypothesis is tested by calculating the vector momentum

$$\vec{p} = \vec{p}_+ + \vec{p}_-$$

and the invariant mass

$$M^{*2} = (E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2$$

of the parent state. Distributions of M^* are shown in Fig. 1; the direction of the decaying parent has not been restricted. The pronounced peak at the K^o mass, even in the case of free decays demonstrates the power of the apparatus to select $K^o \to \pi^+\pi^-$ decays. Candidates for $K^o \to \pi^+\pi^-$ in the mass range $450 < M^* < 550$ MeV are sorted into bins of proper time of $0.5 \cdot 10^{-10}$ s width and plotted against Θ^2. Θ is the angle between the beam direction and the calculated parent direction, $\vec{p}$. Background events due to diffraction scattering and leptonic decays do not in general prefer the beam direction. Transmitted K_L and K_S states are reconstructed with a standard deviation in Θ^2 of $0.9 \cdot 10^{-6}$ rad^2. The background is subtracted with the help of Monte Carlo calculations on the angular distribution in leptonic decays. The mass distribution of these selected events is fitted by a gaussian centred at $M_K = 496.6 \pm 1.5$ MeV with a standard deviation of 3.0 MeV.

Several positions and thicknesses of regenerator were used. They are summarized in Table 1. Regenerators of three different lengths were chosen to explore the interference term with maximum sensitivity at different times $t_i \simeq 2 \cdot \ell n R/\Gamma_S$. The positions were chosen to locate for different regenerators the proper times t_i at the same region in which the acceptance of the detector is largest. The detection efficiency in this region is then common to all data. It was calibrated further by placing the thick regenerator into this region (sample 5). In this way possible uncertainties encountered in calculating detection efficiencies by Monte Carlo methods are reduced.

Normalisation of the 5 samples of data to the same flux of K_L^o transmitted through the regenerator has been achieved by counting reconstructed $K_{\mu 3}$ decays. This method of internal monitoring has two advantages: it is independent of regenerator thickness and therefore of the nuclear attenuation and has the same fate as selected $K^o \rightarrow \pi^+\pi^-$ events in the measuring and reconstruction process. $K_{\mu 3}$ monitor events were selected from a restricted fiducial decay region. Background from $K^o \rightarrow \pi^+\pi^-$ decays, mislabelled $K_{\mu 3}$ because of a finite transmission probability of pions through the iron absorber, has been eliminated by inserting inside the mass region $480 < M^* < 510$ MeV the fraction of events recorded without regenerator.

The monitor rate has been corrected for the detection of diffraction scattered K_L^o to obtain the transmitted K_L^o flux. In leptonic decays the direction of the parent K^o cannot be determined from the momentum vectors of the charged secondaries. For this reason background due to diffracted K_L^o cannot be subtracted by extrapolation to $\Theta^2 = 0$ as in the case of $\pi^+\pi^-$. We have calculated the detection efficiency for diffraction scattered K_L^o decaying in the $K_{\mu 3}$ mode by

Monte Carlo methods. Using for carbon a nuclear radius of $3.24 \cdot 10^{-13}$cm we find a detection efficiency of 0.22 relative to decays of transmitted K^o. The diffraction cross section has been determined from the total cross section, $\sigma_T = 186 \pm 40$ mb and the carbon radius to $\sigma_D = 21.6$ mb. The largest correction then amounts to $(1.2 \pm 0.25) \cdot 10^{-2}$ for 30 cm carbon.

The observed rates are summarized in Table 1 and their time dependence is shown in Figs. 2a and 2b. To compare the time dependence of the observed rates with the expectation eq. (3), the detection efficiency $\epsilon(\tau,p)$ as a function of proper time has been calculated by Monte Carlo methods. The momentum spectrum S(p) of the K_L^o beam has been determined experimentally in two ways. Observed $K_{\mu 3}$ events were compared to the calculated detection efficiencies using an iterative method. The K_L^o spectra obtained with and without regenerator are in good agreement. The momentum spectrum of regenerated K_S decaying into $(\pi^+\pi^-)$ has been determined experimentally from sample 5. Only events in the first two bins which are dominated by the quadratic term $R^2 e^{-\tau \Gamma_S}$, were used. Good agreement between spectra of incident, transmitted and regenerated kaons was found. For comparison with the data the expected time distribution (3) has been properly weighted with the detection efficiencies $\epsilon(\tau, p)$ and the spectrum S(p) and then integrated over all momenta

$$N_{\pi\pi}(\tau) = B \cdot N_{\mu 3} \int dp \, \epsilon(\tau, p) \, S(p) \cdot \Gamma_{+-}(\tau) \qquad (4)$$

$N_{\mu 3}$ is the number of observed $K_{\mu 3}$ monitor events, B is a constant, independent of regenerator thickness and location. It is proportional to the relative rate of $K_L^o \to \pi^+\pi^-$ and $K_L^o \to \pi \mu \nu$. The phase of the regeneration amplitude, φ_ρ, contains two contributions, the nuclear phase and a kinematical phase shift.

$$\varphi_\rho = \arg i\,(f - \bar{f}) + \arg\left(\frac{1 - \exp i\,\Delta M \ell}{\Delta M}\right) = \varphi_{if} + \alpha(\Delta m, \ell) \qquad (5)$$

The kinematical phase shift has been evaluated explicitly in the fitting process.

The parameters determined in the fit are

$$\varphi_{if} - \varphi_{\eta}, \frac{\Delta m}{\Gamma_S} = \frac{Re(\Delta M)}{\Gamma_S}, \tau_S = \frac{1}{\Gamma_S}, R = |\rho|/|\eta_{+-}|$$

for 5.6 cm of carbon of average density 1.785 gr/cm^3 at 4.5 GeV/c and $|\eta_{+-}|$ from the constant B.

The results of the fit are

$$R = |\rho/\eta_{+-}| = 4.22 \pm 0.11$$

$$\varphi = \varphi_{if} - \varphi_{\eta} = -(89.7^{\circ} \pm 6.3^{\circ})$$

$$\Delta m/\Gamma_S = 0.483 \pm 0.020$$

$$\tau_S = 1/\Gamma_S = (0.883 \pm 0.017) \cdot 10^{-10} \text{ s}$$

$$|\eta_{+-}| = (1.90 \pm 0.07) \cdot 10^{-3}$$

$$\chi^2 = 34 \text{ for } 54 \text{ degrees of freedom.}$$

From the regeneration amplitude we extract the modulus of $(f - \bar{f})/K$

$$\frac{|f - \bar{f}|}{K} = (4.67 \pm 0.23) \cdot 10^{-27} \text{ cm}^2.$$

The imaginary part of $(f - \bar{f})$ can be obtained from total cross sections of K^+ and K^- on carbon. One finds at 2.7 GeV/c [2]

$$\frac{Im\ (\bar{f} - f)}{K} = (0.51 \pm 0.024) \cdot 10^{-26} \text{ cm}^2.$$

This has been extrapolated to 4.5 GeV/c with the help of K^+ and K^- total cross sections on protons and neutrons [3] with the result

$$\frac{Im\ (\bar{f} - f)}{K} = (3.72 \pm 0.38) \cdot 10^{-27} \text{ cm}^2 .$$

We then obtain

$$\cos \arg i(f - \bar{f}) = \frac{Im\ (\bar{f} - f)}{|f - \bar{f}|} = \pm (0.795 \pm 0.100)$$

giving $\quad \arg i(f-\bar{f}) \quad = -(37 \pm 10)^{\circ}$, and with $\varphi = \varphi_{if} - \varphi_{\eta}$

$$\arg \eta_{+-} = \quad = 53^{\circ} \pm 12^{\circ} .$$

The sign of $\arg i(f-\bar{f})$ has been chosen in agreement with S. Bennett et al.[4] . The argument of $i(f-\bar{f})$ found here can be compared to the result of an optical model calculation of M. Lusignoli et al[5] who find for carbon at 5 GeV/c $\arg i(f-\bar{f}) = -(31 \pm 15)^{\circ}$.

Table 1

Summary of data samples.

Material	Position	t_i	$\vert K_L^o + \rho K_S^o \vert^2 \to \pi^+\pi^-$	$K_L^o \to \pi^+\pi^-$	$K_{\mu 3}$
Free decay				215	2625
2 cm carbon	1	$1.5\ \tau_S$	705	255	3230
4 cm carbon	1	$2.9\ \tau_S$	1957	315	4500
30cm carbon	2	$5.4\ \tau_S$	4005	395	5439
30cm carbon	3	$5.4\ \tau_S$	5205	57	870

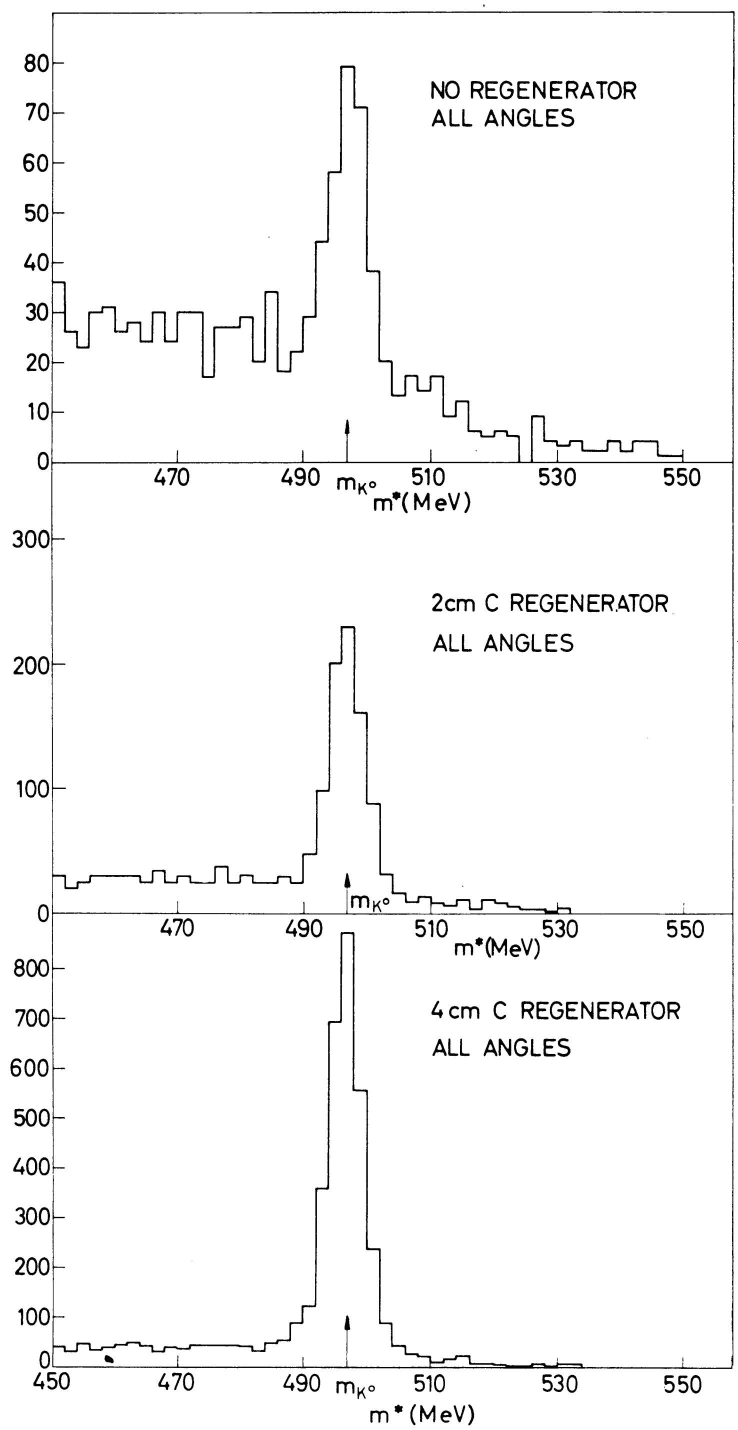

Figure 1 : Mass distributions of events labelled $(\pi^+\pi^-)$ for different beam conditions. Note that no selection of the parent direction has been made.

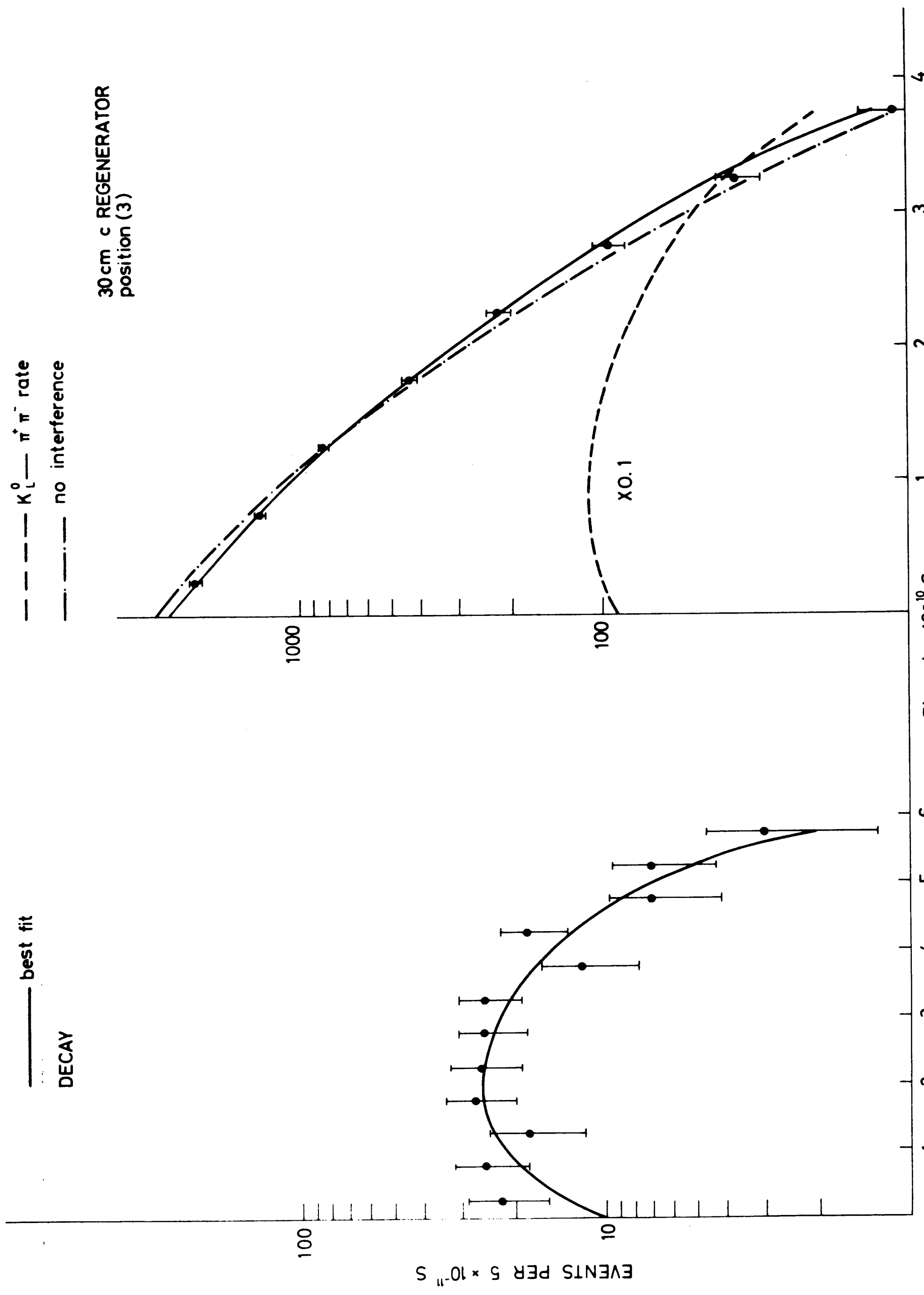

Figure 2a : Decay rate as a function of proper time for free decays and for sample 5. Calculated detection efficiencies are shown.

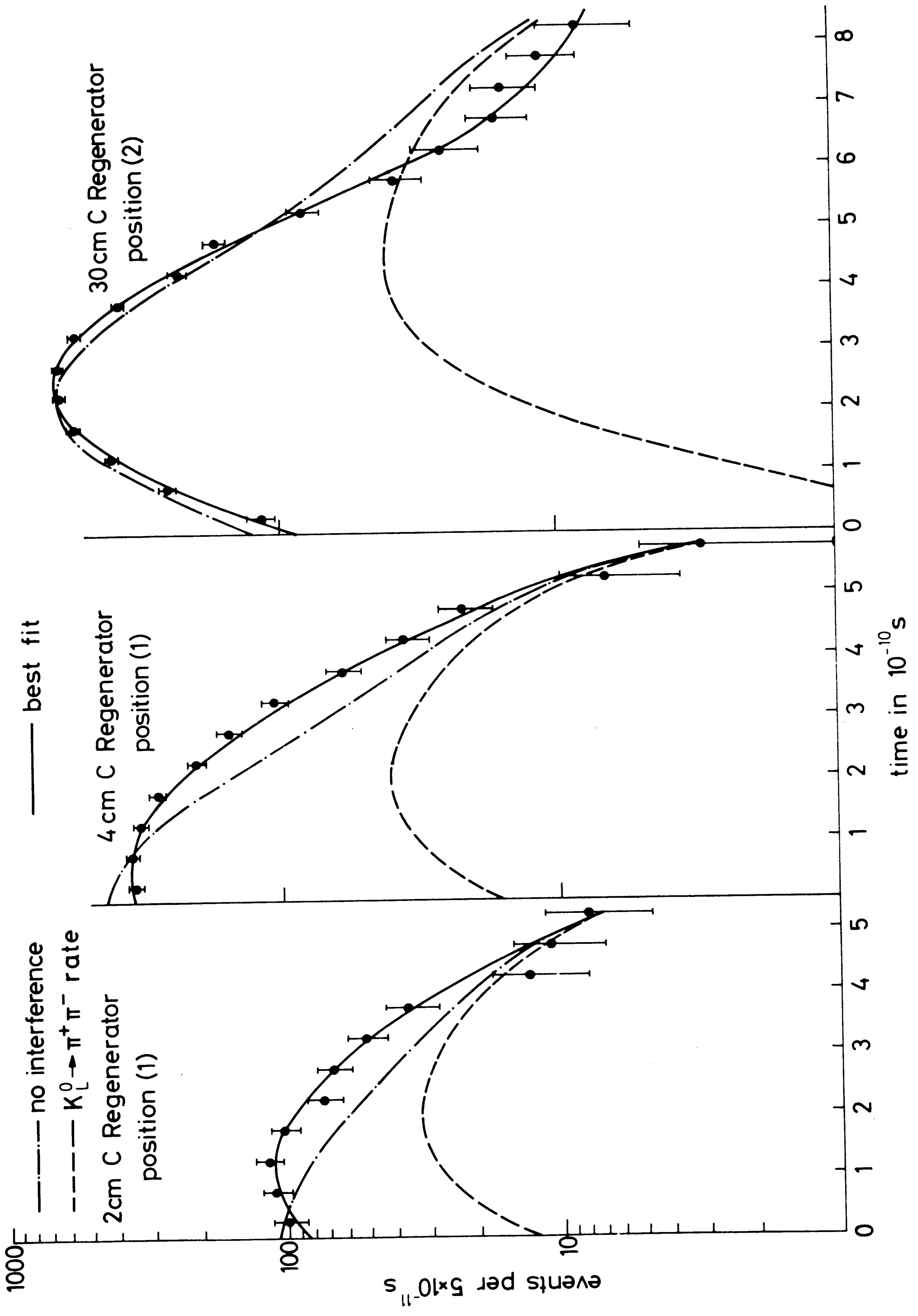

Figure 2b : Decay rate as a function of proper time observed behind different carbon regenerators (samples 2, 3 and 4 of table 1).

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