Resonant *CP* violation in Higgs radiation at an e^+e^- linear collider

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We study resonant *CP* violation in the Higgsstrahlung process $e^+e^- \to H_{1,2,3}(Z \to e^+e^-, \mu^+\mu^-)$ and subsequent decays $H_{1,2,3} \to b\bar{b}, \tau^-\tau^+$, in the minimal supersymmetric extension of the standard model (MSSM) with Higgs-sector *CP* violation induced by radiative corrections. At a high-energy e^+e^- linear collider, the recoil-mass method enables one to determine the invariant mass of a fermion pair produced by Higgs decays with a precision as good as 1 GeV. Assuming an integrated luminosity of 100 fb^{-1}, we show that the production line shape of a coupled system of neutral Higgs bosons decaying into $b\bar{b}$ quarks is sensitive to the *CP*-violating parameters. When the Higgs bosons decay into $\tau^-\tau^+$, two *CP* asymmetries can be defined using the longitudinal and transverse polarizations of the τ leptons. Taking into account the constraints from electric dipole moments, we find that these *CP* asymmetries can be as large as 80%, in a trimixing scenario where all three neutral Higgs states of the MSSM are nearly degenerate and mix significantly.

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I. INTRODUCTION

A future e^+e^- linear collider, such as the projected International Linear Collider (ILC), will have the potential to probe the standard model (SM) Higgs sector with higher precision than its predecessors, the Tevatron collider at Fermilab and the Large Hadron Collider (LHC) at CERN. The present direct search limits from the search for Higgsstrahlung [1] at LEP show that the SM Higgs boson should be heavier than about 114 GeV [2]. This lower limit is within the mass range favored indirectly by precision electroweak measurements [3]. Future refinements of these direct and indirect limits will be very crucial for identifying the underlying structure of the fundamental Higgs sector, within either the SM or some nonminimal model of Higgs physics.

One well-motivated model of physics beyond the SM is the minimal supersymmetric extension of the standard model (MSSM) [4]. The MSSM predicts three neutral Higgs states. In the presence of explicit *CP*-violating sources, such as complex soft squark masses, gaugino masses and trilinear couplings, all the three neutral Higgs bosons mix through *CP*-violating quantum effects [5–9] to mass eigenstates, $H_{1,2,3}$, of indefinite *CP*. In this *CP*-violating MSSM, one interesting possibility is that the lightest neutral Higgs boson could be considerably lighter than 114 GeV [10,11], at moderate values of $\tan\beta \lesssim 10$, for relatively light charged Higgs-boson masses, $M_{H^\pm} \sim 130$–170 GeV. Alternatively, if $\tan\beta \gtrsim 40$, all three neutral Higgs states $H_{1,2,3}$ may have similar masses and strongly mix with each other dynamically, through *CP*-violating off-diagonal absorptive self-energy effects [12,13]. Such a scenario was studied in [13–15] and termed the *CP*-violating trimixing scenario of the MSSM.

The general unconstrained MSSM has dozens of additional *CP*-violating phases beyond the Kobayashi-Maskawa phase of the SM. These phases appear in complex soft supersymmetry-breaking mass parameters of sfermions and gauginos and in complex trilinear Yukawa couplings. They have a wealth of phenomenological implications. In particular, they give rise to signatures of *CP* violation in sparticle production and decay at high-energy colliders [16–18], have observable effects on electric dipole moments (EDMs) [19–21] and *B*-meson decays [22,23], and might constitute the extra ingredients needed for electroweak baryogenesis [24].

In this paper we study resonant *CP*-violating phenomena in the Higgsstrahlung mechanism for producing neutral MSSM Higgs bosons at a high-energy e^+e^- linear collider. Although our study is performed within the radiative *CP*-violating framework of the MSSM [5–9,25–28], the results of our analysis would also be applicable to more general *CP*-violating two-Higgs-doublet models [29] in which all three neutral Higgs bosons mix strongly. This work extends previous studies of the masses, couplings, production and decays of the mixed-*CP* neutral Higgs bosons $H_{1,2,3}$, with a view to searches at LEP [10], the LHC [10,11,15,30–34], the ILC [35], a $\mu^+\mu^-$ collider [36] and a photon linear collider (PLC) [14,37–40]. As in our previous works [13–15], we present a complete treatment of loop-induced *CP* violation and Higgs trimixing, including off-diagonal absorptive effects in the resummed Higgs-boson propagator matrix [12]. Complementary to the previous studies at the ILC [35], our focus here is on analyzing the production line shape of a coupled system of neutral Higgs bosons in the Higgsstrahlung process, as well as the construction of feasible *CP* asymmetries which could be probed experimentally.

Higgs-boson production via the Higgsstrahlung process $e^+e^- \to H_iZ$ [1], where the Z boson decays into electron or muon pairs, offers a unique environment for determining the masses and widths of the neutral Higgs bosons by the recoil-mass method [41,42]. Thanks to the excellent energy and momentum resolution of electrons and muons coming from the Z-boson decay, the recoil mass against the Z boson, $p^2 = s - 2 \cdot \sqrt{s} \cdot E_Z + M_Z^2$, can be reconstructed with a precision as good as 1 GeV. Here s and E_Z are the collider center-of-mass energy squared and the energy of the Z boson, respectively.

As mentioned above, our focus is on the CP-violating trimixing scenario of the MSSM, in which all three neutral Higgs bosons have similar masses and mix strongly with each other. In particular, we examine the production line shape of the coupled system of neutral Higgs bosons in this trimixing scenario. The line shape for $H_{1,2,3} \to \bar{b}b$ decays is sensitive to the CP-violating parameters of the model. Moreover, employing the longitudinal and transverse polarizations of the tau leptons coming from the decays of Higgs bosons, we can measure CP asymmetries which can be as large as 80%, without violating EDM constraints.

The layout of the paper is as follows. In Sec. II, we study the Higgsstrahlung process at an e^+e^- collider and the subsequent Higgs-boson decays, $H_i \to f\bar{f}$ with $f = b$ and τ^-. We define the individual cross sections that depend on the longitudinal and transverse polarizations of the final fermions. In Sec. III, we consider the constraints from the nonobservation of an EDM in the thallium atom on the relevant CP phases in the trimixing scenario. In Sec. IV, we construct two CP asymmetries and present numerical examples for the two final states $f\bar{f} = b\bar{b}$ and $\tau^-\tau^+$. Our conclusions are given in Sec. V.

II. THE PROCESS $e^+e^- \to H_iZ \to [f(\sigma)\bar{f}(\bar{\sigma})]Z$

The general helicity amplitude for the process $e^-(p_1, \omega)e^+(p_2, \bar{\omega}) \to H_i(p)Z(k, \lambda) \to [f(l_1, \sigma)\bar{f}(l_2, \bar{\sigma})]$ $Z(k, \lambda)$, depicted in Fig. 1, may be written as

$$\mathcal{M}^{\lambda\sigma}_{\omega} = \frac{-g^2 g_f M_W}{c_W^3 \sqrt{s}\sqrt{p^2}} D_Z(s)\langle\omega; \lambda\rangle_\Theta \left(\sum_{i,j} \langle\sigma\rangle^f_{ij}\right) \delta_{\omega-\bar{\omega}}\delta_{\sigma\bar{\sigma}}. \tag{1}$$

The four-momenta and helicities of the initial electron and positron are denoted by (p_1, ω) and $(p_2, \bar{\omega})$, respectively,

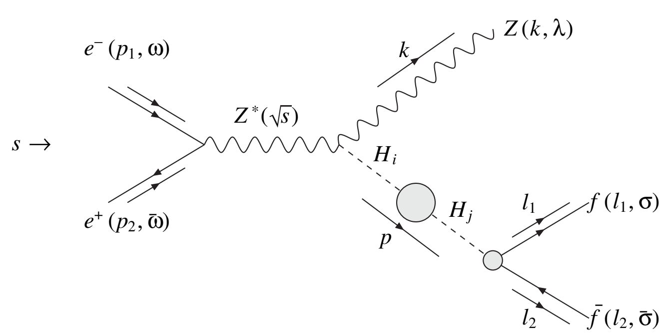

FIG. 1. The dominant mechanism contributing to the process $e^+e^- \to H_iZ \to [f(\sigma)\bar{f}(\bar{\sigma})]Z$.

and $s \equiv (p_1 + p_2)^2$. We denote the helicities of f and $\bar{f}$ by σ and $\bar{\sigma}$, and that of the Z boson by λ. Also, $\sigma = +\,(-)$ stands for a right- (left)-handed particle and $\lambda = \pm$ and $\lambda = 0$ for the transverse (right and left helicities) and longitudinal polarizations, respectively. The four-momentum of the intermediate Higgs boson is denoted by p and those of the final fermions by l_1 and l_2 with $p = l_1 + l_2$. Finally, Θ is the angle between $\mathbf{p}_1$ and $\mathbf{k}$ where the four-momentum of the Z boson is $k = (E_Z, \mathbf{k})$.

In (1), $D_Z(s)$ is the s-normalized Breit-Wigner propagator for the Z boson:

$$D_Z(s) = \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}, \tag{2}$$

and $\langle\omega; \lambda\rangle_\Theta$ and $\langle\sigma\rangle^f_{ij}$ describe the reduced amplitudes

$$\begin{aligned} \langle\omega; \lambda = 0\rangle_\Theta &= -(v_e + \omega a_e)\frac{\omega E_Z}{M_Z} s_\Theta, \\ \langle\omega; \lambda = \pm\rangle_\Theta &= (v_e + \omega a_e)\frac{1 + \omega\lambda c_\Theta}{\sqrt{2}}, \\ \langle\sigma\rangle^f_{ij} &= g_{H_iVV} D_{ij}(p^2)(\sigma\beta g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f}), \end{aligned} \tag{3}$$

where $s_\Theta \equiv \sin\Theta$, $c_\Theta \equiv \cos\Theta$ and $v_e = -1/4 + s_W^2$ and $a_e = 1/4$. This result is consistent with the one given in [43,44]. For the definitions of the couplings, the threshold corrections that are enhanced for large values of $\tan\beta$ for $f = b, \tau$, and the full 3×3 propagator matrix $D_{ij}(p^2)$, we refer to [13,45].

The differential cross section, after integrating over c_Θ, is given by

$$\begin{aligned} p^2\frac{d\sigma}{dp^2} = \sigma^{\rm SM}_{HZ}(p^2) \times \frac{N_f g_f^2 \beta_f(p^2)}{16\pi^2}\{&(1 + P_L\bar{P}_L)C_1^f(p^2) \\ &+ (P_L + \bar{P}_L)C_2^f(p^2) + P_T\bar{P}_T[\cos(\alpha - \bar{\alpha})C_3^f(p^2) \\ &+ \sin(\alpha - \bar{\alpha})C_4^f(p^2)]\}, \end{aligned} \tag{4}$$

where N_f is the color factor of the final-state fermion: $N_f = 3$ for the b quark and 1 for the τ lepton, $\beta^f(p^2) \equiv \sqrt{1 - 4m_f^2/p^2}$. Moreover, $\sigma^{\rm SM}_{HZ}(p^2)$ is the SM cross section for the Higgsstrahlung of an off-shell Higgs boson with mass $\sqrt{p^2}$, i.e.,

$$\sigma^{\rm SM}_{HZ}(p^2) = \frac{g^4(v_e^2 + a_e^2)}{192\pi c_W^4 s}\frac{\lambda^{1/2}(p^2)[\lambda(p^2) + 12M_Z^2/s]}{(1 - M_Z^2/s)^2 + M_Z^2\Gamma_Z^2/s^2}, \tag{5}$$

where $\lambda(p^2) = (1 - p^2/s - M_Z^2/s)^2 - 4p^2M_Z^2/s^2$ is a kinematic phase-space function. Then, with the definition

$$\langle\sigma\rangle^f \equiv \sum_{i,j}\langle\sigma\rangle^f_{ij}, \tag{6}$$

the polarization coefficients C_i^f may conveniently be expressed as follows:

$$\begin{aligned} C_1^f(p^2) &= \tfrac{1}{4}(|\langle+\rangle^f|^2 + |\langle-\rangle^f|^2), \\ C_2^f(p^2) &= \tfrac{1}{4}(|\langle+\rangle^f|^2 - |\langle-\rangle^f|^2), \\ C_3^f(p^2) &= -\tfrac{1}{2}\mathrm{Re}(\langle+\rangle^f\langle-\rangle^{f*}), \\ C_4^f(p^2) &= \tfrac{1}{2}\mathrm{Im}(\langle+\rangle^f\langle-\rangle^{f*}). \end{aligned} \tag{7}$$

In (4), P_L and $\bar{P}_L$ are the longitudinal polarizations of the final fermion f and antifermion $\bar{f}$, respectively, whereas P_T and $\bar{P}_T$ are the degrees of transverse polarization, with α and $\bar{\alpha}$ being the azimuthal angles with respect to the f-$\bar{f}$ production plane. We depict in Fig. 2 the production plane in the Higgs-boson rest frame in the case when $f = \tau^-$ and the τ leptons decay into charged hadrons $h^\pm$ and neutrinos, $\tau^\mp \to h^\mp \nu_\tau(\bar{\nu}_\tau)$ with $h^\pm = \pi^\pm$, $\rho^\pm$, etc.

Identifying the polarization analyzer for $\tau^\mp$ as

$$\hat{a}^{\mp} = \pm\hat{h}^{\mp}, \tag{8}$$

where $\hat{h}^{\mp}$ denote unit vectors parallel to the $h^\mp$ momenta in the $\tau^\mp$ rest frame, we have

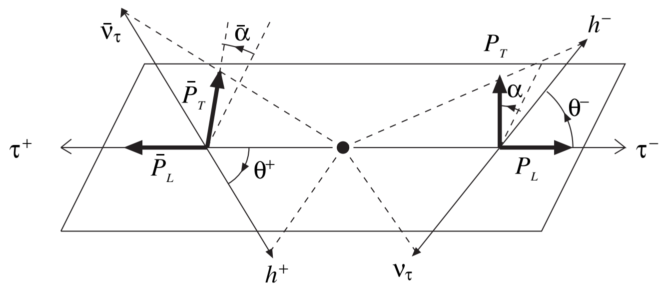

FIG. 2. The $\tau^+\tau^-$ production plane in the Higgs-boson rest frame, in the case when the τ leptons decay into hadrons $h^\pm$ and neutrinos. The longitudinal-polarization vector $P_L(\bar{P}_L)$ and the transverse-polarization vector $P_T(\bar{P}_T)$ with the azimuthal angle $\alpha(\bar{\alpha})$ of $\tau^-(\tau^+)$ are shown.

$$\begin{aligned} P_L &= \cos\theta^-, & P_T &= \sin\theta^-, & \alpha &= \varphi^-; \\ \bar{P}_L &= \cos\theta^+, & \bar{P}_T &= \sin\theta^+, & \bar{\alpha} &= \varphi^+ - \pi, \end{aligned} \tag{9}$$

where $\theta^\pm$ and $\varphi^\pm$ are the polar and azimuthal angles of $h^\pm$, respectively, in the $\tau^\pm$ rest frame. With this identification, the expression in the curly brackets of (4) becomes

$$\begin{aligned} &(1 + P_L\bar{P}_L)C_1^f(p^2) + (P_L + \bar{P}_L)C_2^f(p^2) + P_T\bar{P}_T[\cos(\alpha - \bar{\alpha})C_3^f(p^2) + \sin(\alpha - \bar{\alpha})C_4^f(p^2)] \\ &= (1 + \cos\theta^-\cos\theta^+)C_1^f(p^2) + (\cos\theta^- + \cos\theta^+)C_2^f(p^2) - \sin\theta^+\sin\theta^-[\cos(\varphi^- - \varphi^+)C_3^f(p^2) + \sin(\varphi^- - \varphi^+)C_4^f(p^2)]. \end{aligned} \tag{10}$$

We observe that the polarization coefficients $C_i^\tau(p^2)$ can be determined by examining the angular distributions of the charged hadrons coming from the τ-lepton decays; $\tau^\mp \to h^\mp\nu(\bar{\nu})$ [46].[1]

Finally, for our phenomenological discussion in Sec. IV, it proves more convenient to define the individual cross sections:

$$\hat{\sigma}_i^f(p^2) \equiv \sigma_{HZ}^{\mathrm{SM}}(p^2)\frac{N_f g_f^2 \beta_f(p^2)}{16\pi^2}C_i^f(p^2). \tag{11}$$

III. THE TRIMIXING SCENARIO AND THE THALLIUM EDM

We take the following parameter set for numerical examples in Sec. IV:

$$\begin{gathered} \tan\beta = 50, \qquad M_{H^\pm}^{\mathrm{pole}} = 155\ \mathrm{GeV}, \\ M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = M_{\mathrm{SUSY}} = 0.5\ \mathrm{TeV}, \\ |\mu| = 0.5\ \mathrm{TeV}, \qquad |A_{t,b,\tau}| = 1\ \mathrm{TeV}, \\ |M_2| = |M_1| = 0.3\ \mathrm{TeV}, \qquad |M_3| = 1\ \mathrm{TeV}, \\ \Phi_\mu = 0°, \qquad \Phi_1 = \Phi_2 = 0°. \end{gathered} \tag{12}$$

We refer to this scenario as the trimixing scenario, since the mass differences between three neutral Higgs bosons are comparable to the decay widths and all the three Higgs bosons mix significantly in the presence of nonvanishing *CP* phases. In this scenario, the common third generation phase $\Phi_A = \Phi_{A_t} = \Phi_{A_b} = \Phi_{A_\tau}$ and that of the gluino mass parameter Φ_3 are free parameters, which are constrained by the nonobservation of EDMs of atoms, molecules, and the neutron [21]. The contributions of the first- and second-generation phases, e.g. $\Phi_{A_{e,\mu}}$, $\Phi_{A_{d,s}}$ etc., to EDMs can be drastically reduced either by making these phases sufficiently small, or if the first- and second-generation squarks and sleptons are sufficiently heavy. The impact of these phases on the Higgs sector is negligible.

The EDM of the thallium atom currently provides the best constraint on the MSSM scenarios of our interest. The atomic EDM of ^{205}Tl get main contributions from two terms [47,48]:

$$\begin{aligned} d_{\mathrm{Tl}}[e\,\mathrm{cm}] = -585 \cdot d_e[e\,\mathrm{cm}] - 8.5 \times 10^{-19}[e\,\mathrm{cm}] \\ \cdot (C_S\ \mathrm{TeV}^2) + \cdots, \end{aligned} \tag{13}$$

where d_e denotes the electron EDM and C_S is the coefficient of a *CP*-odd electron-nucleon interaction

[1] In Ref. [46], the *CP*- and $CP\tilde{T}$-odd $C_2^f(p^2)$ coefficient is missing since only one Higgs state was considered.

$\mathcal{L}_{C_S} = C_S \bar{e} i\gamma_5 e \bar{N}N$.[2] The dots denote subdominant contributions from 6-dimensional tensor and higher-dimensional operators. The experimental $2-\sigma$ bound on the thallium EDM is [50]

$$|d_{\rm Tl}| \lesssim 1.3 \times 10^{-24}[e\,{\rm cm}]. \tag{14}$$

In the CPsuperH [45] conventions and notations, the coefficient C_S is given by

$$C_S = -(0.1\ {\rm GeV}) \tan\beta \frac{m_e \pi \alpha_{\rm em}}{s_W^2 M_W^2} \sum_{i=1}^{3} \frac{g_{H_i gg} O_{ai}}{M_{H_i}^2}, \tag{15}$$

where

$$g_{H_i gg} = \sum_{q=t,b} \left\{ \frac{2}{3} g_{H_i qq}^S - \frac{v^2}{12} \sum_{j=1,2} \frac{g_{H_i \tilde{q}_j^* \tilde{q}_j}}{m_{\tilde{q}_j}^2} \right\}. \tag{16}$$

The Higgs-boson two-loop contributions to the electron EDM d_e are [51]

$$\begin{aligned}
\left(\frac{d_e}{e}\right)^{\tilde{q}} &= \frac{3\alpha_{\rm em} Q_q^2 m_e}{32\pi^3} \sum_{i=1}^{3} \frac{g_{H_i e^+ e^-}^P}{M_{H_i}^2} \sum_{j=1,2} g_{H_i \tilde{q}_j^* \tilde{q}_j} F(\tau_{\tilde{q}_j i}),\\
\left(\frac{d_e}{e}\right)^{q} &= -\frac{3\alpha_{\rm em}^2 Q_q^2 m_e}{8\pi^2 s_W^2 M_W^2} \sum_{i=1}^{3} [g_{H_i e^+ e^-}^P g_{H_i \bar{q}q}^S f(\tau_{qi})\\
&\quad + g_{H_i e^+ e^-}^S g_{H_i \bar{q}q}^P g(\tau_{qi})],\\
\left(\frac{d_e}{e}\right)^{\chi^\pm} &= -\frac{\alpha_{\rm em}^2 m_e}{4\sqrt{2}\pi^2 s_W^2 M_W} \sum_{i=1}^{3} \sum_{j=1,2} \frac{1}{m_{\chi_j^\pm}}\\
&\quad \times [g_{H_i e^+ e^-}^P g_{H_i \chi_j^+ \chi_j^-}^S f(\tau_{\chi_j^\pm i})\\
&\quad + g_{H_i e^+ e^-}^S g_{H_i \chi_j^+ \chi_j^-}^P g(\tau_{\chi_j^\pm i})],
\end{aligned} \tag{17}$$

with $q = t, b$ and $\tau_{xi} = m_x^2/M_{H_i}^2$. The total Higgs-mediated two-loop $(d_e/e)^H$ is given by the sum

$$\left(\frac{d_e}{e}\right)^{H} = \left(\frac{d_e}{e}\right)^{\tilde{t}} + \left(\frac{d_e}{e}\right)^{\tilde{b}} + \left(\frac{d_e}{e}\right)^{t} + \left(\frac{d_e}{e}\right)^{b} + \left(\frac{d_e}{e}\right)^{\chi^\pm} + \ldots, \tag{18}$$

and the two-loop functions $F(\tau)$, $f(\tau)$, and $g(\tau)$ may be found in Ref. [51]. The ellipses in (18) denote other subdominant two-loop contributions to EDM that involve charged Higgs and Higgsino effects [52].

We note that C_S in (15) and $(d_e/e)^H$ in (18) are calculated at the electroweak (EW) scale, where the responsible effective interactions are generated. In order to calculate the running from the EW scale to the appropriate low-energy scale, the anomalous dimension (matrix) should be considered. We neglect this effect by observing that it can be absorbed in the evaluation of the matrix element

$$\langle N | \frac{\alpha_S}{8\pi} G^{a,\mu\nu} G_{\mu\nu}^a | N \rangle = -(0.1)\ {\rm GeV} \bar{N}N, \tag{19}$$

which gives (15). We use $\alpha_{\rm em} = 1/137$.

In Fig. 3, we show the rescaled thallium EDM $\hat{d}_{\rm Tl} \equiv d_{\rm Tl} \times 10^{24}$ in units of e cm in the Φ_A-Φ_3 plane (upper left) and as a function of Φ_A (Φ_3) for several values of Φ_3 (Φ_A) in the upper-right (lower-left) frame. The lower-right frame shows the individual contributions as functions of Φ_3 when $\Phi_A = 60°$. We consider contributions from the Higgs-mediated two-loop $(d_e/e)^H$ and C_S, not including other contributions. In the upper-left frame, different ranges of $|\hat{d}_{\rm Tl}|$ are shown explicitly by different shadings. The blank unshaded region around the point $\Phi_3 = \Phi_A = 180°$ is not theoretically allowed since there large threshold corrections to the bottom-quark Yukawa coupling h_b result in a tachyonic sbottom, a complex or negative Higgs mass, a nonperturbative value of the Yukawa coupling $|h_b| > 2$, and/or a failure of the iteration method of calculating the corrections. We note that, in the region $|\hat{d}_{\rm Tl}| < 10$, the thallium EDM constraint can be evaded by assuming cancellations of less than 1 part in 10 between the two-loop contributions considered here and possible one-loop contributions not discussed here. As mentioned above, such cancellations are always possible in a general unconstrained MSSM scenario, where one-loop EDM effects depend on different *CP*-odd phases related to the first and second generations of squarks and sleptons. In our case, cancellations between the contributions from $(d_e/e)^H$ and C_S are shown in Fig. 3 as dips for specific values of Φ_A and Φ_3 in the upper-right and lower-left frames. These are responsible for the narrow region filled with black squares in the upper-left frame in which $|\hat{d}_{\rm Tl}| < 1e$ cm. For example, the lower-right frame clearly shows the cancellation between the contributions from $(d_e/e)^{q,\tilde{q}}$ and C_S when $\Phi_3 \sim 220°$ and $280°$ for $\Phi_A = 60°$, see also the dotted line in the lower-left frame.

In the *CP*-violating trimixing scenarios of the MSSM, the flavor-changing neutral current couplings of the Higgs bosons $H_{1,2,3}$ to down-type quarks are considerably enhanced at large values of $\tan\beta$ [22,53], i.e., for $\tan\beta \gtrsim 40$. These $\tan\beta$-enhanced Higgs couplings can give rise to potentially important constraints on the parameters of the *CP*-violating MSSM, which arise from the nonobservation of the Higgs-mediated B-meson decay $B_{s,d} \to \mu\mu$ at the Tevatron [54]. However, according to a detailed study [23], the derived constraints are highly dependent on detailed aspects of flavor physics, and may be relaxed dramatically for certain choices of the soft supersymmetry-breaking mass spectrum that cause cancellations in the unitarity sum over quark flavors. In view of these and other theoretical uncertainties [55], $B_{s,d} \to \mu\mu$ decays do not yet impose significant constraints on the parameter space relevant to this study.

[2]Our sign convention for C_S follows the one given in [48,49].

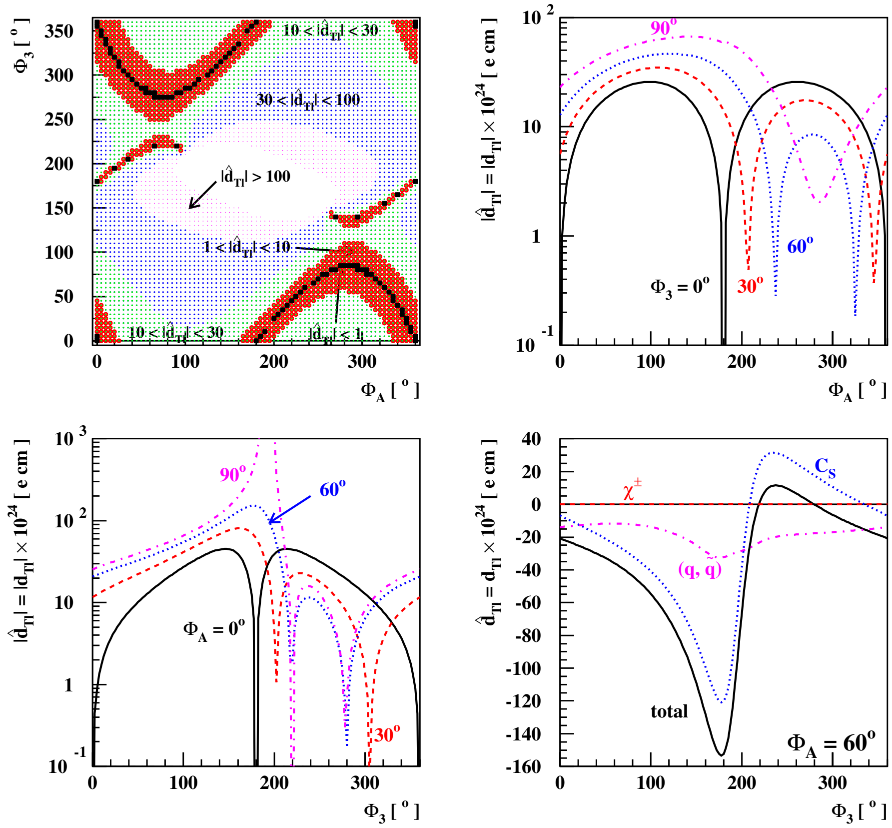

FIG. 3 (color online). The thallium EDM $\hat{d}_{\rm Tl} \equiv d_{\rm Tl} \times 10^{24} e$ cm in the trimixing scenario. The upper-left frame displays $|\hat{d}_{\rm Tl}|$ in the (Φ_A, Φ_3) plane. The unshaded region around the point $\Phi_3 = \Phi_A = 180°$ is not theoretically allowed. The different shaded regions correspond to different ranges of $|\hat{d}_{\rm Tl}|$, as shown: specifically, $|\hat{d}_{\rm Tl}| < 1$ in the narrow region denoted by solid black squares. In the upper-right frame, we show $|\hat{d}_{\rm Tl}|$ as a function of Φ_A for several values of Φ_3. In the lower-left frame, we show $|\hat{d}_{\rm Tl}|$ as a function of Φ_3 for four values of Φ_A. In the lower-right frame, we show the C_S (dotted line) and $(d_e/e)^{q,\tilde{q}}$ (dashed-dotted line) contributions to $\hat{d}_{\rm Tl}$ separately as functions of Φ_3 when $\Phi_A = 60°$. As shown by the dashed line, the chargino contribution is negligible.

IV. NUMERICAL EXAMPLES

For numerical studies, we take $\sqrt{s} = 300$ GeV and consider four different combinations of *CP* phases of (Φ_3, Φ_A) in the trimixing scenario (12) that are chosen to respect the thallium EDM constraint:

$$\begin{aligned}
&\mathbf{P0}: (\Phi_3, \Phi_A) = (0°, 0°): (M_{H_1}, M_{H_2}, M_{H_3}; \Gamma_{H_1}, \Gamma_{H_2}, \Gamma_{H_3}) = (119.2, 123.7, 125.6; 1.42, 2.95, 1.50)\ \text{GeV},\\
&\mathbf{P1}: (\Phi_3, \Phi_A) = (-55°, 30°): (M_{H_1}, M_{H_2}, M_{H_3}; \Gamma_{H_1}, \Gamma_{H_2}, \Gamma_{H_3}) = (118.9, 122.9, 124.6; 1.57, 3.45, 2.60)\ \text{GeV},\\
&\mathbf{P2}: (\Phi_3, \Phi_A) = (-80°, 60°): (M_{H_1}, M_{H_2}, M_{H_3}; \Gamma_{H_1}, \Gamma_{H_2}, \Gamma_{H_3}) = (118.6, 121.1, 123.5; 2.17, 4.77, 4.45)\ \text{GeV},\\
&\mathbf{P3}: (\Phi_3, \Phi_A) = (-80°, 90°): (M_{H_1}, M_{H_2}, M_{H_3}; \Gamma_{H_1}, \Gamma_{H_2}, \Gamma_{H_3}) = (119.0, 119.5, 122.9; 2.57, 5.70, 5.63)\ \text{GeV},
\end{aligned} \tag{20}$$

where the masses and widths of the neutral Higgs bosons are calculated using CPSUPERH [45]. We observe that the three neutral Higgs bosons are almost degenerate with masses around 120 GeV, and large widths comparable to the mass differences. At the *CP*-conserving point **P0**, the second lightest Higgs boson H_2 is *CP* odd and the *CP*-even H_1 and H_3 have strong *two-way* mixing. But, in the presence of nonvanishing *CP* phases, as at points **P1**, **P2** and **P3**, all the three neutral Higgs bosons mix significantly. The three *CP*-violating points **P1**, **P2** and **P3** are chosen to lie along the narrow region filled with black squares in the upper-left panel of Fig. 3 where the two contributions from $(d_e/e)^H$ and C_S to the thallium EDM cancel approximately and we have $|\hat{d}_{\rm Tl}| < 1 e$ cm. Thus, this selection of points complements the information available from low-energy EDM experiments.

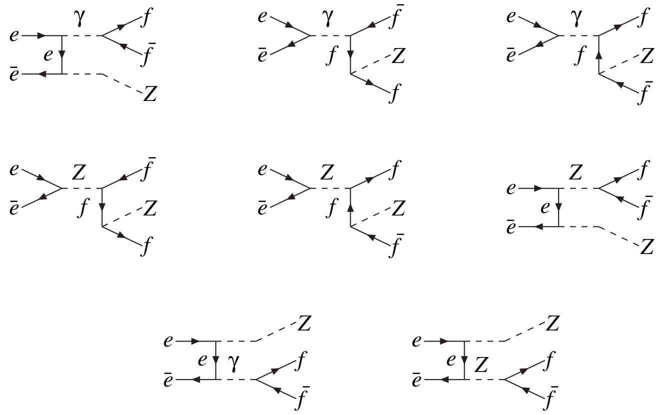

FIG. 4. Diagrams for the SM background process $e^+e^- \to f\bar{f}Z$ with $f = b$ and τ^-.

A. Backgrounds

We consider the two final states $f = b$ and $f = \tau^-$ for the Higgs-boson decays. Therefore, before showing the numerical results for the Higgsstrahlung processes, we first consider the SM background processes $e^+e^- \to f\bar{f}Z$ with $f = b$ and τ^-, omitting the Higgs-mediated diagrams, as seen in Fig. 4. The background cross sections are evaluated using COMPHEP [56]. We show in Fig. 5 the product of the differential background cross section $d\sigma_{\rm bkg}/d\sqrt{p^2}$ and the branching fraction of the Z boson into electrons and muons, $B(Z \to l^+l^-)$, as a function of the invariant mass of the fermion-antifermion pair $\sqrt{p^2}$ in units of fb/GeV. The solid line is for $f = b$ and the dashed line for $f = \tau^-$. We used for the leptonic branching fraction [57]:

$$\begin{aligned} B(Z \to l^+l^-) &= B(Z \to e^+e^-) + B(Z \to \mu^+\mu^-) \\ &= 6.73 \times 10^{-2}. \end{aligned} \tag{21}$$

We note that the product of the background cross section and the branching fraction of the Z boson into light leptons is smaller than $\sim$0.03 (0.01) fb/GeV for $f = b$ (τ^-) when $\sqrt{p^2} > 110$ GeV.

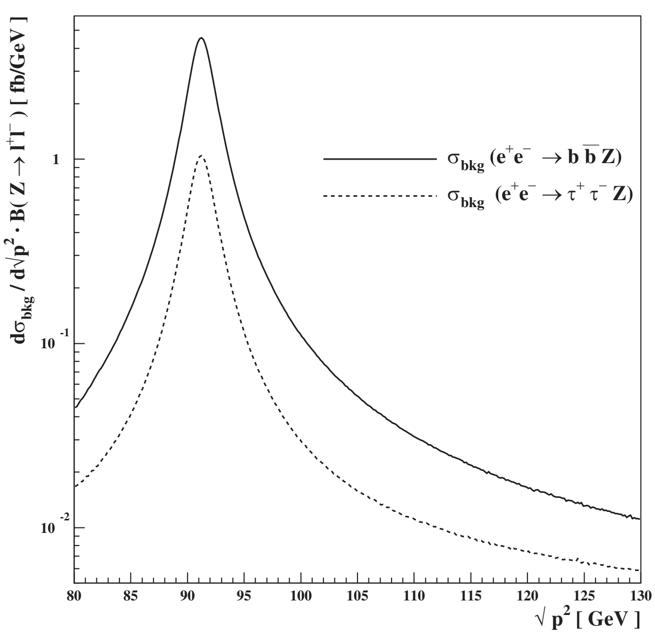

FIG. 5. The product of the differential background cross section and the branching fraction of the Z boson into electrons and muons, $(d\sigma_{\rm bkg}/d\sqrt{p^2}) \times B(Z \to l^+l^-)$, as a function of $\sqrt{p^2}$ in units of fb/GeV. The solid line is for the process $e^+e^- \to b\bar{b}Z$ and the dashed line for $e^+e^- \to \tau^-\tau^+Z$.

B. *CP*-conserving cross sections

The differential total cross section can be obtained by summing over the polarizations P and $\bar{P}$ in (4). We have

$$\frac{d\sigma^f_{\rm tot}}{d\sqrt{p^2}} = \frac{8\hat{\sigma}^f_1(p^2)}{\sqrt{p^2}}, \tag{22}$$

where $\hat{\sigma}^f_1$ is defined as in (11). We show in Fig. 6 the differential total cross section multiplied by $B(Z \to l^+l^-)$ when the produced Higgs bosons decay into b quarks: $(d\sigma^b_{\rm tot}/d\sqrt{p^2}) \times B(Z \to l^+l^-)$. The cross sections are significantly larger (0.1–5 fb/GeV) than that of the SM background (< 0.02 fb/GeV) around the peaks. In the

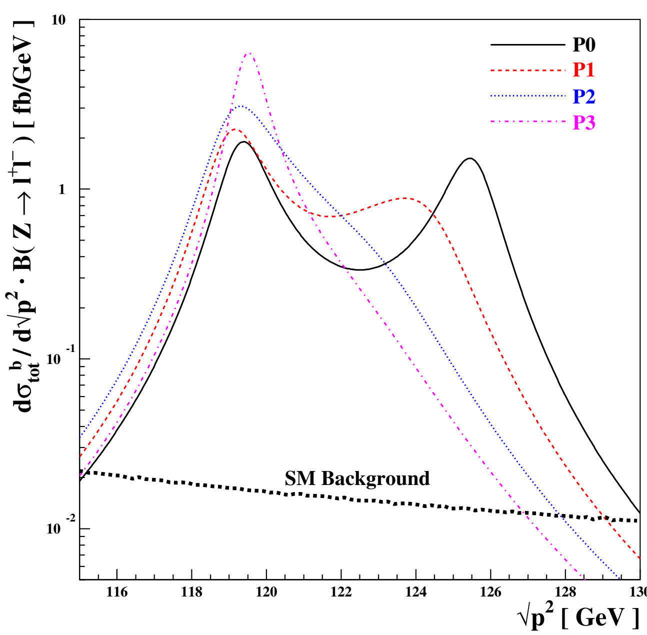

FIG. 6 (color online). The differential total cross section multiplied by $B(Z \to l^+l^-)$ when the produced Higgs bosons decay into b quarks: $(d\sigma^b_{\rm tot}/d\sqrt{p^2}) \times B(Z \to l^+l^-)$. The *CP*-conserving two-way mixing (**P0**) and the *CP*-violating trimixing (**P1**-**P3**) scenarios have been taken, see (12) and (20). The solid line is for **P0**, the dashed line for **P1**, the dotted line for **P2**, and the dashed-dotted line for **P3**. The SM background cross section from Fig. 5 is also shown.

CP-conserving case **P0**, we clearly see two peaks of the *CP*-even Higgs bosons at $\sqrt{p^2} = 119.2$ GeV (H_1) and at $\sqrt{p^2} = 125.6$ GeV (H_3), see also (20). The second lightest *CP*-odd H_2 does not contribute. However, in the *CP*-violating cases this two-peak structure becomes less clear as the phase Φ_A increases: see the dashed (**P1**: $\Phi_A = 30°$), dotted (**P2**: $\Phi_A = 60°$), and dashed-dotted (**P3**: $\Phi_A = 90°$) lines. The disappearance of the two-peak structure is the combined effect of increasing (decreasing) H_2 (H_3) coupling to Z bosons, $g^2_{H_2VV}$ ($g^2_{H_3VV}$), and decreasing mass difference between H_1 and H_2 without visible changes in M_{H_1} around 119 GeV. The sensitivity of this *CP*-conserving quantity to the *CP*-violating phases will be measurable by examining the production line shape at the ILC. For example, an integrated luminosity larger than $\sim$100 fb^{-1} would enable a difference of $\sim$0.1 fb/GeV in cross sections to be distinguished easily.

As was emphasized in [13], the resonance line shape of a coupled system of neutral Higgs bosons is not a process-independent quantity, but crucially depends on its production and decay channels. A combined analysis of the different production and decay channels at the LHC, ILC and PLC can shed light on whether one is dealing with a single, two- or multicomponent system of Higgs bosons. Such an extensive analysis is beyond the scope of the present paper and may be given elsewhere. As we demonstrate explicitly below, the possible observation of nonzero *CP* asymmetries could give further insight into the *CP* composition of such a resonant Higgs-boson system.[3]

When the produced Higgs bosons decay into τ leptons, we can construct another *CP*-conserving cross section in addition to the total cross section, by measuring the transverse polarizations of tau leptons or, equivalently, by examining the polar and azimuthal angle distributions of the charged hadrons coming from the τ-lepton decays:

$$\frac{d\sigma_3^\tau}{d\sqrt{p^2}} = \frac{8\hat{\sigma}_3^\tau(p^2)}{\sqrt{p^2}}. \tag{23}$$

This is related to the polarization coefficient $C_3^f(p^2)$ in (4) and (10). The *CP*-conserving total and transverse differential cross sections multiplied by $B(Z \to l^+l^-)$ are shown in the left and right panels of Fig. 7, respectively. The cross sections are smaller than that of the b-quark case ($\sim$ 0.01–1 fb/GeV), but the transverse cross section $d\sigma_3^\tau/d\sqrt{p^2}$ provides extra sensitivity to the *CP*-violating phases in the τ-lepton case, in addition to the total cross section.

[3]We should note that performing an overall fit to the production line shape becomes more challenging in the presence of *CP* violation. Specifically, one has three masses of neutral Higgs bosons, six widths including off-diagonal absorptive effects, and two independent Higgs-boson couplings to Z bosons. Therefore, the analysis of other observables in addition to the total cross section would be very useful for the complete determination of the parameters.

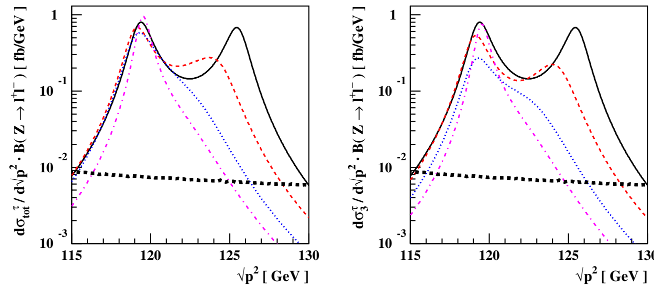

FIG. 7 (color online). Two differential *CP*-conserving cross sections [multiplied by $B(Z \to l^+l^-)$] that are observable when the produced Higgs bosons decay into τ leptons: $(d\sigma^\tau_{\rm tot}/d\sqrt{p^2}) \times B(Z \to l^+l^-)$ (left panel) and $(d\sigma_3^\tau/d\sqrt{p^2}) \times B(Z \to l^+l^-)$ (right panel). The line styles are as in Fig. 6.

C. *CP*-violating cross sections and asymmetries

When the produced Higgs bosons decay into τ leptons, there are two *CP*-violating cross sections which are defined using the longitudinal and transverse polarizations of τ leptons:

$$\frac{d\Delta\sigma_L^\tau}{d\sqrt{p^2}} = \frac{8\hat{\sigma}_2^\tau(p^2)}{\sqrt{p^2}}, \qquad \frac{d\Delta\sigma_T^\tau}{d\sqrt{p^2}} = \frac{8\hat{\sigma}_4^\tau(p^2)}{\sqrt{p^2}}, \tag{24}$$

where $\Delta\sigma_L^\tau$ and $\Delta\sigma_T^\tau$ are related to the polarization coefficients $C_2^f(p^2)$ and $C_4^f(p^2)$, respectively. The polarization coefficients $C_2^f(p^2)$ and $C_4^f(p^2)$ can be determined by measuring the longitudinal and transverse polarizations of τ leptons, respectively [cf. (4) and (10)]. We define the corresponding longitudinal and transverse *CP* asymmetries as follows:

$$\begin{aligned} a_L^\tau(p^2) &\equiv \frac{d\Delta\sigma_L^\tau/d\sqrt{p^2}}{d\sigma^\tau_{\rm tot}/d\sqrt{p^2}} = \frac{\sigma_2^\tau(p^2)}{\sigma_1^\tau(p^2)}, \\ a_T^\tau(p^2) &\equiv \frac{d\Delta\sigma_T^\tau/d\sqrt{p^2}}{d\sigma^\tau_{\rm tot}/d\sqrt{p^2}} = \frac{\sigma_4^\tau(p^2)}{\sigma_1^\tau(p^2)}. \end{aligned} \tag{25}$$

We show in Fig. 8 the *CP*-violating cross sections (left column) and *CP* asymmetries (right column). The *CP*-violating cross sections are large enough to be measured, assuming luminosity of >100 fb^{-1}, for each choice of the *CP*-violating phases, and the *CP* asymmetry can be as large as 80%. Analysis of the production line shape would enable the cases with different *CP*-violating phases to be distinguished from each other. Since the *CP*-violating scenarios chosen respect the low-energy EDM constraints, these examples show that linear-collider measurements are complementary, and large *CP*-violating effects cannot be excluded *a priori*.

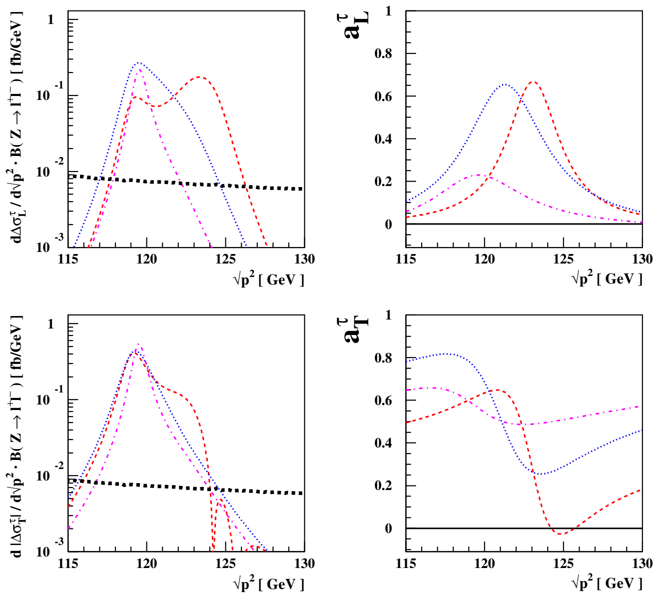

FIG. 8 (color online). Two differential *CP*-violating cross sections [multiplied by $B(Z \to l^+ l^-)$] observable when the produced Higgs bosons decay into τ leptons: $(d\Delta\sigma_L^\tau / d\sqrt{p^2}) \times B(Z \to l^+ l^-)$ (upper-left panel) and $(d\Delta\sigma_T^\tau / d\sqrt{p^2}) \times B(Z \to l^+ l^-)$ (lower-left panel). The two corresponding *CP*-violating asymmetries are shown in the right column: a_L^τ (upper-right panel) and a_T^τ (lower-right panel). The lines styles are the same as in Fig. 6.

V. CONCLUSIONS

We have shown that the Higgsstrahlung process $e^+e^- \to H_{1,2,3}(Z \to e^+e^-, \mu^+\mu^-)$, with subsequent decays $H_{1,2,3} \to b\bar{b}, \tau^-\tau^+$ is potentially a useful channel of searching for radiative Higgs-sector *CP* violation in the MSSM. The recoil-mass method would enable one to measure the invariant mass of a $b\bar{b}$ or $\tau^-\tau^+$ pair produced in Higgs decay with a precision as good as 1 GeV. In trimixing scenarios where all three neutral Higgs states of the MSSM are nearly degenerate and mix significantly, this accuracy would enable details of complicated line shapes to be disentangled. An integrated luminosity of 100 fb^{-1} would already be sufficient to measure *CP*-violating parameters via their influences on the production line shape of a coupled system of neutral Higgs bosons decaying into $b\bar{b}$ quarks. Measurements of the Higgs bosons decaying into $\tau^-\tau^+$ would enable an additional *CP*-conserving cross section and two *CP*-violating asymmetries to be defined in terms of the longitudinal and transverse polarizations of the τ leptons.

We find that these *CP* asymmetries could be as large as 80%, even after taking into account the constraints from the thallium electric dipole moment, and different *CP*-violating models compatible with the thallium data could be distinguished. Thus, measurements of *CP* violation in Higgsstrahlung at an e^+e^- linear collider would complement high-precision low-energy measurements, and might provide a signal, even if none is visible in the low-energy experiments.

Finally, we should stress that the analysis presented in this paper is general and applies equally well to extended *CP*-violating Higgs sectors with similar phenomenological features. *CP* violation and its relation to the cosmological baryon asymmetry are among the outstanding questions in possible physics beyond the standard model, and measurements of the Higgsstrahlung process could provide a unique window that could shine valuable light on attempts to relate these two puzzles.

ACKNOWLEDGMENTS

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