

## Radiative Corrections to Bremsstrahlung in Radiative Return\*

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Radiating a photon from the initial state provides a useful tool for studying a range of low energy physics using a high-energy  $e^+e^-$  accelerator. Accurate results require careful calculation of the first order virtual photon corrections. We compare exact results for initial state radiative corrections, finding agreement to an order of  $10^{-5}$  or better as a fraction of the Born cross-section for most of the range of photon energies, at CMS energies relevant in both high-energy collision and radiative return experiments.

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### 1. Introduction

Radiative return [1–3] provides a mechanism for exploring a wide range of CMS energies in hadron production in a high luminosity  $e^+e^-$  collider. Radiating a hard photon from the initial state (ISR) reduces the effective energy of the collision, allowing a range of energies to be scanned by observing different values of the hard photon energy. A precise calculation of this process requires including the  $\mathcal{O}(\alpha^2)$  contributions arising from an

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additional virtual or soft photon. These effects were integrated into a MC generator PHOKHARA designed to calculate radiative return at DAΦNE, CLEO-C and  $B$  factories. [4–6] The PHOKHARA MC is discussed by H. Kühn in these proceedings. [7] Both the processes  $e^+e^- \rightarrow \pi^+\pi^-\gamma$  and  $e^+e^- \rightarrow \mu^+\mu^-\gamma$  are implemented in PHOKHARA. The results of [6] include mass corrections needed in the limit when the photon is emitted at small angles. The inclusion of such photons in the radiative return cross-section is advantageous due to the enhanced rate for collinear emission.

The process  $e^+e^- \rightarrow f\bar{f} + n\gamma$  is also implemented in the KK Monte Carlo. [8,9] In particular, the initial state radiative correction to the process  $e^+e^- \rightarrow \mu^+\mu^-\gamma$  was calculated exactly [10] at order  $\alpha^2$ . The KKMC was designed for high energy  $e^+e^-$  annihilation at LEP and LEP2, so the energies tested in ref. [10] were higher than for PHOKHARA, but the initial state radiation was calculated to the same level of exactness in each case. Previous results [11] (BVNB) and [12] (IN) for the virtual correction to initial state bremsstrahlung in this process have been compared to the results of [10] (JMWY), but the results of BVNB are not fully differential, and the results of IN do not include mass corrections.

The comparison of the virtual corrections of JMWY to those of [5,6] (KR) is the closest presently available. Since both are calculated with special attention to small photon angles, electron mass corrections are included in each expression, but by different means. This comparison is a component of a Monte Carlo comparison, reported by S. Jadach in these proceedings, [18] of the KKMC and PHOKHARA for muon pair or pion pair final states. In that comparison, agreement to within 0.2% was found for muon pair final states with pure initial state radiative corrections.

## 2. Comparison of Virtual Corrections

In this note, we will compare an implementation of the initial state virtual corrections of JMWY and KR directly in the context of the KKMC. This comparison [15–17] is of particular interest because the published matrix elements have very different forms, making an analytic comparison non-trivial. This is most evident in the appearance of mass terms proportional to  $(p_i \cdot k)^{-2}$  and  $(p_i \cdot k)^{-3}$  in the expressions of KR, and the absence of such terms in the expressions of JMWY, where  $p_i$  is an incoming fermion momentum, and  $k$  is the emitted photon momentum. We have verified that, in fact, all such terms cancel exactly in the expressions of KR, leaving a leading collinear factor of  $(p_i \cdot k)^{-1}$ . This cancellation should be implemented analytically to obtain a stable evaluation in a MC program.

We have also verified that in the massless limit, the two expressions for the virtual correction agree in the NLL limit, where the photon is taken to

be collinear with an incoming fermion. [17] This comparison also makes use of a careful expansion of the two expressions in the collinear limit, which is needed to cancel apparent extra powers of  $p_i \cdot k$  in the denominators in the expressions of KR.

The previously-available comparisons, refs. [11,12], for the virtual photon correction to ISR were earlier shown to agree with [10] in the collinear limits. In the case of [11], this includes the mass corrections. However, these two comparisons are less complete. In the case of [11], the direction of the photon has been integrated, and in the case of [12], the mass corrections needed for high precision in the collinear limit are not included.

Since the four expressions JMWY, BVNB, IN and KR all agree analytically to NLL order in the massless limit, it is useful to compare the residual NNLL contribution after subtracting the common collinear limit of each expression. In practice, this was done by calculating the YFS residual [19,20]  $\bar{\beta}_1^{(2)}$  for single hard photon emission at order  $\alpha^2$ , where a standard IR contribution has been subtracted. These residuals are used in the implementation of the KKMC. [8,9] In the collinear (NLL) limit, this residual can be related to  $\bar{\beta}_1^{(1)}$  at order  $\alpha$  (without the virtual photon) via a form factor  $f_{\text{NLL}}$  such that

$$\bar{\beta}_1^{(2)} = \bar{\beta}_1^{(1)} \left( 1 + \frac{\alpha}{2\pi} \langle f_{\text{NLL}} \rangle \right) \quad (1)$$

with spin-averaged NLL form factor [10]

$$\begin{aligned} \langle f_{\text{NLL}} \rangle = & 2 \left\{ \ln \left( \frac{s}{m_e^2} \right) - 1 \right\} + \frac{r_1(1-r_1)}{1+(1-r_1)^2} + \frac{r_2(1-r_2)}{1+(1-r_2)^2} \\ & + 2 \ln r_1 \ln(1-r_2) + 2 \ln r_2 \ln(1-r_1) - \ln^2(1-r_1) - \ln^2(1-r_2) \\ & + 3 \ln(1-r_1) + 3 \ln(1-r_2) + 2\text{Sp}(r_1) + 2\text{Sp}(r_2) + \langle f_{\text{NLL}}^m \rangle. \end{aligned} \quad (2)$$

Here,  $s = (p_1 + p_2)^2$ ,  $r_i = 2p_i \cdot k/s$ , and  $\text{Sp}(z)$  is the Spence dilogarithm function. The NLL limit of the mass correction is taken to be [10]

$$\begin{aligned} \langle f_{\text{NLL}}^m \rangle = & \frac{2m_e^2}{s} \left( \frac{r_1}{r_2} + \frac{r_2}{r_1} \right) \frac{1-r_1-r_2}{(1-r_1)^2 + (1-r_2)^2} \\ & \times \left\{ \langle f_{\text{NLL}}^{m=0} \rangle + \left[ \ln(1-r_1) + \ln(1-r_2) - 1 \right] \ln \left( \frac{s}{m_e^2} \right) - \frac{3}{2} \ln(1-r_1) \right. \\ & \left. - \frac{3}{2} \ln(1-r_2) + \frac{1}{2} \ln^2(1-r_1) + \frac{1}{2} \ln^2(1-r_2) + 1 \right\}. \end{aligned} \quad (3)$$

After subtracting this expression from each of the results, we obtain the NNLL contribution to be compared.

The four expressions were implemented in the EEX3 option of the KK MC (the YFS3ff generator), [9] and compared for muon pair production

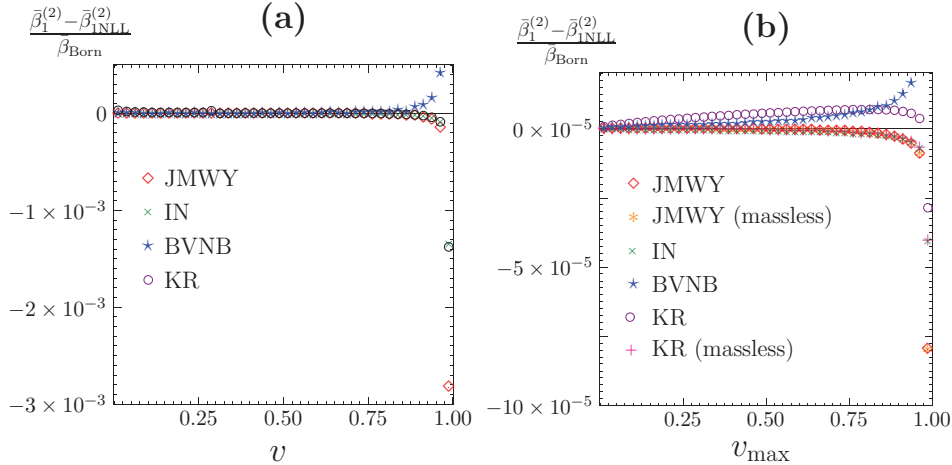


Fig. 1. Comparison of the NNLL contributions to ISR for muon pair production at a CMS energy of 200 GeV. The expressions are compared for  $10^8$  events generated by the EEX3 option of the KK Monte Carlo as a function of the fraction  $v$  of the beam energy radiated to the photon. The results is in units of the Born cross-section. Fig. (a) is differential in  $v$ , and fig. (b) is integrated up to a cut  $v_{\text{max}}$ .

with pure initial state radiation (ISR). Fig. 1 compares the NNLL results for  $10^8$  generated events. The CMS energy was chosen as 200 GeV to match the earlier comparisons in ref. [10]. Fig. 1(a) shows a differential distribution in the photon energy fraction  $v = 1 - Q^2/s$ , where  $Q$  is the effective CMS momentum for the radiative return process. In fig. 1(b), the cross-section has been integrated up to an energy cut  $v_{\text{max}}$ , as in the original comparisons in ref. [10]. In order to compare both the size of the NNLL effects and the mass corrections separately, fig. 1(b) includes runs with and without the mass terms in the JMWY and KR expressions. Comparisons of the type in fig. 1(b) have been discussed in ref. [17], and with a different choice of NLL limit (equivalent up to collinear terms) in refs. [15, 16].

In the integrated cross-section, it is found that all of the results agree to within  $0.5 \times 10^{-5}$  in units of the Born cross-section ( $e^+e^- \rightarrow \mu^+\mu^-$  without radiation) up to a cut of  $v_{\text{max}} = 0.95$ , except for the BVNB result, which is not fully differential in the photon momenta. For the last data point, with a maximum  $v = 0.975$ , a larger departure is seen, with the difference between JMWY and KR results reaching  $5.2 \times 10^{-5}$  units of the Born cross-section.

Differences in mass correction account for  $0.6 \times 10^{-5}$  units of this difference. This shows that in spite of the apparent difference in the analytic

expressions for the mass terms, they are essentially equivalent. This is a non-trivial result, since the mass corrections of KR were calculated by applying FeynCalc [21] to the exact expression for the leptonic tensor, while the mass corrections of JMWY were calculated using the methods of ref. [22], after verifying that these methods reproduce the exact mass corrections up to terms of order  $m_e^2/s$  in the fully integrated cross-section. This technique leads to a compact expression for the essential contribution from the mass correction in the collinear limit which can be evaluated without potential numerical difficulties which can arise from higher powers of collinear factors in the denominators.

In the differential plot, fig. 1(a), the difference between the JMWY and KR results at  $v = 0.975$  is found to be 1.4 per mil in units of the Born cross-section, with the KR result in agreement with the IN result. This difference is consistent with fig. 1(b), since it is due mostly to the last bin, which includes 1/40 of the  $v$  range. In fact, the difference between the results in the next next to the last bin, at  $v = 0.95$ , is only  $3 \times 10^{-5}$  in units of the Born cross-section.

The parameters for fig. 1 were chosen to match earlier comparisons [10] for the LEP2 final data analysis, when the KR result was not yet available.

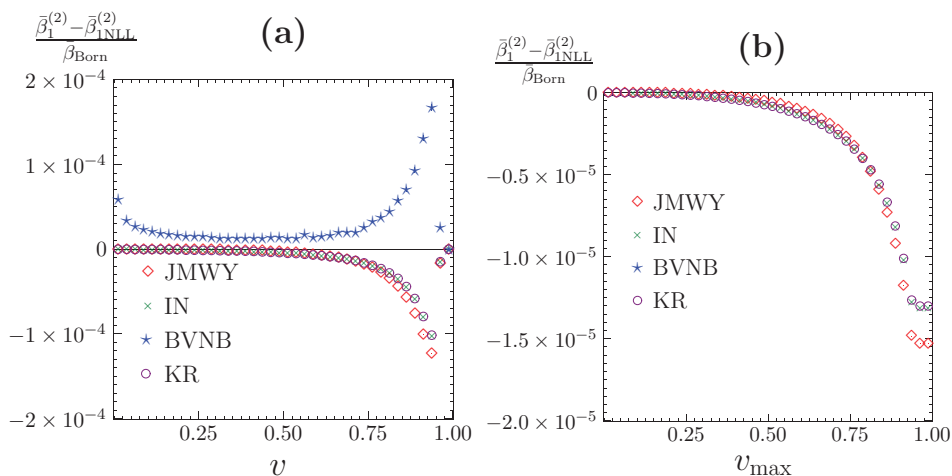


Fig. 2. Comparison of the NNLL contributions to ISR for muon pair production at a CMS energy of 1.0 GeV. The expressions are compared for  $10^8$  events generated by the EEX3 option of the KK Monte Carlo as a function of the fraction  $v$  of the beam energy radiated to the photon. The results is in units of the Born cross-section. Fig. (a) is differential in  $v$ , and fig. (b) is integrated up to a cut  $v_{\text{max}}$ .

For radiative return, a low-energy comparison would be more appropriate. In fig. 2, we have chosen 1 GeV as a representative  $e^+e^-$  CMS energy. No cuts were applied on the fermion directions. The BVNB result, which is not differential in the photon directions, has a greater difference from the other results at low CMS energy, but the JMWY and KR results agree more closely, reaching a maximum of  $2 \times 10^{-5}$  units of the Born cross-section in the differential distribution 2(a), and  $2.3 \times 10^{-6}$  in the integrated distribution 2(b). For  $v < 0.85$ , all of the differential results except BVNB agree to within  $10^{-5}$  of the Born cross-section.

### 3. Comparison of Virtual Corrections

Our results show that the results of JMWY and KR for the virtual corrections used in the calculation of radiative return agree to within  $5 \times 10^{-5}$  units of the Born cross-section for the full range of photon energies in the integrated distribution of fig. 1(b), or within 1.4 per mil in the differential distribution of fig. 1(a) at a CMS energy of 200 GeV. Over most of the range of photon energies, the agreement is on the order of  $10^{-5}$  or better.

Excellent agreement is also found at a CMS energy of 1.0 GeV, an energy scale more relevant for radiative return experiments at, for example, DAΦNE. Here, both the differential and integrated distributions in fig. 2 show agreement on the order of  $10^{-5}$  or better for the JMWY and KR results over the entire range of photon energies.

The comparison of the effect of mass corrections is of particular interest, since Differences in the treatment of mass corrections are the most obvious distinction between the expressions of JMWY and KR at an analytic level, but The MC results show that in fact, the difference between the mass corrections is insignificant, less than  $0.6 \times 10^{-5}$  even in the large  $v$  limit.

These results show that we have a clear understanding of the precision for the hard photon plus virtual photon contribution to the order  $\alpha^2$  radiative correction to  $f\bar{f}$  production, an important process not just in radiative return, but also in the final LEP2 data analysis and any anticipated future linear collider physics. [16]

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