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CP-odd invariants in models with several Higgs doublets

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Abstract

We present CP-odd Higgs-basis invariants, which can be used to signal CP violation in a multi-Higgs system, written in an arbitrary Higgs basis. It is shown through specific examples how these CP-odd invariants can also be useful to determine the character of CP breaking (i.e. whether it is hard or soft CP breaking) in a given Higgs Lagrangian. We analyse in detail the cases of two and three Higgs doublets.

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1 Introduction

One of the most fundamental open questions in particle physics is discovering what the mechanism responsible for gauge-symmetry breaking is. In the Standard Model (SM), the breaking of the $SU(2) \times U(1) \times SU(3)_c$ symmetry is achieved by the Higgs mechanism, realized through the introduction of one Higgs doublet. However, there is no fundamental reason for having only one Higgs doublet, and multi-Higgs systems are required by a large class of models, including supersymmetric models as well as models where CP is spontaneously broken [1] [2]. In the SM with only one Higgs doublet, the Higgs sector does not contain new sources of CP violation, but this is no longer true in models with more than one Higgs doublet. This has prompted several recent analyses on the impact of multi-Higgs in the physics at future colliders [3].

In this paper, we study CP violation in multi-Higgs extensions of the SM, addressing in particular the question of finding the conditions for a given Higgs potential to violate CP at the Lagrangian level. This question is not trivial, since in a model with more than one Higgs doublet, one has the freedom to make Higgs-basis transformations that do not change the physical content of the model, but do change both the quadratic and the quartic couplings. Couplings that are complex in one Higgs basis may become real in another basis. A similar difficulty arises when one investigates whether, in a given model, there is soft or hard CP breaking. Indeed a given model may have complex quartic couplings in one Higgs basis, while they may all become real in another basis, with only the quadratic couplings now complex, thus indicating that in that particular model CP is only softly broken.

The above intrinsic ambiguities motivate the study of conditions for CP invariance expressed in terms of CP-odd Higgs-basis invariants. We analyse in detail the cases of two and three Higgs doublets and briefly describe the case of an arbitrary number of Higgs doublets.

2 The Higgs Lagrangian

Let us consider the Standard $SU(2) \times U(1) \times SU(3)_c$ Model with n_d Higgs doublets. For the moment we consider only the Higgs sector. The most general renormalizable polynomial consistent with the gauge invariance is given by:

$$\mathcal{L}_\phi = Y_{ab}\ \phi_a^\dagger \phi_b + Z_{abcd}\ \left(\phi_a^\dagger \phi_b\right)\left(\phi_c^\dagger \phi_d\right), \tag{1}$$

where repeated indices are summed. Hermiticity of $\mathcal{L}_\phi$ implies:

$$Y_{ab}^* = Y_{ba} \qquad ; \qquad Z_{abcd}^* = Z_{badc}\ . \tag{2}$$

Furthermore, by construction it is obvious that:

$$Z_{abcd} = Z_{cdab}\ . \tag{3}$$

It is important to note that one can make a Higgs-basis transformation (HBT) defined by:

$$\phi_a \xrightarrow{\text{HBT}} \phi'_a = V_{ai}\ \phi_i \qquad , \qquad \phi^\dagger_a \xrightarrow{\text{HBT}} (\phi')^\dagger_a = V^*_{ai}\ (\phi')^\dagger_i\ , \tag{4}$$

where V is an $n_d \times n_d$ unitary matrix acting in the space of Higgs doublets. Under a HBT, the physics content of $\mathcal{L}_\phi$ does not change, but the couplings Y and Z transform as:

$$\begin{aligned} Y_{ab} &\xrightarrow{\text{HBT}} Y'_{ab} = V_{am}\ Y_{mn}\ V^\dagger_{nb} \\ Z_{abcd} &\xrightarrow{\text{HBT}} Z'_{abcd} = V_{am}\ V_{cp}\ Z_{mnpq}\ \ V^\dagger_{nb}\ V^\dagger_{qd}. \end{aligned} \tag{5}$$

We now address the following question: What are the necessary and sufficient conditions for $\mathcal{L}_\phi$ to be CP-invariant? Note that an entirely analogous question has been asked in the framework of the SM, for the Yukawa sector, where it has been shown [4] that a necessary condition for CP invariance at the Lagrangian level in the SM is given by:

$$\text{Tr}\ [\ g_u g_u^\dagger,\ g_d g_d^\dagger\ ]^3 = 0, \tag{6}$$

where g_u, g_d denote the Yukawa couplings involving u_R and d_R respectively; after gauge symmetry breaking, these lead to the quark masses for the up and down quarks. The condition of Eq. (6) is a necessary condition for CP invariance for an arbitrary number of generations and, for three generations, is also a sufficient condition for CP invariance.

In order to answer the above question for a multi-Higgs sector, we follow the general method proposed in [4], applied to specific models in [5] and described in detail in [6]. The simplest approach consists in considering the most general CP transformation of the Higgs fields, which leaves the Higgs kinetic energy term invariant, and investigate what restrictions are implied for the couplings Y and Z, by CP invariance. The most general CP transformation that leaves the kinetic energy invariant is:

$$\phi_a \xrightarrow{\text{CP}} U_{ai}\ \phi^*_i \qquad ; \qquad \phi^\dagger_a \xrightarrow{\text{CP}} U^*_{ai}\ \phi^T_i \tag{7}$$

where U is an $n_d \times n_d$ unitary matrix operating in Higgs doublets space. It can be readily seen, using Eqs. (1) and (7), that the necessary and sufficient condition for $\mathcal{L}_\phi$ to conserve CP is that the following relations be satisfied:

$$\begin{aligned} (Y^*)_{ab} &= U^\dagger_{am}\ Y_{mn}\ U_{nb} \\ (Z^*)_{abcd} &= U^\dagger_{am}\ U^\dagger_{cp}\ \ Z_{mnpq}\ U_{nb}\ U_{qd}. \end{aligned} \tag{8}$$

In other words, for a given $\mathcal{L}_\phi$, written as in Eq. (1), the necessary and sufficient condition for $\mathcal{L}_\phi$ to be CP-invariant is the existence of an n_d dimensional unitary matrix U, satisfying Eqs. (8). Obviously, the above criterion is HBT-invariant. In order to show that this is indeed the case, let us assume that a solution U exists for $\mathcal{L}_\phi$, written in a certain basis. Then a solution U' will also exist in another basis related to the initial one through Eq. (4). From Eqs. (4), (5) and (8) it follows that, in the primed basis, the solution is given by:

$$U' = V \ U \ V^T. \tag{9}$$

Although Eqs. (8) provide necessary and sufficient conditions for CP invariance in the Higgs sector, valid for an arbitrary number of Higgs doublets, they are not very practical, since they require the search for a solution for the unitary matrix U. It is thus useful to consider quantities which, on the one hand are HBT-invariant and, on the other hand, are constrained to vanish by CP invariance. In the case of the Higgs sector, an analysis [7] has been done in terms of HBT invariants which involve vacuum expectation values (vev's) of Higgs fields. These HBT invariants are very useful at low energies in the analysis of various CP-violating phenomena, but they do not answer directly the question we address in this paper, to wit, what the HBT-invariant conditions for CP invariance of $\mathcal{L}_\phi$, prior to gauge symmetry breaking are. The relevance of this question has been emphasized in [8]. Furthermore, at high energies, $SU(2) \times U(1) \times SU(3)_c$ is not broken, and therefore HBT invariants involving vev's are no longer relevant. The CP-odd HBT invariants, which we consider in this paper, are expressed directly in terms of the couplings Y and Z of the Lagrangian and thus continue to be relevant at high energies. Note that CP-odd HBT invariants that do not vanish at high energies are specially useful in the analysis of baryogenesis, including baryogenesis through leptogenesis.

3 Construction of CP-odd Higgs-basis transformation invariants

In the construction of HBT invariants, it is useful to adopt a more compact notation, which renders more transparent the invariance of various quantities under HBT. We define:

$$\begin{aligned} Y_a^b &\equiv Y_{ab} \\ Z_{ac}^{bd} &\equiv Z_{abcd} \ , \end{aligned} \tag{10}$$

where the upper indices are those that transform with $V^\dagger$ under a HBT, as specified in Eqs. (5).

Next we consider separately the cases of two and three Higgs doublets.

3.1 Two Higgs doublets

3.1.1 The general case

For later convenience, and in order to settle the notation, we write explicitly the most general Higgs potential for two Higgs doublets as:

$$
\begin{aligned}
V_{H_2} &= m_1\ \phi_1^\dagger\phi_1 + p\ e^{i\varphi}\ \phi_1^\dagger\phi_2 + p\ e^{-i\varphi}\ \phi_2^\dagger\phi_1 + m_2\ \phi_2^\dagger\phi_2 + \\
&+ a_1\ \left(\phi_1^\dagger\phi_1\right)^2 + a_2\ \left(\phi_2^\dagger\phi_2\right)^2 + b\ \left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + b'\ \left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \\
&+ c_1\ e^{i\theta_1}\ \left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_1\right) + c_1\ e^{-i\theta_1}\ \left(\phi_1^\dagger\phi_1\right)\left(\phi_1^\dagger\phi_2\right) + \\
&+ c_2\ e^{i\theta_2}\ \left(\phi_2^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + c_2\ e^{-i\theta_2}\ \left(\phi_2^\dagger\phi_2\right)\left(\phi_1^\dagger\phi_2\right) + \\
&+ d\ e^{i\delta}\ \left(\phi_1^\dagger\phi_2\right)^2 + d\ e^{-i\delta}\ \left(\phi_2^\dagger\phi_1\right)^2 .
\end{aligned}
\tag{11}
$$

We have written the most general Higgs potential for two Higgs doublets, displaying the phase dependence explicitly. It is clear that this potential contains an excess of parameters. This is entirely analogous to the situation encountered in the fermion sector where, for example, the identification of the correct number of independent parameters contained in Yukawa couplings requires that one go to the weak basis, where one of the Yukawa coupling matrices is diagonal and real, while the other one is Hermitian, with only one rephasing-invariant phase. Similarly in the Higgs sector a basis can be choosen, without loss of generality, where the quadratic terms are diagonal and furthermore, of the three remaining phases θ_1, θ_2, δ, one can still eliminate one by rephasing one of the Higgs fields. This means that in a two Higgs doublets system there are only two independent CP-violating phases.

We want to find the necessary and sufficient conditions for CP invariance in terms of CP-odd HBT invariants. From the above counting, we expect to find two such conditions. As previously mentioned, in Ref. [7] CP-odd invariants were constructed involving $v_a \equiv \langle 0\,|\phi_a|\,0\rangle$. The simplest invariants of this type are:

$$\begin{aligned}
\tilde{I}_1 &\equiv Y_a^b \; Z_{bc}^{cd} \; W_d^a \\
\tilde{I}_2 &\equiv W_a^b \; Y_b^c \; W_d^e \; Y_e^f \; Z_{cf}^{ad} \; ,
\end{aligned} \tag{12}$$

where $W_a^b \equiv W_{ab} \equiv v_a v_b^*$. CP invariance requires $\mathrm{Im}(\tilde{I}_i)$ to vanish. One could be tempted to consider HBT invariants analogous to $\tilde{I}_1$ and $\tilde{I}_2$, simply replacing W_i^j by Y_i^j. However, it can be readily verified that these new HBT invariants are automatically real. In fact, with this replacement both $\tilde{I}_i$ become traces of products of two Hermitian matrices and such traces are always real, independently of the number of Higgs doublets.

A set of necessary and sufficient conditions for CP invariance: Next we show that in the case of two Higgs doublets, necessary and sufficient conditions for $\mathcal{L}_\phi \equiv V_{H_2}$ to be CP-invariant are:

$$\begin{aligned}
I_1 &\equiv \mathrm{Tr}[Y \; Z_Y \; \widehat{Z} - \widehat{Z} \; Z_Y \; Y] = 0 \\
I_2 &\equiv \mathrm{Tr}[Y \; Z_2 \; \widetilde{Z} - \widetilde{Z} \; Z_2 \; Y] = 0 \; ,
\end{aligned} \tag{13}$$

where we have defined the following $n_d \times n_d$ Hermitian matrices[5].:

$$\begin{aligned}
\widehat{Z}_a^b &\equiv Z_{am}^{bm} & \qquad \widetilde{Z}_a^b &\equiv Z_{am}^{mb} \\
(Z_Y)_a^b &\equiv Z_{an}^{bm} Y_m^n & \qquad (Z_2)_a^b &\equiv Z_{an}^{pm} Z_{mp}^{nb}
\end{aligned} \tag{14}$$

The conditions of Eqs. (13) are a chosen set of equations, which can be derived from Eq. (8), by using the property of invariance of the trace under similarity transformations. Therefore, they are necessary conditions for CP invariance for an arbitrary number of Higgs doublets. In order to prove that they are also sufficient in the case of two Higgs doublets, let us consider the explicit form of the potential $\mathcal{L}_\phi = V_{H_2}$, written above for the case of two Higgs doublets, where all phases allowed by hermiticity are written explicitly. Since the conditions of Eq. (13) are written in terms of HBT invariants, they can be computed in any appropriately chosen HB. In the HB basis, where the quadratic terms are diagonal, these conditions can be written as:

$$\begin{aligned}
I_1 &\equiv \tfrac{i}{2} c_1 c_2 (m_1 - m_2)^2 \sin(\theta_2 - \theta_1) = 0 \\
I_2 &\equiv \tfrac{i}{2} \; (m_1 - m_2) \; [\; d c_1^2 \sin(\delta + 2\theta_1) + d c_2^2 \sin(\delta + 2\theta_2) + \\
&+ 2 d c_1 c_2 \sin(\delta + \theta_1 + \theta_2) + c_1 c_2 (a_2 - a_1) \sin(\theta_1 - \theta_2) \;] = 0 \; .
\end{aligned} \tag{15}$$

[5]For this particular case, $n_d = 2$

Note that, for simplicity, we have omitted the primes in the coefficients, which obviously assume new values after the HBT that diagonalizes the quadratic terms. As mentioned above, by making a phase redefinition of the ϕ_i, one of the three phases θ_1, θ_2, δ can be eliminated. We choose $\delta = 0$. From the first of Eqs. (15) we obtain:

$$\theta_2 = \theta_1 \qquad \text{or} \qquad \theta_2 = \theta_1 + \pi \ , \tag{16}$$

where we have assumed non-degeneracy of m_i. Taking for definiteness $\theta_2 = \theta_1$, the condition $I_2 = 0$, implies:

$$I_2 \equiv \frac{i}{2}\ (m_1 - m_2)\ \ d(c_1 + c_2)^2 \sin(2\theta_1) = 0, \tag{17}$$

leading to two solutions:

$$\theta_1 = \theta_2 = 0 \qquad \text{or} \qquad \theta_1 = \theta_2 = \pi/2 \ . \tag{18}$$

The case $\theta_1 = \theta_2 = 0$ obviously corresponds to $\mathcal{L}_\phi$ CP-invariant, with ϕ_1, ϕ_2 transforming trivially under CP, i.e. the matrix U in Eq. (7) is a 2×2 identity matrix. It can be easily checked that the solution $\theta_1 = \theta_2 = \pi/2$ also corresponds to a CP-invariant $\mathcal{L}_\phi$ with the matrix U given by:

$$U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{19}$$

Therefore this solution corresponds to two Higgs doublets with opposite CP parities. In our proof, we have assumed that there is no degeneracy of m_i. It is clear from Eqs. (15) that in the degenerate limit $m_1 = m_2$, both I_1, I_2 vanish and yet there is the possibility of CP violation. One can easily construct a HBT invariant that does not trivially vanish in the degenerate limit; an example is:

$$I_3 = \mathrm{Tr}[Z_2\ Z_3\ \widehat{Z} - \widehat{Z}\ Z_3\ Z_2] \ , \tag{20}$$

where Z_3 is also an $n_d \times n_d$ Hermitian matrix given by:

$$(Z_3)_a^b = Z_{ar}^{mp} Z_{mn}^{rs} Z_{ps}^{nb} \ . \tag{21}$$

The explicit form of I_3 in terms of parameters of the Lagrangian is given by:

$$\begin{aligned} I_3 = {} & A\sin(\theta_1 - \theta_2) + B\sin(2(\theta_1 - \theta_2)) + C\sin(\delta + \theta_1 + \theta_2) + \\ & + \sum_{ij=1,2} C_{ij}\sin(2\delta + \theta_i + 3\theta_j) + D_{ij}\sin(\delta + 3\theta_i - \theta_j) \ , \end{aligned} \tag{22}$$

where the A, B, C's and D's are polynomials of degree 6 in a's, b's, c's and d, the parameters of the quartic terms of the two Higgs potential in Eq. (11).

3.1.2 Soft CP breaking with two Higgs doublets

Let us now consider the soft CP-breaking case, in which CP is explicitly broken and there is a Higgs basis where all quartic couplings of the potential are real and the coefficient $Y_{12} \equiv Y_1^2 = p\ e^{i\varphi}$ complex. This case may be interesting in the context of models with explicit CP violation and suppressed Higgs flavour-neutral currents [9]. Next, we show that the CP-odd HBT invariants that we have considered are also useful to find out whether, in a given model, there is hard or soft CP breaking. It is clear that if a CP-odd invariant, built exclusively from quartic couplings, does not vanish, then one has hard CP breaking. On the other hand, one may have complex Z couplings in a given HB and yet CP be only softly broken. In this case HBT invariants such as I_3 would vanish and only HBT invariants including combinations of Y's and Z's could signal CP violation. We illustrate such a situation through a simple example. Let us consider the following Higgs potential:

$$
\begin{aligned}
V'_{H_2} \;=\;& m_1\ \phi_1^\dagger \phi_1 + m_2\ \phi_2^\dagger \phi_2 + \\
+\;& a \left[\left(\phi_1^\dagger \phi_1\right)^2 + \left(\phi_2^\dagger \phi_2\right)^2 \right] + b\ \left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) + \\
+\;& c\ \left[e^{i\theta}\ \left(\phi_1^\dagger \phi_1\right)\left(\phi_1^\dagger \phi_2\right) + e^{i\theta}\left(\phi_2^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right) + \text{h.c.} \right] + \\
+\;& d\ \left[e^{i\delta}\ \left(\phi_1^\dagger \phi_2\right)^2 + \text{h.c.} \right] .
\end{aligned}
\tag{23}
$$

An explicit evaluation of I_i (i = 1,2,3) gives:

$$
\begin{aligned}
&I_1 \equiv \text{Tr}[Y\ Z_Y\ \widehat{Z} - \widehat{Z}\ Z_Y\ Y] = 0 \\
&I_2 \equiv \text{Tr}[Y\ Z_2\ \widetilde{Z} - \widetilde{Z}\ Z_2\ Y] = 2c^2 d\ (m_1 - m_2)\ \sin(\delta + 2\theta) \\
&I_3 \equiv \text{Tr}[Z_2\ Z_3\ \widehat{Z} - \widehat{Z}\ Z_3\ Z_2] = 0\ .
\end{aligned}
\tag{24}
$$

The fact that $I_3 = 0$, while $I_2 \neq 0$, provides a hint that CP is only softly broken. It can be verified that this is indeed the case, because the quartic couplings, by themselves, conserve CP, provided one chooses in Eq. (7) the following U matrix:

$$
U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\ .
\tag{25}
$$

One may say that $\mathcal{L}_\phi = V'_{H_2}$ of Eq. (23) corresponds to "hidden" soft CP breaking.

3.2 Three Higgs doublets

3.2.1 The general case

Three Higgs doublets were considered in an attempt to introduce CP violation in an extension of the SM with natural flavour conservation (NFC) [10] in the Higgs sector. If one introduces NFC in a two Higgs doublet system, through an exact symmetry of the Lagrangian under which each Higgs doublet transforms differently, there is no longer the possibility of CP violation in the Higgs sector. Is is clear from Eq. (11) that such a symmetry, in the case of two Higgs doublets, would forbid all terms in c_i as well as the quadratic non-diagonal term, so that the phase of the term in d could then be rephased away. On the other hand, it was shown that with three Higgs doublets it is possible to violate CP in the Higgs sector either at the Lagrangian level [11] or spontaneously [2], while having NFC. The general three Higgs doublets system, without extra symmetries, is rather complicated, due to the large number of parameters involved. It can be readily verified that for n_d Higgs doublets the total number of CP-violating phases contained in the Higgs potential is:

$$N_{\text{phases}} = \frac{1}{4}\left[n_d^2(n_d^2-1)\right] - (n_d - 1). \tag{26}$$

The second term in Eq. (26) results from the fact that one can eliminate $(n_d - 1)$ phases by rephasing the Higgs fields. From Eq. (26) it follows that the number of independent CP-violating phases grows fast with increasing n_d. One has 0, 2, 16 independent phases for $n_d = 1, 2, 3$ respectively. In view of this, we only point out some of the novel generic features which arise in the case of three Higgs doublets and then consider specific models. It is clear that the CP-odd invariants I_1, I_2 defined in Eqs. (13) provide necessary conditions for CP invariance for an arbitrary number of generations. However, there are simpler CP-odd invariants, which are irrelevant for the case of two Higgs doublets (since they trivially vanish in that case) but are useful in the case of three Higgs doublets. A simple CP-odd invariant of this class is:

$$I_s = \operatorname{Tr}\left([Y\ ,\ \widetilde{Z}]^3\right)\ . \tag{27}$$

An equivalent invariant can be obtained by replacing $\tilde{Z}$ by $\hat{Z}$ as defined in Eq. (14). The vanishing of I_s is a non-trivial necessary condition for CP invariance in the case of three or more Higgs doublets. It is analogous to the condition obtained in the quark sector, given in Eq. (6). In the HB where Y is diagonal, one obtains:

$$I_s = 6i\ (Y_{22} - Y_{11})\ (Y_{33} - Y_{11})\ (Y_{33} - Y_{22})\ \operatorname{Im}(\widetilde{Z}_{12}\widetilde{Z}_{23}\widetilde{Z}_{31})\ . \tag{28}$$

3.2.2 Weinberg three Higgs doublets model

In the three Higgs doublets model proposed by Weinberg [11], a $Z_2 \times Z_2 \times Z_2$ symmetry, under separate reflections of the Higgs doublets of the form $\phi_i \to -\phi_i$, together with an appropriately chosen transformation for the quark fields, ensures NFC and leads to a strong reduction in the number of parameters. The Higgs potential is given by:

$$
\begin{aligned}
V = \sum_{i=123} m_i \ \phi_i^\dagger \phi_i + a_{ii} \ \left(\phi_i^\dagger \phi_i\right)^2 + \sum_{i<j} 2b_{ij} \ \left(\phi_i^\dagger \phi_i\right) \ \left(\phi_j^\dagger \phi_j\right) + \\
+2c_{ij} \ \left(\phi_i^\dagger \phi_j\right) \ \left(\phi_j^\dagger \phi_i\right) + \left[d_{ij} \ e^{i\theta_{ij}} \ \left(\phi_i^\dagger \phi_j\right)^2 + h.c.\right] .
\end{aligned}
\tag{29}
$$

There are three different $d_{ij} \ e^{i\theta_{ij}}$ terms, and only these can be complex. In this case, the matrices Y, $\tilde{Z}$ and $\hat{Z}$ are diagonal, hence the invariant written for the general case in Eq. (27) is automatically zero. Yet, as it was pointed out by Weinberg, there can be CP violation, since in general one cannot rotate away simultaneously the three phases θ_{ij} through phase redefinitions of the three Higgs doublets. This implies that, in this case, a CP-odd invariant will have to include terms of the form $Z_{1212}Z_{2323}Z_{3131}$ so as to be sensitive to the CP-violating phase that cannot be rotated away. As a result, the simplest HBT invariant relevant to this model must contain $Z^{nq}_{ak}Z^{rs}_{nq}Z^{ka}_{rs}$. However, such a simple invariant is real. The simplest way out is to insert the two diagonal matrices Y and $\hat{Z}$ between Z's. Thus, a relevant CP-odd invariant is given by:

$$
I_2^W = \mathrm{Im}[Z^{nq}_{ak} \ Y^m_n \ Z^{rs}_{mq} \ \widehat{Z}^t_r \ Z^{ka}_{ts}] \ . \tag{30}
$$

Its explicit form is:

$$
\begin{aligned}
I_2^W = d_{12}d_{13}d_{23} \ [(\ m_3 - m_2)(a_{11} - b_{23}) - (\ m_3 - m_1)(a_{22} - b_{13}) + \\
+(\ m_2 - m_1)(a_{33} - b_{12})] \sin(\theta_{12} - \theta_{13} + \theta_{23}) \ .
\end{aligned}
\tag{31}
$$

Assuming non-degenerate values for the m_i, a non-vanishing I_2^W indicates a non-vanishing $(\theta_{12} - \theta_{13} + \theta_{23})$.

3.2.3 Three Higgs doublets with NFC and soft CP breaking

Let us assume that the $Z_2 \times Z_2 \times Z_2$ symmetry, which implements NFC in the Higgs potential, is softly broken by quadratic terms in the potential. In this case, one can also have soft CP breaking. In other words, CP can be violated even if all quartic terms are real. Obviously, in this case the matrix

Y is no longer diagonal, in this basis, and the invariant I_s defined in Eq. (27) does not automatically vanish, being given by:

$$I_s = 6(a_{22} - a_{33} + c_{12} - c_{13})(a_{33} - a_{11} + c_{23} - c_{12})(a_{22} - a_{11} + c_{23} - c_{13}) \cdot \\ \cdot\, y_{12} y_{13} y_{23} \ \sin[\varphi_{12} - \varphi_{13} + \varphi_{23}] \ , \tag{32}$$

where we have written the off-diagonal terms of the Hermitian matrix Y as $Y_{ij} = y_{ij}\ e^{i\varphi_{ij}}$. On the other hand, in this model with soft CP breaking, the CP-odd invariant I_2^W vanishes. The reason for the vanishing of I_2^W in this case is the following: this invariant is also the imaginary part of the trace of a multiplication of the Y matrix and a matrix made of combinations of Z's. In the Weinberg model this combination of Z's is diagonal and, in addition, it is real in the soft CP-breaking case. The matrix Y is Hermitian, hence the trace of the product of these two matrices is real. This illustrates the usefulness of CP-odd HBT invariants in determining the character of CP breaking (hard or soft) when the Higgs potential is written in an arbitrary basis, where that character may not be transparent.

4 Summary and conclusions

We have shown how CP-odd HBT invariants can play an important rôle in the study of CP properties of Higgs systems. In order to derive these invariants we first consider the most general CP transformation, which leaves the Higgs kinetic-energy term invariant and then derive the restrictions implied by CP invariance on the quadratic and quartic scalar couplings. The method used in this paper to analyse CP invariance of the Higgs potential does not require finding the actual CP transformation under which the scalar potential is invariant. This renders this method particularly useful, since such a transformation may not be trivial and may become quite complicated as the number of Higgs doublets increases. The HBT invariants offer the advantage that they can be directly evaluated in any Higgs basis. We have also illustrated how the HBT invariants can be used to find out whether there is soft or hard CP breaking in a given Higgs potential. We have studied in detail only the case of two and three Higgs doublets, but our approach can be readily extended to an arbitrary number of these. One last comment is in order: throughout the paper, we have only discussed the Higgs sector. It is clear that one can readily extend the analysis to the remainder of the Lagrangian, in particular the fermion sector. More specifically, one can construct quantities involving both Yukawa and Higgs couplings which are invariant under basis transformations comprising both HBT and fermion basis transformations.

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References

[1] T. D. Lee, Phys. Rev. D **8** (1973) 1226.

[2] G. C. Branco, Phys. Rev. Lett. **44** (1980) 504; G. C. Branco, Phys. Rev. D **22** (1980) 2901; G. C. Branco, A. J. Buras and J. M. Gerard, Nucl. Phys. B **259** (1985) 306.

[3] M. Boz and N. K. Pak, Phys. Rev. D **65** (2002) 075014; E. O. Iltan, Phys. Rev. D **65** (2002) 073013 [arXiv:hep-ph/0111038]; M. Carena, J. R. Ellis, S. Mrenna, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **659** (2003) 145 [arXiv:hep-ph/0211467]; W. Khater and P. Osland, Nucl. Phys. B **661** (2003) 209 [arXiv:hep-ph/0302004]; J. L. Diaz-Cruz, R. Noriega-Papaqui and A. Rosado, arXiv:hep-ph/0410391; G. Weiglein *et al.* [LHC/LC Study Group], arXiv:hep-ph/0410364, and references therein; C. P. W. Group *et al.*, arXiv:hep-ph/0412251, and references therein; A. Sopczak, arXiv:hep-ph/0502002.

[4] J. Bernabeu, G. C. Branco and M. Gronau, Phys. Lett. B **169** (1986) 243.

[5] G. C. Branco and M. N. Rebelo, Phys. Lett. B **173** (1986) 313; M. Gronau, A. Kfir and R. Loewy, Phys. Rev. Lett. **56** (1986) 1538; G. C. Branco and V. A. Kostelecky, Phys. Rev. D **39** (1989) 2075; G. C. Branco, M. N. Rebelo and J. W. F. Valle, Phys. Lett. B **225** (1989) 385; F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B **386** (1996) 241 [arXiv:hep-ph/9605418]; F. del Aguila, J. A. Aguilar-Saavedra and M. Zralek, Comput. Phys. Commun. **100** (1997) 231 [arXiv:hep-ph/9607311]; J. A. Aguilar-Saavedra, J. Phys. G **24** (1998) L31 [arXiv:hep-ph/9703461]; A. Pilaftsis, Phys. Rev. D **56** (1997) 5431

[arXiv:hep-ph/9707235]; O. Lebedev, Phys. Rev. D **67** (2003) 015013 [arXiv:hep-ph/0209023]; S. Davidson and R. Kitano, JHEP **0403** (2004) 020 [arXiv:hep-ph/0312007]; F. J. Botella, M. Nebot and O. Vives, arXiv:hep-ph/0407349.

[6] G. C. Branco, L. Lavoura and J. P. Silva, "CP violation," International Series of Monographs on Physics, No. 103, Oxford University Press. Oxford, UK: Clarendon (1999), 511 pp.

[7] L. Lavoura and J. P. Silva, Phys. Rev. D **50** (1994) 4619 [arXiv:hep-ph/9404276]; F. J. Botella and J. P. Silva, Phys. Rev. D **51** (1995) 3870 [arXiv:hep-ph/9411288].

[8] I. F. Ginzburg and M. Krawczyk, arXiv:hep-ph/0408011. J. Gunion and H. Haber in: Talk by J. Gunion at the Workshop "CP studies and non-standard Higgs physics", CERN, 2-3 December 2004, http://kraml.home.cern.ch/kraml/cpnsh/

[9] G. C. Branco and M. N. Rebelo, Phys. Lett. B **160** (1985) 117.

[10] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958.

[11] S. Weinberg, Phys. Rev. Lett. **37** (1976) 657.