

CERN LHC signatures of resonant  $CP$  violation in a minimal supersymmetric Higgs sectorJohn Ellis,<sup>1</sup> Jae Sik Lee,<sup>2</sup> and Apostolos Pilaftsis<sup>2</sup><sup>1</sup>Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland<sup>2</sup>Department of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom  
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We present the general formalism for studying  $CP$ -violating phenomena in the production, mixing and decay of a coupled system of  $CP$ -violating neutral Higgs bosons at high-energy colliders. Considering the minimal supersymmetric standard model (MSSM) Higgs sector in which  $CP$  violation is radiatively induced by phases in the soft supersymmetry-breaking third-generation trilinear squark couplings and gaugino masses, we apply our formalism to neutral Higgs production via  $\bar{b}b$ ,  $gg$  and  $W^+W^-$  collisions at the CERN LHC. We discuss  $CP$  asymmetries in the longitudinal and transverse polarizations of  $\tau^+\tau^-$  pairs. The signatures of  $CP$  violation are more prominent in the production via  $gg$  and  $W^+W^-$  than via  $\bar{b}b$ , and are resonantly enhanced when two (or all three) neutral Higgs bosons are nearly degenerate with mass differences comparable to their decay widths. Such scenarios occur naturally in the MSSM for values of  $\tan\beta \gtrsim 5(30)$  and large (small) charged Higgs-boson masses. We analyze representative examples with large mixing between the three neutral Higgs bosons weighing about 120 GeV, that may exhibit observable  $CP$  asymmetries even as large as 80%.

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## I. INTRODUCTION

If supersymmetry (SUSY) turns out to be realized at low energies  $\lesssim 1$  TeV [1], the following interesting questions will then arise: does SUSY make observable contributions to the violation of either flavour or  $CP$ ? Even in the minimal supersymmetric extension of the standard model (MSSM), the soft SUSY-breaking sector may include about a hundred parameters that violate these symmetries. However, if one imposes flavour conservation on the soft SUSY-breaking parameters  $m_0$ ,  $m_{1/2}$  and  $A$ , and assumes that they are universal, then only two physical  $CP$ -violating phases remain: one in the gaugino masses  $m_{1/2}$  and one in the trilinear couplings  $A$ .

These  $CP$ -violating phases may in principle be measured directly in the production cross sections and decay widths of sparticles at high-energy colliders [2,3], or indirectly via their radiative effect on the Higgs sector [4]<sup>1</sup>. The Higgs sector of the MSSM is affected at the one-loop level by the trilinear phase [4,10–17] and at the two-loop level by the gaugino mass phase [12–15]. This loop-induced  $CP$  violation mixes the  $CP$ -even Higgses  $h, H$  with the  $CP$ -odd Higgs boson  $A$ . Many studies have been made of the masses and couplings of the resulting mixed- $CP$  Higgs bosons  $H_{1,2,3}$ , and some of their phenomenological consequences for searches at LEP and future colliders have also been considered [18–26].

More complete studies of  $CP$ -violating Higgs bosons will require a careful treatment of the resonant mixing of multiple Higgs bosons that couple to the same initial and

final states. In general, one could expect that the  $CP$ -violating mixing of the heavier MSSM Higgs bosons  $H, A$  may be more important than their mixings with the lightest  $CP$ -even Higgs boson  $h$ . However, non-negligible mixing among all three neutral Higgs states is also possible in a general  $CP$ -violating MSSM. Such a scenario naturally emerges from a parameter space where  $\tan\beta$  is large, i.e.  $\tan\beta \gtrsim 30$ , and the charged Higgs bosons  $H^\pm$  are relatively light with  $M_{H^\pm} \lesssim 160$  GeV.

In this paper, we develop the general formalism for describing the dynamics that governs the production, mixing and decay of a coupled system of  $CP$ -violating neutral Higgs bosons. Our formalism makes use of the field-theoretic resummation approach developed in [27] to treat unstable particle-mixing transitions. Within the context of gauge theories, it is important that resummation approaches to unstable particles consistently maintain crucial field-theoretic properties, such as gauge invariance, analyticity and unitarity [28]. It has been shown in [28,29] that all these properties are preserved within the framework of the pinch technique (PT) [30]. Here, using the PT, we compute the gauge-mediated diagonal as well as off-diagonal absorptive parts in the resummed Higgs-boson propagator matrix. Finally, an essential ingredient of our formalism is the inclusion of the  $CP$ -violating loop corrections in the production and decay vertices of the Higgs bosons.

We illustrate our general formalism for the coupled-channel  $H_{1,2,3}$  mixing by explicit treatments of the production processes  $gg, b\bar{b}^2$  and  $W^+W^- \rightarrow H_{1,2,3} \rightarrow \tau^+\tau^-$ . These are the most important production mechanisms for

<sup>1</sup>Additional indirect constraints on the soft SUSY-breaking phases and the MSSM mass spectrum may be obtained from experimental limits on electric dipole moments (EDMs) [5–7] and  $B$ -meson observables [8,9].

<sup>2</sup>We note that the  $b\bar{b}$  fusion process may become the leading production channel at large  $\tan\beta$  at the LHC, as has recently been shown in [31].

neutral Higgs bosons at the LHC, while the decay channel that seems the most promising for studies of  $CP$  violation is that into  $\tau^+\tau^-$  pairs. To quantify the genuine signatures of  $CP$  violation, we calculate  $CP$  asymmetries that are defined in terms of longitudinal and transverse polarizations of the  $\tau^\pm$  leptons. When  $\tan\beta$  is large and/or the charged Higgs-boson mass is large, so that two or more Higgs bosons are nearly degenerate, even small  $CP$ -violating phases could induce sizeable  $CP$ -violating mixing. However, as we demonstrate, there are systematic cancellations due to  $CPT$ -preserving rescattering effects in the process  $b\bar{b} \rightarrow H_{1,2,3} \rightarrow \tau^+\tau^-$  that suppress the  $CP$ -violating signatures in this case. There are no such cancellations in  $gg$  and  $W^+W^- \rightarrow H_{1,2,3} \rightarrow \tau^+\tau^-$ , which could have much larger  $CP$  asymmetries at the LHC. We analyze representative examples with large threeway mixing to show that these  $CP$  asymmetries might well exceed the 10% level and could even reach values up to 80%.

Although our predictions are obtained in the MSSM with explicit  $CP$  violation, it is important to stress that large  $CP$ -violating effects could also occur in a general  $CP$ -violating 2-Higgs-doublet model with a similar Higgs-boson mass spectrum. Our presentation is organized in such a way that the formalism may easily be extended to Higgs production at other colliders. For instance, our formalism can be applied to  $\gamma\gamma$  colliders [25,32], which are analogous to  $gg$  collisions at the LHC, to  $\mu^+\mu^-$  colliders [26], which have formal similarities with  $b\bar{b}$  collisions at the LHC, and to  $WW$ -fusion and Higgsstrahlung processes at  $e^+e^-$  linear colliders [33].

Section II presents the general formalism for the coupled-channel analysis of Higgs bosons, including explicit formulae for the absorptive parts of the Higgs-boson propagator matrix and the vertex corrections. In Sec. III, we apply the results of our formalism to the production channels  $gg, b\bar{b}, W^+W^- \rightarrow \tau^+\tau^-$  at the LHC. In Sec. 7 we present numerical estimates of particular  $CP$ -violating MSSM scenarios that exhibit large  $CP$  asymmetries. Our numerical estimates are based on the Fortran code CPsuperH [34]. Finally, Sec. V contains our conclusions and discusses the prospects for pursuing studies of Higgs-sector  $CP$  violation at future colliders beyond the LHC.

## II. FORMALISM FOR COUPLED-CHANNEL ANALYSES OF HIGGS-SECTOR $CP$ VIOLATION

We consider situations where two or more MSSM Higgs bosons contribute simultaneously to the production of some fermion-antifermion pair whose polarization states can be measured. We treat explicitly the example of  $H_{1,2,3} \rightarrow \tau^+\tau^-$ , but the formalism could easily be adapted to other cases such as  $t\bar{t}, \chi_i^+\chi_j^-$  or  $\chi_i^0\chi_j^0$ . There have been extensive discussions of the masses and couplings of MSSM Higgs bosons mixed by loop-induced  $CP$

violation [10–13]. To account properly for the constraints that  $CPT$  invariance and unitarity imposes on the cross sections [27], we must consider the full off-shell propagator matrix for mixed MSSM Higgs bosons, including off-diagonal absorptive parts.

The absorptive part of the Higgs-boson propagator matrix receives contributions from loops of fermions, vector bosons, associated pairs of Higgs and vector bosons, Higgs-boson pairs, and sfermions:

$$\begin{aligned} \Im\hat{\Pi}_{ij}(s) &= \Im\hat{\Pi}_{ij}^{ff}(s) + \Im\hat{\Pi}_{ij}^{VV}(s) + \Im\hat{\Pi}_{ij}^{HV}(s) \\ &+ \Im\hat{\Pi}_{ij}^{HH}(s) + \Im\hat{\Pi}_{ij}^{\tilde{f}\tilde{f}}(s). \end{aligned} \quad (2.1)$$

The contributions of the exchanges of the bottom and top quarks,  $\tau$  leptons, neutralinos  $\chi_i^0$  and charginos  $\chi_i^\pm$  are summed in  $\Im\hat{\Pi}_{ij}^{ff}(s)$ . The latter may conveniently be cast into the form

$$\begin{aligned} \Im\hat{\Pi}_{ij}^{ff}(s) &= \frac{s}{8\pi} \sum_{f,f'=b,t,\tau,\tilde{\chi}^0,\tilde{\chi}^-} K_f(s)g_f^2\Delta_{ff'}N_{CL}^f[(1-\kappa_f \\ &- \kappa_{f'})(g_{H_i\tilde{f}f}^S g_{H_j\tilde{f}'f}^{S*} + g_{H_i\tilde{f}'f}^P g_{H_j\tilde{f}f}^{P*}) \\ &- 2\sqrt{\kappa_f\kappa_{f'}}(g_{H_i\tilde{f}f}^S g_{H_j\tilde{f}'f}^{S*} \\ &- g_{H_i\tilde{f}'f}^P g_{H_j\tilde{f}f}^{P*})]\lambda^{1/2}(1,\kappa_f,\kappa_{f'})\Theta[s-(m_f \\ &+ m_{f'})^2], \end{aligned} \quad (2.2)$$

where  $K_{b,t}(s) \simeq 1 + 5.67 \frac{\alpha_s(s)}{\pi}$ ,  $\Delta_{ff'} = \delta_{ff'}(f, f' = b, t, \tau)$ ,  $\frac{4}{1+\delta_{ff'}}(f, f' = \tilde{\chi}_{1,2,3,4}^0)$ , or  $1(f, f' = \tilde{\chi}_{1,2}^\pm)$ , and  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$  with  $\kappa_x \equiv m_x^2/s$ . Here and subsequently, we follow the convention of CPsuperH [34] for the couplings of the Higgs bosons to fermions, vector bosons, Higgs bosons, and sfermions. For the calculation of the bottom- and top-quark contributions, the running quark masses at the scale  $\sqrt{s}$  have been used in the couplings  $g_{b,t} = gm_{b,t}(\sqrt{s})/2M_W$ . Specifically, we use  $m_b(m_b^{\text{pole}}) = 3$  GeV, where  $m_t^{\text{pole}} = 175$  GeV.

The vector-boson loop contributions are

$$\begin{aligned} \Im\hat{\Pi}_{ij}^{VV}(s) &= \frac{g^2 g_{H_i VV} g_{H_j VV} \delta_V}{128\pi M_W^2} \beta_V [-4M_V^2(2s - 3M_V^2) \\ &+ 2M_V^2(M_{H_i}^2 + M_{H_j}^2) + M_{H_i}^2 M_{H_j}^2] \Theta(s \\ &- 4M_V^2), \end{aligned} \quad (2.3)$$

where  $\beta_V = (1 - 4\kappa_V)^{1/2}$  and  $\delta_W = 2$ ,  $\delta_Z = 1$ .

Correspondingly, the exchanges of Higgs and vector-boson pairs give

$$\begin{aligned}
\Im\widehat{\Pi}_{ij}^{HV}(s) &= \frac{g^2}{64\pi M_W^2} \sum_{k=1,2,3} g_{H_i H_k Z} g_{H_j H_k Z} \lambda^{1/2}(1, \kappa_Z, \kappa_{H_k}) [-4sM_Z^2 + (M_Z^2 - M_{H_k}^2)^2 + (M_Z^2 - M_{H_k}^2)(M_{H_i}^2 + M_{H_j}^2) \\
&\quad + M_{H_i}^2 M_{H_j}^2] \Theta[s - (M_Z + M_{H_k})^2] + \frac{g^2}{32\pi M_W^2} \Re(g_{H_i H^+ W^-} g_{H_j H^+ W^-}^*) \lambda^{1/2}(1, \kappa_W, \kappa_{H^\pm}) [-4sM_W^2 \\
&\quad + (M_W^2 - M_{H^\pm}^2)^2 + (M_W^2 - M_{H^\pm}^2)(M_{H_i}^2 + M_{H_j}^2) + M_{H_i}^2 M_{H_j}^2] \Theta[s - (M_W + M_{H^\pm})^2]. \tag{2.4}
\end{aligned}$$

In deriving (2.3) and (2.4), we apply the PT to the MSSM Higgs sector following a procedure very analogous to the one given in [35] for the SM Higgs sector. As a consequence, the PT self-energies  $\Im\widehat{\Pi}_{ij}^{VV}(s)$  and  $\Im\widehat{\Pi}_{ij}^{VH}(s)$  depend linearly on  $s$  at high energies. This differs crucially from the bad high-energy dependence  $\propto s^2$  that one usually encounters when the Higgs-boson self-energies are calculated in the unitary gauge. In fact, if the Higgs-boson self-energies are embedded in a truly gauge-independent quantity such as the  $S$ -matrix element of a  $2 \rightarrow 2$  process, the badly high-energy-behaved  $s^2$ -dependent terms cancel against corresponding  $s^2$  terms present in the vertices and boxes order by order in perturbation theory. In this context, PT provides a self-consistent approach to extract those  $s^2$ -dependent terms from boxes and vertices, thus giving rise to effective Higgs self-energies that are independent of the gauge-fixing parameter and  $s^2$ . More details on the PT may be found in [28–30,35].

Finally, the contributions of the MSSM Higgs bosons and sfermions are

$$\Im\widehat{\Pi}_{ij}^{HH}(s) = \frac{v^2}{16\pi} \sum_{k \geq l=1,2,3} \frac{S_{ij;kl}}{1 + \delta_{kl}} g_{H_i H_k H_l} g_{H_j H_k H_l} \lambda^{1/2}(1, \kappa_{H_k}, \kappa_{H_l}) \Theta[s - (M_{H_k} + M_{H_l})^2], \tag{2.5}$$

$$\Im\widehat{\Pi}_{ij}^{\tilde{f}\tilde{f}}(s) = \frac{v^2}{16\pi} \sum_{f=b,t,\tau} \sum_{k,l=1,2} N_C^f g_{H_i \tilde{f}_k \tilde{f}_l} g_{H_j \tilde{f}_k \tilde{f}_l}^* \lambda^{1/2}(1, \kappa_{\tilde{f}_k}, \kappa_{\tilde{f}_l}) \Theta[s - (M_{\tilde{f}_k} + M_{\tilde{f}_l})^2]. \tag{2.6}$$

Note that the symmetry factor  $S_{ij;kl}$  has to be calculated appropriately. When  $i = j = 1$  and  $k = l = 2$ , for example, the symmetry factor for the squared self-coupling  $g_{H_1 H_2 H_2}^2$  is  $S_{11;22} = 4$ .

When considering any specific production process and decay channel, the Higgs-boson propagator matrix must be combined with the appropriate vertices, that themselves receive  $CP$ -violating loop corrections. Since the main decay channel we consider for the LHC is  $\tau^+ \tau^-$ , and since many of the interesting Higgs production and other decay mechanisms also involve fermions such as  $b\bar{b}$ , we also summarize relevant aspects of the loop-induced corrections to the  $H_{1,2,3} f\bar{f}$  vertices.

The exchanges of gluinos and charginos give finite loop-induced corrections to the  $H_{1,2,3} b\bar{b}$  Yukawa coupling

with the structure

$$h_b = \frac{\sqrt{2}m_b}{v \cos\beta} \frac{1}{1 + (\delta h_b/h_b) + (\Delta h_b/h_b) \tan\beta}. \tag{2.7}$$

The  $\tan\beta$ -enhanced threshold correction  $(\Delta h_b/h_b)$  has terms proportional to the strong coupling  $\alpha_s$  and the top-quark Yukawa coupling  $|h_t|^2$ . See Eqs. (2.4) and (2.5) in [20] for the analytic forms of  $(\delta h_b/h_b)$  and  $(\Delta h_b/h_b)$ , respectively. In addition, there are contributions to  $(\Delta h_b/h_b)$  coming from the exchanges of binos and winos which are proportional to the electromagnetic fine-structure constant  $\alpha_{\text{em}}$  [36]. Taking  $CP$  violation into account, these additional contributions read

$$\begin{aligned}
(\Delta h_b/h_b)_{\text{em}} &= -\frac{\alpha_{\text{em}} \mu^* M_2^*}{4\pi s_W^2} \left[ |U_{L1}^{\tilde{t}}|^2 I(m_{\tilde{t}_1}^2, |M_2|^2, |\mu|^2) + |U_{L2}^{\tilde{t}}|^2 I(m_{\tilde{t}_2}^2, |M_2|^2, |\mu|^2) + \frac{1}{2} |U_{L1}^{\tilde{b}}|^2 I(m_{\tilde{b}_1}^2, |M_2|^2, |\mu|^2) \right. \\
&\quad \left. + \frac{1}{2} |U_{L2}^{\tilde{b}}|^2 I(m_{\tilde{b}_2}^2, |M_2|^2, |\mu|^2) \right] - \frac{\alpha_{\text{em}} \mu^* M_1^*}{12\pi c_W^2} \left[ \frac{1}{3} I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, |M_1|^2) + \frac{1}{2} |U_{L1}^{\tilde{t}}|^2 I(m_{\tilde{b}_1}^2, |M_1|^2, |\mu|^2) \right. \\
&\quad \left. + \frac{1}{2} |U_{L2}^{\tilde{t}}|^2 I(m_{\tilde{b}_2}^2, |M_1|^2, |\mu|^2) + |U_{R1}^{\tilde{b}}|^2 I(m_{\tilde{b}_1}^2, |M_1|^2, |\mu|^2) + |U_{R2}^{\tilde{b}}|^2 I(m_{\tilde{b}_2}^2, |M_1|^2, |\mu|^2) \right], \tag{2.8}
\end{aligned}$$

where

$$I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a-b)(b-c)(a-c)}. \tag{2.9}$$

We follow the convention of CPsuperH [34] for the mixing matrices of the stops  $U^{\tilde{t}}$ , sbottoms  $U^{\tilde{b}}$  and staus  $U^{\tilde{\tau}}$ .

There are formulae analogous to those above for the loop corrections to the  $H_{1,2,3} \tilde{t}\tilde{t}$  vertices, which would be relevant for  $CP$  violation measurements in  $e^- e^+ \rightarrow \nu \tilde{\nu} \tilde{t}\tilde{t}$  [33], for example.

Analogous exchanges of binos and winos give finite loop-induced corrections to the  $H_{1,2,3}\tau^+\tau^-$  coupling, which have a similar structure:

$$h_\tau = \frac{\sqrt{2}m_\tau}{v \cos\beta} \frac{1}{1 + (\Delta h_\tau/h_\tau) \tan\beta}, \quad (2.10)$$

where

$$\begin{aligned} (\Delta h_\tau/h_\tau) = & -\frac{\alpha_{\text{em}}\mu^* M_2^*}{4\pi s_W^2} \left[ I(m_{\tilde{\nu}_\tau}^2, |M_2|^2, |\mu|^2) + \frac{1}{2}|U_{L1}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_1}^2, |M_2|^2, |\mu|^2) + \frac{1}{2}|U_{L2}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_2}^2, |M_2|^2, |\mu|^2) \right] \\ & + \frac{\alpha_{\text{em}}\mu^* M_1^*}{4\pi c_W^2} \left[ I(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2, |M_1|^2) + \frac{1}{2}|U_{L1}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_1}^2, |M_1|^2, |\mu|^2) + \frac{1}{2}|U_{L2}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_2}^2, |M_1|^2, |\mu|^2) \right. \\ & \left. - |U_{R1}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_1}^2, |M_1|^2, |\mu|^2) - |U_{R2}^{\tilde{\tau}}|^2 I(m_{\tilde{\tau}_2}^2, |M_1|^2, |\mu|^2) \right]. \end{aligned} \quad (2.11)$$

The threshold corrections modify the couplings of the neutral Higgs bosons to the scalar and pseudoscalar fermion bilinears as follows [20]:

$$\begin{aligned} g_{H_i \bar{f} f}^S &= \frac{O_{\phi_1 i}}{\cos\beta} \Re\left(\frac{1}{1 + \kappa_f \tan\beta}\right) + \frac{O_{\phi_2 i}}{\cos\beta} \Re\left(\frac{\kappa_f}{1 + \kappa_f \tan\beta}\right) + O_{ai} \Im\left[\frac{\kappa_f(\tan^2\beta + 1)}{1 + \kappa_f \tan\beta}\right], \\ g_{H_i \bar{f} f}^P &= \frac{O_{\phi_1 i}}{\cos\beta} \Im\left(\frac{\kappa_f \tan\beta}{1 + \kappa_f \tan\beta}\right) - \frac{O_{\phi_2 i}}{\cos\beta} \Im\left(\frac{\kappa_f}{1 + \kappa_f \tan\beta}\right) - O_{ai} \Re\left(\frac{\tan\beta - \kappa_f}{1 + \kappa_f \tan\beta}\right), \end{aligned} \quad (2.12)$$

where  $f = b$  and  $\tau^-$  and

$$\kappa_b = \frac{(\Delta h_b/h_b)}{1 + (\delta h_b/h_b)}, \quad \kappa_\tau = (\Delta h_\tau/h_\tau). \quad (2.13)$$

There are similar formulae for the  $H_{1,2,3}\mu^+\mu^-$  vertices that would be relevant for  $\mu^+\mu^-$  colliders. The analogous corrections to the  $H_{1,2,3}e^+e^-$  vertices may be neglected.

Additional contributions to Higgs-boson vertices may arise from absorptive effects due to the opening of third-generation sfermion pair production channels. However, if the  $H_{1,2,3}$ -boson masses are well below the kinematic threshold of such production channels, these absorptive effects are small and can be neglected. Finally, we remind the reader that detailed analytic expressions for the effective Higgs-boson couplings to the photon, the gluon, the  $W^\pm$  bosons and SUSY particles are given in [34].

### III. TAU PAIR PRODUCTION AT THE LHC

To further elucidate the formalism presented in Sec. II, we now discuss in more detail the production, mixing and decay of Higgs bosons into polarized  $\tau^+\tau^-$  pairs at the LHC. We will study individually the three most significant production channels for Higgs bosons in the MSSM at the LHC: (i)  $b\bar{b}$  fusion, (ii)  $gg$  fusion and (iii)  $W^+W^-$  fusion.

#### A. $b\bar{b}$ Fusion

At large  $\tan\beta$ , an important mechanism for producing neutral Higgs bosons at the LHC is  $b\bar{b}$  fusion [31,37–39].

Figure 1 illustrates how the matrix element  $\mathcal{M}^{b\bar{b}}$  for  $b\bar{b} \rightarrow \tau^+\tau^-$  receives contributions from the  $s$ -channel exchanges of the neutral Higgs bosons. The loop-corrected propagator matrix and vertices calculated in the previous section are indicated by shaded circles. The matrix element can be written as

$$\begin{aligned} \mathcal{M}^{b\bar{b}} = & -\frac{g^2 m_b m_\tau}{4M_W^2 \hat{s}} \sum_{i,j=1,2,3} \sum_{\alpha,\beta=\pm} \{(g_{H_i \bar{b} b}^S \\ & + i\alpha g_{H_i \bar{b} b}^P) \bar{v}(k_2, \bar{\lambda}) P_\alpha u(k_1, \lambda) D_{ij}(\hat{s}) \\ & \times (g_{H_j \tau^+ \tau^-}^S + i\beta g_{H_j \tau^+ \tau^-}^P) \bar{u}(p_1, \sigma) P_\beta v(p_2, \bar{\sigma})\}, \end{aligned} \quad (3.1)$$

where  $P_\alpha = (1 + \alpha\gamma_5)/2$  and the running bottom-quark mass at the scale of  $\sqrt{\hat{s}}$  is used. We denote the helicities of  $\tau^-$  and  $\tau^+$  by  $\sigma$  and  $\bar{\sigma}$  and those of the  $b$  and  $\bar{b}$  by  $\lambda$  and  $\bar{\lambda}$ ,

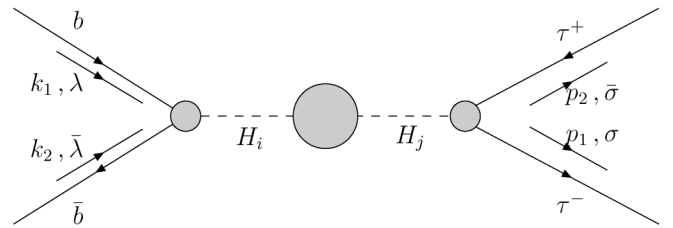


FIG. 1. Mechanisms contributing to the process  $b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-$ , including off-diagonal absorptive parts in the Higgs-boson propagator matrix.

respectively, with  $\sigma, \lambda = +$  and  $-$  standing for right- and left-handed particles. The four-momenta of the  $\tau^-$  and  $\tau^+$  are  $p_1$  and  $p_2$ , respectively, those of the  $b$  and  $\bar{b}$  are  $k_1$  and  $k_2$ , respectively, and  $\hat{s}$  is the center-of-mass energy squared of the  $b\bar{b}$  pair that fuses into a Higgs boson:  $\hat{s} = (k_1 + k_2)^2 = (p_1 + p_2)^2$ .

An important element of our formalism is the consideration of the ‘‘full’’  $3 \times 3$  Higgs-boson propagator matrix  $D(\hat{s})$  in (3.1)<sup>3</sup>. This is given by

$$D(\hat{s}) = \hat{s} \begin{bmatrix} \hat{s} - M_{H_1}^2 + i\Im\hat{\Pi}_{11}(\hat{s}) & i\Im\hat{\Pi}_{12}(\hat{s}) & i\Im\hat{\Pi}_{13}(\hat{s}) \\ i\Im\hat{\Pi}_{21}(\hat{s}) & \hat{s} - M_{H_2}^2 + i\Im\hat{\Pi}_{22}(\hat{s}) & i\Im\hat{\Pi}_{23}(\hat{s}) \\ i\Im\hat{\Pi}_{31}(\hat{s}) & i\Im\hat{\Pi}_{32}(\hat{s}) & \hat{s} - M_{H_3}^2 + i\Im\hat{\Pi}_{33}(\hat{s}) \end{bmatrix}^{-1}, \quad (3.2)$$

where the inversion of the  $3$  by  $3$  matrix is carried out analytically. In (3.2), the absorptive parts of the Higgs self-energies  $\Im\hat{\Pi}_{ij}(\hat{s})$  are given in Sec. II and  $M_{H_{1,2,3}}$  are the one-loop Higgs-boson pole masses, where higher-order absorptive effects on  $M_{H_{1,2,3}}$  have been ignored [15]. In the same context, the off-shell dispersive parts of the Higgs-boson self-energies in the Higgs-boson propagator matrix  $D(\hat{s})$  have also been neglected, since these are formally higher-order effects and very small in the relevant Higgs-boson resonant region. Finally, we include in (3.1) the finite loop-induced corrections to the couplings of Higgs bosons to  $b$  quarks,  $g_{H_i\bar{b}b}^{S,P}$ , and  $\tau$  leptons,  $g_{H_i\tau^+\tau^-}^{S,P}$ , due to the exchanges of gauginos and Higgsinos, as has been discussed in Sec. II.

In the center-of-mass coordinate system for the  $b\bar{b}$  pair, the helicity amplitudes are given by

$$\mathcal{M}^{b\bar{b}}(\sigma\bar{\sigma}; \lambda\bar{\lambda}) = -\frac{g^2 m_b m_\tau}{4M_W^2} \langle \sigma; \lambda \rangle_b \delta_{\sigma\bar{\sigma}} \delta_{\lambda\bar{\lambda}}, \quad (3.3)$$

where

$$\langle \sigma; \lambda \rangle_b \equiv \sum_{i,j=1,2,3} (\lambda\beta_b g_{H_i\bar{b}b}^S + i g_{H_i\bar{b}b}^P) D_{ij}(\hat{s}) (\sigma\beta_\tau g_{H_j\tau^+\tau^-}^S - i g_{H_j\tau^+\tau^-}^P), \quad (3.4)$$

with  $\beta_f = \sqrt{1 - 4m_f^2/\hat{s}}$ . Note that the cross sections for general (longitudinal or transverse)  $\tau^\pm$  polarizations can be computed from the helicity amplitudes by a suitable rotation [40] from the helicity basis to a general spin basis.

The  $\tau$ -polarization-weighted squared matrix elements are given by

$$\begin{aligned} |\overline{\mathcal{M}^{b\bar{b}}}|^2 &= \frac{1}{12} \sum_{\lambda=\pm} \left( \sum_{\sigma\sigma'\bar{\sigma}\bar{\sigma}'} \mathcal{M}_{\sigma\bar{\sigma}}^{b\bar{b}} \mathcal{M}_{\sigma'\bar{\sigma}'}^{b\bar{b}*} \bar{\rho}_{\sigma'\bar{\sigma}'} \rho_{\sigma\bar{\sigma}} \right) \\ &= \frac{1}{12} \sum_{\lambda=\pm} \text{Tr}[\mathcal{M}^{b\bar{b}} \bar{\rho}^T \mathcal{M}^{b\bar{b}\dagger} \rho], \end{aligned} \quad (3.5)$$

<sup>3</sup>Strictly speaking, the complete propagator matrix  $D(\hat{s})$  is a  $4 \times 4$ -dimensional matrix spanned by the basis  $(H_1, H_2, H_3, G^0)$  [27]. However, to a good approximation, we may neglect the small off-resonant self-energy transitions of the Higgs bosons  $H_{1,2,3}$  to the neutral would-be Goldstone boson  $G^0$ .

where  $\rho$  and  $\bar{\rho}$  are  $2 \times 2$  polarization density matrices for the  $\tau^-$  and  $\tau^+$ , respectively:

$$\begin{aligned} \rho &= \frac{1}{2} \begin{pmatrix} 1 + P_L & P_T e^{-i\alpha} \\ P_T e^{i\alpha} & 1 - P_L \end{pmatrix}, \\ \bar{\rho} &= \frac{1}{2} \begin{pmatrix} 1 + \bar{P}_L & -\bar{P}_T e^{i\bar{\alpha}} \\ -\bar{P}_T e^{-i\bar{\alpha}} & 1 - \bar{P}_L \end{pmatrix}. \end{aligned} \quad (3.6)$$

Evaluating the trace in (3.5) yields

$$\begin{aligned} |\overline{\mathcal{M}^{b\bar{b}}}|^2 &= \frac{1}{12} \left( \frac{g^2 m_b m_\tau}{4M_W^2} \right)^2 \{ C_1^b (1 + P_L \bar{P}_L) + C_2^b (P_L \\ &+ \bar{P}_L) + P_T \bar{P}_T [C_3^b \cos(\alpha - \bar{\alpha}) \\ &+ C_4^b \sin(\alpha - \bar{\alpha})] \}. \end{aligned} \quad (3.7)$$

The  $\tau^+\tau^-$  production plane is depicted schematically in Fig. 2, where the transverse-polarization angles  $\alpha$  and  $\bar{\alpha}$  are also defined.

The coefficients  $C_n^b$  ( $n = 1 - 4$ ) in (3.7) are defined in terms of the helicity amplitudes by

$$\begin{aligned} C_1^b &\equiv \frac{1}{4} \sum_{\lambda=\pm} (|\langle +; \lambda \rangle_b|^2 + |\langle -; \lambda \rangle_b|^2), \\ C_2^b &\equiv \frac{1}{4} \sum_{\lambda=\pm} (|\langle +; \lambda \rangle_b|^2 - |\langle -; \lambda \rangle_b|^2), \\ C_3^b &\equiv -\frac{1}{2} \sum_{\lambda=\pm} \Re(\langle +; \lambda \rangle_b \langle -; \lambda \rangle_b^*), \\ C_4^b &\equiv \frac{1}{2} \sum_{\lambda=\pm} \Im(\langle +; \lambda \rangle_b \langle -; \lambda \rangle_b^*). \end{aligned} \quad (3.8)$$

Under  $CP$  and  $CPT$ <sup>4</sup> transformations, the helicity amplitudes transform as follows:

$$\begin{aligned} \langle \sigma; \lambda \rangle_b CP &\leftrightarrow +\langle -\sigma; -\lambda \rangle_b, \\ \langle \sigma; \lambda \rangle_b CPT &\leftrightarrow +\langle -\sigma; -\lambda \rangle_b^*. \end{aligned} \quad (3.9)$$

Hence, the  $CP$  and  $CPT$  parities of the coefficients  $C_n^b$  defined in (3.8) are given by

<sup>4</sup>We define  $\tilde{T}$  as the naive  $T$ -reversal transformation, under which the spins and 3-momenta of the asymptotic states reverse sign, without interchanging initial to final states. In addition, under the operation of  $\tilde{T}$ , the matrix element gets complex conjugated.

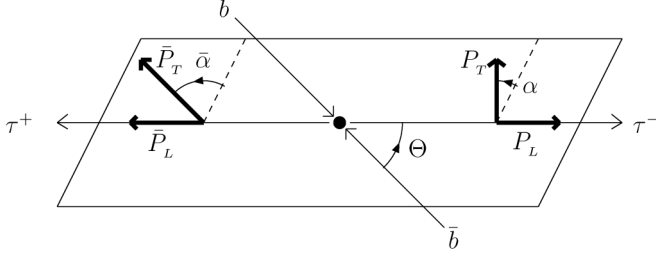


FIG. 2. The  $\tau^+\tau^-$  production plane with definitions of the scattering angle  $\Theta$ . The transverse-polarization vectors  $P_T$  and  $P_{\bar{T}}$  have azimuthal angles  $\alpha$  and  $\bar{\alpha}$ , respectively, with respect to the event plane.

$$C_1^b[++], \quad C_2^b[--], \quad C_3^b[++], \quad C_4^b[-+], \quad (3.10)$$

where the first and second symbols in the square brackets are the  $CP$  and  $CPT$  parities, respectively. Consequently, the coefficients  $C_2^b$  and  $C_4^b$  signify genuine phenomena of  $CP$  violation, whereas  $C_1^b$  and  $C_3^b$  are  $CP$ -conserving. Here, we should remark that a nonzero value for the  $CPT$ -odd coefficient  $C_2^b$  can only be induced by nonvanishing absorptive effects. In our case, such effects mainly originate from the absorptive parts of the Higgs-boson self-energies.

Finally, for our phenomenological discussion in Sec. 7, we define the parton-level cross sections

$$\hat{\sigma}_i(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-) \equiv \frac{\beta_\tau}{192\pi\hat{s}} \left( \frac{g^2 m_b m_\tau}{4M_W^2} \right)^2 C_i^b, \quad (3.11)$$

where the intermediate state  $H$  collectively denotes all the  $H_i \rightarrow H_j$  resonant transitions with  $i, j = 1, 2, 3$ .

### B. $gg$ Fusion

The matrix element  $\mathcal{M}^{gg}$  for the process  $gg \rightarrow H \rightarrow \tau^+\tau^-$ , depicted in Fig. 3, can be written as

$$\begin{aligned} \mathcal{M}^{gg} &= \frac{g\alpha_s m_\tau \delta^{ab}}{8\pi\nu M_W} \times \sum_{i,j=1,2,3} \sum_{\alpha=\pm} G_{H_i}(k_1, \epsilon_1; k_2, \epsilon_2) \\ &\quad \times D_{ij}(\hat{s})(g_{H_j\tau^+\tau^-}^S + i\alpha g_{H_j\tau^+\tau^-}^P) \bar{u}(p_1, \sigma) P_\alpha v(p_2, \bar{\sigma}). \end{aligned} \quad (3.12)$$

In the above,  $a$  and  $b$  are indices of the  $SU(3)$  generators in the adjoint representation and  $k_{1,2}$  and  $\epsilon_{1,2}$  are the four-momenta and wave vectors of the two gluons, respectively. Again, we denote the helicities of  $\tau^-$  and  $\tau^+$  by  $\sigma$  and  $\bar{\sigma}$  with  $\sigma = +$  and  $-$  standing for right- and left-handed particles. The four-momenta of  $\tau^-$  and  $\tau^+$  are  $p_1$  and  $p_2$ , respectively, and  $\hat{s} = (k_1 + k_2)^2 = (p_1 + p_2)^2$ . The Higgs-boson propagator matrix  $D(\hat{s})$  was given in (3.2) and the loop-induced couplings of the Higgs bosons  $H_i$  to two gluons are given by

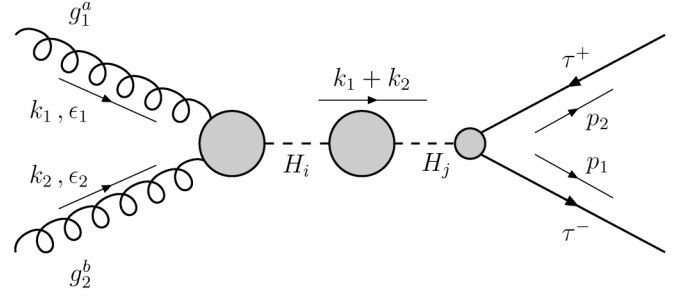


FIG. 3. Mechanisms contributing to the process  $gg \rightarrow H \rightarrow \tau^+\tau^-$  via the three neutral Higgs bosons  $H_{1,2,3}$ .

$$\begin{aligned} G_{H_i}(k_1, \epsilon_1; k_2, \epsilon_2) &= iS_i^g(\sqrt{\hat{s}}) \left( \epsilon_1 \cdot \epsilon_2 - \frac{2}{\hat{s}} k_1 \cdot \epsilon_2 k_2 \cdot \epsilon_1 \right) \\ &\quad - iP_i^g(\sqrt{\hat{s}}) \frac{2}{\hat{s}} \epsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma, \end{aligned} \quad (3.13)$$

with  $\epsilon_{0123} = 1$ . For the loop functions  $S_i^g$  and  $P_i^g$ , we follow the definitions of [34].

In the two-gluon center-of-mass coordinate system with  $\mathbf{k}_1$  along the positive  $z$  direction and  $\mathbf{k}_2$  along the negative  $z$  direction, the wave vectors of two photons are given by

$$\begin{aligned} \epsilon_1^\mu(\lambda_1) &= \frac{1}{\sqrt{2}}(0, -\lambda_1, -i, 0), \\ \epsilon_2^\mu(\lambda_2) &= \frac{1}{\sqrt{2}}(0, -\lambda_2, i, 0), \end{aligned} \quad (3.14)$$

where  $\lambda = +1$  and  $-1$  denote the right and left gluon helicities, respectively. The helicity amplitudes are given by

$$\mathcal{M}^{gg}(\sigma\bar{\sigma}; \lambda_1\lambda_2) = \frac{g\alpha_s m_\tau \sqrt{\hat{s}} \delta^{ab}}{8\pi\nu M_W} \langle \sigma; \lambda_1 \rangle_g \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2}, \quad (3.15)$$

where the amplitude  $\langle \sigma; \lambda \rangle_g$  is defined as

$$\begin{aligned} \langle \sigma; \lambda \rangle_g &\equiv \sum_{i,j=1,2,3} [S_i^g(\sqrt{\hat{s}}) + i\lambda P_i^g(\sqrt{\hat{s}})] D_{ij}(\hat{s}) (\sigma\beta_\tau g_{H_j\tau^+\tau^-}^S \\ &\quad - i g_{H_j\tau^+\tau^-}^P). \end{aligned} \quad (3.16)$$

We note that the amplitude (3.15) has the same structure as the amplitude (3.3) for  $b\bar{b} \rightarrow \tau^+\tau^-$ , except for the overall constant. We obtain from the helicity amplitudes the polarization-weighted squared matrix elements given by

$$\begin{aligned} |\overline{\mathcal{M}^{gg}}|^2 &= \frac{1}{32} \left( \frac{g\alpha_s m_\tau \sqrt{\hat{s}}}{8\pi\nu M_W} \right)^2 \{ C_1^g(1 + P_L \bar{P}_L) + C_2^g(P_L \\ &\quad + \bar{P}_L) + P_T \bar{P}_T [C_3^g \cos(\alpha - \bar{\alpha}) \\ &\quad + C_4^g \sin(\alpha - \bar{\alpha}) \}], \end{aligned} \quad (3.17)$$

where the coefficients  $C_n^g$  are obtained by replacing

$\langle\sigma; \lambda\rangle_b \rightarrow \langle\sigma; \lambda\rangle_g$  and interpreting  $\lambda$  as the gluon helicity in (3.8).

Under the  $CP$  and  $CPT$  transformations, the helicity amplitudes transform as follows:

$$\begin{aligned}\langle\sigma; \lambda\rangle_g CP &\leftrightarrow -\langle-\sigma; -\lambda\rangle_g, \\ \langle\sigma; \lambda\rangle_g CPT &\leftrightarrow -\langle-\sigma; -\lambda\rangle_g^*,\end{aligned}\quad (3.18)$$

where the  $CP$  and  $CPT$  parities of the coefficients  $C_i^g$  are the same as those of the  $C_i^b$ . Finally, we define the parton-level cross sections as:

$$\hat{\sigma}_i(gg \rightarrow H \rightarrow \tau^+\tau^-) \equiv \frac{\beta_\tau}{512\pi\hat{s}} \left( \frac{g\alpha_s m_\tau \sqrt{\hat{s}}}{8\pi v M_W} \right)^2 C_i^g. \quad (3.19)$$

Note that the  $CP$ - and  $CPT$ -odd cross section  $\hat{\sigma}_2$  receives contributions from the absorptive parts of the  $H_{1,2,3}gg$  vertices and Higgs-boson self-energies as well.

### C. $W^+W^-$ Fusion

The last important mechanism for the production of the MSSM neutral Higgs bosons at the LHC is  $W^+W^-$  fusion [41–44]. The matrix element  $\mathcal{M}^{WW}$  for this process,  $W^-(k_1)W^+(k_2) \rightarrow H \rightarrow \tau^-(p_1)\tau^+(p_2)$  with  $\hat{s} = (k_1 + k_2)^2$ , is given by

$$\begin{aligned}\mathcal{M}^{WW} &= \frac{g^2 m_\tau}{2\hat{s}} \sum_{i,j=1}^3 \sum_{\alpha=\pm} g_{H_i V V} \epsilon_1 \cdot \epsilon_2 D_{ij}(\hat{s}) (g_{H_i \tau^+ \tau^-}^S \\ &\quad + i\alpha g_{H_i \tau^+ \tau^-}^P) \bar{u}(p_1, \sigma) P_\alpha v(p_2, \bar{\sigma}),\end{aligned}\quad (3.20)$$

where  $\epsilon_1$  and  $\epsilon_2$  are the polarization vectors of two vector bosons and  $g_{H_i V V}$  denotes the coupling of the Higgs boson  $H_i$  with a pair of gauge bosons, as defined through the interaction Lagrangian

$$\mathcal{L}_{HVV} = gM_W \left( W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} Z_\mu Z^\mu \right) \sum_{i=1}^3 g_{H_i V V} H_i. \quad (3.21)$$

In the  $W^+W^-$  center-of-mass coordinate system with  $\mathbf{k}_1$  along the positive  $z$  direction and  $\mathbf{k}_2$  along the negative  $z$  direction, the polarization vectors of two vector bosons are given by

$$\begin{aligned}\epsilon_1^\mu(\lambda_1 = \pm 1) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0), \\ \epsilon_1^\mu(\lambda_1 = 0) &= \frac{1}{\sqrt{k_1^2}}(|\mathbf{k}_1|, 0, 0, k_1^0), \\ \epsilon_2^\mu(\lambda_2 = \pm 1) &= \frac{1}{\sqrt{2}}(0, \mp 1, i, 0), \\ \epsilon_2^\mu(\lambda_2 = 0) &= \frac{1}{\sqrt{k_2^2}}(|\mathbf{k}_2|, 0, 0, -k_2^0),\end{aligned}\quad (3.22)$$

where the polarization vectors are normalized by  $\epsilon_i(\lambda) \cdot \epsilon_i^*(\lambda') = -\delta_{\lambda\lambda'}$ , and  $\lambda = \pm 1$  and  $\lambda = 0$  denote the transverse (right and left helicities) and longitudinal polarizations, respectively. In this frame, the helicity amplitude is given by

$$\mathcal{M}^{WW}(\sigma\bar{\sigma}; \lambda_1\lambda_2) = \frac{g^2 m_\tau}{2\sqrt{\hat{s}}} \langle\sigma; \lambda_1\rangle_W \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2}, \quad (3.23)$$

where the amplitude  $\langle\sigma; \lambda\rangle_W$  is defined by

$$\begin{aligned}\langle\sigma; \lambda\rangle_W &\equiv \sum_{i,j=1,2,3} \omega(\lambda) g_{H_i V V} D_{ij}(\hat{s}) (\sigma\beta_\tau g_{H_i \tau^+ \tau^-}^S \\ &\quad - i g_{H_i \tau^+ \tau^-}^P),\end{aligned}\quad (3.24)$$

with

$$\omega(\pm) = 1 \quad \text{and} \quad \omega(0) = -k_1 \cdot k_2 / \sqrt{k_1^2 k_2^2}. \quad (3.25)$$

The factor  $\omega(0)$  becomes  $1 - \hat{s}/2M_W^2$  for on-shell vector bosons and dominates the amplitude for  $\hat{s} \gg M_W^2$ .

One can then obtain the following averaged amplitude squared:

$$\begin{aligned}|\overline{\mathcal{M}^{WW}}|^2 &= \frac{1}{9} \left( \frac{g^2 m_\tau}{2\sqrt{\hat{s}}} \right)^2 \{ C_1^W (1 + P_L \bar{P}_L) + C_2^W (P_L + \bar{P}_L) \\ &\quad + P_T \bar{P}_T [C_3^W \cos(\alpha - \bar{\alpha}) + C_4^W \sin(\alpha - \bar{\alpha})] \},\end{aligned}\quad (3.26)$$

where the coefficients  $C_n^W$  can be obtained by replacing  $\langle\sigma; \lambda\rangle_b \rightarrow \langle\sigma; \lambda\rangle_W$  and summing over  $\lambda = \pm, 0$  in (3.8). The  $CP$  and  $CPT$  parities of the coefficients  $C_n^W$  are the same as those of  $C_n^b$  or  $C_n^g$ , and the parton-level cross sections are defined similarly as

$$\hat{\sigma}_i(W_{T,L}^+ W_{T,L}^- \rightarrow H \rightarrow \tau^+\tau^-) \equiv \frac{\beta_\tau}{144\pi\hat{s}} \left( \frac{g^2 m_\tau}{2\sqrt{\hat{s}}} \right)^2 C_i^W. \quad (3.27)$$

In kinematic situations where the longitudinal  $W_L^+ W_L^-$  contributions can be neglected, the average factor  $1/144$  should be replaced by  $1/64$ . Finally, we note that it is straightforward to calculate  $ZZ$ -fusion processes in a similar fashion, although their cross sections are smaller approximately by a factor of 4 than  $W^+W^-$  collisions at the hadron level.

## IV. NUMERICAL EXAMPLES

We now present some numerical examples of  $CP$ -violating Higgs signatures in  $\tau^+\tau^-$  production at the LHC. As already mentioned, these signatures may be enhanced at large  $\tan\beta$ , and three-way mixing is potentially important for small charged Higgs-boson masses. Since the prospects for observing  $H_{1,2,3} \rightarrow \tau^+\tau^-$  at the LHC are best for light Higgs bosons, we present in this section some numerical analyses in a specific scenario in which all the three-Higgs states mix significantly.

Explicitly, we take the following parameter set:

$$\begin{aligned}
 \tan\beta &= 50, & M_{H^\pm}^{\text{pole}} &= 155 \text{ GeV}, \\
 M_{\tilde{Q}_3} &= M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = M_{\text{SUSY}} \\
 &= 0.5 \text{ TeV}, \\
 |\mu| &= 0.5 \text{ TeV}, & |A_{t,b,\tau}| &= 1 \text{ TeV}, \\
 |M_2| &= |M_1| = 0.3 \text{ TeV}, & |M_3| &= 1 \text{ TeV}, \\
 \Phi_\mu &= 0^\circ, & \Phi_A &= \Phi_{A_t} = \Phi_{A_b} = \Phi_{A_\tau} = 90^\circ, \\
 \Phi_1 &= \Phi_2 = 0^\circ,
 \end{aligned} \tag{4.1}$$

and we consider two values for the phase of the gluino mass parameter  $M_3$ :  $\Phi_3 = -90^\circ, -10^\circ$ . For  $\Phi_3 = -10^\circ$ , CPsuperH yields for the masses and widths of the neutral Higgs bosons:

$$\begin{aligned}
 M_{H_1} &= 120.2 \text{ GeV}, & M_{H_2} &= 121.4 \text{ GeV}, \\
 M_{H_3} &= 124.5 \text{ GeV}, & \Gamma_{H_1} &= 1.19 \text{ GeV}, \\
 \Gamma_{H_2} &= 3.42 \text{ GeV}, & \Gamma_{H_3} &= 3.20 \text{ GeV},
 \end{aligned} \tag{4.2}$$

and for  $\Phi_3 = -90^\circ$ :

$$\begin{aligned}
 M_{H_1} &= 118.4 \text{ GeV}, & M_{H_2} &= 119.0 \text{ GeV}, \\
 M_{H_3} &= 122.5 \text{ GeV}, & \Gamma_{H_1} &= 3.91 \text{ GeV}, \\
 \Gamma_{H_2} &= 6.02 \text{ GeV}, & \Gamma_{H_3} &= 6.34 \text{ GeV},
 \end{aligned} \tag{4.3}$$

respectively.

In Figs. 4 and 5, we show the parton-level cross sections  $\hat{\sigma}_i(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-)$ ,  $\hat{\sigma}_i(gg \rightarrow H \rightarrow \tau^+\tau^-)$  and  $\hat{\sigma}_i(WW \rightarrow H \rightarrow \tau^+\tau^-)$  defined in (3.11), (3.19) and (3.27), respectively, as functions of the  $\tau^+\tau^-$  invariant mass  $\sqrt{\hat{s}}$ . The solid lines are for  $\Phi_3 = -90^\circ$  and the dashed (red) ones for  $\Phi_3 = -10^\circ$ . We recall that non-vanishing of  $\hat{\sigma}_2$  and  $\hat{\sigma}_4$  are direct signals of  $CP$  violation in longitudinally and transversally polarized  $\tau^+\tau^-$  pairs, respectively.

The parton-level cross sections  $\hat{\sigma}_i(WW \rightarrow \tau^+\tau^-)$  have been computed by neglecting the contribution of the longitudinally-polarized  $W^\pm$ , i.e., setting  $\omega(0) = 0$ . For the MSSM scenario defined in (4.1), this is a plausible approximation for Higgs-boson masses below the  $WW$  threshold. Possible uncertainties that such a treatment may introduce largely cancel when we consider ratios of the cross sections  $\hat{\sigma}_i(WW \rightarrow \tau^+\tau^-)$ , such as the  $CP$  asymmetries to be defined later in this section.

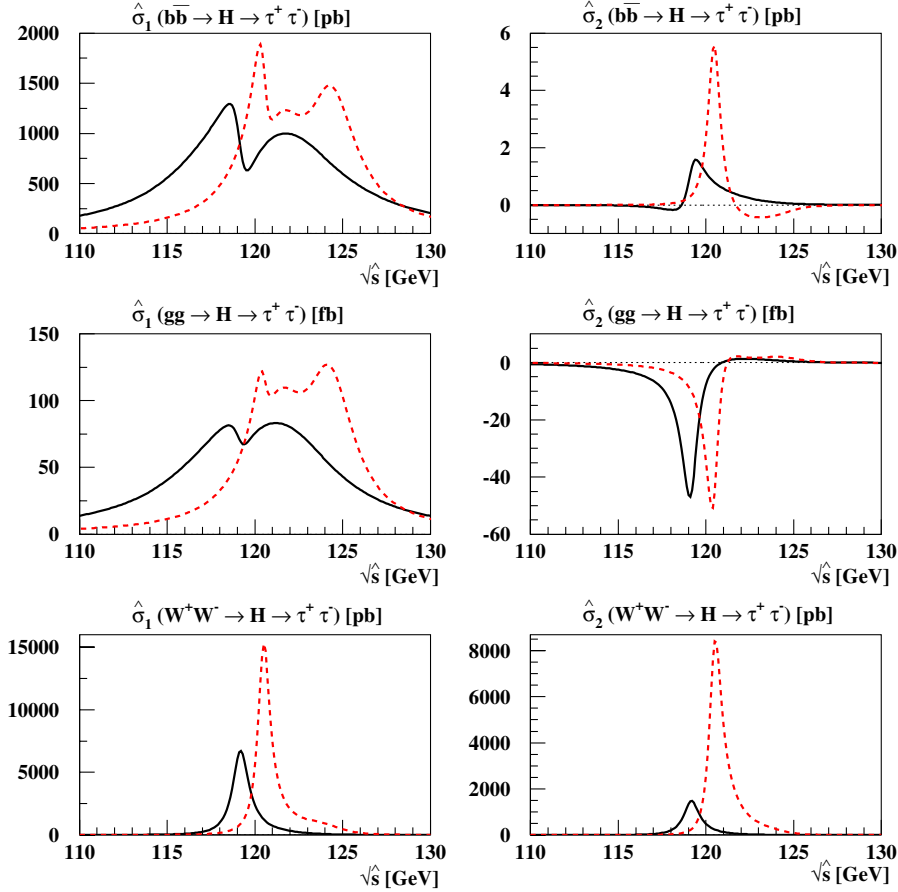


FIG. 4 (color online). The parton-level cross sections  $\hat{\sigma}_{1,2}(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-)$  in  $pb$ ,  $\hat{\sigma}_{1,2}(gg \rightarrow H \rightarrow \tau^+\tau^-)$  in  $fb$ , and  $\hat{\sigma}_{1,2}(W^+W^- \rightarrow H \rightarrow \tau^+\tau^-)$  in  $pb$  as functions of  $\sqrt{\hat{s}}$ . The solid lines are for the three-Higgs mixing scenario with  $\Phi_3 = -90^\circ$  and the dashed ones with  $\Phi_3 = -10^\circ$ .



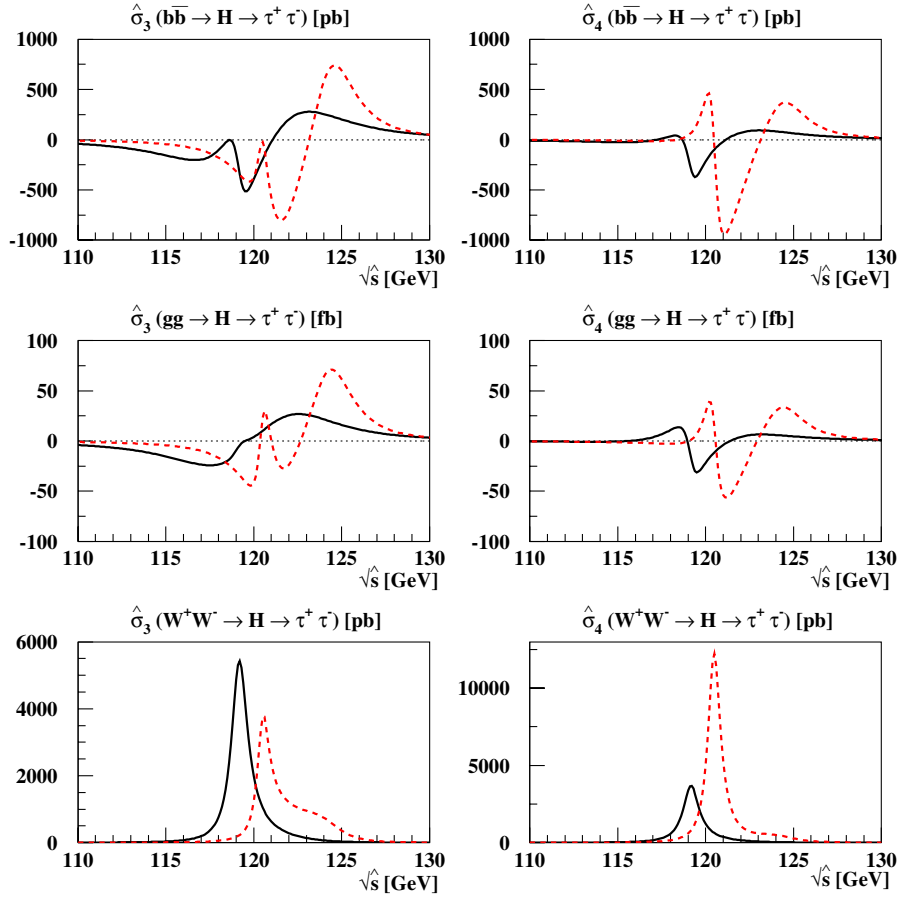


FIG. 5 (color online). The parton-level cross sections  $\hat{\sigma}_{3,4}(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-)$  in  $pb$ ,  $\hat{\sigma}_{3,4}(gg \rightarrow H \rightarrow \tau^+\tau^-)$  in  $fb$ , and  $\hat{\sigma}_{3,4}(W^+W^- \rightarrow H \rightarrow \tau^+\tau^-)$  in  $pb$  as functions of  $\sqrt{\hat{s}}$ . The solid lines are for the three-Higgs mixing scenario with  $\Phi_3 = -90^\circ$  and the dashed lines with  $\Phi_3 = -10^\circ$ .

In Fig. 4, we observe that the cross section  $\hat{\sigma}_2$ , which quantifies  $CP$  violation in the production of longitudinally-polarized  $\tau$ -lepton pairs at the parton-level, is comparable to the spin-averaged cross section  $\hat{\sigma}_1$  in  $WW$  and  $gg$  collisions. This implies that  $CP$  violation can be very large in these channels. Instead, in  $b\bar{b}$  fusion, the ratio  $\hat{\sigma}_2/\hat{\sigma}_1$  is always less than 1%, so  $CP$ -violating effects in the production of longitudinally-polarized  $\tau$  leptons are unobservably small in this case.

The smallness of  $\hat{\sigma}_2$  in  $b\bar{b}$  fusion is a result of an intriguing interplay between unitarity and  $CPT$  invari-

ance [27]. In detail, the  $CP$ -violating cross section  $\hat{\sigma}_2$  may be calculated by

$$\hat{\sigma}_2(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-) = \frac{1}{4}[\hat{\sigma}(b\bar{b} \rightarrow H \rightarrow \tau_R^+\tau_R^-) - \hat{\sigma}(b\bar{b} \rightarrow H \rightarrow \tau_L^+\tau_L^-)], \quad (4.4)$$

where  $\hat{\sigma}$  denotes the usual subprocess cross section. For the scenario under study, unitarity cuts of  $b\bar{b}$  pairs dominate the absorptive part of the Higgs-boson self-energies. Employing this fact and the optical theorem, we obtain

$$\begin{aligned} \sum_{\lambda=L,R} \hat{\sigma}_2(b_\lambda \bar{b}_\lambda \rightarrow H \rightarrow \tau^+\tau^-) &= C_{\text{PS}}[\Im \mathcal{T}(\tau_R^+\tau_R^- \rightarrow H \rightarrow \tau_R^+\tau_R^-) - \Im \mathcal{T}(\tau_L^+\tau_L^- \rightarrow H \rightarrow \tau_L^+\tau_L^-)] \\ &+ \mathcal{O}[\hat{\sigma}'_2 B(H_{1,2,3} \rightarrow \tau^+\tau^-)], \end{aligned} \quad (4.5)$$

where  $C_{\text{PS}}$  is a phase-space correction factor and  $\mathcal{T}(\tau_{L,R}^+\tau_{L,R}^- \rightarrow H \rightarrow \tau_{L,R}^+\tau_{L,R}^-)$  denote the usual matrix elements. In (4.5),  $\hat{\sigma}'_2$  is the  $CP$ -violating cross section  $\hat{\sigma}_2$  calculated by omitting the off-diagonal absorptive parts in the Higgs-boson propagator matrix  $D(\hat{s})$ . The size of  $\hat{\sigma}'_2$  is smaller at least by a factor ten than the spin-averaged

cross section  $\hat{\sigma}_1$ . On the other hand,  $CPT$  invariance imposes the constraint

$$\mathcal{T}(\tau_R^+\tau_R^- \rightarrow H \rightarrow \tau_R^+\tau_R^-) = \mathcal{T}(\tau_L^+\tau_L^- \rightarrow H \rightarrow \tau_L^+\tau_L^-). \quad (4.6)$$

With the aid of (4.6), it is not difficult to see using (4.5) that the  $CP$ -violating cross section  $\hat{\sigma}_2(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-)$  vanishes up to  $CP$ -violating terms suppressed by extra factors of order  $B(H_{1,2,3} \rightarrow \tau^+\tau^-)$ .

Our numerical estimates presented in Fig. 5 show that the  $CP$ -violating transverse-polarization cross section  $\hat{\sigma}_4$  may be quite sizeable for all production channels. However,  $\hat{\sigma}_4$  generically exhibits an alternating sign for  $b\bar{b}$  and  $gg$  collisions, and  $CP$  violation becomes very small when we integrate over the whole Higgs-boson resonance region. Moreover, the transverse  $\tau^\pm$  polarizations will be difficult to measure at the LHC because of the experimental conditions, notably the large boosts of the  $\tau^\pm$ . On the other

hand, analogous asymmetries might be observable in  $t\bar{t}$  production and/or in  $\tau^+\tau^-$  production at a  $\mu^+\mu^-$  collider.

The Higgs production channels via  $b\bar{b}$  and  $gg$  fusion processes can be separated from the  $W^+W^-$  fusion channel by applying a number of kinematic cuts [45] including the imposition of a veto on any hadronic activity between jets [46,47]. Therefore, we treat the contributions from  $b\bar{b}$  and  $gg$  collisions to the physical Higgs-exchange process  $pp \rightarrow H \rightarrow \tau^+\tau^-X$  separately from those coming from  $WW$  fusion. More explicitly, the physical  $\tau^+\tau^-$  cross section can be computed by integrating the parton-level cross sections with the distribution of  $b$  quarks, gluons and  $W$ -bosons in the proton,

$$\tau \frac{d\sigma_{\text{tot}}}{d\tau} [pp(b\bar{b}, gg) \rightarrow \tau^+\tau^-X] = 4\hat{\sigma}_1(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-)\tau \frac{d\mathcal{L}^{bb}}{d\tau} + 4K\hat{\sigma}_1(gg \rightarrow H \rightarrow \tau^+\tau^-)\tau \frac{d\mathcal{L}^{gg}}{d\tau}, \quad (4.7)$$

$$\tau \frac{d\sigma_{\text{tot}}}{d\tau} [pp(W^+W^-) \rightarrow \tau^+\tau^-X] = 4\hat{\sigma}_1(W^+W^- \rightarrow H \rightarrow \tau^+\tau^-)\tau \frac{d\mathcal{L}^{WW}}{d\tau}, \quad (4.8)$$

where  $\tau$  is the Drell-Yan variable  $\tau = \hat{s}/s$  and  $s$  is the invariant squared center-of-mass energy of the LHC. In (4.7), we use the value  $K = 1 + \frac{\alpha_s(\hat{s})}{\pi}(\pi^2 + 11/2)$ , ignoring the small difference between the  $K$ -factors for  $CP$ -even and  $CP$ -odd Higgs states. The effective luminosities for  $b\bar{b}$  and  $gg$  collisions,  $\mathcal{L}^{bb}$  and  $\mathcal{L}^{gg}$ , may be determined by

$$\begin{aligned} \tau \frac{d\mathcal{L}^{bb}}{d\tau} &= \int_\tau^1 dx \left[ \frac{\tau}{x} b(x, Q) \bar{b}\left(\frac{\tau}{x}, Q\right) + (b \leftrightarrow \bar{b}) \right], \\ \tau \frac{d\mathcal{L}^{gg}}{d\tau} &= \int_\tau^1 dx \frac{\tau}{x} g(x, Q) g\left(\frac{\tau}{x}, Q\right), \end{aligned} \quad (4.9)$$

where  $b(x, Q)$ ,  $\bar{b}(x, Q)$  and  $g(x, Q)$  are the  $b$ ,  $\bar{b}$  and gluon distribution functions in the proton and  $Q$  is the factorization scale. In our numerical analysis, we use the leading-order CTEQ6L [48] parton distribution functions for  $b(x, Q)$  and  $\bar{b}(x, Q)$ , and the CTEQ6M parton distribution function for  $g(x, Q)$ . We choose the factorization scale  $Q = \sqrt{\hat{s}}/4$  for the  $b$ -quark fusion process as suggested and confirmed in [38].

Correspondingly, in (4.8), the effective luminosities for the transverse and longitudinal  $W$ -bosons, denoted as  $W_{T,L}^\pm$ , can be computed in terms of effective densities  $F_{W_{T,L}}^p(x, Q)$  in the colliding protons, which are in turn calculated in terms of the quark parton distribution functions  $q(x, Q)$  in the proton:<sup>5</sup>

<sup>5</sup>Here we consider identical polarizations for the  $W^\pm$  bosons in the  $W^+W^-$  fusion process.

$$\begin{aligned} \tau \frac{d\mathcal{L}^{W_P W_P}}{d\tau} &= \int_\tau^1 dx \left[ \frac{\tau}{x} F_{W_P^+}^p(x, Q) F_{W_P^-}^p\left(\frac{\tau}{x}, Q\right) \right. \\ &\quad \left. + (W_P^+ \leftrightarrow W_P^-) \right], \end{aligned} \quad (4.10)$$

$$F_{W_P^+}^p(x, Q) = \sum_{q=u,d,c,s} \int_x^1 \frac{dy}{y} q(y, Q) F_{W_P^+}^q(x/y, Q),$$

where the transverse ( $P = T$ ) and longitudinal ( $P = L$ ) effective densities  $F_{W_{T,L}}^q$  in the quark  $q$  are given by [42,43]

$$\begin{aligned} F_{W_T^+}^q(x, Q) &= \frac{\alpha_{\text{em}}}{8\pi s_W^2} \ln\left(\frac{Q^2}{M_W^2}\right) \frac{1 + (1-x)^2}{x}, \\ F_{W_L^+}^q(x, Q) &= \frac{\alpha_{\text{em}}}{4\pi s_W^2} \frac{1-x}{x}. \end{aligned} \quad (4.11)$$

Note that the summation over quark flavours  $q$  in the expression for  $F_{W_{T,L}}^p(x, Q)$  includes  $q = \bar{u}, d, \bar{c}, s$ . Moreover, we take  $Q = \sqrt{\hat{s}}$  in our numerical estimates.

To analyze the signatures of  $CP$  violation in the production of longitudinally-polarized  $\tau$ -leptons, we first define the physical observables

$$\begin{aligned} \sigma_{RR} &= \sigma(pp \rightarrow H \rightarrow \tau_R^+ \tau_R^- X), \\ \sigma_{LL} &= \sigma(pp \rightarrow H \rightarrow \tau_L^+ \tau_L^- X). \end{aligned} \quad (4.12)$$

Evidently, the total cross section for Higgs production and decay into  $\tau^+\tau^-$  pairs is given in terms of  $\sigma_{RR}$  and  $\sigma_{LL}$  by

$$\sigma_{\text{tot}}(pp \rightarrow H \rightarrow \tau^+\tau^-X) = \sigma_{RR} + \sigma_{LL}. \quad (4.13)$$

Although the initial state  $pp$  is not symmetric under  $CP$ , it can, however, be shown that, up to negligible higher-

order  $CP$ -violating electroweak effects, the effective luminosities for  $gg$ ,  $b\bar{b}$  and  $W^+W^-$  densities will be practically the same for  $pp$  and  $\bar{p}\bar{p}$  collisions. Therefore, the difference of cross sections

$$\Delta\sigma_{CP} = \sigma_{RR} - \sigma_{LL} \quad (4.14)$$

is a measure of genuine  $CP$  violation at the LHC. In analogy with (4.7) and (4.8), the  $CP$ -violating cross section  $\Delta\sigma_{CP}$  can be computed by

$$\tau \frac{d\Delta\sigma_{CP}}{d\tau} [pp(b\bar{b}, gg) \rightarrow \tau^+\tau^- X] = 4\hat{\sigma}_2(b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-) \tau \frac{d\mathcal{L}^{b\bar{b}}}{d\tau} + 4K\hat{\sigma}_2(gg \rightarrow H \rightarrow \tau^+\tau^-) \tau \frac{d\mathcal{L}^{gg}}{d\tau}, \quad (4.15)$$

$$\tau \frac{d\Delta\sigma_{CP}}{d\tau} [pp(W^+W^-) \rightarrow \tau^+\tau^- X] = 4\hat{\sigma}_2(W^+W^- \rightarrow H \rightarrow \tau^+\tau^-) \tau \frac{d\mathcal{L}^{WW}}{d\tau}. \quad (4.16)$$

To gauge the sizes of the signatures of  $CP$  violation at the LHC, we define the following two  $CP$  asymmetries:

$$a_{CP}(\tau) \equiv \frac{\tau \frac{d\Delta\sigma_{CP}}{d\tau}}{\tau \frac{d\sigma_{\text{tot}}}{d\tau}}, \quad \mathcal{A}_{CP} \equiv \frac{\Delta\sigma_{CP}}{\sigma_{\text{tot}}}, \quad (4.17)$$

pertinent to the hadron-level processes  $pp(b\bar{b}, gg, WW) \rightarrow H \rightarrow \tau^+\tau^- X$ .

We plot in Fig. 6 the differential cross sections  $\tau \frac{d\sigma_{\text{tot}}}{d\tau}$  and  $\tau \frac{d\Delta\sigma_{CP}}{d\tau}$  as functions of  $\sqrt{\hat{s}}$ . The upper two frames are

for the process  $b\bar{b} \rightarrow \tau^+\tau^-$ , the frames in the middle for  $gg \rightarrow \tau^+\tau^-$ , and the lower ones for  $W^+W^- \rightarrow \tau^+\tau^-$ . We observe that the main production mechanism is  $b\bar{b}$  fusion, which gives a cross section about 5 times larger than that due to gluon fusion for the scenario under consideration. However, as has been mentioned above, the  $W^+W^-$ -fusion production mechanism, albeit much smaller, can be experimentally distinguished from that due to  $b\bar{b}$  and  $gg$  collisions. Therefore, in Fig. 7 we display the  $CP$  asymmetry  $a_{CP}$  defined in (4.17) sepa-

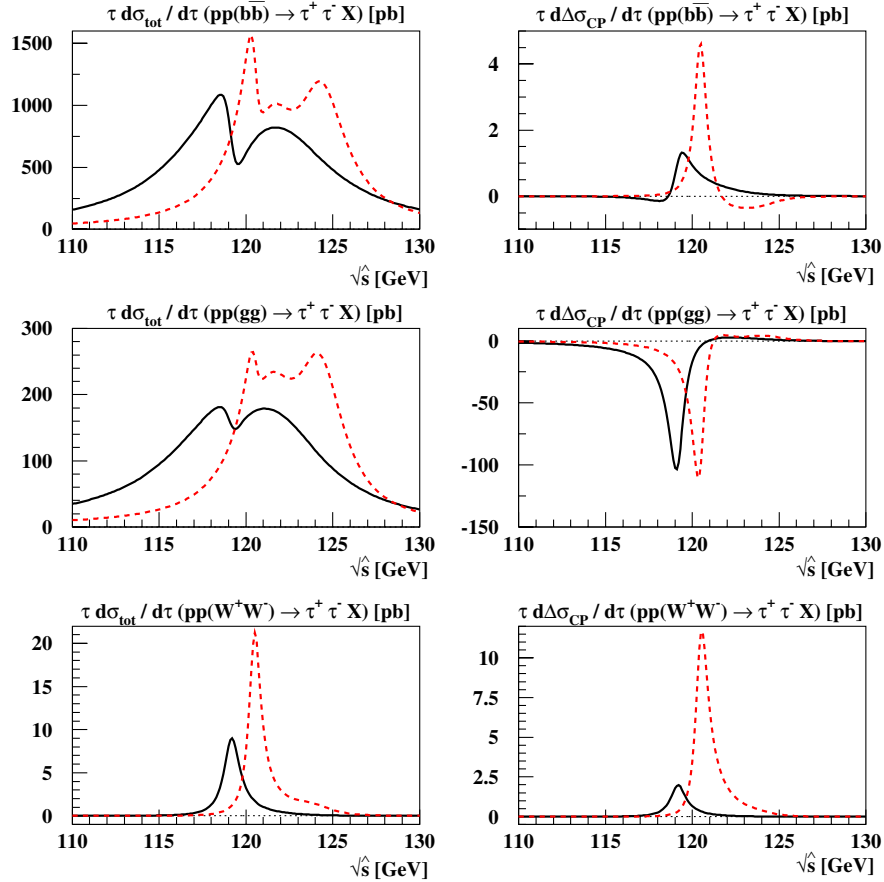


FIG. 6 (color online). The differential cross sections  $\tau \frac{d\sigma_{\text{tot}}}{d\tau}$  and  $\tau \frac{d\Delta\sigma_{CP}}{d\tau}$  as functions of  $\sqrt{\hat{s}}$ . The upper frames are for the process  $b\bar{b} \rightarrow H \rightarrow \tau^+\tau^-$ , the middle frames for  $gg \rightarrow H \rightarrow \tau^+\tau^-$  and the lower frames for  $W^+W^- \rightarrow H \rightarrow \tau^+\tau^-$ . The solid lines are for the three-Higgs mixing scenario with  $\Phi_3 = -90^\circ$  and the dashed lines with  $\Phi_3 = -10^\circ$ .

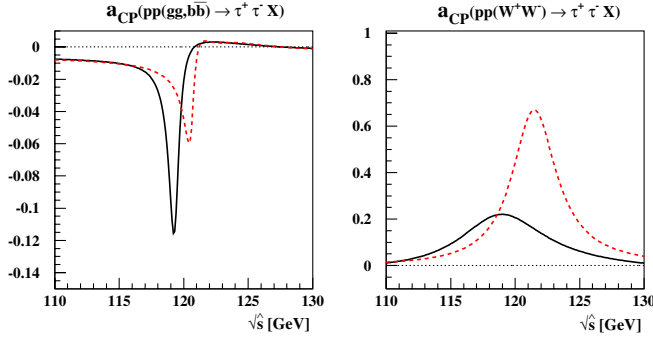


FIG. 7 (color online). Numerical estimates of differential  $CP$  asymmetries  $a_{CP}$  defined in (4.17) as functions of  $\sqrt{s}$ . The solid line corresponds to the three-Higgs mixing scenario with  $\Phi_3 = -90^\circ$  and the dashed one to  $\Phi_3 = -10^\circ$ .

rately for the  $b\bar{b} + gg$  and  $W^+W^-$  subprocesses. We note that the large  $CP$  asymmetry in  $gg$  subprocess is diluted by the dominant cross section via  $b\bar{b}$  fusion<sup>6</sup>.

It is important to emphasize here that the  $CP$ -violating observable  $\tau \frac{d\Delta\sigma_{CP}}{d\tau}$  for the  $WW$ -fusion process does not change sign as the  $\tau^+\tau^-$ -system energy  $\sqrt{s}$  varies over the entire Higgs-boson resonant region. Such a kinematic behavior is ensured by the presence of the off-diagonal absorptive parts of the Higgs-boson self-energies. Instead, if these off-diagonal absorptive parts are neglected, we find the erroneous result that the  $CP$ -violating observable flips sign in the resonant region, thereby leading to unobservably small  $CP$  asymmetries when averaged over the energy  $\sqrt{s}$ .

Although  $CP$  violation in the  $WW$  and  $gg$  production channels may be sizeable, it is difficult to measure the differential  $CP$  asymmetry  $a_{CP}$  at the LHC because of the low-energy resolution of the reconstructed  $\tau^+\tau^-$  invariant mass. This last fact also limits our ability to reconstruct with sufficient accuracy the line shape of the decaying coupled Higgs-boson system at the LHC. This is unfortunate since one would miss the very interesting feature shown in Fig. 6 that, unlike the case of a single resonance, the locations of the various maxima in the resonant line shapes described by  $\tau \frac{d\sigma_{tot}}{d\tau}$  crucially depend on the production and decay channels of the coupled Higgs-boson system. Therefore, the extra analyzing power of  $e^+e^-$  and  $\mu^+\mu^-$  colliders would be highly valuable for unravelling the existence of a strongly-mixed Higgs-boson system and studying in more detail its dynamical properties.

Motivated by the large differential  $CP$  asymmetry in the  $W^+W^-$ -fusion process, we perform a numerical analysis of the total  $CP$  asymmetry for the reaction

<sup>6</sup>Specifically, the total  $CP$  asymmetry in the  $gg$  subprocess is  $\mathcal{A}_{CP}^{gg} = -8.4(-6.2)\%$ , for  $\Phi_3 = -90^\circ(-10^\circ)$ . However, after the inclusion of  $bb$  collisions, the combined  $CP$  asymmetry  $\mathcal{A}_{CP}^{bb+gg}$  reduces to  $-1.4(-1.0)\%$ .

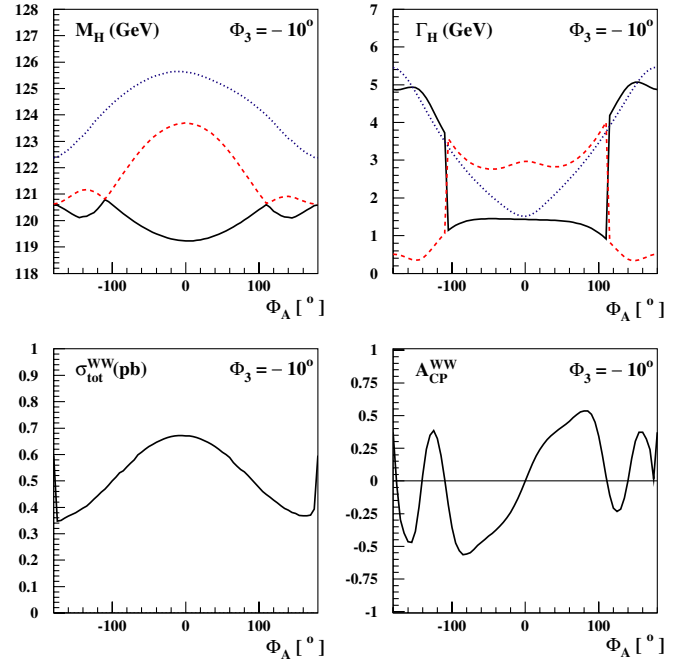


FIG. 8 (color online). Numerical estimates of Higgs-boson masses and decay widths, the cross section  $\sigma_{tot}[pp(WW) \rightarrow H \rightarrow \tau^+\tau^-X]$  and its associated total  $CP$  asymmetry  $\mathcal{A}_{CP}^{WW}$  defined in (4.17) as functions of  $\Phi_A = \Phi_{A_i} = \Phi_{A_b} = \Phi_{A_r}$ , for  $\Phi_3 = -10^\circ$ . In the upper two frames, the solid, dashed, and dotted lines refer to the  $H_1$ ,  $H_2$ , and  $H_3$  bosons, respectively.

$pp(WW) \rightarrow H \rightarrow \tau^+\tau^-$ , integrated over the Higgs resonance peaks. We present in Figs. 8, 9 and 10 the predicted values for the cross section  $\sigma_{tot}[pp(WW) \rightarrow H \rightarrow \tau^+\tau^-X]$  and its associated total integrated  $CP$  asymmetry  $\mathcal{A}_{CP}^{WW}$  defined in (4.17) as functions of  $\Phi_A = \Phi_{A_i} = \Phi_{A_b} = \Phi_{A_r}$ , for  $\Phi_3 = -10^\circ$ ,  $-70^\circ$ , and  $-90^\circ$ , respectively. In the upper two frames of the figures, we display the dependence of the Higgs-boson masses and their decay widths on the  $CP$ -violating phase  $\Phi_A$ , where the solid, dashed and dotted lines refer to the  $H_1$ ,  $H_2$  and  $H_3$  bosons, respectively. In our numerical analysis, we fix the remaining parameters of the MSSM as in (4.1). Unlike in Figs. 8, 9, and 10, we present in Fig. 11 numerical estimates by fixing the value of  $\Phi_A$  to  $-90^\circ$ , but varying the  $CP$ -violating phase  $\Phi_3$ . For the scenario under study, all three Higgs bosons mix among themselves significantly, giving rise to level crossings as the  $CP$ -odd phases vary. These effects of level crossing lead to a nontrivial behavior in  $\Gamma_{H_i}$ , which is between 1 GeV and 10 GeV<sup>7</sup>, and in  $\mathcal{A}_{CP}^{WW}$ . We find that the total cross section is between 0.1 pb and 0.7 pb and is comparable to the corresponding SM cross section 0.3 pb for  $M_{H_{SM}} = 120$  GeV [45]. We

<sup>7</sup>In Fig. 11, the widths of the  $H_1$  and  $H_2$  become larger than 10 GeV when  $\Phi_3 > 100^\circ$  or  $\Phi_3 < -140^\circ$ , where  $M_{H_1}$  decreases very rapidly and  $H_1$  decouples from the  $H_2 - H_3$  mixing system.

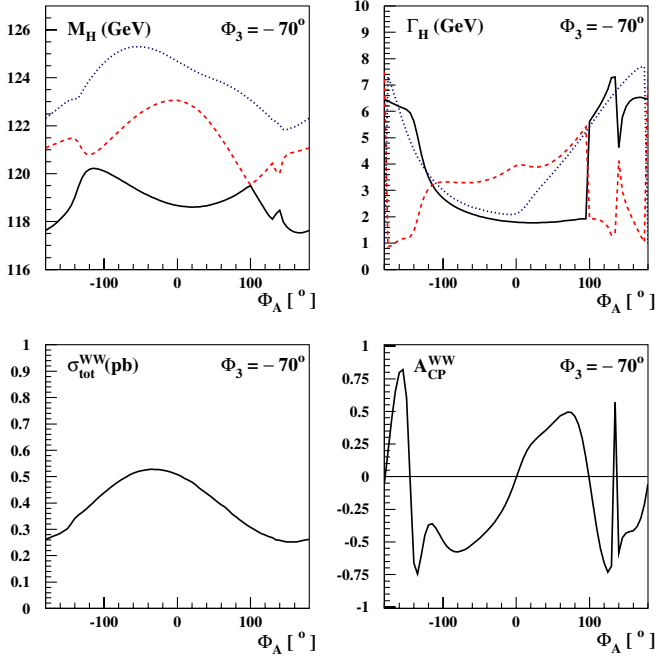


FIG. 9 (color online). The same as in Fig. 8, but for  $\Phi_3 = -70^\circ$ .

observe that the  $CP$  asymmetry  $\mathcal{A}_{CP}^{WW}$  is large for a wide range of  $CP$  phases and can even be as large as 80% for  $\Phi_3 = -70^\circ$ . Even for small  $CP$ -violating phases,  $\Phi_3 = -10^\circ$  and  $(180^\circ - |\Phi_A|) < 20^\circ$ , the  $CP$  asymmetry can be  $\sim 50\%$ , as shown in Fig. 8. Again, we note that possible

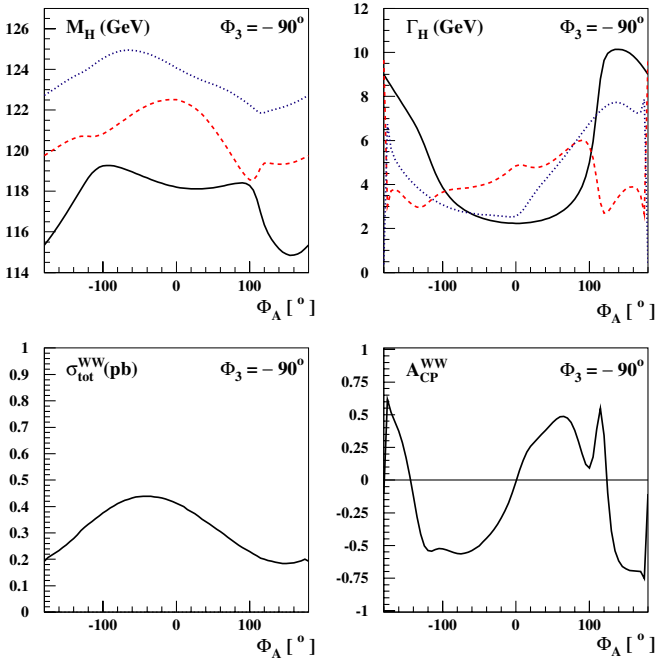


FIG. 10 (color online). The same as in Fig. 8, but for  $\Phi_3 = -90^\circ$ .

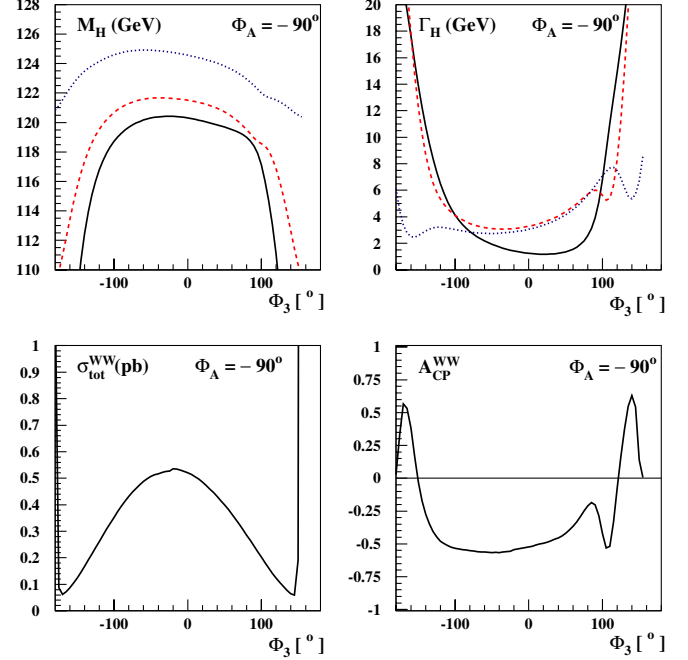


FIG. 11 (color online). Numerical values for  $M_{H_{1,2,3}}$  and  $\Gamma_{H_{1,2,3}}$ ,  $\sigma_{\text{tot}}[pp(WW) \rightarrow H \rightarrow \tau^+ \tau^- X]$  and  $\mathcal{A}_{CP}^{WW}$  as functions of  $\Phi_3$ , for  $\Phi_A = \Phi_{A_t} = \Phi_{A_b} = \Phi_{A_\tau} = -90^\circ$ . We follow the same line conventions as in Fig. 8.

uncertainties in the calculation of the cross sections largely cancel in the  $CP$  asymmetry  $\mathcal{A}_{CP}$ .

Finally, we comment briefly on the possible impact of low-energy constraints on the  $CP$  asymmetries, especially those arising from the nonobservation of the electron and neutron EDMs and the absence of the Higgs-mediated  $B$ -meson decay  $B_{s,d} \rightarrow \mu\mu$  at the Tevatron [49]. The EDM constraints may be considerably relaxed if we consider scenarios with the first two generations of squarks heavier than about 3 TeV, and if we allow some degree of cancellations [6] between the one- and higher-loop EDM contributions [50]. For the scenarios under study, we have estimated that the required degree of cancellation is always smaller than 80%, where 100% corresponds to complete cancellation. Therefore, a full implementation of EDM constraints will not alter the results of the present analysis in a significant way. On the other hand, the lack of observation of  $B_{s,d} \rightarrow \mu\mu$  at the Tevatron [49] imposes further constraints on the parameters of the  $CP$ -violating MSSM. However, the derived constraints are highly flavour-dependent and can be dramatically relaxed for certain choices of the soft SUSY-breaking mass spectrum that enable unitarity cancellations in the flavour space. For a detailed study, see [9].

## V. CONCLUSIONS AND PROSPECTS

We have presented the general formalism for analyzing  $CP$ -violating phenomena in the production, mixing and decay of a coupled system of multiple  $CP$ -violating neu-

tral Higgs bosons. Our formalism, which is developed from [27], can be applied to models with an extended  $CP$ -violating Higgs sector, including the highly predictive framework of the MSSM with radiative Higgs-sector  $CP$  violation. An important element of the formalism is the consideration of the full  $s$ -dependent  $3 \times 3$  Higgs-boson propagator matrix, where the gauge-mediated contributions to self-energies have been calculated in the framework of the pinch technique [30,35].

As an application of our formalism, we have studied in detail the production of  $CP$ -violating MSSM  $H_{1,2,3}$  bosons via  $b\bar{b}$ ,  $gg$  and  $W^+W^-$  collisions and their subsequent decays into  $\tau^+\tau^-$  pairs at the LHC. In addition to the Higgs self-energy effects, we have also given explicitly the relevant formulae in the MSSM with loop-induced  $CP$  violation in the production and decay vertices. We have considered specific MSSM scenarios that predict three nearly degenerate, strongly-mixed Higgs bosons with  $M_{H_{1,2,3}} \sim 120$  GeV. Such scenarios naturally occur in a general  $CP$ -violating MSSM when  $\tan\beta$  is larger than 30 and the charged Higgs boson is lighter than about 160 GeV.

We have analyzed  $CP$  asymmetries in both longitudinally- and transversely-polarized  $\tau^+\tau^-$  pairs.  $CP$  asymmetries that make use of the transverse-polarization of the  $\tau$ -lepton, although being intrinsically very large in the  $CP$ -violating MSSM scenarios mentioned above, generically exhibit an alternating sign and become unobservably small after averaging over the entire Higgs-boson resonant region. Also, reconstruction of transversely-polarized  $\tau$  leptons appears rather difficult at the LHC. However, such  $CP$  asymmetries might ideally be tested at a  $\mu^+\mu^-$  collider, where a high-energy resolution can be achieved.

At the LHC, more promising are  $CP$  asymmetries based on the longitudinal  $\tau$ -lepton polarization. In particular, the  $CP$  asymmetry in the production channel  $W^+W^- \rightarrow H_{1,2,3} \rightarrow \tau^+\tau^-$  may well exceed the 10% level and reach values up to 80%. It is important to stress again that the  $WW$  production channel can be cleanly isolated from the  $gg$  and  $b\bar{b}$  channels, mainly by vetoing any hadronic activity between jets (for details, see [45]). Hence, depending on the efficiency of longitudinal  $\tau$ -lepton polarization techniques [51], the production channel  $W^+W^- \rightarrow H_{1,2,3} \rightarrow \tau^+\tau^-$  may become the ‘‘golden’’ channel for studying signatures of Higgs-sector  $CP$  violation at the LHC.

The formalism presented in this paper may easily be applied to other colliders as well, most notably to  $e^+e^-$ ,  $\gamma\gamma$  and  $\mu^+\mu^-$  colliders. At  $e^+e^-$  linear colliders, Higgs bosons can copiously be produced via the Higgsstrahlung or  $W^+W^-$  fusion processes. At  $\gamma\gamma$  and  $\mu^+\mu^-$  colliders, the polarizations of the colliding beams may also be varied, thereby providing additional probes of Higgs-sector  $CP$  violation. The aforementioned colliders can provide cleaner experimental conditions than the LHC. Consequently, even if the  $CP$  asymmetries discussed here prove difficult to observe at the LHC, the formalism and the analysis techniques developed here to investigate Higgs-sector  $CP$  violation will be directly applicable to such future colliders as well.

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