

**The anomalous magnetic moment of the muon in the MSSM —
recent developments**Sven Heinemeyer^a, Dominik Stöckinger^b, and Georg Weiglein^b^a *CERN, TH Division, Dept. of Physics, 1211 Geneva 23, Switzerland*^b *Institute for Particle Physics Phenomenology, University of Durham, UK*

We present recent results of two interesting classes of supersymmetric two-loop contributions to $(g-2)_\mu$. Two-loop diagrams involving either a closed sfermion loop or a closed chargino/neutralino loop can amount to 5×10^{-10} , which is almost one standard deviation of the current experimental uncertainty. We discuss the dependence of these two classes on the unknown supersymmetric parameters and their impact on the supersymmetric prediction of $(g-2)_\mu$.

1 Introduction

After continuous improvement in the experimental ¹ and Standard Model-theoretical ^{2,3,4,5,6,7} determination of the anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ of the muon, there remains a tantalizing discrepancy ^a

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} = (24.5 \pm 9) \times 10^{-10} \quad (1)$$

between the experimental value and the Standard Model prediction.

It is an interesting question whether the observed deviation (1) is due to supersymmetric effects. The supersymmetric one-loop contribution is approximately given by ⁸

$$a_\mu^{\text{SUSY,1L}} \approx 13 \times 10^{-10} \frac{\tan \beta \text{sign}(\mu)}{(M_{\text{SUSY}}/100 \text{ GeV})^2}, \quad (2)$$

if all supersymmetric particles (the relevant ones are the smuon, sneutrino, chargino and neutralino) have a common mass M_{SUSY} .

This formula shows that supersymmetric effects can easily account for a $(20 \dots 30) \times 10^{-10}$ deviation, if μ is positive and M_{SUSY} lies roughly between 100 GeV (for small $\tan \beta$) and 600 GeV (for large $\tan \beta$). On the other hand, the precision of the measurement places strong bounds on the supersymmetric parameter space.

Here we review the results of Refs. ^{9,10} for the Minimal Supersymmetric Standard Model (MSSM) two-loop contributions of

^aHere we use the evaluations from ^{3,7} for the hadronic contributions. Other e^+e^- data driven evaluations result in similar deviations of $2-3\sigma$. Recent analyses concerning τ data indicate that uncertainties due to isospin breaking effects may have been underestimated earlier ⁴. We thank F. Jegerlehner for discussions on this point.

- two-loop diagrams involving a closed subloop of sfermions (stops, sbottoms, staus, and tau-sneutrinos)
- two-loop diagrams involving a closed subloop of charginos and/or neutralinos

These contributions constitute the class of two-loop contributions to a_μ , where a supersymmetric loop is inserted into a SM (or more precisely a two-Higgs-doublet model) one-loop diagram.

These diagrams are particularly interesting since they can depend on other parameters than the supersymmetric one-loop diagrams and can therefore change the qualitative behaviour of the supersymmetric contribution to a_μ . In particular, they could even be large if the one-loop contribution is suppressed, e.g. due to heavy smuons and sneutrinos.

Calculational details and remarks to the regularization and the γ_5 problem can be found in Refs.^{9,10,11}. Essentially we evaluate the two-loop and corresponding counterterm diagrams using standard large mass expansion and integral reduction techniques. A major difficulty stems from the large number of different mass scales and the involved structure of the MSSM Feynman rules.

2 Parameter dependence and discussion

The results for the supersymmetric contributions to a_μ are functions of all MSSM parameters. The values of the MSSM parameters are unknown, but the parameter space is strongly restricted by several experimental constraints. It turns out the parameter dependence and the corresponding phenomenological discussion shows important differences between the sfermion and the chargino/neutralino loop contributions.

- The sfermion loop contributions depend on the Higgs sector parameters μ and $\tan\beta$ and the sfermion mass parameters in a rather complicated way. It also turns out that experimental constraints on the MSSM parameter space significantly restrict the possible sfermion loop contributions⁹.
- In contrast, the chargino/neutralino loop contributions depend on μ , $\tan\beta$ and the gaugino mass parameter M_2 in a quite straightforward way, and experimental constraints on the parameter space have not much impact¹⁰.

2.1 Sfermion contributions

In Fig. 1 we show the full results for the sfermion contributions as functions of the lightest sfermion mass for universal sfermion mass parameters. They

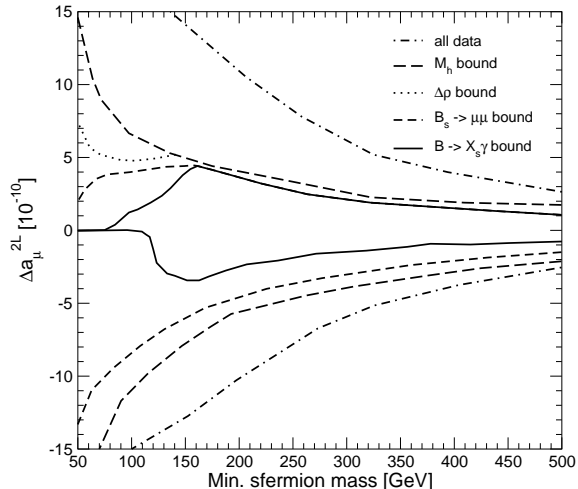


Figure 1: Maximum contributions of the two-loop diagrams with a closed sfermion loop to a_μ as a function of the lightest sfermion mass. No constraints on the MSSM parameters are taken into account for the outermost curve. Going to the inner curves additional constraints (see text) have been applied.

are obtained from a scan over the supersymmetric parameter space and display clearly the impact of taking into account experimental constraints on the parameter space.

The outer lines show the maximum possible results for $\tan\beta = 50$ if all MSSM mass parameters are varied universally up to 3 TeV (for the \mathcal{CP} -odd Higgs-boson mass we use $M_A > 150$ GeV) ignoring all experimental constraints on the parameter space. The next lines show the maximum possible results if only parameter points are used that are in agreement with the experimental limit on M_h ^b. As indicated above we find that the maximum results are drastically reduced. For a lightest sfermion mass of 100 GeV, the results are reduced from more than 15×10^{-10} to about 5×10^{-10} . The inner lines correspond to taking into account more experimental constraints on $\Delta\rho$, $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ and $\text{BR}(B \rightarrow X_s\gamma)$. They reduce the maximum contributions further.

As discussed in detail in Refs.^{9,11}, the restriction of universal sfermion mass parameters used in Fig. 1 is indirectly responsible for the significant impact of the M_h -bound. If the ratio between sbottom and stop masses is very large and one stop mass remains light, larger contributions to a_μ become possible without violating the experimental bounds.

^bFor a full list of references on the experimental constraints see Ref.⁹.

2.2 Chargino/neutralino contributions

The chargino/neutralino two-loop contributions have a more straightforward parameter dependence. They depend on $\tan\beta$ and the mass parameters for the Higgsinos, μ , the gauginos, M_2 , and the \mathcal{CP} -odd Higgs boson, M_A . For the simple case that all these mass parameters are equal to a common mass scale M_{SUSY} , we obtain the approximation

$$a_\mu^{\chi,2\text{L}} = 11 \times 10^{-10} \frac{(\tan\beta/50) \text{sign}(\mu)}{(M_{\text{SUSY}}/100 \text{ GeV})^2}. \quad (3)$$

If all the masses are even equal to the smuon and sneutrino masses, this formula can be immediately compared to the one-loop contributions (2). In this case the chargino/neutralino two-loop contributions amount to about 2% of the one-loop contributions.

If the smuon and sneutrino masses are heavier than the chargino and neutralino masses, the one-loop contributions are suppressed and the two-loop contributions can have a larger impact. Fig. 2 shows the sum $a_\mu^{\text{SUSY},1\text{L}} + a_\mu^{\chi,2\text{L}}$ in comparison to the one-loop result $a_\mu^{\text{SUSY},1\text{L}}$ alone as a contour plot in the μ - M_2 -plane. The smuon and sneutrino masses are fixed to 1 TeV and $\tan\beta = 50, 25$, $M_A = 200$. We find that in this case the two-loop corrections from the chargino/neutralino loop diagrams can modify the $1\sigma, 2\sigma, \dots$ contours significantly.

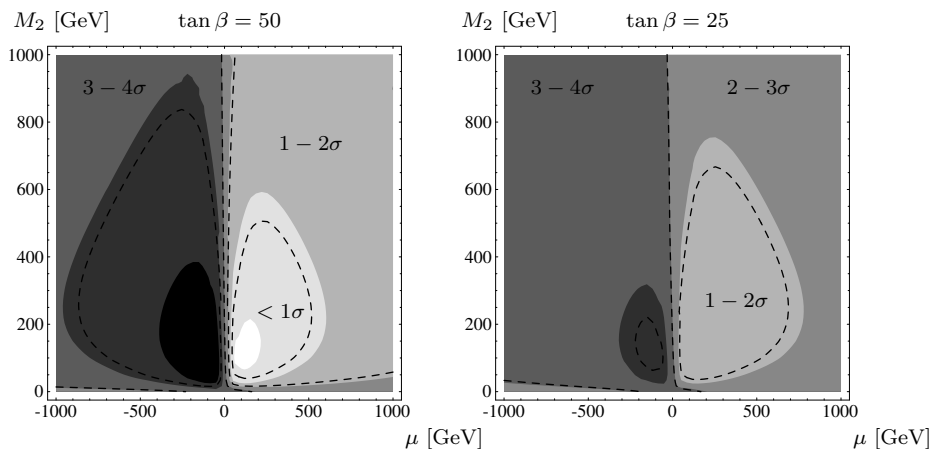


Figure 2: Contour plots of $a_\mu^{\text{SUSY},1\text{L}} + a_\mu^{\chi,2\text{L}}$ (fully drawn areas) and $a_\mu^{\text{SUSY},1\text{L}}$ (dashed contours) in the μ - M_2 -plane. In the left plot we choose $\tan\beta = 50$, in the right plot $\tan\beta = 25$. The borders of the regions and the contours correspond to $1\sigma, 2\sigma, \dots$ deviation from the observed value according to eq. (1).

3 Outlook

Supersymmetric contributions to a_μ could easily account for the observed $(20 \dots 30) \times 10^{-10}$ deviation between SM theory and experiment. Conversely, the precision of the experiment places stringent bounds on the MSSM parameter space.

The two-loop contributions presented here can substantially modify the supersymmetric one-loop contribution, and their knowledge reduces the theoretical uncertainty of the supersymmetric prediction for a_μ . Apart from the magnitude of these contributions (of order $0.5 \dots 1\sigma$), it is interesting how significantly the experimental constraints on the supersymmetric parameter space influence the possible results.

In Refs.^{9,10} also the SM/two-Higgs-doublet model-like contributions of two-loop diagrams with fermion loops and with purely bosonic loops have been computed. The difference of the diagrams in the MSSM and the SM is smaller than 1×10^{-10} . The remaining task is to complete the full two-loop calculation of a_μ in the MSSM. The missing diagrams are the two-loop corrections to the supersymmetric one-loop diagrams with smuon or sneutrino exchange. In order to calculate them, the full one-loop renormalization of the MSSM will be needed.

1. [The Muon $g-2$ Collaboration], *Phys. Rev. Lett.* **92** (2004) 161802.
2. M. Davier, S. Eidelman, A. Höcker and Z. Zhang, *Eur. Phys. J. C* **31** (2003) 503.
3. K. Hagiwara, A. Martin, D. Nomura and T. Teubner, *Phys. Rev. D* **69** (2004) 093003.
4. S. Ghozzi and F. Jegerlehner, *Phys. Lett. B* **583** (2004) 222.
5. J. de Troconiz and F. Yndurain, hep-ph/0402285.
6. M. Knecht and A. Nyffeler, *Phys. Rev. D* **65** (2002) 073034; M. Knecht, A. Nyffeler, M. Perrottet and E. De Rafael, *Phys. Rev. Lett.* **88** (2002) 071802; I. Blokland, A. Czarnecki and K. Melnikov, *Phys. Rev. Lett.* **88** (2002) 071803; M. Ramsey-Musolf and M. Wise, *Phys. Rev. Lett.* **89** (2002) 041601; J. Kuhn, A. Onishchenko, A. Pivovarov and O. Veretin, *Phys. Rev. D* **68** (2003) 033018.
7. K. Melnikov and A. Vainshtein, hep-ph/0312226.
8. T. Moroi, *Phys. Rev. D* **53** (1996) 6565 [Erratum-ibid. **D 56** (1997) 4424].
9. S. Heinemeyer, D. Stöckinger and G. Weiglein, *Nucl. Phys. B* **690** (2004) 62.
10. S. Heinemeyer, D. Stöckinger and G. Weiglein, hep-ph/0405255.
11. D. Stöckinger, *Proceedings of "Loops&Legs2004"*, hep-ph/0406306.