

# Maximizing the spin correlation of top quark pairs produced at the Large Hadron Collider

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## Abstract:

The measurement of top quark spin correlation is an important tool for precise studies of top quark interactions. In this letter I construct a quantization axis maximizing the spin correlation at the LHC within the Standard Model. Using this axis a spin correlation of 48% or even more, on applying additional cuts, can be reached. This represents a significant improvement compared to the helicity bases studied thus far.

# 1. Introduction

At the Large Hadron Collider (LHC) at CERN a huge number of top quark pairs will be produced. In the low luminosity run, production of around 8 million top quark pairs per year can be anticipated. This large number of top quarks allows very precise measurements in the top sector. In particular, we can verify to high accuracy that the top quark has indeed the quantum numbers predicted by the Standard Model. Furthermore, given the high energy scale involved in top quark reactions, top quark physics is also an ideal laboratory to search for new physics. For example, we may search for new  $s$ -channel resonances which may couple strongly to the top quark. To study the properties of such a hypothetical resonance the top quark spin correlation is a suitable tool. In particular, this may help to disentangle the nature of the intermediate resonance. It is important to keep in mind that the top quark is unique among the quarks because it decays before it can hadronize [1]. The spin information is thus not diluted by hadronization. In the Standard Model where the top decays predominantly via the parity violating weak interaction, the spin information is transferred to the angular distribution of the decay products. The top polarization is thus a ‘good observable’ in the sense that it is experimentally accessible through a detailed study of the decay products [2]. The aforementioned spin correlation of top quark pairs can be defined by [3]

$$C = \frac{\sigma_{t\bar{t}}(\uparrow\uparrow) + \sigma_{t\bar{t}}(\downarrow\downarrow) - \sigma_{t\bar{t}}(\uparrow\downarrow) - \sigma_{t\bar{t}}(\downarrow\uparrow)}{\sigma_{t\bar{t}}(\uparrow\uparrow) + \sigma_{t\bar{t}}(\downarrow\downarrow) + \sigma_{t\bar{t}}(\uparrow\downarrow) + \sigma_{t\bar{t}}(\downarrow\uparrow)}, \quad (1.1)$$

where  $\sigma_{t\bar{t}}(\uparrow/\downarrow \uparrow/\downarrow)$  denotes the cross section for the production of a top quark pair ( $+X$ , beyond leading-order) with spins up or down with respect to a specific quantization axis. In fact, given that already in the Standard Model the spins of the top and antitop are correlated, the spin correlation is also an interesting observable to test the details of Standard Model top quark interactions with high accuracy. The main production processes in the Standard Model for top quark pair production in hadronic collisions are the quark-antiquark annihilation process and the gluon fusion process. While the first dominates top quark pair production at the Tevatron, the latter dominates top quark pair production at the LHC. For top quark pairs produced in quark-antiquark annihilation it is well known that an optimal quantization axis exists — the so-called ‘off-diagonal’ axis — for which the top spins are 100% correlated [4]. Given that at the Tevatron roughly 80% of the top quark pairs are produced in quark-antiquark annihilation it is thus sufficient to choose this axis to obtain a large value for the spin correlation. For the gluon process — as we will see later — no such optimal quantization axis exists. Although no optimal axis exists it is still useful to find an axis for which the correlation is at least ‘maximal’. Such an axis might be used to improve the significance with which the spin correlation can be established at the LHC. In this letter I describe the construction of such an axis in detail. Note that in the following I will restrict myself to the top quark final state. Details on how to measure the spin correlation at the level of the observable decay products can be found for example in [5, 6, 7, 3, 8].

## 2. Maximizing the spin correlation

In this section I discuss how to maximize the top quark spin correlation in the Standard Model. To study spin effects in quantum mechanics a convenient tool is the spin density matrix at the parton level [9]. The most general form of the spin density matrix  $\rho$  for top quark pair production is given by

$$\rho = A \mathbb{1} \otimes \mathbb{1} + B_i^t \sigma_i \otimes \mathbb{1} + B_i^{\bar{t}} \mathbb{1} \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j \quad (2.1)$$

where  $\sigma_i$  are the Pauli matrices. The operator  $\frac{\sigma_i}{2} \otimes \mathbb{1}$  ( $\mathbb{1} \otimes \frac{\sigma_i}{2}$ ) denotes the spin operator of the top (anti-)quark in its rest frame. The different contributions to the spin density matrix have a very simple interpretation. The first contribution is essentially nothing but the differential cross section for top quark pair production at the parton level:

$$\frac{d\sigma_{t\bar{t}}}{d\cos(\vartheta)} = \frac{\beta}{8\pi s} A, \quad (2.2)$$

where  $s$  describes the partonic center of mass energy and  $\beta$  is the velocity of the top quark in the partonic center of mass system. The scattering angle of the top quark with respect to the beam is given by  $\vartheta$ . The second and third terms in Eq. (2.1) describe the polarization of the top and antitop quark. The last term may parameterize a correlation between the spins of the top and the antitop. Note that a non-vanishing  $C_{ij}$  does not necessarily mean that the spins are correlated. Only in the absence of polarization does a non-vanishing  $C_{ij}$  directly signify spin correlation. The spin density matrix as given in Eq. (2.1) above is not normalized. This has to be taken into account when calculating the expectation values of spin observables:

$$\langle O \rangle_\rho = \frac{\text{Tr}[O\rho]}{\text{Tr}[\rho]} = \frac{\text{Tr}[O\rho]}{\int d\text{Lips} 4A}. \quad (2.3)$$

Note that taking the trace also includes a phase space integration  $\int d\text{Lips}$  over the lorentz-invariant phase space. If the interaction responsible for the production of the top quark pairs satisfies additional symmetries, the explicit form of the spin density matrix can be further constrained [9]. For top quark pair production at a hadron collider, where the responsible interaction is QCD, we can immediately conclude that at leading-order no polarization is allowed ( $B_i^t = B_i^{\bar{t}} = 0$ ) due to the parity invariance of QCD. At the one-loop level, a tiny polarization transverse to the scattering plane is induced by absorptive parts [10]. Also the explicit form of the matrix  $C$  is constrained by the symmetries of QCD [9]:

$$C_{ij} = c_0 \delta_{ij} + \hat{p}_i \hat{p}_j c_4 + \hat{k}_i \hat{k}_j c_5 + (\hat{k}_i \hat{p}_j + \hat{p}_i \hat{k}_j) c_6. \quad (2.4)$$

Here  $\hat{\mathbf{p}}$  is the direction of the incoming beam and  $\hat{\mathbf{k}}$  is the direction of the outgoing top quark. Other structures one could think of, for example

$$\epsilon_{ijk} (c_1 \hat{p}_k + c_2 \hat{k}_k + c_3 \hat{n}_k), \quad (2.5)$$

where  $\hat{\mathbf{n}}$  is given by

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{p}} \times \hat{\mathbf{k}}}{|\hat{\mathbf{p}} \times \hat{\mathbf{k}}|}, \quad (2.6)$$

are forbidden in QCD due to discrete symmetries. In leading-order QCD the spin density matrix is thus completely determined by the functions  $A, c_0, c_4, c_5, c_6$ . For quark-antiquark annihilation they are given by [9]:

$$A^q = \kappa_q (2 - (1 - z^2)\beta^2), \quad (2.7)$$

$$c_0^q = -\kappa_q (1 - z^2)\beta^2, \quad (2.8)$$

$$c_4^q = 2\kappa_q, \quad (2.9)$$

$$c_5^q = 2\kappa_q \left( (1 - z^2)\beta^2 + 2z^2 \left[ 1 - \sqrt{1 - \beta^2} \right] \right), \quad (2.10)$$

$$c_6^q = -2\kappa_q z \left( 1 - \sqrt{1 - \beta^2} \right), \quad (2.11)$$

with

$$\kappa_q = \pi^2 \alpha_s^2 \frac{N^2 - 1}{N^2} \stackrel{N=3}{=} \frac{8}{9} \pi^2 \alpha_s^2, \quad (2.12)$$

where  $\alpha_s$  is the QCD coupling constant and  $N$  denotes the number of colours. For the gluon fusion process the functions  $A, c_0, c_4, c_5, c_6$  are given by [9]:

$$A^g = 2\kappa_g (1 + 2\beta^2(1 - \beta^2)(1 - z^2) - z^4\beta^4), \quad (2.13)$$

$$c_0^g = -2\kappa_g (1 - 2\beta^2 + 2(1 - z^2)\beta^4 + z^4\beta^4), \quad (2.14)$$

$$c_4^g = 4\kappa_g (1 - z^2)\beta^2, \quad (2.15)$$

$$c_5^g = 4\kappa_g \beta^2 \left( -2z^2(1 - z^2)\sqrt{1 - \beta^2} + 2(z^2 + \beta^2)(1 - z^2) - 1 + \beta^2 z^4 \right), \quad (2.16)$$

$$c_6^g = -4\kappa_g z (1 - z^2)\beta^2 \left( 1 - \sqrt{1 - \beta^2} \right), \quad (2.17)$$

with

$$\kappa_g = \frac{\pi^2 \alpha_s^2}{(1 - z^2\beta^2)^2} \frac{(N^2 - 2 + N^2 z^2 \beta^2)}{N(N^2 - 1)} \stackrel{N=3}{=} \frac{1}{24} \pi^2 \alpha_s^2 \frac{7 + 9z^2\beta^2}{(1 - z^2\beta^2)^2}. \quad (2.18)$$

In the absence of polarization, the spin correlation at the parton level as defined in Eq. (1.1) is just given by

$$C = 4 \langle (\mathbf{a} \cdot \mathbf{s}_t)(\mathbf{b} \cdot \mathbf{s}_{\bar{t}}) \rangle = \frac{\int \text{Lips } a_i C_{ij} b_j}{\int \text{Lips } A}, \quad (2.19)$$

where the quantization axis of the (anti)top quark is described by the normalized vector  $\mathbf{a}$  ( $\mathbf{b}$ ). It is now clear how one can maximize the spin correlation: just determine the maximal eigenvalue of the matrix  $C$  and choose  $\mathbf{a}$  and  $\mathbf{b}$  equal to the corresponding eigenvector, including an additional sign if the eigenvalue is negative. Note that the matrix  $C$  is symmetric so that this procedure can always be carried out. Without loss of generality we may choose for the moment a coordinate frame in which  $\hat{\mathbf{p}}$  and  $\hat{\mathbf{k}}$  are given by:

$$\hat{\mathbf{p}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{k}} = \begin{pmatrix} \sqrt{1 - z^2} \\ 0 \\ z \end{pmatrix}, \quad (2.20)$$

with  $z = \cos(\vartheta)$ . Using this specific coordinate frame the matrix  $C$  reads:

$$C = \begin{pmatrix} c_0 + (1 - z^2)c_5 & 0 & z\sqrt{1 - z^2}c_5 + \sqrt{1 - z^2}c_6 \\ 0 & c_0 & 0 \\ z\sqrt{1 - z^2}c_5 + \sqrt{1 - z^2}c_6 & 0 & c_0 + c_4 + z^2c_5 + 2zc_6 \end{pmatrix}. \quad (2.21)$$

It is straightforward to determine the eigenvalues:

$$c_0, c_0 + \frac{1}{2}c_4 + \frac{1}{2}c_5 + zc_6 \pm \frac{1}{2}\sqrt{c_5^2 + c_4^2 + 4c_6^2 - 2c_4c_5 + 4zc_5c_6 + 4zc_4c_6 + 4z^2c_5c_4}. \quad (2.22)$$

The corresponding eigenvectors are given by

$$\mathbf{e}_1 = \hat{\mathbf{p}} \times \hat{\mathbf{k}}, \quad (2.23)$$

and

$$\begin{aligned} \mathbf{e}_\pm &= \left( \frac{1}{2}c_4 - \frac{1}{2}c_5 \pm \frac{1}{2}\sqrt{c_5^2 + c_4^2 + 4c_6^2 - 2c_4c_5 + 4zc_5c_6 + 4zc_4c_6 + 4z^2c_5c_4} \right) \hat{\mathbf{p}} \\ &+ (zc_5 + c_6)\hat{\mathbf{k}} \end{aligned} \quad (2.24)$$

Note that the eigenvectors are not normalized to one. The only thing that remains to be done is to determine which of the eigenvalues is the largest. Clearly this will depend on the initial state. For the quark-antiquark annihilation process the largest eigenvalue in the entire kinematical region is given by the one where the square root enters with a plus sign. Using the explicit form for  $c_0^q - c_6^q$  I find

$$\begin{aligned} \lambda_{\max}^q &= c_0^q + \frac{1}{2}c_4^q + \frac{1}{2}c_5^q + zc_6^q \\ &+ \frac{1}{2}\sqrt{c_5^{q2} + c_4^{q2} + 4c_6^{q2} + 4zc_5^qc_6^q - 2c_4^qc_5^q + 4zc_4^qc_6^q + 4z^2c_5^qc_4^q} \\ &= A^q. \end{aligned} \quad (2.25)$$

I thus reproduce the well known result that the maximal axis yield 100% spin correlation for the quark-antiquark annihilation sub-process [4]. The corresponding eigenvector is than given by

$$\mathbf{e}_+^q \sim \frac{\mathbf{p} + (\gamma - 1)z\hat{\mathbf{k}}}{\sqrt{1 + z^2(\gamma^2 - 1)}} \quad (2.26)$$

in agreement with ref. [4]. Given that at the Tevatron most of the top quark pairs are produced in quark-antiquark annihilation this axis will produce an almost optimal value for the spin correlation. At the LHC, as mentioned earlier, gluon fusion is the dominant process. Unfortunately, for the gluon channel no such compact expression for the axis maximizing the spin correlation exists. Nevertheless, the axis can be constructed on an event by event basis. To do so one first calculates the eigenvalues of the  $C$  matrix for the gluon fusion process for the event. One then determines which one has the largest absolute value. The quantization axis is then given by the corresponding eigenvector to that eigenvalue. If the eigenvalue is negative one introduces an additional sign in the quantization axis of the top or the antitop quark. The quantization bases constructed in this way will yield an ‘optimal’ value for the spin correlation at the LHC. By explicitly calculating the eigenvalues in terms of  $z$  and  $\beta$  one can also show that none of them is equal to  $A_g$ . This implies that for the gluon fusion process no optimal axis for which the spins are 100% correlated exists.

### 3. Numerical results

In this section I present results for the spin correlation at the LHC using the maximal axis derived above. At the LHC it is known that QCD corrections do not significantly change the spin correlation [11], therefore I will only discuss leading-order predictions in what follows. As input I use  $m_t = 178$  GeV. Note that the spin correlation only depends on the QCD coupling constant  $\alpha_s$  through the parton distribution functions. There is no explicit dependence on  $\alpha_s$ . For the parton distribution functions I use CTEQ6.1L [12]. The factorization scale  $\mu_f$  is set to  $\mu_f = m_t$ . The results are shown in Table 1. Using the proposed axis a spin cor-

$C_{\text{hel}}$	$C_{\text{max}}$	$C_{\text{hel}}$ with cut	$C_{\text{max}}$ with cut
0.318	0.484	0.453	0.502

Table 1: Top quark spin correlation at the LHC using different quantization axes.

relation of almost 50% is obtained. It is known [13, 14] that applying an additional cut on the  $t\bar{t}$  invariant mass can improve the observed spin correlation significantly. In the two last columns the influence of a cut,

$$(k_t + k_{\bar{t}})^2 < 550 \text{ GeV}, \quad (3.1)$$

is studied. A further increase of the correlation is observed, although in that case it might be easier to use the helicity bases. Given that the spin correlation is defined as a ratio of two cross sections it can be expected that the factorization scale dependence cancels to a large extent. Indeed varying the factorization scale from  $\mu_f = m_t/10$  to  $\mu_f = 10m_t$  the value for  $C_{\text{max}}$  changes only from 50.2% to 46.6%. The scale dependence could be reduced further by including the next-to-leading order corrections [11].

### 4. Conclusion

In this letter I constructed a quantization axis for which the spin correlation of top quark pairs produced by gluon fusion is maximal. Given that around 90% of the top quark pairs at the LHC are produced via gluon fusion, this axis will yield an almost maximal value at the LHC. In leading-order using the CTEQ6.1L the proposed axis yields a spin correlation of 48%. An additional cut on the  $t\bar{t}$  invariant mass can be used to further increase the correlation. Using  $(k_t + k_{\bar{t}})^2 < 550$  GeV increases the spin correlation  $C_{\text{max}}$  by 2%. The use of the proposed axis is an important improvement which might help to establish top quark spin correlation at the LHC.

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see also <http://agenda.cern.ch/fullAgenda.php?ida=a044275>.