Deflections of cosmic rays in a random component of the Galactic magnetic field

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We express the mean square deflections of the ultra-high energy cosmic rays (UHECR) caused by the random component of the Galactic magnetic field (GMF) in terms of the GMF power spectrum and use recent measurements of the GMF spectrum to estimate them quantitatively. We find that deflections due to the random field typically constitute $0.03-0.3$ of the deflections which are due to the regular component and depend on the direction on the sky. They are small enough not to preclude the identification of UHECR sources, but large enough to be detected in the new generation of UHECR experiments.

I. INTRODUCTION

The Galactic magnetic field (GMF) plays an important role in the propagation of cosmic rays even at highest energies. Expected deflections — of order few degrees or larger — are comparable or exceed the angular resolution of the existing cosmic ray experiments. Such deflections may therefore be observable. Their understanding is crucial when searching for sources of the highest-energy cosmic rays if the latter are charged particles.

The detailed study of deflections of ultra-high energy proton primaries in the GMF is of importance provided the deflections in extra-galactic magnetic fields are small. According to the results of Refs. [1] this is likely to be the case (see, however, Ref. [2]).

The Galactic magnetic field has been shown to have both regular and turbulent components. The regular component is thought to have a spiral structure reminiscent of the Galactic arms with one or more reversals toward inner (and probably also outer) Galaxy and the magnitude of order 3 μG in the vicinity of the Earth [3]. This means that protons with energy 4×10^{19} eV can be deflected in the regular GMF by $\sim 5°$. There are indications that such a coherent deflections may indeed be present [4, 5, 6, 7] in the cosmic ray data.

The random component of GMF causes the spread of arrival directions of UHECR around the mean position, thus diluting (and potentially destroying) the information about the actual location of the source. Under certain conditions on the magnetic field it may also lead to the "lensing" of cosmic rays [8, 9] provided the number of sources contributing to the observed UHECR flux is small, as is favored by the statistics of clustering [11]. Both effects may give useful information on the cosmic rays and GMF itself.

Observationally, the magnitude of the random component of GMF is comparable to the magnitude of the regular one. However, the deflections of cosmic ray primaries in the random field depend upon the details of its power spectrum. If the correlation length L_c of the random component is much smaller than the propagation distance D, the deflections caused by the random field are proportional to $\sqrt{DL_c}$, see e.g. Ref. [12]. The deflections in the regular field are then much larger as they are proportional to the distance D instead.

If the correlation length diverges or is simply larger than the propagation distance, the deflections are no longer proportional to $\sqrt{DL_c}$. In fact, several measurements of the GMF power spectrum indicate that the correlation length is large or infinite in some directions on the sky. In those regions a dedicated study of UHECR deflections is necessary.

In this paper we study propagation of ultra-high energy cosmic ray primaries in the turbulent component of the Galactic magnetic field and obtain estimates of expected deflections using the observational knowledge of GMF. To this end we derive the relation between the mean square deflection and the power spectrum of the GMF fluctuations. The result is most conveniently represented through the factor R defined as

$$\frac{\delta_r}{\delta_u} = \frac{B_r}{B_{u,\perp}} R\,, \qquad (1)$$

where δ_r and δ_u are deflections in the random and uniform components of GMF, respectively, B_r is the rms value of a random magnetic field strength and $B_{u,\perp}$ is a projection of a uniform field onto direction orthogonal to the line of sight. The factor R varies between 0 and 1 and is expressed in terms of the power spectrum of the random field by Eq. (22). We show that existing observations give a typical value of R in our Galaxy in the range $R \sim 0.03-0.3$. This implies typical deflections of a 4×10^{19} eV proton in the random field of order $0.2° - 1.5°$ depending on the direction.

The paper is organized as follows. In Sect. II we recall the relations between the power spectrum and the correlation length and introduce the notations. In Sec. III we describe existing observations of the random magnetic field, in particular, the random-to-uniform ratio and the parameters of the power spectrum. In Sect. IV we turn to the deflections of UHECR in the random magnetic field and derive the expression for the coefficient R. Sect. V summarizes our results.

II. CORRELATION LENGTH AND POWER SPECTRUM OF GMF

To introduce notations, consider the statistical properties of a random magnetic field $B_a(\mathbf{r})$, where $a = 1, 2, 3$. We define the Fourier components of the magnetic field according to

$$B_a(\mathbf{r}) = \int d^3q B_a(\mathbf{q}) e^{i\mathbf{rq}}.$$

Here B_a refers to the fluctuating component of the total magnetic field; the regular part of GMF has to be treated separately. In what follows we assume that the fluctuations obey Gaussian statistics and are spatially homogeneous and isotropic.

The last two assumptions deserve a comment. The statistical characteristics of the magnetic field are different in different sky patches. Our analysis and results should be applied to each of these patches separately. We assume that statistical properties of GMF are (approximately) constant over a single patch. Also, within one patch the magnetic field fluctuations may not be isotropic, with the preferred direction being set by the regular component of GMF. The present data are not sufficient to establish or rule out the isotropy of GMF fluctuations. With the more precise data this assumption may need to be reconsidered, and the analysis may need to be refined.

With the above assumptions, all correlators of the magnetic field can be expressed in terms of the two-point correlation function, which can be written as

$$\langle B_a(\mathbf{q}) B_b^*(\mathbf{q}')\rangle = \frac{\mathcal{B}(q)}{2q^3}(\delta_{ab} - n_a n_b)\delta^3(\mathbf{q} - \mathbf{q}'), \qquad (2)$$

where $n_a = q_a/q$ is a unit vector in the direction of $\mathbf{q}$ and the projection tensor ensures the divergence-free nature of the magnetic field, $q_a B^a(\mathbf{q}) = 0$. The dimensionfull normalization factors are chosen in such a way that the power spectrum $\mathcal{B}(q)$ has physical units of B^2, i.e. in our case it is measured in the units of $(\text{Gauss})^2$. The correlation function of the magnetic field fluctuations is defined as

$$\begin{aligned} \xi(r) = & \langle B_a(\mathbf{r_0}) B^a(\mathbf{r_0} + \mathbf{r})\rangle = \int \frac{d^3q}{q^3}\, \mathcal{B}(q)\, e^{-i\mathbf{qr}} \\ = & 4\pi \int_0^\infty \frac{dq}{q}\mathcal{B}(q)\, \frac{\sin(qr)}{qr} \; . \end{aligned} \qquad (3)$$

It determines the rms value of the field amplitude B_r,

$$B_r^2 \equiv \langle B_a B^a \rangle = \xi(0) \; , \qquad (4)$$

and the correlation length L_c,

$$L_c \equiv \frac{\int_0^\infty dr\, \xi(r)}{\xi(0)} \; . \qquad (5)$$

The energy density contained in a random component of the magnetic field is related to the field variance as $\rho_B = B_r^2/8\pi$.

As will be discussed in Sect. III B, observations support the power-law behavior of power spectrum of the magnetic field fluctuations in a certain range of momenta,

$$\mathcal{B}(q) \propto \frac{1}{q^{\alpha-1}} \; . \qquad (6)$$

Since the energy density in the magnetic field is finite, there has to be a break in the pure power-law behavior, which can be parameterized as

$$\mathcal{B}(q) = \begin{cases} A\left(\dfrac{q_c}{q}\right)^{\alpha_1 - 1} & \text{at} \quad q < q_c \\ A\left(\dfrac{q_c}{q}\right)^{\alpha_2 - 1} & \text{at} \quad q > q_c \; , \end{cases} \qquad (7)$$

where A is a normalization constant and q_c is the momentum scale at which the break of the spectrum occurs. (Note that abrupt ultraviolet and infrared cut-off can be modeled as $\alpha_2 \to \infty$ and $\alpha_1 \to -\infty$, respectively.) The variance B_r converges if $\alpha_2 > 1$ and $\alpha_1 < 1$. Summing up contributions from both parts of the spectrum one finds

$$B_r^2 = 4\pi A\, \frac{\alpha_2 - \alpha_1}{(\alpha_2 - 1)(1 - \alpha_1)} \; . \qquad (8)$$

Finiteness of the correlation length requires stronger constraint, $\alpha_1 < 0$. One then has

$$L_c = \frac{\pi}{2q_c}\, \frac{(\alpha_1 - 1)(\alpha_2 - 1)}{\alpha_1 \alpha_2} \; . \qquad (9)$$

If $0 < \alpha_1 < 1$, the correlation length diverges at small momenta and is dominated by the largest possible distance scale in the problem. Deflections of cosmic rays are most significant in this case.

III. OBSERVATIONS OF RANDOM COMPONENT OF GMF

Current knowledge of the Galactic magnetic field is based on: (i) Faraday rotation measurements of Galactic and extragalactic radio sources, (ii) starlight polarization data, and (iii) observations of diffuse Galactic synchrotron emission. Different methods are sensitive to the magnetic field in regions with different physical conditions. Faraday rotation is sensitive to a field in a warm ionized medium, stellar polarization measurements sample the field in regions occupied by interstellar dust grains (e.g., neutral media), while synchrotron radiation originates from regions containing fast electrons. The volume occupied by the dust exceeds that of the warm ionized medium; therefore, the stellar polarization data are likely to be more close to the true volume average as compared to RM. The synchrotron data sample even larger volume.

Faraday rotation measure (RM) is sensitive to the projection of the magnetic field on the line of sight. The field direction along the line of sight is given by the sign of RM. The magnitude of the random field B_r can be estimated by analyzing the deviations of RM from the uniform field along different directions.

Stellar and synchrotron polarization data contain information about the field perpendicular to the line of sight. This property is convenient for our purposes since the plane-of-the-sky component of the magnetic field determines also the deflections of UHECR primary particles. The ratio of the amplitudes of the random to the uniform magnetic field components can be estimated along a single direction. To extract the power spectrum one has to study angular correlation function of the polarization data.

A. The relative strength of uniform and random fields

Synchrotron emission. The total magnetic field strength is related to the synchrotron emissivity, while the polarization of the Galactic diffuse synchrotron background offers a method for determining the ratio of uniform to random field strengths. Namely, the observed fractional polarization $p_{\rm obs}$ along a given direction obeys [13]

$$\frac{p_{\rm obs}}{p_{\rm max}} = \frac{B^2_{u,\perp}}{B^2_{u,\perp} + B^2_{r,\perp}} , \qquad (10)$$

where the subscript $\perp$ on B_u and B_r refers to the plane-of-the-sky components of uniform and random field, respectively. In Eq. (10) $p_{\rm max}$ is the fractional polarization that would be observed for a perfectly uniform field, $p_{\rm max} \approx 0.72$. The fractional polarization of $p_{\rm obs} \approx 35\%$ was found in Ref. [14] to be a typical maximum for our Galaxy. This implies

$$B_{u,\perp}/B_{r,\perp} \approx 1 . \qquad (11)$$

Note that this ratio depends upon direction. For instance, in the direction of the Galactic anti-center $p_{\rm obs} \approx 20\%$ (being averaged over $-20^\circ < b < 20^\circ$), which corresponds to $B_{u,\perp}/B_{r,\perp} \approx 0.62$. The typical coherence length was estimated in Ref. [14] to be less than 75 pc, while the distance to the region where polarized emission originates was found to be about ~ 0.5 kpc.

Starlight polarization. Polarization in starlight appears because of selective absorption by interstellar dust grains whose minor axis is aligned with the magnetic field $\mathbf{B}$. The same expression, Eq. (10), is valid for the starlight polarization data as well (with $p_{\rm max}$ being related to dust extinction). The resulting magnitude of the random component of magnetic field derived from the starlight polarization data is consistent with Eq. (11). For example, the estimate of Ref. [15] reads $B_{u,\perp}/B_{r,\perp} \approx 0.8$.

Faraday rotation. Unlike polarization data, the Faraday rotation measure is sensitive to the magnetic field component parallel to the line of sight. Another disadvantage of this method is that it does not allow to find the ratio of random to uniform components of the magnetic field along a given direction. However, this information can be extracted from the residuals of a fit to a uniform field provided RMs in many neighboring directions are known. For instance, such kind of study for a particular region of the sky of about $10^\circ \times 10^\circ$ centered at $(l, b) \approx (140^\circ, -40^\circ)$ was carried out in Ref. [16]. It was found that $B_r \approx B_u$, in agreement with Eq. (11). The power spectrum of magnetic field fluctuations was also determined. The scale of the break of the Kolmogorov turbulence, Eq. (7), was found to be $2\pi/q_c = (3.6 \pm 0.2)$ pc.

B. The power spectrum of magnetic field fluctuations

Three-dimensional power spectrum of the magnetic field fluctuations, $\mathcal{B}(q)$, is not measured directly. Instead, one measures the two-dimensional angular correlation function $K(\theta)$ of some physical observable (e.g., the intensity of the synchrotron radiation) or the corresponding two-dimensional power spectrum C_l. (The multipole l corresponds to a typical angular scale of $\theta = \pi/l$.) In the case of the power-law behavior, Eq. (6), the angular correlation functions K and C_l also follow power laws, $K \propto \theta^\beta$ and $C_l \propto l^{-\gamma}$. In the absence of observational systematic effects (e.g., Faraday depolarization and finite beam width) these exponents are related as

$$\begin{aligned} \beta &= \alpha, \\ \gamma &= \alpha + 2 , \end{aligned} \qquad (12)$$

provided the relevant integrals converge.

The momentum scale q_c at which the break in $\mathcal{B}(q)$ occurs is not observed directly either. Instead, one observes the break in the power-law behavior of $K(\theta)$ (or of C_l) at some angular scale θ_c. This scale q_c is estimated as

$$q_c = \frac{2\pi}{\theta_c D} , \qquad (13)$$

where D is the distance over which the magnetic field extends along a given line of sight. It is convenient to express the results in terms of the observable parameter θ_c.

Synchrotron emission. The statistical properties of polarized synchrotron emission depend upon direction on the sky and are different for different observables. In Ref. [17] the angular power spectra (APS) of the Parkes survey of the Southern Galactic plane at 2.4 GHz was analyzed. It was found that in the multipole range $40 < l < 250$ $(0.7^\circ < \theta < 5^\circ)$ the APS of the polarized intensity is fitted by $\gamma = 2.37$, while the power spectrum of E and B components of the polarized signal has the slope $\gamma \approx 1.5$, and the power spectrum of polarization angle corresponds to $\gamma \approx 1.7$. At the multipole order $l > 250$ ($\theta < 0.7^\circ$) the derived power spectra were affected by the beam cut-off. Similar results were found in Refs. [18, 19] for other Galactic latitudes. In particular, while being close to 1.5 on average, the slope of E and B components was found [18] to be in the range $1 < \gamma < 2.7$ depending on the particular region of the sky and the survey used in the multipole range $l < 1000$ ($\theta > 10'$).

Negative values of α (derived with the use of Eq.(12), if applicable) indicate that the correlation length of magnetic field exists and is small in many sky patches.

Starlight polarization. The angular power spectrum of the starlight polarization for the Galactic plane data ($|b| < 10^\circ$) is consistent with $\gamma \approx 1.5$ for all angular scales $\theta > 10'$ (or $l < 1000$), see Ref. [15].

Faraday rotation. Small-scale variations of the rotation measure of extragalactic radio sources were studied in Refs. [16, 20, 21]. Three different sky patches were considered in Ref. [20]. In two patches the index β of the correlation function of rotation measure was found to be consistent with zero (or slightly negative) on large angular scales $> 2^\circ$, while in the third positive β was observed. There is a clear drop in the correlation function on small angular scales $\theta < 0.1^\circ$, [21]. Therefore, the break in the spectrum has to be at angular scales $0.1^\circ < \theta_c < 2^\circ$ (θ_c cannot be quantified more precisely as there are no data points at these intermediate angular scales).

Fluctuations in electron density were factored out in Ref. [16] and the power spectrum of fluctuating magnetic field was determined for a particular region with previously mapped emission measure of warm ionized medium. The spectrum of random magnetic field derived in Ref. [16] can be parameterized by Eq. (7) with $A \approx 4.5 \times 10^{-2}\ \mu G^2$. At large scales the angular correlation function is consistent with the two-dimensional turbulence, $\alpha_1 = 2/3$, while at small scales $q > q_c$ the spectrum coincides with the Kolmogorov turbulence $\alpha_2 = 5/3$. The break in the spectrum occurs at $\theta_c \sim 0.07^\circ$ which corresponds to $2\pi/q_c \approx 3.6$ pc assuming $D = 3$ kpc. With the parameters found in Ref. [16], Eq. (8) gives $B_r \approx 1.6\ \mu G$. Note that for the uniform component of the magnetic field in the same region one has $B_u \approx 2.2\ \mu G$ and $B_{u,||} \approx -0.8\ \mu G$. The slope of $\alpha_1 = 2/3$ was measured up to $2\pi/q_c \sim 80$ pc. Thus, in this particular sky patch the correlation length of magnetic field fluctuations either diverges or is larger than 80 pc.

The steepening of APS at small angular scales was recently detected in high resolution studies of Refs. [22, 23] for several sky patches near the Galactic plane. The exponent γ for correlation function of RM derived in Ref. [22] is consistent with zero or slightly negative, while $\theta_c \approx 0.3^\circ$. However, in this case the synchrotron emission was observed at low frequencies, so that the observation length is small, $D \sim 600$ pc, resulting in $2\pi/q_c \approx 3.9$ pc, consistent with Ref. [16]. (Note that with the full propagation distance θ_c would have been smaller than 0.3°.) Finally, the parameters derived in Ref. [23] are $\gamma = 0.2$ and $\theta_c = 0.07^\circ$.

IV. UHECR DEFLECTIONS IN THE RANDOM MAGNETIC FIELD

In this section we show that the knowledge of the ratio $B_{u,\perp}/B_r$, the exponents α_1 and α_2 and the angular scale θ_c is sufficient to quantify the spread of deflections of UHECR primaries caused by the random component of GMF. As we have seen in Sect. III A, existing observations suggest that the magnitudes of the random and uniform components of GMF are comparable. In what follows it will be convenient to normalize the deflection due to the random field to the deflection δ_u which would occur in the uniform field over the same distance and at the same particle energy and charge. After traveling the distance D in a uniform magnetic field, a particle with the electric charge Ze and energy E is deflected by an angle

$$\delta_u = \frac{ZeD}{E} B_{u,\perp} \ . \tag{14}$$

This has to be compared to the mean square deflection angle δ_r in the random component of the Galactic magnetic filed.

A. Mean square deflection in terms of magnetic field power spectrum

Propagation and deflections of UHECR primaries by random magnetic field were studied in many papers, see e.g. Refs. [24, 25, 26, 27, 28, 29, 30, 31]. However, usually the main focus is the diffusive regime in the extra-galactic magnetic field. Deflections in turbulent component of GMF were studied in Refs. [8, 9], but with the emphasis on the possibility of magnetic field reconstruction with future high statistics cosmic ray data and assuming small correlation length. The turbulent component of GMF with a simplifying assumption of a cell-like structure was also included in Monte-Carlo simulations of Ref. [32]. To our knowledge, the ballistic regime of small deflections in the situation when the coherence length might be not small, relevant for the case of realistic GMF, was not studied in detail.

Propagation of UHECR primaries is quasi-rectilinear, with typical deflection angles not exceeding $10^\circ - 20^\circ$ even for lowest energies. The contribution of turbulent field in these deflections is expected to be even smaller. Therefore, a ballistic approximation gives a good description of UHECR propagation. In this regime, the deflection angles are characterized by the following line integrals,

$$\delta_i = \frac{Ze}{E} \int_0^D dz\ \epsilon_{ik} B_k(z) \ , \tag{15}$$

where the axis z is chosen along the particle trajectory and indices $i,k = 1,2$ label two orthogonal directions. The mean square deflections are

$$\delta_r^2 \equiv \langle \delta_i \delta^i \rangle = \frac{Z^2 e^2}{E^2} \iint_0^D dz\, dz'\ \epsilon_{ik}\epsilon_{jp} \langle B_k(z) B_p(z') \rangle \ . \tag{16}$$

Here the average is taken over the ensemble of different realizations of the turbulent magnetic field $B_a(x)$. For a statistically homogeneous random field the correlator in Eq. (16) is the function of $r = z' - z$

$$\epsilon_{ik}\epsilon_{jp} \langle B_k(z) B_p(z') \rangle = \xi_{11}(r) + \xi_{22}(r) \equiv \xi_\perp(r) \ , \tag{17}$$

where $\xi_{ii}(r) \equiv \langle B_i(z) B_i(z+r) \rangle$ (no summation over i). Using Eq. (2) which enforces the divergence-free constraint one finds

$$\xi_\perp(r) = 4\pi \int_0^\infty \frac{dq}{q} \mathcal{B}(q) \left[\frac{\sin(qr)}{qr} + \frac{\cos(qr)}{q^2 r^2} - \frac{\sin(qr)}{q^3 r^3} \right] .$$

This relation implies

$$\int_0^\infty dr\ \xi_\perp(r) = \frac{1}{2} \int_0^\infty dr\ \xi(r) \ . \tag{18}$$

Note that the assumption of chaotically oriented magnetic cells, which is often made, would give instead $\xi_\perp(r) = (2/3)\ \xi(r)$. However, this assumption is inconsistent with the divergence-free nature of the magnetic field, $q_a B^a(\mathbf{q}) = 0$.

Changing variables in Eq. (16) from z, z' to r and $u = (z + z')$ one obtains

$$\delta_r^2 = \frac{2 Z^2 e^2}{E^2}\ B_r^2 \int_0^D du \int_0^u dr\ \frac{\xi_\perp(r)}{\xi(0)} \ , \tag{19}$$

where Eq. (4) was used. It is convenient to represent the result as a ratio of rms deflections in random field to the deflection in the uniform field δ_u given by Eq. (14). Thus, we arrive at Eq. (1) where the dimensionless factor R is

$$R^2 \equiv \frac{2}{D^2} \int_0^D du \int_0^u dr\ \frac{\xi_\perp(r)}{\xi(0)} \ . \tag{20}$$

This factor varies between zero and one.

If the correlation length L_c defined by Eq. (5) is much smaller than the propagation distance D, the upper limit in the integral over r can be extended to infinity. One then finds

$$R^2 = \frac{L_c}{D} \ . \tag{21}$$

In the general case, the expression Eq. (20) can be brought to the form

$$R^2 = \frac{4\pi}{D \xi(0)} \int_0^\infty \frac{dq}{q^2} \mathcal{B}(q)\ f(Dq) \ , \tag{22}$$

where

$$f(x) = \mathrm{Si}(x) + \frac{\cos x}{x} - \frac{\sin x}{x^2} \ , \tag{23}$$

and $\mathrm{Si}(x) = \int_0^x dy\ \sin(y)/y$ is the integral sine function. At small arguments the function $f(x)$ grows linearly as $f(x) = 2x/3 + O(x^3)$, while at $x \gtrsim 2\pi$ it rapidly converges to the asymptotic value $\pi/2$.

B. Mean square deflections in the random component of GMF

In those Galactic regions where the correlation length exists and satisfies $L_c \ll D$, Eq. (21) can be used. In the case $0 < \alpha_1 < 1$ when the correlation length L_c defined by Eq. (5) diverges, or when $\alpha_1 < 0$ but the condition $L_c \ll D$ does not hold, Eq. (21) is not applicable. In these cases Eq. (22) has to be used instead. Assuming that the power-law spectrum of the turbulent magnetic field is given by Eq. (7) as supported by the existing observations, we obtain

$$R^2 = \frac{(\alpha_2 - 1)(1 - \alpha_1)}{(\alpha_2 - \alpha_1)} \times \tag{24}$$
$$\left[(Dq_c)^{\alpha_1 - 1} \int_0^{Dq_c} \frac{dy f(y)}{y^{1+\alpha_1}} + (Dq_c)^{\alpha_2 - 1} \int_{Dq_c}^\infty \frac{dy f(y)}{y^{1+\alpha_2}} \right] ,$$

where $f(y)$ is defined in Eq. (23). As one can see, the final result depends on the product Dq_c. Therefore, with the use of Eq. (13) it can be rewritten in terms of the single directly observable scale $\theta_c = 2\pi/Dq_c$.

In the case $Dq_c \gg \pi$ and $\alpha_1 < 0$ (when both integrals are saturated at $y = Dq_c$), we recover Eq. (21),

$$R = \sqrt{\frac{L_c}{D}} = \sqrt{\frac{\theta_c}{4}\ \frac{(\alpha_1 - 1)(\alpha_2 - 1)}{\alpha_1 \alpha_2}} \ . \tag{25}$$

For α_1 varying within $0.2 < \alpha_1 < 0.8$ and $\alpha_2 = 5/3$ (which corresponds to the Kolmogorov turbulence), there exists an approximate analytic expression for R which holds with an accuracy of about 10%,

$$R \approx (Dq_c)^{(\alpha_1 - 1)/2} = (\theta_c/2\pi)^{(1-\alpha_1)/2} \ . \tag{26}$$

In the general case the factor R has to be calculated numerically. Its dependence on α_1 for $\alpha_2 = 5/3$ in three cases $\theta_c = 6^\circ$, $\theta_c = 0.6^\circ$ and $\theta_c = 0.06^\circ$ is shown in Fig. 1 by the dotted, dashed and solid lines, respectively. We recall now that in many "low" resolution studies the steepening of APS is not detected, up to large multipoles, $l \sim 1000$. This suggests that $\theta_c < 10'$ and the dashed line in Fig. 1 should serve as a good upper limit for R.

The diamonds and bars on the same figure represent the coefficient R for the cases where the break in the APS was (possibly) detected, as discussed in Section III B. The corresponding survey regions are shown in Fig. 2 as

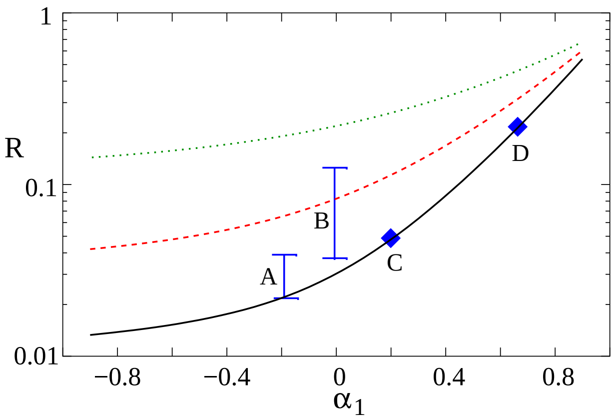

FIG. 1: The coefficient R as a function of α_1 is shown by dotted, dashed and solid curves for $\theta_c = 6°,\ 0.6°$ and $0.06°$ respectively. The data-points correspond to the APS derived for sky regions A-D, as discussed in the text. These regions are displayed in Fig. 2.

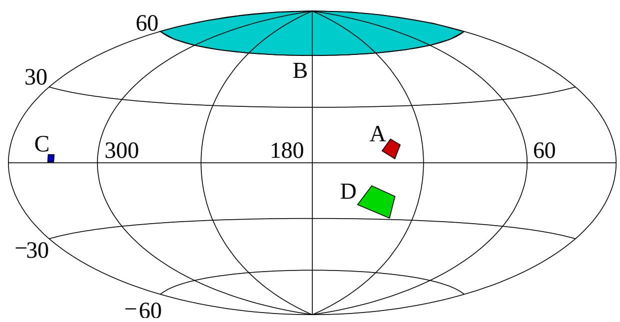

FIG. 2: Regions A-D in Galactic coordinates where the break in the APS was detected. Corresponding values of R for these regions are shown as data-points in Fig. 1.

colored patches A (Ref. [22]), B (Ref. [21]), C (Ref. [23]) and D (Ref. [16]).

Small observed values of (or upper limits for) θ_c indicate that either the scale $2\pi/q_c$ is small, or the extent of GMF along given direction, D, is large. In either case the resulting coefficient R is small, $0.02 < R < 0.2$.

V. CONCLUSIONS

Deflections of UHECR in the random component of the Galactic magnetic field are usually discussed in the limit when the correlation length exists and is much smaller than the propagation distance. However, avaliable GMF data suggest that these assumptions may not be valid uniformly all over the sky. We have calculated deflections in a more general approach which does not require correlation length to exist and relies directly on the power spectrum of GMF fluctuations. We have shown that the ratio of the deflections in the random and uniform components of GMF, Eq. (1), is expressed in terms of the factor R which depends on the spectrum of the magnetic field fluctuations as given by Eq. (22). We have calculated this factor for the power law spectrum with a single break, Fig. 1.

Using the measurements of the GMF power spectrum in the sky regions where it is available, we have shown that the deflections in the random component are small, $0.03 - 0.3$ of the deflections in the uniform field. This is sufficiently small not to preclude identification of sources of UHECR using methods of Refs. [4, 7]. For instance, the deflection of a proton with energy $E = 4 \times 10^{19}$ eV due to the random component of GMF is expected to be about $0.2° - 1.5°$. This is below the resolution of the AGASA experiment, but can be above the resolution of the HiRes detector in the stereo mode and the expected resolution of the Pierre Auger experiment. Thus, the detailed study of the random component of GMF is particularly important for the interpretation of data which will be collected by the new generation of UHECR experiments. To this end, the all-sky map of the essential parameters determining the power spectra of GMF is highly desirable. These maps may be obtained from the measurements of Faraday rotation and maps of diffuse polarized synchrotron Galactic emission. The wealth of relevant information is expected to be provided by the WMAP satellite.

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