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J. Proriol
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DEFINITION OF INVARIANTS IN 3-D

APPLICATION: STUDY OF THE NUMBER OF JETS IN E^+E^- EVENTS

J. PRORIOL
Laboratoire de Physique Corpusculaire
Clermont-Ferrand

Abstract. We define invariants for 3D objects with a spherical symmetry. These new invariants are used to study the number of jets in e^+e^- events.

1-INTRODUCTION

In pattern recognition, some 2D invariants were used to solve some rotation invariant problems[1].

But in e^+e^- events, the jets have a spherical 3D symmetry. The usual method to tag the number of jets is the clusterization using classical algorithms. An attempt to use a neural network was done by Jousset[3] using an Hopfield neural network.

In this paper we propose a set of 3D invariants which can be used to study the number of jets in e^+e^- events.

2-INVARIANTS IN 3D ROTATION

The angles used in 3D rotation are ϑ and ϕ; we know the spherical orthogonal functions $Y_{lm}(\vartheta,\phi)$[3].

A function $f(\vartheta,\phi)$ can be developped using $Y_{lm}(\vartheta,\phi)$ functions.

$$f(\vartheta,\phi)=\sum_l\sum_m \alpha_{lm}\, Y^*_{lm}(\vartheta,\phi) \quad .$$

If the axis are rotated with α,β,γ Euler angles the rotation angles become ϑ' and ϕ'. The new development is

$$f(\vartheta,\phi)=\sum_l\sum_m \alpha'_{lm}\, Y^*_{lm}(\vartheta',\phi') .$$

But we know a relation between ϑ,ϕ and ϑ',ϕ' angles[3].

$$Y_{lm}(\vartheta',\phi')=\sum_{m'} \mathcal{D}^l_{m'm}(\alpha,\beta,\gamma)\, Y_{lm'}(\vartheta,\phi),$$

where the $\mathcal{D}$ functions are the usual orthogonal functions with Euler angles.

The relation between α' and α is then:

$$\alpha'_{lm}=\sum_{m''}\mathcal{D}^{l}_{m''m}(\alpha,\beta,\gamma)\ \alpha_{lm''}.$$

Using the orthogonal relation[3]

$$\sum_{m}\mathcal{D}^{l*}_{m''m}(\alpha,\beta,\gamma)\ \mathcal{D}^{l}_{m'm}(\alpha,\beta,\gamma)=\delta_{m'm''},$$

we see that the combination of the α values

$$\beta(l)=\sum_{m}\alpha_{lm}\ \alpha^{*}_{lm}$$

is invariant for a 3D rotation.

The α parameters are computed with the relation

$$\alpha_{lm}=\int f(\vartheta,\phi)\ Y_{lm}(\vartheta,\phi)\ d\Omega,$$

using the associated Legendre functions $P^{l}_{m}(\cos\vartheta)$ we write for the α coefficients:

$$\alpha_{lm}=(-)^{m}\left[\frac{(2l+1)(l-m)!}{4\pi\ (l+m)!}\right]\int P^{l}_{m}(\cos\vartheta)e^{im\phi}\ f(\vartheta,\phi)\ d\Omega\ .$$

The contributions to $\beta(l)$ of opposite m values are identical.

3-SIMULATION OF 3 CLASSES

We have simulated 3 classes of different simple objects with dots on a sphere.

Classe 1:2 dots at $\vartheta_1=0,\phi_1=0$ and $\vartheta_2=\pi,\phi_2=0$.

Classe 2:3 dots at $\vartheta_1=0,\phi_1=0$;$\vartheta_2=2\pi/3,\phi_2=0$ and $\vartheta_3=2\pi/3,\phi_3=\pi$.

Classe 3:4 dots at $\vartheta_1=0,\phi_1=0$;$\vartheta_2=\pi,\phi_2=0$;

$\vartheta_3=\pi/2,\phi_3=0;\vartheta_4=\pi/2,\phi_4=\pi$.

We have verified the invariance of $\beta(l)$ values with random Euler angles.When we introduice a noise in the positions of the dots,the $\beta(0)$ value is discriminating for the classes,because

$$\beta(0)=\frac{1}{4\pi}(\sum_{dots}P^{0}_{0})^{2},$$

this value is independant of the noise.

4-THE NUMBER OF JETS IN e^+e^- EVENTS

In LEP detectors,the tracks of an hadronic event are given by the momentum p_i,the ϑ_i and ϕ_i angular positions.

We choose as f function the following relation:

$$f(\vartheta,\phi)=\sum_{i\ \text{tracks}} p_i\ \delta(\cos\vartheta-\cos\vartheta_i)\delta(\phi-\phi_i)$$

then

$$\alpha_{lm}=(-)^m\left[\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}\right]\sum_{i\ \text{tracks}} p_i P_m^l(\cos\vartheta_i)e^{im\phi_i}.$$

We have generated 2,3 and n jets events with the ALEPH Monte-Carlo simulation.The jets were clustered with the JADE algorithm using ycut=(6/E)**2.

The beta variables are interesting to study the shape of an e^+e^- event,but other shape variables have already been used in high energy physics[4,5].The sphericity product,the aplanarity,Fox-Wolfram coefficients can be used.

In our study,we have used 23 variables:sphericity product,aplanarity,Fox-Wolfram coefficients H40 and H50.A F-test method[5] applied to these variables,showed that their discriminating power for the study of the jets is important.To these 4 variables we have added 19 β_i coefficienits :i=0..18.

We have feeded a (23-23-8-3) MLP neural network:the inputs are the 23 variables,we use 2 hidden layers,the 3 outputs are the 3 classes:2 jets events,3 jets events,n jets events.

The learning was done with a set of 3*2400 events.The test was done on a set of 73376 events:48948 2 jets events,21385 3 jets events,3033 n jets events.

In the test,85.3% of the events were correcty classified.

The classification given by the MLP network gives:

The purity of a 2 jets sample is 95.9%,

The purity of a 3 jets sample is 75.3%,

The purity of a n jets sample is 41.6%.

The percentage of true 2 jets tagged as 2 jets is 89.6%,

The percentage of true 3 jets tagged as 3 jets is 75.4%,

The percentage of true n jets tagged as n jets is 85.0%.

In figure 1 we give the outputs of the neurones of the 4th layer.In figure 2,we give the curves of purity/efficiency[4] for the 3 classes of the problem.

5-CONCLUSION

We have computed a new family of 3D invariants.We have given some results of a study of the number of jets in e^+e^- events.

The study of the jets can be improved,because we have supposed that the JADE algorithm is perfect.

6-AKNOWLEDGMENTS

The simulation in high energy physics,were done with ALEPH full Monte-Carlo simulations.I would like to thank the physicists from ALEPH for the quality of the software and for their constant support.

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Figure captions

Figure 1

Outputs of the MLP neural network

1-2 jets events

2-3 jets events

3-n jets events

4-not 2 jets events

5-not 3 jets events

6-not n jets events

Figure 2

Purity versus efficiency

stars: 2 jets events

lozenge:3 jets events

circle: n jets events

Figure 1

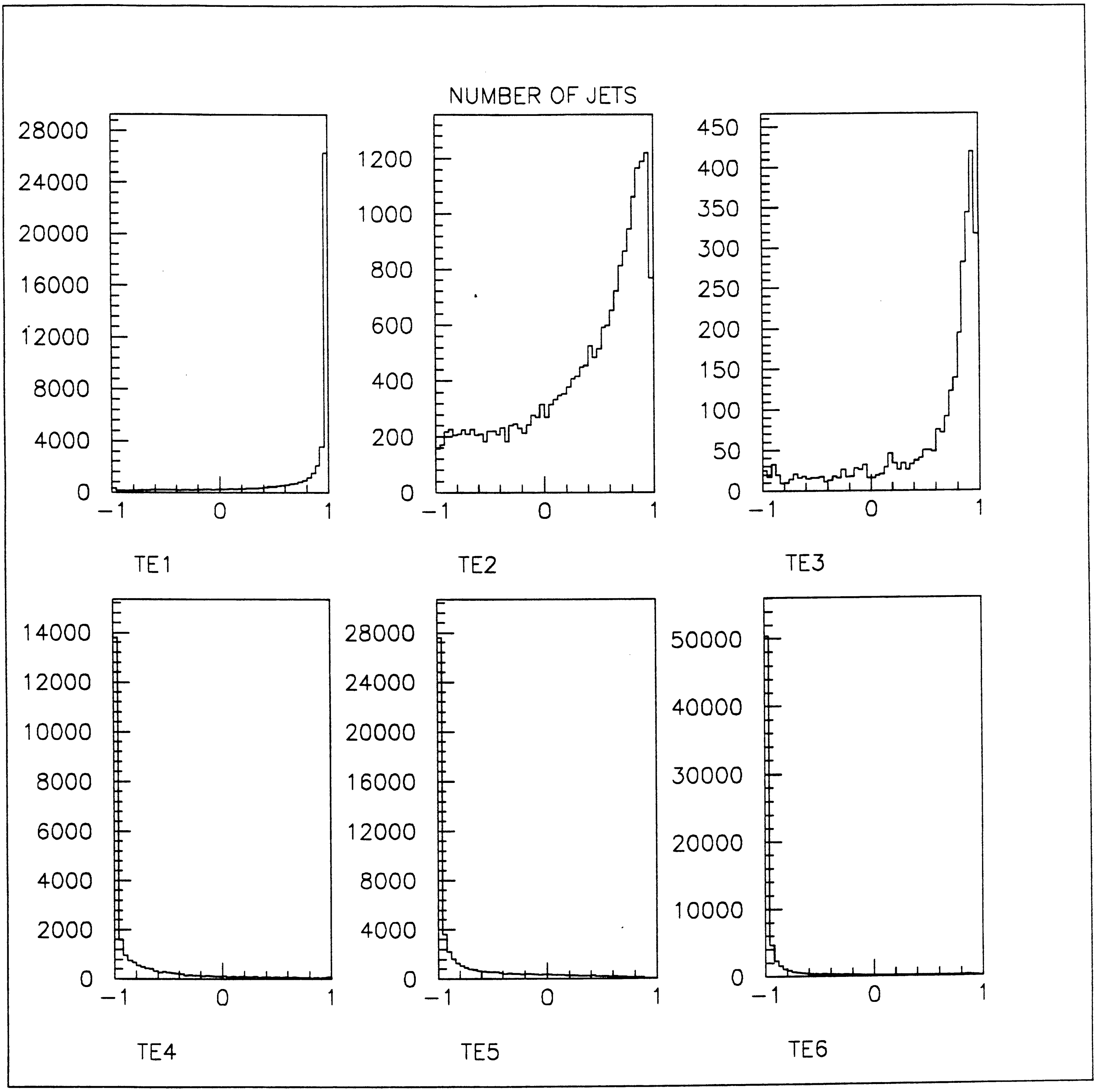
NUMBER OF JETS
28000
24000
20000
16000
12000
8000
4000
0
−1
0
1
TE1
1200
1000
800
600
400
200
0
−1
0
1
TE2
450
400
350
300
250
200
150
100
50
0
−1
0
1
TE3
14000
12000
10000
8000
6000
4000
2000
0
−1
0
1
TE4
28000
24000
20000
16000
12000
8000
4000
0
−1
0
1
TE5
50000
40000
30000
20000
10000
0
−1
0
1
TE6

Figure 2