ALEPH 93-177
PHYSIC 93-152
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16.11.1993

Search for $\mathcal{CP}$ – violation in the decay $Z^0 \to \tau^+\tau^-$

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November 10, 1993

Abstract

The data collected in the years 1990, 1991 and 1992 have been used to search for $\mathcal{CP}$ – violation in the decay of the Z^0 into $\tau^+\tau^-$. Considering the τ decay modes e, μ, π, ρ and a_1 a measurement of the weak dipole form factor of the τ – lepton was performed. No signal of $\mathcal{CP}$ – violation was found. The result is $d_\tau = (+0.13 \pm 0.60_{stat} \pm 0.25_{sys}) \cdot 10^{-17} ecm$ obtained with 19628 identified $\tau^+\tau^-$ – events. This gives an upper limit on the dipole form factor of $|d_\tau| < 1.4 \cdot 10^{-17} ecm$ at the 95% confidence level.

1 Introduction

The decay of the Z^0 – boson into $\tau^+\tau^-$ is an appropriate process to search for signals of $\mathcal{CP}$ – violation caused by new interactions. Since in the Standard Model $\mathcal{CP}$ – violation occurs only in the couplings of the charged current, $\mathcal{CP}$ – odd contributions to the reaction $Z^0 \rightarrow \tau^+\tau^-$ are at most of the order of 10^{-7}[1] compared to the electroweak amplitude at the Born level. On the other hand many extensions of the Standard Model, such as models with leptoquarks or with extended Higgs – sectors[2,3], but also left – right or supersymmetric interactions[4] can generate sizeable $\mathcal{CP}$ – violating effects at the $Z\tau\tau$ – vertex. The parametrisation of these contributions by the weak dipole form factor of the τ – lepton at the Z – resonance gives a model independent description of the on – shell amplitude, which in this case consists of the coherent sum of the electroweak amplitude and the amplitude proportional to the dipole form factor d_τ^Z.

In order to measure the dipole form factor we use the following $\mathcal{CP}$ – odd tensor observable [5,6]:

$$\hat{T}_{ij} = (\hat{\mathrm{p}}_+ - \hat{\mathrm{p}}_-)_i \cdot \frac{(\hat{\mathrm{p}}_+ \times \hat{\mathrm{p}}_-)_j}{|\hat{\mathrm{p}}_+ \times \hat{\mathrm{p}}_-|} \quad + \quad (i \longleftrightarrow j) \qquad \mathrm{i,j} = 1,2,3 \tag{1}$$

$\hat{\mathrm{p}}_+$, $\hat{\mathrm{p}}_-$ denote the normalized momentum vectors of the charged decay particle of τ^+ and τ^-, respectively, i, j are cartesian coordinates. The mean value of the distribution of every component $\hat{T}_{ij}$ is connected to the dipole form factor through the relation[5,6]:

$$\left\langle \hat{T}_{ij} \right\rangle_{AB} = \frac{M_Z}{e} \cdot \mathrm{d}_\tau^Z \cdot \hat{c}_{AB} \cdot s_{ij} \tag{2}$$

with $s_{ij} = \mathrm{diag}(-\frac{1}{6}, -\frac{1}{6}, \frac{1}{3})$. The constants $\hat{c}_{AB}$[1] depend in sign and magnitude explicitly on the decay mode of τ^- (A) and τ^+ (B). Therefore the greatest sensitivity will be obtained if the different event classes A – B are separated. This analysis uses the major decay modes e, μ, π, ρ and a_1, which have also sizeable sensitivities. The dependencies and the systematics of the constants $\hat{c}_{AB}$ will be discussed in detail in chapter 3.

As can be seen from the matrix s_{ij}, $\hat{T}_{33}$ is the most sensitive component and will give the largest signal if $d_\tau \neq 0$. Furthermore the systematic studies of the 33 – component are easier to be done (see chapter 4). Therefore only $\hat{T}_{33}$ is used in this analysis.

2 Event selection

The event selection is mainly based on the neural net method developped for the analysis of the τ – polarization. The neural net used for particle identification is described in [8,9] and the cuts applied to classify the hemispheres and to reject background are listed in detail in [10].

Modifications concern the e – e – class and the decay $a_1 \rightarrow 2\pi^0\pi$. In order to reintroduce the e – e – class the background rejection was changed in the case of both hemispheres being adjoined to the decay $\tau \rightarrow e\nu_e\nu_\tau$. The following cuts are applied, especially to suppress bhabha – background:

[1] In the following they are called sensitivities.

Event class	Selection-efficiency	Background from τ - events	Background from other processes
e - e	36.4 ± 0.6	2.8 ± 0.3	3.6 ± 1.2
e - μ	67.9 ± 0.4	3.2 ± 0.2	1.0 ± 0.3
e - π	39.3 ± 0.5	9.9 ± 0.5	0.3 ± 0.3
e - ρ	33.3 ± 0.4	7.4 ± 0.4	0.2 ± 0.2
e - 3π	41.7 ± 0.7	5.5 ± 0.5	0.2 ± 0.2
e - $2\pi^0\pi$	25.2 ± 0.6	26.8 ± 1.0	0.4 ± 0.3
$\mu - \mu$	60.3 ± 0.6	3.2 ± 0.2	2.5 ± 0.7
$\mu - \pi$	55.5 ± 0.5	8.9 ± 0.4	0.3 ± 0.2
$\mu - \rho$	42.3 ± 0.4	7.2 ± 0.3	< 0.1
$\mu - 3\pi$	50.4 ± 0.7	5.1 ± 0.4	0.3 ± 0.3
$\mu - 2\pi^0\pi$	32.8 ± 0.6	27.9 ± 0.9	0.3 ± 0.3
$\pi - \pi$	47.1 ± 1.0	14.3 ± 0.9	1.0 ± 0.7
$\pi - \rho$	36.4 ± 0.5	13.6 ± 0.5	0.2 ± 0.2
$\pi - 3\pi$	42.8 ± 0.8	13.7 ± 0.8	0.2 ± 0.2
$\pi - 2\pi^0\pi$	27.6 ± 0.7	32.3 ± 1.2	0.5 ± 0.5
$\rho - \rho$	28.3 ± 0.4	12.4 ± 0.6	0.2 ± 0.2
$\rho - 3\pi$	30.9 ± 0.5	10.8 ± 0.4	0.3 ± 0.2
$\rho - 2\pi^0\pi$	22.2 ± 0.5	30.3 ± 0.9	< 0.1

Table 1: *Efficencies and background with statistical errors for the event classes used in the analysis. All numbers are given in per cent.*

- $|\cos \vartheta_{thrust}| < 0.75$
- $\sum_{hemi}(E_{hemisphere}^{TPC+photons})/E_{cms} < 0.45$
- $\max(E_{hemisphere}^{TPC+photons}/E_{beam}) > 0.2$

The remaining background from bhabhas is smaller than 1 %. To identify the decay $\tau \rightarrow a_1 \rightarrow 2\pi^0\pi$ further mass cuts are applied:

- One $\pi\pi^0$ - or $\pi\gamma$ - combination should be compatible with M_ρ:
  $|M_{\pi\pi^0} - 0.77\, GeV/c^2| \leq 0.2\, GeV/c^2$
- The invariant mass of the $2\pi^0\pi$ - system should be compatible with the a_1 - mass:
  $|M_{invariant} - 1.27\, GeV/c^2| \leq 0.5\, GeV/c^2$

The resulting efficencies and the background fractions in all event classes, which are used in this analysis, are shown in table 1.

3 Systematic studies of the sensitivities $\hat{c}_{AB}$

In order to extract the dipole form factor from the measurement of the distributions of $\hat{T}_{33}$ the sensitivities $\hat{c}_{AB}$ should be known with adequate precision. For this purpose a Monte – Carlo – generator[11] exists, which also includes the $\mathcal{CP}$ – violating amplitude due to a nonvanishing weak dipole form factor[2], of which the magnitude can be chosen. According to relation (1.2) the sensitivities are computed from the mean value of the generated distributions of $\hat{T}_{33}$ with known $d_\tau \neq 0$.

Detector resolution

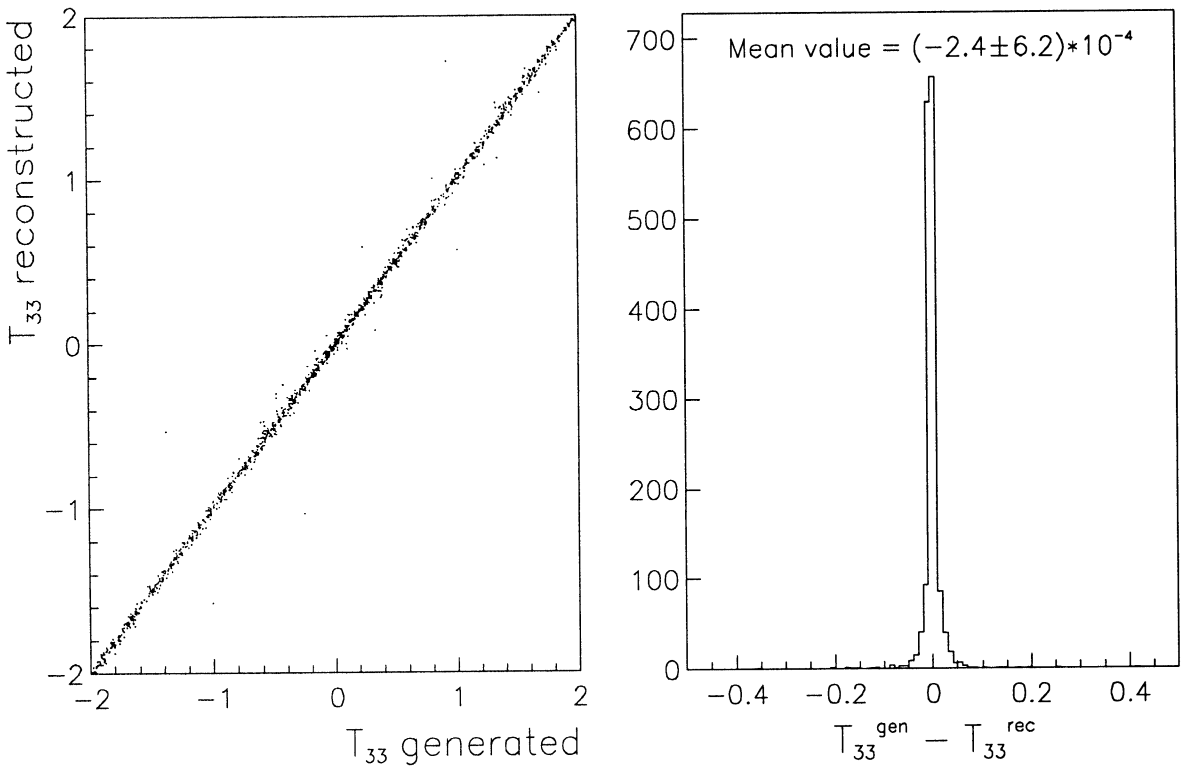

Figure 1: *Detector resolution: a) $\hat{T}_{33}$(generated) vs. $\hat{T}_{33}$(reconstructed), b) The difference between these two variables.*

The values of the sensitivities $\hat{c}_{AB}$ may depend on the resolution of the detector. To check this, 3000 events with both τ's decaying into $\pi\nu_\tau$ were generated including the GALEPH – simulation. Figure 1 shows a very strong correlation between the generated and reconstructed values of $\hat{T}_{33}$. Assuming that the deviation of the reconstructed values from the generated ones follows a normal distribution, the influence of the resolution on the sensitivities was determined. The resulting relative error $\Delta\,\hat{c}_{AB}/\hat{c}_{AB} = 0.005$ is about a factor of 10 smaller

[2]The Standard generator for τ – events KORALZ[12] does not have the possibility to produce these $\mathcal{CP}$ – odd contributions.

than the statistical error. Therefore the detector simulation is omitted for the computation of the $\hat{c}_{AB}$'s.

In the case of the a_1 decaying into three charged pions, the measurement of the absolute momentum of the charged tracks, which is worse than the measurement of angles, enters the direction of the a_1 itself. This yields a bigger error on the sensitivity of $\Delta \hat{c}_{AB}/\hat{c}_{AB} \approx 0.07$, which was taken into account.

Effects of the event selection

If the experimental cuts are considered, the values of the $\hat{c}_{AB}$'s will be changed, because these constants include an integration over the accessible phase space of the τ decay particles. To get the correct values the effects of the selection are accounted for in two ways. First, the resulting cuts in all event classes were determined and all cuts using quantities already defined at gernerator level were implemented in the Monte-Carlo-program. For example cuts on the Ecal-energy of electrons or π^0's have been included, but cuts on Hcal-energies or on tracks in the neighbourhood of Ecal-cracks, which reduce the phase space only by a very small amount, have been omitted. Moreover, it must be noticed that the distribution of the selected events over the remaining phase space doesn't correspond to the distribution of all events, because the efficiency of the selection is not the same for all regions of phase space. Low energetic ρ's, e.g. are identified quite poorly, since there is a certain probability to misidentify the charged pion as lepton or to loose the π^0. To take this into account a weighting procedure is adopted. Every generated event is weighted using two dimensional histograms, which contain the efficiency of the event selection in dependence on $\cos\theta_{thrust}$ and the momentum of the particle used to construct the observable $\hat{T}_{33}$. These histograms were computed with the official τ-MC including the detector simulation. Since events, which are not reconstructed in the detector enter the normalisation, the generated momenta and $\cos\theta_{thrust}$ have to be used.

Every event class used in the analysis needs its own weighting histograms, because the efficiency depends strongly on the class. Furthermore there are correlations between the two hemispheres of an event due to the cuts of the preselection and of the background rejection removing the whole event or retaining it. To disentangle these correlations every hemisphere requires two histograms, one containing the efficiency to find the event and a second histogram, which gives the efficiency to identify the hemisphere if the event is selected.

A neural net trained with Monte-Carlo-events is used for particle identification. Extensive studies have been performed[8,9] to check the systematics of the neural net. A comparison between the efficencies computed with MC-events and the efficencies determined from data samples of kinematically identified electrons, muons and pions shows no significant differences. Therefore the efficencies in the weighting histograms include no systematic effect originating from the neural net.

Background

Table 3 shows that the sensitivities depend in magnitude and sign on the decay modes of τ^- and τ^+. Therefore background events have in general sensitivities, which are different from

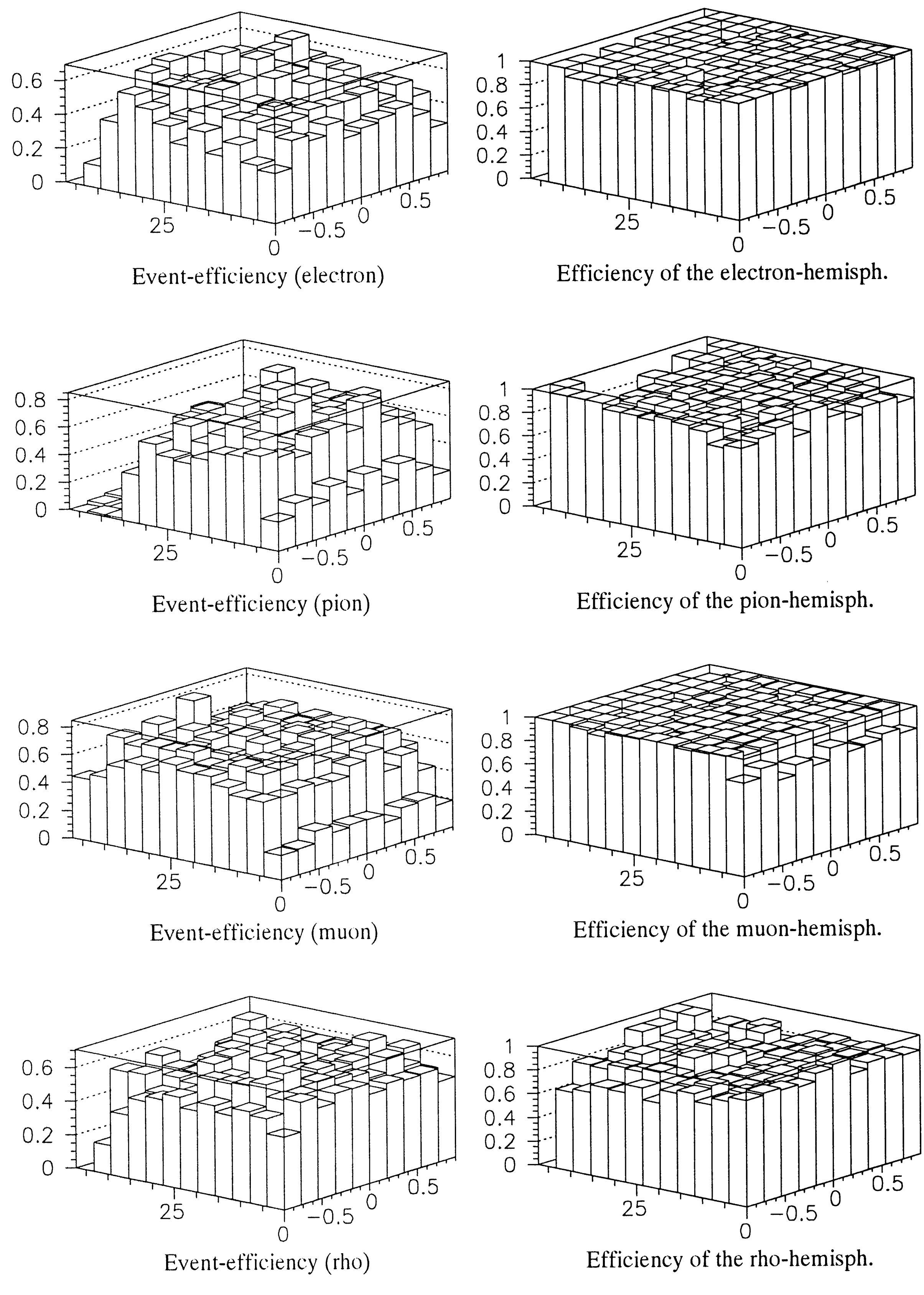

Figure 2: *The efficiency histograms for the classes* $e-\pi$ *(figures 1-4) and* $\mu-\rho$ *(figures 5-8).*

Ereignis – klasse	Schwerpunktsenergie		
	88.25 GeV	91.25 GeV	94.25 GeV
$\pi-\pi$	-1.76 ± 0.05	-1.83 ± 0.05	-1.89 ± 0.05
Lepton – Lepton	$+0.69\pm0.05$	$+0.73\pm0.05$	$+0.75\pm0.05$

Table 2: *Dependence of the sensitivites on the cms – energy.*

the $\hat{c}_{AB}$ of a certain event class. These modifications are considered by computing effective values with the following formula:

$$\hat{c}_{AB}^{eff} = p_{AB} \cdot \hat{c}_{AB} + \sum_i p_i \cdot \hat{c}_i$$

The sum runs over all combinations of τ – decays, which contribute to the background and over all sources of non – τ – background. p_i gives the fraction of a background class and c_i the corresponding sensitivity.

A few τ – decays, especially with K^* – mesons, have unknown sensitivity. Using the relation $\hat{c}_{AB} \approx \hat{c}_A + \hat{c}_B$ [6,7], in these cases the unknown values are chosen to be $\hat{c}_B \approx 0.0 \pm 1.0$, where the error is determined from the decay with the greatest sensitivity, the decay into $\pi\nu_\tau$. The 3 – π final state may contain a fraction of non – resonant τ – decays. This was taken into account using a result from ARGUS[13]; they measured that the non – resonant fraction is smaller than 6 % with 95 % c.l.. Since such a 4 – body – decay contains almost no information about the spin of the τ – lepton, an additional background in the a_1 – classes with $p_i = 0.0 \pm 0.06$ and $\hat{c}_B = 0.0 \pm 0.2$ was assumed.

Kaons from $\tau \to K\nu_\tau$ are not considered as background, but they are associated with the decay $\tau \to \pi\nu_\tau$, because these two particles are almost indistinguishable in the detector. According to the fraction of kaons in the π – classes the sensitivity was slightly reduced, since the sensitivity of the kaon is a little bit smaller than that of the pion due to the larger mass.

Non – τ – background hasn't any sensitivity for the τ – dipole – form factor. Therefore the values $\hat{c}_{AB}$ are assumed to be zero within an appropriate error estimated from the behaviour of τ – events.

Some smaller effects

The relation between the mean value of the observable and the weak dipole form factor is linear only for small values of d_τ. In order to linearize the relation for computation of the $\hat{c}_{AB}$'s, $\langle \hat{T}_{33} \rangle$ has to be scaled with the ratio $\Gamma_{Z\to\tau\tau}\,(d_\tau \neq 0)\ /\ \Gamma^{SM}_{Z\to\tau\tau}$. This correction is not necessary for the data, because the experimental value $\Gamma^{exp}_{Z\to\tau\tau} = 82.76 \pm 1.02\,MeV$[14] is in good agreement with the Standard – Model value $\Gamma^{SM}_{Z\to\tau\tau} = 83.7 \pm 0.4\,MeV$[14].

In 1990 and 1991 the data was collected at cms – energies between 88.25 GeV and 94.25 GeV, whereof 77.5 % of the events were taken at the peak energy. The values in table 2 show

Class	$\hat{c}^{eff}_{AB}$	$\Delta\hat{c}^{stat}_{AB}$	$\Delta\hat{c}^{sys}_{AB}$	$\Delta\hat{c}^{His}_{AB}$	$\Delta\hat{c}_i$	Δp^{stat}_i	Δp^{sys}_i
e − e	0.590	0.065	0.035	0.006	0.002	0.002	0.001
e − μ	0.720	0.054	0.006	0.002	0.002	0.001	0.002
e − π	-0.890	0.060	0.023	0.003	0.027	0.003	0.001
e − ρ	0.355	0.060	0.031	0.005	0.024	0.002	0.001
$\mu-\mu$	0.904	0.098	0.005	0.007	0.003	0.002	0.004
$\mu-\pi$	-0.573	0.046	0.040	0.002	0.018	0.005	0.002
$\mu-\rho$	0.388	0.047	0.025	0.007	0.018	0.002	0.001
$\pi-\pi$	-1.883	0.053	0.019	0.010	0.031	0.019	0.013
$\pi-\rho$	-1.531	0.065	0.032	0.011	0.022	0.009	0.025
$\rho-\rho$	-0.954	0.077	0.009	0.007	0.038	0.007	0.017

Class	$\hat{c}^{eff}_{AB}$	$\Delta\hat{c}^{stat}_{AB}$	$\Delta\hat{c}^{sys}_{AB}$	$\Delta\hat{c}^{sys}_{a_1}$	$\Delta\hat{c}_i$	Δp^{stat}_i	Δp^{sys}_i	$\Delta\hat{c}^{nr}_{a_1}$	$\Delta\hat{c}^{P}_{3\pi}$
e − 3π	0.485	0.053	0.003	0.1	0.008	0.003	0.016	0.015	–
e − $2\pi^0\pi$	0.442	0.053	0.013	0.1	0.063	0.006	0.002	0.011	–
$\mu-3\pi$	0.489	0.053	0.002	0.1	0.017	0.003	0.021	0.016	–
$\mu-2\pi^0\pi$	0.629	0.053	0.021	0.15	0.068	0.008	0.004	0.016	–
$\pi-3\pi$	-1.540	0.053	0.006	0.2	0.026	0.014	0.068	0.059	0.122
$\pi-2\pi^0\pi$	-1.450	0.053	0.011	0.2	0.061	0.026	0.009	0.024	–
$\rho-3\pi$	-0.702	0.053	0.007	0.15	0.029	0.014	0.060	0.017	0.051
$\rho-2\pi^0\pi$	-0.722	0.053	0.017	0.15	0.066	0.010	0.007	0.014	–

Table 3: *The effective sensitivities and the errors. The specification of the errors is described in the text.*

a slight increase of the sensitivities with energy. Since the scan points are symmetric around the peak and the number of events at opposite points is similar, the relative change of the $\hat{c}_{AB}$'s is smaller than 0.2 % and can be neglected compared to the statistical error on $\hat{c}_{AB}$ of about 10 %.

The effective sensitivities

In event classes with the resonances ρ and a_1 the sensitivities depend on the momentum vector used for construction of $\hat{T}_{33}$. If both hemispheres contain hadrons, the reconstructed momentum vector of ρ and a_1 itself shall be used to maximize the sensitivity. On the other hand, in the lepton − ρ and lepton − a_1 − classes the sensitivity will be greater if one uses the momentum of the charged pion from the ρ − decay and in the case of the a_1 the pion, which differs in charge from the two other pions.

Due to technical reasons[3], it was not possible to compute weighting histograms for the

[3]The TCL − format doesn't include generated momenta except for the decays into e, μ, π and ρ. To compute

a_1 – classes. To account for this, an additional error was determined, using the behaviour of other classes.

Table 3 presents the effective sensitivities and the errors, of which the sources are explained in the following:

- $\Delta\hat{c}_{AB}^{stat}$: statistical error of the generated events.
- $\Delta\hat{c}_{AB}^{sys}$: variation of cuts including changed weighting histograms.
- $\Delta\hat{c}_{AB}^{His}$: statistical error of the histograms.
- $\Delta\hat{c}_i$: uncertainty of the sensitivities of the background.
- Δp_i^{stat}: statistical error of the background fractions.
- Δp_i^{sys}: systematic error of the background fractions.
- a_1 – classes:
  - $\Delta\hat{c}_{a_1}^{sys}$: missing weighting histograms.
  - $\Delta\hat{c}_{a_1}^{nr}$: possible non – resonant fraction of the 3 – π final state.
- $\pi - a_1(3\pi)$ and $\rho - a_1(3\pi)$: $\Delta\hat{c}_{3\pi}^{p}$: error from the measurement of the absolute momentum.

4 Systematic studies of $\left\langle \hat{T}_{33} \right\rangle$

Event selection

To discuss the effects of the event selection on the sensitivities, it was assumed that the selection criteria are $\mathcal{CP}$ – blind. This assumption cannot be tested just looking at the cuts applied, because the neural net may introduce a $\mathcal{CP}$ – violating effect. Therefore another weighting procedure was employed. It is based on one – dimensional histograms, which contain the selection efficiency over angular variables of the particle entering directly the representation of $\hat{T}_{33}$ in polar coordinates (see section about tracking devices). These are $\sin(\varphi_+ - \varphi_-)$, the difference $(\cos\vartheta_+ - \cos\vartheta_-)$, $\sin\vartheta_+ \cdot \sin\vartheta_-$ and the acollinearity $\sin\psi$. Events generated with $d_\tau = 0$ are weighted with these histograms. If the selection is not $\mathcal{CP}$ – blind, this will appear in the angular variables, of which $\hat{T}_{33}$ is constructed, and therefore in the corresponding efficiency histograms. This would yield an observable shift of $\langle\hat{T}_{33}\rangle$ of the weighted distribution, since the mean value of the unweighted distribution is compatible with zero. As a result of this procedure, a shift wasn't found and the statistical error was taken as systematic error on $\langle\hat{T}_{33}\rangle$.

the probability of an event to be found in dependence on momentum, also particles that are not reconstructed are needed.

	Lepton – Lepton	Lepton – Hadron	Hadron – Hadron
Without radiative corrections	$+0.0090 \pm 0.0050$	$+0.0010 \pm 0.0026$	-0.0045 ± 0.0024
With initial state radiation	-0.0036 ± 0.0051	$+0.0007 \pm 0.0027$	$+0.0010 \pm 0.0024$
With initial and final state rad.	$+0.0092 \pm 0.0052$	-0.0020 ± 0.0027	$+0.0018 \pm 0.0026$

Table 4: *Comparison of the mean values of $\hat{T}_{33}$ with and without radiative corrections for the three event classes .*

Radiative corrections

The MC – generator, which is used to compute the sensitivities, doesn't include QED – corrections. Therefore some considerations are suitable. The emission of photons in the initial state affects the cms – energy. But the smearing of $\sqrt{s}$ is strongly cut off by th Z – line shape. Since the dependence of the sensitivities on $\sqrt{s}$ is very small (see section 3), the change in energy doesn't influence the value of $\hat{c}_{AB}$.

The correlation between the spin vectors of the τ – leptons is destroyed by photon emission in the final state. A possible influence of such a spin – flip on the $\hat{c}_{AB}$'s is nevertheless negligible, because these corrections are of the order of α/π and therefore only a small amount of events is affected.

The influence of QED – corrections on the directions of the momentum vectors of the τ – decay particles can be studied quantitativly using KORALZ – events, because possible effects must affect the distribution of the observable $\hat{T}_{33}$ and furthermore should be independent of the value of d_τ. For this purpose 50000 KORALZ – events without radiative correections, with ISR and with all corrections were generated and divided into threse classes, namely lepton – lepton, lepton – hadron and hadron – hadron to account for the different momentum vectors used for construction of $\hat{T}_{33}$. Table 4 shows the results indicating that there is no systematic effect within the precision of the statistical error, which is taken as systematic error on the mean values $\langle \hat{T}_{33} \rangle_{AB}$.

In the case of electrons bremsstrahlung in the detector material must be considered. To study possible effects 3000 e – e – events including the complete detector simulation have been generated. The result for the mean value of the difference between generated and reconstructed $\hat{T}_{33}$ was:

$$\left\langle \hat{T}_{33}^{gen} - \hat{T}_{33}^{rec} \right\rangle = -0.0040 \pm 0.0042$$

This statistical error was taken as systematic error on $\langle \hat{T}_{33} \rangle_{AB}$ in event classes with electrons.

Tracking devices

The most important subdetectors for this analysis are the tracking devices. Therefore it is necessary to check if these devices may simulate a $\mathcal{CP}$ – violating effect. An appropriate data sample for this study are μ – pairs from the process $Z^0 \to \mu^+\mu^-$. Since the muons are back to back they are very sensitive on possible errors in the measurement of angles. The momenta of μ^+ and μ^- are used to construct the observable $\hat{T}_{33}$. Any non – zero mean value of the distribution is then introduced by the tracking devices, because a possible dipole form factor of the muon must be much smaller than that of the τ[4] and because the spin analysing decay of the μ is not observed in the detector. The following cuts are applied to identify the

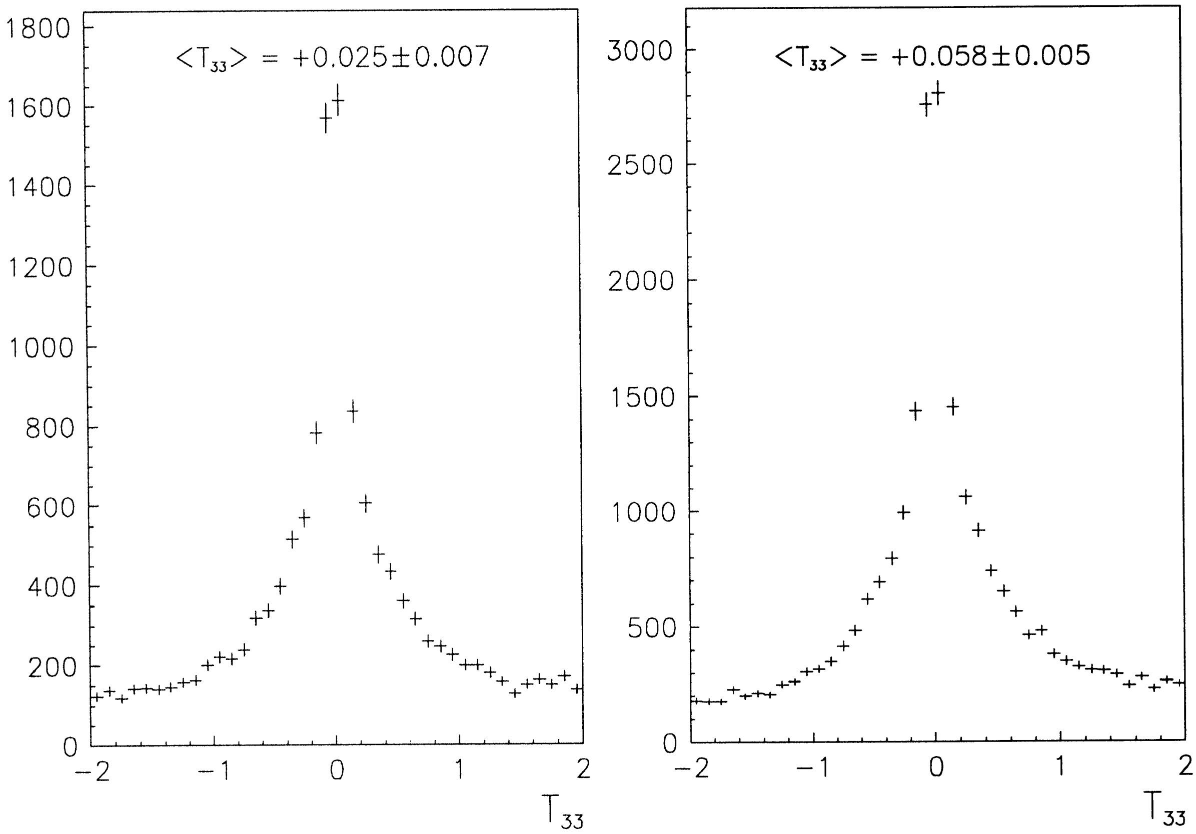

Figure 3: *The distribution of the observable $\hat{T}_{33}$ for the μ – pairs of 1990 and 1991 (left), and of 1992(right), respectively.*

μ – pairs:

- Exactly 1 good track in each hemisphere
- For both hemispheres:
  - $|\cos\vartheta_{track}| < 0.9$
  - track identified as muon by the neural net

[4]The dipole form factor is at least proportional to the fermion mass.

- $E^{\mu}_{TPC}/E_{beam} > 0.9$ and $E^{\mu}_{TPC}/E_{beam} < 1.2$
- $E^{hemisphere}_{Ecal} < 15GeV$
- No GAMPEC – photons

The efficiency is $(65.5 \pm 0.2)\,\%$ with a purity of more than $99.9\,\%$.

In figure 3 the results are plotted for the years 1990, 1991 and 1992, respectively. Both distributions show an asymmetry. The deviation in 1990 and 1991 is $3.6\,\sigma$ and in 1992 it is a $11.3\,\sigma$ effect. To study the source of the effect the representation of $\hat{T}_{33}$ in polar coordinates is helpful:

$$\hat{T}_{33} = \frac{2}{|\sin\psi|} \cdot (\cos\vartheta_+ - \cos\vartheta_-) \cdot \sin\vartheta_+ \cdot \sin\vartheta_- \cdot \sin(\varphi_+ - \varphi_-) \qquad (3)$$

Figure 4 shows a strong correlation between $\hat{T}_{33}$ and $\sin(\varphi_+ - \varphi_-)$ in all $\cos\vartheta$ – bins, since $\hat{T}_{33}$ depends also on the sign of $\cos\vartheta_+$. The effect in 1992 is also visible in other measured quantities, e.g. the absolute momentum calibration. Therefore the shift in $\langle\hat{T}_{33}\rangle_{AB}$ is only due to a detector effect, which is significant in 1992. In 1990 and 1991 the same effect might be present, since the behaviour of the variables seems to be similar but less significant.

The source of this effect is unknown. It is not envoked by some bad runs, e.g. with TPC – shorts, because the effect is visible over the whole run period 1992. Possibly there are still residuals from the alignment procedure or some small contributions from remaining local inhomogenities of the fields [15].

Since a method [16,17] exists to eliminate a shift due to a detector effect in the final result (see next section) no ad hoc corrections are applied.

Electromagnetic calorimeter

In event classes with hadrons in both hemispheres the resonances $\rho \to \pi^0\pi$ and $a_1 \to 2\pi^0\pi$ are reconstructed with π^0's, of which the momenta are computed using photons in the Ecal. In these classes the Ecal as well may introduce a $\mathcal{CP}$ – violating effect.

One possibility would be to look at the energy deposition of μ – pairs in the Ecal. But in this case it is not clear how to propagate the error to τ – events, because the Ecal – information enters only with the ratio of charged momentum to π^0 – momentum the direction of the resonances. Therefore a direct method is used to estimate the error taking the τ – decays into ρ and a_1 itself as test sample. To do this the observable $\hat{T}_{33}$ is built from combinations of ρ^+ and ρ^- and a_1^+ and a_1^-, respectively, originating from different events. The only condition besides opposite charge is a minimal acollinearity of 170^0 for ρ – mesons and of 160^0 for a_1 – mesons due to the lower statistics.

Before this method can be applied two questions should be answered: Is it possible to see $\mathcal{CP}$ – violating effects from the Ecal and is it possible to destroy correlations between the hemispheres coming from a nonvanishing dipole form factor?

To answer the first question a twist of one endcap of $10\,mrad$ against the barrel was simulated. The result is:

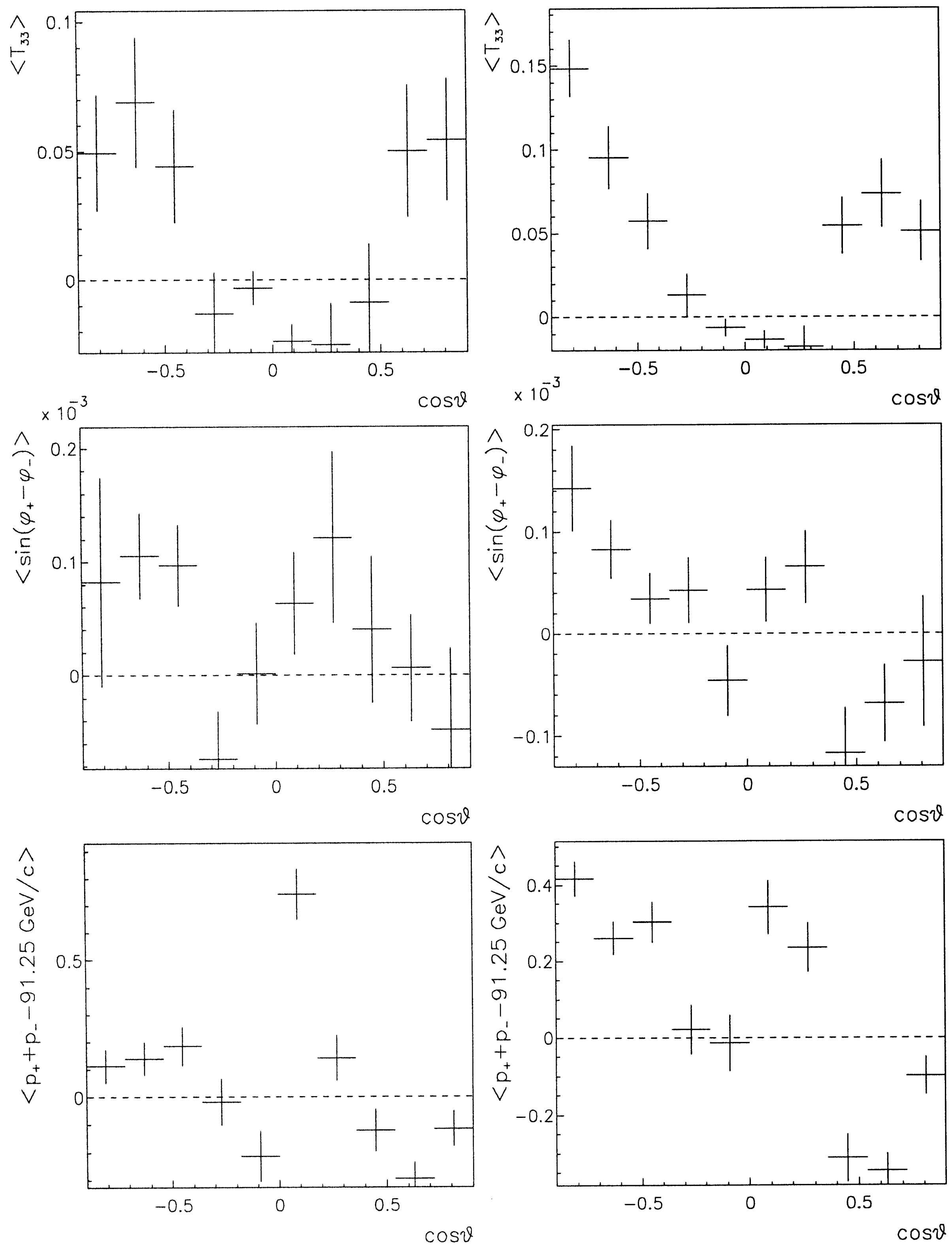

Figure 4: *In dependence on* $\cos\vartheta$ *of the positive muon the variables* $\hat{T}_{33}$, $\sin(\varphi_+ - \varphi_-)$ and $(p_{\mu^+} + p_{\mu^-} - 91.25\,GeV/c)$ *are plotted. On the left side the results of 1990/1991* **are shown** *and on the right of 1992.*

Mixing	1990, 1991	1992
$\rho^+ - \rho^-$	-0.0116±0.0056	0.0026±0.0034
$\pi^0 - \pi^0$	-0.0093±0.0056	0.0130±0.0034
$\pi^+ - \pi^-$	-0.0050±0.0056	-0.0054±0.0034
$a_1^+ - a_1^-$	-0.0013±0.0074	-0.0155±0.0049
$\sum\pi^0 - \sum\pi^0$	-0.0033±0.0074	-0.0187±0.0049
$\pi^+ - \pi^-$	-0.0050±0.0074	-0.0010±0.0049

Table 5: *The results of the event mixing for ρ and a_1 in the years 1990, 1991 and 1992.*

$$\rho^+\rho^-: \quad \langle\hat{T}_{33}\rangle = (0.24 \pm 0.32)\cdot 10^{-2} \qquad \langle\hat{T}_{33}^{Twist}\rangle = (0.75 \pm 0.32)\cdot 10^{-2}$$
$$\pi^0\pi^0: \quad \langle\hat{T}_{33}\rangle = (-0.14 \pm 0.32)\cdot 10^{-2} \qquad \langle\hat{T}_{33}^{Twist}\rangle = (0.81 \pm 0.32)\cdot 10^{-2}$$

The difference in the mean values $\langle\hat{T}_{33}\rangle$ and $\langle\hat{T}_{33}^{Twist}\rangle$ is not a statistical fluctuation, but completely due to the twist, because exactly the same $\rho^+\rho^-$ – combinations have been used. Moreover the effect is bigger for the corresponding $\pi^0\pi^0$ – combinations as expected.

Correlations between the mesons are destroyed completely by the event mixing. This can be shown using $\rho^+\rho^-$ – events generated with a dipole form factor of $d_\tau = 0.1\,\frac{e}{M_Z}$. This yields $\left\langle\hat{T}_{33}\right\rangle = (-0.524 \pm 0.033)\cdot 10^{-1}$ and after the event mixing one obtains $\left\langle\hat{T}_{33}\right\rangle = (0.009 \pm 0.041)\cdot 10^{-1}$. The relativ large region of acollinearity reduces the sensitivity for possible effects compared to μ – pairs, but it yields a limit on the error still small enough for the present data sample. Since effects of the tracking devices are known to be of the order of 10^{-4}, these effects cannot cause a fake in the result of the event mixing.

Table 5 presents the results for the data collected in all three years. In 1992 in both cases the $\pi^0\pi^0$ – combinations show a 3.8σ deviation which vanishes for $\rho^+\rho^-$ and reduces to 3σ for $a_1^+a_1^-$. To account for a possible effect, the 3σ interval was taken as systematic error on $\langle\hat{T}_{33}\rangle$ in the four classes concerned.

Results of $\langle\hat{T}_{33}\rangle$

Table 6 lists the results for the years 1990, 1991 and separately for 1992 with the following errors:

- $\Delta\langle\hat{T}_{33}\rangle_{AB}^{stat}$: statistical error of the data
- $\Delta\langle\hat{T}_{33}\rangle_{AB}^{sel}$: error from the test of $\mathcal{CP}$ – invariance of the event selection
- $\Delta\langle\hat{T}_{33}\rangle_{AB}^{rad}$: error estimating the influence of radiative corrections
- $\Delta\langle\hat{T}_{33}\rangle_{AB}^{Ecal}$: error resulting from the check of $\mathcal{CP}$ – invariance of the Ecal.

The errors $\Delta\langle\hat{T}_{33}\rangle_{AB}^{sel}$ and $\Delta\langle\hat{T}_{33}\rangle_{AB}^{rad}$ are the same for both data samples and therefore they are quoted only once.

Event class	Number of events	1990 and 1991				
		$\langle\hat{T}_{33}\rangle_{AB}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{stat}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{sel}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{rad}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{Ecal}$
e − e	238	−0.107	0.064	0.006	0.013	–
e − μ	897	−0.068	0.037	0.002	0.013	–
e − π	442	+0.048	0.051	0.006	0.012	–
e − ρ	636	+0.057	0.043	0.008	0.012	–
$\mu-\mu$	473	+0.010	0.051	0.004	0.005	–
$\mu-\pi$	574	+0.061	0.047	0.004	0.003	–
$\mu-\rho$	877	+0.066	0.038	0.009	0.003	–
$\pi-\pi$	202	+0.087	0.077	0.003	0.003	–
$\pi-\rho$	533	−0.038	0.045	0.006	0.003	0.017
$\rho-\rho$	442	−0.073	0.050	0.010	0.003	0.017
e − 3π	307	+0.060	0.065	0.012	0.012	–
e − $2\pi^0\pi$	233	+0.049	0.074	0.007	0.012	–
$\mu-3\pi$	338	+0.033	0.058	0.008	0.003	–
$\mu-2\pi^0\pi$	275	+0.071	0.067	0.005	0.003	–
$\pi-3\pi$	207	−0.004	0.071	0.010	0.003	–
$\pi-2\pi^0\pi$	176	+0.010	0.077	0.009	0.003	0.043
$\rho-3\pi$	338	−0.113	0.060	0.012	0.003	–
$\rho-2\pi^0\pi$	268	+0.014	0.064	0.013	0.003	0.043
$\sum$ = 7456 events						

Event class	Number of events	1992		
		$\langle\hat{T}_{33}\rangle_{AB}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{stat}$	$\Delta\langle\hat{T}_{33}\rangle_{AB}^{Ecal}$
e − e	430	+0.078	0.050	–
e − μ	1552	−0.021	0.028	–
e − π	659	−0.022	0.043	–
e − ρ	1165	−0.019	0.032	–
$\mu-\mu$	787	+0.046	0.039	–
$\mu-\pi$	889	+0.046	0.036	–
$\mu-\rho$	1414	−0.002	0.029	–
$\pi-\pi$	277	+0.004	0.061	–
$\pi-\rho$	819	+0.022	0.037	0.010
$\rho-\rho$	658	+0.005	0.041	0.010
e − 3π	497	−0.044	0.049	–
e − $2\pi^0\pi$	327	+0.027	0.060	–
$\mu-3\pi$	599	−0.025	0.046	–
$\mu-2\pi^0\pi$	455	−0.029	0.053	–
$\pi-3\pi$	357	−0.048	0.057	–
$\pi-2\pi^0\pi$	291	−0.050	0.062	0.029
$\rho-3\pi$	551	−0.040	0.047	–
$\rho-2\pi^0\pi$	445	+0.041	0.052	0.029
$\sum$ = 12172 events				

Table 6: *The mean values $\langle\hat{T}_{33}\rangle_{AB}$ in the various event−classes and the errors for the years 1990, 1991 and 1992, respectively.*

Event class	1990 und 1991					1992				
	d_τ	σ	Δd_τ^{sys}	Δd_τ^{c}	Δd_τ^{Ecal}	d_τ	σ	Δd_τ^{sys}	Δd_τ^{c}	Δd_τ^{Ecal}
e − e	−0.54	0.31	0.07	0.01	−	+0.39	0.25	0.07	0.05	−
e − μ	−0.28	0.15	0.05	0.02	−	−0.09	0.12	0.05	0.01	−
e − π	−0.16	0.17	0.05	0.01	−	+0.07	0.14	0.05	0.01	−
e − ρ	+0.48	0.36	0.12	0.10	−	−0.16	0.27	0.12	0.03	−
$\mu - \mu$	+0.03	0.16	0.02	0.003	−	+0.15	0.13	0.02	0.02	−
$\mu - \pi$	−0.32	0.25	0.03	0.04	−	−0.24	0.19	0.03	0.03	−
$\mu - \rho$	+0.51	0.29	0.07	0.15	−	−0.02	0.22	0.02	0.002	−
$\pi - \pi$	−0.14	0.12	0.01	0.01	−	−0.01	0.10	0.01	0.001	−
$\pi - \rho$	+0.07	0.09	0.01	0.004	0.03	−0.04	0.07	0.01	0.003	0.02
$\rho - \rho$	+0.23	0.15	0.03	0.02	0.05	−0.02	0.13	0.03	0.002	0.03
e − 3π	+0.37	0.40	0.10	0.09	−	−0.27	0.30	0.10	0.07	−
e − $2\pi^0\pi$	+0.33	0.50	0.09	0.10	−	+0.18	0.41	0.09	0.05	−
$\mu - 3\pi$	+0.20	0.36	0.05	0.06	−	−0.15	0.28	0.05	0.04	−
$\mu - 2\pi^0\pi$	+0.34	0.32	0.03	0.09	−	−0.14	0.25	0.03	0.04	−
$\pi - 3\pi$	+0.01	0.14	0.02	0.001	−	+0.09	0.11	0.02	0.01	−
$\pi - 2\pi^0\pi$	−0.02	0.16	0.02	0.003	0.09	+0.10	0.13	0.02	0.02	0.06
$\rho - 3\pi$	+0.48	0.26	0.05	0.12	−	+0.17	0.20	0.05	0.04	−
$\rho - 2\pi^0\pi$	−0.06	0.27	0.06	0.01	0.18	−0.17	0.22	0.06	0.04	0.12

Table 7: *The dipole form factors and the errors. The specification of the errors is given in the text. All numbers are in units* $[e/M_Z]$.

5 Results

The results for the dipole form factor in the various channels are presented in table 7 with the following errors considered:

- σ: statistical error of the data
- Δd_τ^{sys}: systematic error, resulting from $\Delta\langle\hat{T}_{33}\rangle_{AB}^{sel}$ and $\Delta\langle\hat{T}_{33}\rangle_{AB}^{rad}$
- Δd_τ^{c}: error from the combined error of the sensitivities $\hat{c}_{AB}$
- Δd_τ^{Ecal}: error from $\Delta\langle\hat{T}_{33}\rangle_{AB}^{Ecal}$

Elimination of the detector shift

The method developped in [17] and described briefly in the following not only eliminates the detector shift, but also reduces the systematical error, which would be dominated by systematic effects for the tracking devices, substantially. This method is based on the different signs of the sensitivities in different event classes. Instead of computing the weighted mean of

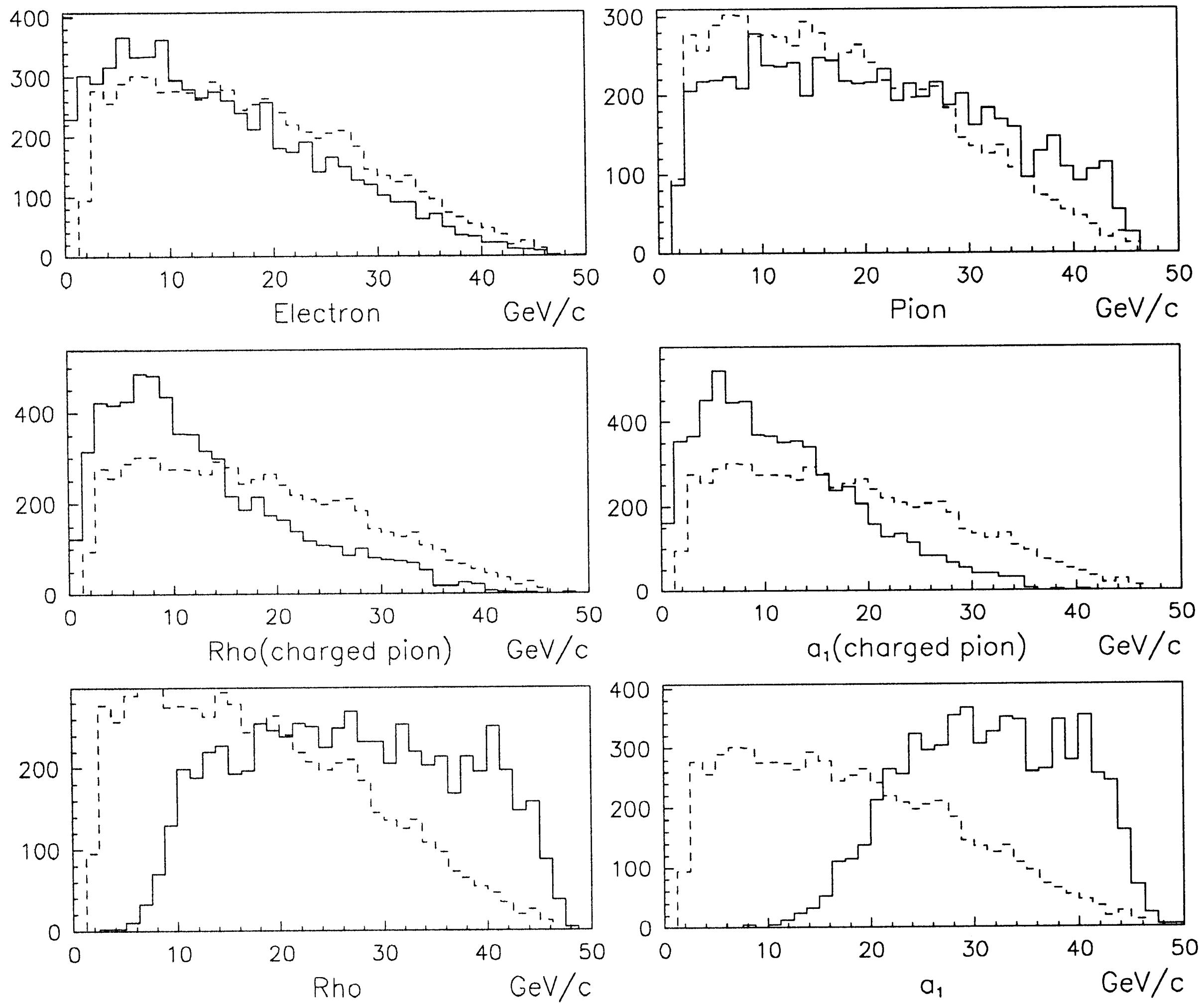

Figure 5: *The momentum spectra of the reconstructed particles in 1992. The number of entries is normalized to the number of muons, of which the spectrum is used as reference (dashed lines).*

the dipole form factors of all classes, two sums are constructed from the classes with positive $\hat{c}_{AB}$ and negative $\hat{c}_{AB}$, respectively:

$$d_\pm = \sigma_\pm^2 \cdot \sum\nolimits_\pm \left(\frac{d_i}{\sigma_i^2}\right) \quad \text{with} \quad \sigma_\pm^2 = \frac{1}{\sum_\pm 1/\sigma_i^2}$$

The value of d_+ is the sum of the physical dipole form factor d_τ and a part originating from a detector shift $\Delta\hat{T}$. Correspondingly d_- is the difference:

$$d_\pm = d_\tau \pm 3 \cdot \frac{\Delta\hat{T}}{|c_\pm|} \quad \text{with} \quad c_\pm = \left(\sigma_\pm^2 \cdot \sum \frac{1}{\sigma_i^2 c_i}\right)^{-1}$$

From the weighted difference of d_+ and d_- one obtains the shift $\Delta\hat{T}$:

$$\Delta\hat{T} = \frac{1}{3} \cdot \frac{d_+ - d_-}{\frac{1}{|c_+|} - \frac{1}{|c_-|}}$$

and the appropriate weighted sum yields the physical dipole form factor:

$$d_\tau = \frac{|c_+|\, d_+ + |c_-|\, d_-}{|c_+| + |c_+|}$$

The result for the shifts computed with the above formula are:

$$\Delta\hat{T}_{90/91} = -0.001 \pm 0.045_{stat} \pm 0.011_{sys} \pm 0.010_{\Delta\hat{c}}$$

$$\Delta\hat{T}_{92} = -0.001 \pm 0.036_{stat} \pm 0.010_{sys} \pm 0.004_{\Delta\hat{c}}$$

In both years there isn't any significant shift observable.

Figure 5 shows the momentum spectra of the particles after selection. All spectra are normalized to the number of μ's from $\tau-$decays, of which the spectrum is plotted upon all other histograms (dashed curves). Even if a detector shift has a strong momentum dependence, $\Delta\hat{T}$ will be the same for electrons, muons, pions and charged pions from $\rho-$ or a_1-decays within the statistical precision, since all spectra are comparable. To check that in the case of reconstructed ρ's and a_1's the assumption of a constant shift $\Delta\hat{T}$ is correct, the weighted mean d_- is splitted into two values d_-^1 and d_-^2. The first sum d_-^1 runs over the classes $e-\pi$, $\mu-\pi$ and $\pi-\pi$, whereas d_-^2 is constructed using the remaining classes with reconstructed resonances ρ and a_1. d_-^1 and d_-^2 together with the unchanged value of d_+ are then used to compute two shifts $\Delta\hat{T}^1$ and $\Delta\hat{T}^2$, which will be the same if the shift is constant in all classes. Since the $\mu-$pairs show an effect in 1992, the shifts $\Delta\hat{T}^1$ and $\Delta\hat{T}^2$ are computed for this year showing no discrepancy:

$$\Delta\hat{T}_{92}^1 = +0.009 \pm 0.044_{stat} \quad \text{and} \quad \Delta\hat{T}_{92}^2 = -0.002 \pm 0.032_{stat}$$

A shift introduced by the Ecal cannot be eliminated with this method, because this error appears only in a few classes with negative sensitivity. The remaining systematic error on the dipole form factor is:

$$\Delta d_\tau^{Ecal} = \frac{|c_-|\, \sigma_-^2}{|c_+| + |c_-|} \cdot \sqrt{\sum_i \left(\frac{\Delta d_i^{Ecal}}{\sigma_i^2}\right)^2}$$

Final results

Using the method described above one obtains for 1990 and 1991 the following results:

$$d_+ = (+0.23 \pm 1.82_{stat} \pm 0.46_{sys} \pm 0.38_{\Delta c}) \cdot 10^{-17}\, ecm$$
$$\text{with} \quad c_+ = +0.625 \pm 0.020_{\Delta c_i}$$

$$d_- = (+0.30 \pm 1.10_{stat} \pm 0.19_{sys} \pm 0.13_{\Delta c}) \cdot 10^{-17}\, ecm$$
$$\text{with} \quad c_- = -1.155 \pm 0.050_{\Delta c_i}$$

This yields:

$$d_\tau = (+0.27 \pm 0.96_{stat} \pm 0.20_{sys} \pm 0.16_{\Delta c_i} \pm 0.22_{Ecal}) \cdot 10^{-17}\, ecm$$

and an upper limit of:

$$|d_\tau| < 2.3 \cdot 10^{-17}\, ecm \quad (95\,\%\ c.l.)$$

The results for 1992 are:

$$d_+ = (+0.01 \pm 1.45_{stat} \pm 0.43_{sys} \pm 0.20_{\Delta c}) \cdot 10^{-17}\, ecm$$
$$\text{with} \quad c_+ = +0.611 \pm 0.019_{\Delta c_i}$$

$$d_- = (+0.05 \pm 0.87_{stat} \pm 0.18_{sys} \pm 0.08\Delta c) \cdot 10^{-17}\, ecm$$
$$\text{with} \quad c_- = -1.094 \pm 0.042_{\Delta c_i}$$

The dipole form factor is:

$$d_\tau = (+0.03 \pm 0.76_{stat} \pm 0.19_{sys} \pm 0.09_{\Delta c_i} \pm 0.13_{Ecal}) \cdot 10^{-17}\, ecm$$

$$|d_\tau| < 1.6 \cdot 10^{-17}\, ecm \quad (95\,\%\ c.l.)$$

Combining the results of all three years gives:

$$\boxed{\begin{array}{c} d_\tau = (+0.13 \pm 0.60_{stat} \pm 0.19_{sys} \pm 0.11_{\Delta c_i} \pm 0.12_{Ecal}) \cdot 10^{-17}\, ecm \\ |d_\tau| < 1.4 \cdot 10^{-17}\, ecm \quad (95\,\%\ c.l.) \end{array}}$$

6 Conclusion

Using 19628 reconstructed and identified $\tau^+\tau^-$ – events collected in 1990, 1991 and 1992, a search for $\mathcal{CP}$ – violation was performed. No signal of $\mathcal{CP}$ – violation in the decay $Z^0 \rightarrow \tau^+\tau^-$ was observed. The measurement using the decays of the τ into e, μ, π, ρ and a_1 yields:

$$d_\tau = (+0.13 \pm 0.60_{stat} \pm 0.25_{sys}) \cdot 10^{-17} ecm$$

The resulting upper limit on the weak dipole form factor of the τ – lepton is:

$$|d_\tau| < 1.4 \cdot 10^{-17} ecm \quad (95\,\%\ c.l.)$$

The result might be compared to the value published last year [17]:
Exclusive measurement with classes consisting of the τ – decays into electron, muon, pion and rho(3845 events):

$$|d_\tau| < 3.7 \cdot 10^{-17} ecm \quad (95\,\%\ c.l.)$$

The improvement is not only due to 1992 statistics, but also to a more efficient selection and the use of the a_1 – decay modes.

Acknowledgements

We wish to thank John Conway, John Harton, Michael Schmitt and Michael Walsh placing the TCL – data format, which allows a fast and uncomplicated working, at our disposal.

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