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SCOT, a Monte Carlo Generator to Study the Lorentz Structure of $e^+e^- \to Z \to \tau^+\tau^-$

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Abstract

Together with the vector and axial vector couplings, the general Lorentz structure of leptonic Z decays, ($e^+e^- \to Z \to \tau^+\tau^-$), allows various additional types of interaction. In the present study a Monte Carlo generator, SCOT, is developed which simulates the general Lorentz structure of Z boson production and decay, and ultimately the τ decay. The program uses spin amplitudes to take the τ polarisation and the spin correlation into account and may be used for simulations at LEP and the SLC.

Title of the program: SCOT version: 1.0
Author of the program: Ulrich Stiegler (STIEGLER at CERNVM)
Computer: IBM3090; Installation: CERN
Programing language used: FORTRAN 77
No. Of bits in a word: 32
Peripherals used: Line printer
Libraries used: TAUOLA 2.4; PHOTOS 2.0; JETSET 7.3.
No. of lines (without libraries): 1702
Keywords: Lorentz Structure, Z decay, magnetic moment, dipole moment, heavy leptons, tau, Monte Carlo simulation, spin polarisation, spin correlation
Nature of Physical problem: Beyond the standard model Z boson production and decay allows a large number of additional coupling constants like weak magnetic and weak dipole moments. These can be studied by measuring the final spin density of $e^+e^- \to Z(\gamma) \to \tau^+\tau^-$.
Typical running time: 4 events per second (IBM 168 units), in case of all decay modes populated according to their branching ratio (this time includes some analysis routines) and 64 events per second for $\tau^\pm \to \pi^\pm\nu$.

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1 Introduction

Measurements of the weak neutral current coupling constants are important for detailed tests of the standard model of electroweak interactions. To analyse the decay $Z \rightarrow \tau\tau$ is a major advantage compared to other Z decay modes because the τ decay can be used as a polarimeter. Furthermore, the τ is the heaviest known lepton and most of the existing models show mass dependent deviations from the standard model. Last but not least, the interpretation of observed anomalies are not affected by QCD corrections.

The vector and axial vector coupling constants, v_f, a_f, in Z decays have been measured with high precision at LEP [1]. A method to determine the weak electric dipole moment, which violates CP symmetry, was developed [2], and measurements have been performed [3, 4, 5]. Furthermore, a method to measure the weak magnetic moment was recently proposed [6]. The measurement of the imaginary part of the vector and axial vector coupling constants is discussed in [7]. A more general calculation which takes all the possible various coupling constants of flavor conserving neutral current into account is given in [8]. These measurements can be performed at LEP and the SLC.

The basic concept of these methods consists of measuring the polarisation and spin correlation of the final state leptons. Due to parity violation in τ decays, the energy spectra and angular distributions of the decay products reflect the τ polarisation, thus allowing a determination of the spin density of the $\tau^+\tau^-$ system.

In practice, these measurements are performed by selecting τ events from the recorded data according to certain selection criteria. After this step the τ decay modes are identified and various kinematical variables like the energy of the particles are reconstructed which are than used to measure the coupling constants. Since the final result can be affected by the selection criteria and by the finite resolution and misalignment of the detector it is worthwhile to simulate the analysis with artificially generated events: The events are generated with a given set of coupling constants using the Monte Carlo technique. These generated events are analysed with the same analysis procedure which is applied to the measured data. The coupling constants which are extracted from the generated events must agree with the set of coupling constants used as input parameters of the Monte Carlo program.

The Monte Carlo generator presented here generates the process $e^+e^- \rightarrow \tau^+\tau^-$ and the decay of the τ lepton. The program is based on the formulas of [8] and simulates the general Lorentz structure of flavor conserving neutral current including spin correlation.

2 The Lorentz structure for $ee \rightarrow \tau\tau$

In the current study, the spin density of the $\tau^+\tau^-$ state is programmed in the frame work of a general Lorentz structure for $e^+e^- \rightarrow \tau^+\tau^-$, neglecting flavor changing currents which might be neutral or of charge two.

The general Lorentz structure for the process $e^+(k_1)e^-(k_2) \rightarrow \tau^+(p_1)\tau^-(p_2)$ can be described by the spin amplitude $M_{\lambda_1\lambda_2\alpha_1\alpha_2}$

$$M_{\lambda_1\lambda_2\alpha_1\alpha_2} = \sum_{(i)} g_{(i)}\bar{v}(k_1,\lambda_1)\Gamma_e^{(i)}u(k_2,\lambda_2)\bar{u}(p_2,\alpha_2)\Gamma_\tau^{(i)}v(p_1,\alpha_1) \quad , \tag{1}$$

where $\Gamma_f^{(i)}$, with $i = \gamma, Z$, stands for the photon and Z boson contribution respectively. They are described by effective couplings shown below. The quantities λ_1, $-\lambda_2$, α_1 and $-\alpha_2$ are the helicities of the e^+, e^-, τ^+ and τ^- and $g_{(i)}$ are functions of the c.m.s. energy, $q = \sqrt{s}$, and of t.

The photon exchange is described by:

$$\Gamma_f^{(\gamma)} = iQ_f e(q^2)\gamma^\mu \qquad g_{(\gamma)} = 1/q^2 \quad , \tag{2}$$

where $Q_f e(q^2)$ is the effective charge at the c.m.s. energy [9].

In the case of Z exchange the vertex $\Gamma_f^{(Z)}$ is given by:

$$\Gamma_f^{(Z)} = ie\Big[v_f\gamma^\mu - a_f\gamma^\mu\gamma_5 + i\frac{\mu_f}{\Lambda}\sigma^{\mu\nu}q_\nu + \frac{d_f}{\Lambda}\gamma_5\sigma^{\mu\nu}q_\nu - i\frac{s_f}{\Lambda}q^\mu + \frac{p_f}{\Lambda}\gamma_5 q^\mu\Big] \tag{3}$$

with the propagator

$$g_{(Z)} = \frac{1}{q^2 - M_Z^2 + i\Gamma q^2/M_Z} \quad . \tag{4}$$

In this expression, e is the electric charge and Λ an arbitrary scale. The vector and axial vector coupling v_f and a_f are given within $SU(2)\times U(1)$ by [9]

$$v_f = x_f(I_3^f - 2Q_f s_{f,eff}^2) \qquad a_f = x_f I_3^f \qquad x_f = \sqrt{\frac{\sqrt{2}G_\mu M_Z^2\rho_f}{e^2}} = \frac{\sqrt{\rho_f}}{2s_{f,eff}c_{f,eff}} \tag{5}$$

parameterized using ρ_f and $s_{f,eff}^2$, which, in the Standard Model, are given by $\rho_f = 1$, $s_{f,eff} = \sin\theta_W$ and $c_{f,eff} = \cos\theta_W$ in the Born approximation. θ_W is the weak mixing angle. The parameters μ_f and d_f denote the weak anomalous magnetic and weak electric dipole moments in analogy to QED. They are given in units of e/Λ. Possible additional couplings are parameterized in terms of s_f and p_f. The phase of the coupling constants are chosen such that the Hamiltonian is hermitian if all constants are real.

3 The Spin Structure

The spin amplitude $M_{\lambda_1\lambda_2\alpha_1\alpha_2}$ can be calculated as described in ref. [8, 10].

For an unpolarised beam the spin density is

$$\rho_{\alpha_1\bar{\alpha}_1\alpha_2\bar{\alpha}_2} = \frac{1}{4}\sum_{\lambda_1\lambda_2} M_{\lambda_1\lambda_2\alpha_1\alpha_2}M^*_{\lambda_1\lambda_2\bar{\alpha}_1\bar{\alpha}_2} \quad . \tag{6}$$

As discussed in ref. [10], the double spin indices in the density matrix can be transformed into vector indices, $a, b = 0, 1, 2, 3$, resulting in a differential cross section:

$$d\sigma = \left(R_{00} + \sum_{a=1,3} R_{a0} w_1^a + \sum_{a=1,3} R_{0a} w_2^a + \sum_{a,b=1,3} R_{ab} w_1^a w_2^b \right) dLips \quad . \tag{7}$$

The vectors $\vec{w}_1$ and $\vec{w}_2$ are the quantisation axes for the spin measurement.

The first term in (7) describes the total cross section, the second and third term the polarisations along the directions $\vec{w}_1$ and $\vec{w}_2$, and the last term the spin correlation.

The form of the matrix R_{ab} is simplified if the coordinate system is chosen with the three axes $a = 1, 2, 3$ as those represented by $\hat{\tau}_a$; $\hat{\tau}_3 = \hat{\tau}$, $\hat{\tau}_1 = (\hat{e} \times \hat{\tau})/s$ and $\hat{\tau}_2 = \hat{\tau} \times (\hat{e} \times \hat{\tau})/s$ where $\hat{\tau}$ ($\hat{e}$) is the $\tau^+(e^+)$ direction of flight and $c = \cos\theta$, $s = \sin\theta$, θ being the angle between the τ^+ and the initial e^+. They can be written as: $\tau_1 = (1, 0, 0)$, $\hat{\tau}_2 = (0, c, s)$, $\hat{\tau}_3 = (0, -s, c)$ and $\hat{e} = (0, 0, 1)$. Therefore the $\hat{\tau}_3$-axis is identified with the τ^+ direction of flight and the $\hat{\tau}_2$-axis is in the τ production plane. In the following we always refer to this coordinate system. Within this system, e.g. R_{03} describes the longitudinal polarisation, R_{02} the transverse polarisation measured in the τ production plane and R_{01} the transverse polarisation measured perpendicular to the τ production plane which is therefore also called the normal polarisation.

3.1 The τ Decay

The quantisation axes cannot be chosen by the experimentator, e.g. for the pion decay mode the vectors $\vec{w}_1$ and $\vec{w}_2$ are parallel to the τ neutrino directions of flight because of their well defined helicity [11]. Therefore the τ decay can be considered as a Stern-Gerlach apparatus where the quantisation axis is given by the decay itself[1]. To each event one can assign two polarimeter vectors, $\vec{h}^\pm$, which are functions of the momenta of the $\tau^\pm$ decay products, describing the "quantisation axis". The correlation between the decay products is than given by

$$d\sigma = \sum_{a,b=0,3} r_{ab} h_a^+ h_b^- d\Omega d\Omega_+^* d\Omega_-^* \quad , \tag{8}$$

with $r_{ab} = R_{ab}/R_{00}$ and $h^\pm = (1, \vec{h}^\pm)$. $d\Omega, d\Omega_+^*$ and $d\Omega_-^*$ are the solid angles of the τ^+ in the c.m.s. and of the decay products in the τ rest frames. The formulas for the polarimeter vectors are given in section 5 and in the description of the TAUOLA library [12].

4 The Monte Carlo Program

4.1 τ Production

The exact calculation of the spin density is shown in [8] where the notation used in the present note is also defined. A short summary of the formulae is given in appendices

[1]Since the quantisation axis is not chosen by the experimentator it is not possible to perform incompatible measurements. Therefore the locality along the lines of Bells inequality cannot be tested.

A and B. The program essentially contains the formulae of these two appendices. It is an advantage not to explicitly program the spin density, but to use the so-called helicity factors (cf. Appendix B). As an example, to add the photon exchange including the full spin dynamics and interference between the photon and Z requires only three lines of FORTRAN code (cf. Appendix B.2). Therefore, the generator can be easily modified to add other particles and different kinds of couplings.

4.2 τ Decay

The TAUOLA library is used to simulate the τ decay [12]. There are two ways to call TAUOLA: firstly as in KORALZ [13] where the polarisation vector of the τ is an input to TAUOLA and secondly as in KORALB [14] where the polarimeter vector is an output of TAUOLA. In the SCOT generator, the TAUOLA library is called as in KORALB to have the polarimeter vector as an output.

4.3 Rejection Procedure

The generator provides unweighted events by rejecting events: In the initialisation step hundred events are generated and $d\sigma$ is calculated according to formula (8). A normalisation factor w_0 is than defined by 1.2 times $d\sigma$ of the event with the largest $d\sigma$. During event generation the program calculates for each generated event i a weight w_i, given by $w_i = d\sigma/w_0$. After this step a random generator is called with flat random numbers r between 0 and 1. The event is rejected if w_i is less then r.

The factor 1.2 was chosen to optimise the efficiency and the number of events with $w_i > 1$ (over weighted events).

4.4 Radiative Correction

The electroweak corrections are implemented by use of effective coupling constants [9]. No specific q^2 dependence is used. This should be sufficient for most applications.

Initial radiation of photons is not included. To add initial radiation would mean a major change. For most applications, a simulation of initial radiation is not required as the radiation is damped by the small width of the Z.

Final state radiation in Z decay strongly changes the kinematic of the event. This is implemented by a leading log approximation using the PHOTOS library [15]. PHOTOS is called after the generation of the τ pair including the decay and the rejection with r_{ab}. Therefore, spin flip due to photon radiation is not implemented. This is a negligible contribution [10, 11].

Since some of the QED corrections affect the selection efficiency, background and reconstruction, the SCOT generator should be used in conjunction with KORALZ. For instance, the reconstruction of the τ axis can be affected by initial and final radiation and by spin correlations. Therefore the effects of radiation should be studied by KORALZ whereas effects due to spin correlation and, partly from final state radiation, can be studied using SCOT.

4.5 Structure of the Program

The structure of the SCOT generator is as follows: The τ production routine generates a τ pair (routine GENWIG), calculates the corelation matrix, r_{ab}, using the formulae of Appendix A and B (routine DENSI), calls TAUOLA to obtain the τ decay and the polarimeter vectors $\vec{h}^{\pm}$ (routine DEKAY of TAUOLA) and performs the rejection according to section 4.3 (routine GENWIG). If the event is rejected, then a new τ pair is generated and TAUOLA is called again. If the event is accepted all particle momenta are boosted in the laboratory system (routine SCOT) and PHOTOS is called which simulates final state radiation.

5 Application of the Program

This section describe how the various coupling constants can be measured. The formulae shown here are also used to test the generator (cf. section 6).

5.1 Structure of the Spin Density

To simplify the discussion it is useful to order the various contributions described in section 2 in powers of some ε:

$$p_e \approx s_e \approx d_e \approx \mu_e \approx p_\tau \approx s_\tau \approx d_\tau \approx \mu_\tau \approx \mathcal{O}(\varepsilon \cdot e/M_Z) \qquad m^2 \approx v_\tau^2 \approx v_e^2 \approx \mathcal{O}(\varepsilon) \quad . \tag{9}$$

The mass m refers to the τ mass in units of the τ energy whereas the electron mass, is of the order of $\mathcal{O}(\varepsilon^2)$.

Neglecting terms of the order of $\mathcal{O}(\varepsilon^{3/2})$, which are difficult to measure because of large statistical errors, the spin density for Z exchange is given by [8]:

$$\begin{array}{rcl}
R_{00} = +R_{33} &=& \omega_\tau \left((1+c^2) + 2cRe(\gamma_{ne})Re(\gamma_{n\tau})\right) \\
R_{11} = -R_{22} &=& s^2\bar{\omega}_\tau \\
R_{01} = -R_{10} &=& -2sc\beta^2 Re(a_\tau d_\tau^*) \\
R_{23} = -R_{32} &=& -2sc\beta^2 Im(a_\tau d_\tau^*) \\
R_{02} = +R_{20} &=& +2sc\beta Re(a_\tau \delta_3^*) \\
R_{13} = +R_{13} &=& +2sc\beta Im(a_\tau \delta_3^*) \\
R_{03} = +R_{30} &=& +\omega_\tau \left((1+c^2)Re(\gamma_{n\tau}) + 2cRe(\gamma_{ne})\right) \\
R_{12} = +R_{21} &=& -\omega_\tau s^2 Im(\gamma_{n\tau})
\end{array} \tag{10}$$

with

$$\begin{array}{rclcrcl}
\omega_e &=& |a_e|^2 + |v_e|^2 & & \omega_\tau &=& \beta^2|a_\tau|^2 + |\delta_1|^2 \\
 & & & & \bar{\omega}_\tau &=& \beta^2|a_\tau|^2 - |\delta_1|^2 \\
\gamma_{ne} &=& 2v_e a_e^*/\omega_e & & \gamma_{n\tau} &=& 2\beta v_\tau a_\tau^*/\omega_\tau \\
\delta_1 &=& v_\tau + m\mu_\tau & & \delta_3 &=& mv_\tau + \mu_\tau
\end{array} \tag{11}$$

and m, the τ mass in units of the τ energy, β the τ velocity in the c.m.s. and $s = \sin\theta_\tau$, $c = \cos\theta_\tau$ where θ_τ is the angle between the τ^+ and positron direction of flight. An overall normalisation N of:

$$N = 2\omega_e N_{ZZ} \qquad N_{ZZ} = \left|\frac{16\sqrt{2}G_\mu M_Z^2 s_{f,eff}^2 c_{f,eff}^2}{q^2 - M_Z^2 + i\Gamma q^2/M_Z}\right|^2 \approx \left|\frac{4e^2}{q^2 - M_Z^2 + i\Gamma q^2/M_Z}\right|^2 \tag{12}$$

has been dropped.

To simplify the formula, the scale factor was chosen to be $\Lambda = q$ which may easily be changed by the substitutions $\mu_\tau \to \mu_\tau\frac{q}{\Lambda}$ and $d_\tau \to d_\tau\frac{q}{\Lambda}$. Formulae including the interference between Z and photon exchange are given in [8].

The density shows that the real part of the product of vector and axial vector coupling may be measured by the longitudinal polarisation (see R_{03}) whereas the imaginary part affects the correlation of the transverse polarisation (see R_{12}). The real parts of the weak magnetic and electric dipole moment may be measured by the polarisations along $\hat{\tau}_1$ and $\hat{\tau}_2$ (see R_{01} and R_{02}) and their imaginary parts by the correlations R_{23} and R_{13}. There seems to be no way to measure s_τ and p_τ if the beam is unpolarised. This measurement would be interesting however, because it directly tests whether the Z boson is a gauge boson or not.

5.2 Extraction of the Spin Density

Assume in an experiment we want to measure the polarisation $P_{\vec{n}}$ of the τ^+ along the direction $\vec{n}$. The differential cross-section can be obtained by integrating eq. (8):

$$d\sigma = (1 + P_{\vec{n}}\vec{n}\cdot\vec{h})d\Omega d\Omega^* \qquad P_{\vec{n}} = \sum_{a=1,3} r_{a0}\vec{n}_a \quad , \tag{13}$$

where $\vec{h}$ is again the polarimeter vector which can be calculated from the measured momenta of the τ^+ decay products. Therefore $P_{\vec{n}}$ can be measured by measuring the polarimeter vector of each event and applying a maximum likelihood fit. The maximum likelihood fit results in [16]:

$$\left\langle \frac{\vec{n}\cdot\vec{h}}{1 + P_{\vec{n}}\vec{n}\cdot\vec{h}} \right\rangle = 0 \quad , \tag{14}$$

which can be approximated for small $P_{\vec{n}}$ by:

$$P_{\vec{n}} \approx \frac{\left\langle \vec{n}\cdot\vec{h} \right\rangle}{\left\langle (\vec{n}\cdot\vec{h})^2 \right\rangle} \qquad \sigma^2 = \left\langle (\vec{n}\cdot\vec{h})^2 \right\rangle / N \quad , \tag{15}$$

where σ is the statistical error of $P_{\vec{n}}$ if N events are analysed.

In a similar way, the correlation $C_{\vec{n}\vec{m}}$ between the polarisations of the τ^+ along the direction $\vec{n}$ and the τ^- along the direction $\vec{m}$ is given by

$$C_{\vec{n}\vec{m}} \approx \frac{\left\langle (\vec{n}\cdot\vec{h}^+)(\vec{m}\cdot\vec{h}^-) \right\rangle}{\left\langle (\vec{n}\cdot\vec{h}^+)^2(\vec{m}\cdot\vec{h}^-)^2 \right\rangle} \qquad \sigma^2 = \left\langle (\vec{n}\cdot\vec{h}^+)^2(\vec{m}\cdot\vec{h}^-)^2 \right\rangle / N \quad . \tag{16}$$

It turns out that this approximation is rather good, even if one wants to measure the correlations r_{11} and r_{22} which are of the order of 1/2. This was tested by comparing the approximations with the results of a full maximum likelihood fit.

Of course, the statistical errors of the various polarisations and correlations are themselves correlated and can be calculated by the exact maximum likelihood fit.

5.3 The Polarimeter Vector

For the pion decay mode, the polarimeter vector is given by

$$\vec{h}^{\pm} = \mp \hat{p} \quad , \tag{17}$$

where $\hat{p}$ is the pion direction of flight in the τ rest frame. Since the neutrino momentum cannot be measured, an integration has to be performed for the polarimeter vector of leptonic decay modes:

$$\vec{h}^{\pm} = \pm \hat{p} \cdot \frac{4x-1}{3-4x} \quad , \tag{18}$$

where x is the lepton energy in the τ rest frame normalised to the τ mass. In case of the ρ decay mode the polarisation vector is given by

$$\vec{h}^{\pm} = \mp \frac{2(q_0 + \vec{q} \cdot \hat{p}_\rho)\vec{q} + (m_\rho^2 - 4m_\pi^2)\hat{p}_\rho}{2q_0(q_0 + \vec{q} \cdot \hat{p}_\rho) + m_\rho^2 - 4m_\pi^2} \quad , \quad (q_0, \vec{q}) = (E_{\pi^\pm} - E_{\pi^0}, \vec{p}_{\pi^\pm} - \vec{p}_{\pi^0}) \quad , \tag{19}$$

where $\hat{p}_\rho$ is the ρ direction of flight and p_π are the pion momenta in the τ rest frame. Another polarisation vector could also be used

$$\vec{h}^{\pm} = \mp \hat{p}_\rho \frac{m_\tau^2 - 2m_\rho^2}{m_\tau^2 + 2m_\rho^2} \quad , \tag{20}$$

resulting in a lower analysing power because of the integration over $\vec{q}$. The formula for the polarimeter vector of the a_1 decay is rather lengthy and can be found in [12] where the polarisation vectors of all the most important τ decay modes are calculated[2]. The absolute value of $\vec{h}^{\pm}$ is one if no integration over unknown particles is performed (hadronic decay modes) and if the hadron current is real up to a global phase (which is not the case for the a_1 decay mode).

The analysing power of the various decay modes, normalised to the pion decay mode, can be described by:

$$S_X = \sqrt{\frac{\left\langle (\vec{n} \cdot \vec{h}_{\tau \to \pi\nu})^2 \right\rangle}{\left\langle (\vec{n} \cdot \vec{h}_{\tau \to X\nu})^2 \right\rangle}} \quad . \tag{21}$$

The Monte Carlo program shows an analysing power of 0.48, 1.00, 0.71 for the lepton, ρ and a_1 decay modes respectively. If the polarimeter vector of eq. (20) is used, the

[2]In fact for the polarimeter vector of the a_1 decay mode the appropriated routines are copied from the TAUOLA library because they are also used for generating the τ decay.

analysing power of the ρ is 0.5. It is clear that the π and ρ decay modes are the most efficient. The expected statistical error σ_X and σ_{XY} of the polarisation for $\tau \to X\nu$ and of the correlation for $\tau^+ \to X\nu$, $\tau^- \to Y\nu$ can than easily be estimated by

$$\sigma_X = S_X \sigma_\pi \sqrt{\frac{Br(\tau \to X\nu)}{Br(\tau \to \pi\nu)}} \quad , \quad \sigma_{XY} = S_X S_Y \sigma_{\pi\pi} \sqrt{\frac{Br(\tau \to X\nu) Br(\tau \to Y\nu)}{(Br(\tau \to \pi\nu))^2}} \quad , \tag{22}$$

Where Br are the branching ratios of the corresponding τ decay modes. Experimental effects due to inefficiency and background are ignored. The statistical errors σ_π and $\sigma_{\pi\pi}$ are discussed in the next section.

5.4 Measurement of the Lorentz Structure

After data taking, the τ events can be selected and the decay mode identified. For each event the polarimeter vector has to be calculated using the formulae above. The spin density and Lorentz structure can be extracted by a least square fit or by the approximations shown previously.

Of course, the method requires a reconstruction of the τ axis which is possible if both τ leptons decay into hadrons. From the hadron energy, the angle between the hadron and the τ axis can be calculated, resulting in two cones around the hadron giving the τ axis at their cross over. Of course if the τ life time is zero, one is left with a two fold ambiguity. However, the reconstruction is unique due to the finite life time of the τ because the two cones have no common top point [17, 18]. This method needs a tracking device with a spatial resolution of several μm in space.

Inspection of equation (10) shows that the following quantities are relevant for the weak moments:

$$\begin{aligned} P_N &= (r^f_{10} - r^b_{10} - r^f_{01} + r^b_{10})/4 \quad , \quad & P_T &= (r^f_{20} - r^b_{20} + r^f_{02} - r^b_{20})/4 \\ C_{NL} &= (r^f_{13} - r^b_{13} + r^f_{31} - r^b_{31})/4 \quad , \quad & C_{TL} &= (r^f_{23} - r^b_{23} + r^f_{32} - r^b_{32})/4 \end{aligned} \tag{23}$$

with

$$r^f_{ab} = \frac{\int_0^{c_0} dc R_{ab}}{\int_0^{c_0} dc R_{00}} \quad , \quad r^b_{ab} = \frac{\int_{-c_0}^0 dc R_{ab}}{\int_{-c_0}^0 dc R_{00}} \quad , \tag{24}$$

where c_0 restrict the geometrical acceptance by requiring $|\cos\theta_\tau| < c_0$. The quantities r_{ab} are measurable using:

$$r^{f,b}_{a0} \approx \frac{\left\langle \hat{\tau}_a \cdot \vec{h}^+ \right\rangle_{f,b}}{\left\langle (\hat{\tau}_a \cdot \vec{h}^+)^2 \right\rangle_{f,b}} \quad , \quad r^{f,b}_{ab} \approx \frac{\left\langle (\hat{\tau}_a \cdot \vec{h}^+)(\hat{\tau}_b \cdot \vec{h}^-) \right\rangle_{f,b}}{\left\langle (\hat{\tau}_a \cdot \vec{h}^+)^2 (\hat{\tau}_b \cdot \vec{h}^-)^2 \right\rangle_{f,b}} \tag{25}$$

where $\hat{\tau}_a$ is defined in section 3 and $\langle x \rangle_{f,b}$ stands for the mean value of x in the forward or backward region. The moments are than given by use of the formulae:

$$\begin{aligned} P_N &= \tfrac{2Re(a_\tau d_\tau^*)}{\omega_\tau} X_0 \quad , \quad & P_T &= \tfrac{2Re(a_\tau d_3^*)}{\omega_\tau} X_0 \quad , \\ C_{NL} &= \tfrac{2Im(a_\tau d_3^*)}{\omega_\tau} X_0 \quad , \quad & C_{TL} &= -\tfrac{2Im(a_\tau d_\tau^*)}{\omega_\tau} X_0 \quad , \end{aligned} \tag{26}$$

with

$$X_0 = \frac{1-\sqrt{(1-c_0^2)^3}}{c_0(1+c_0^2/3)} \quad , \quad \frac{2Re(a_\tau d_\tau^*)}{\omega_\tau} \approx -3.7 Re(d_\tau) \quad etc. \quad . \tag{27}$$

For $c_0 = 0.9(0.5)$ the phase space factor X_0 is 0.80(0.65) respectively[3].

The expected statistical error on the polarisations P_N, P_T and on the correlations C_{NL} and C_{TL} are $1.2/\sqrt{N_\pi}$ and $2.1/\sqrt{N_{\pi\pi}}$, respectively, for the case where both τ decay into pion and neutrino (cf. final output). Therefore the real and imaginary part of the weak electric dipole and magnetic moment can be measured up to a precision of several percent in units of e/m_Z for 10^6 hadronic Z decays.

The requirement on the detector are on one side a spatial resolution of the tracking device of the order of several μm and on the other side a high granular electromagnetic calorimeter to identify photons coming from the π^0 decay in the ρ decay mode.

5.5 Some Remarks on Systematic Errors

For the pion decay mode the quantities $\hat{r}_a \cdot \vec{h}^\pm$ are given by

$$\hat{r}_1 \cdot \vec{h}^\pm = \mp det(\hat{p}_\pm, \hat{e}, \hat{\tau})/s \quad , \quad \hat{r}_2 \cdot \vec{h}^\pm = \mp\left((\hat{e}\cdot\hat{p}_\pm) - (\hat{e}\cdot\hat{\tau})(\hat{\tau}\cdot\hat{p}_\pm)\right)/s \tag{28}$$

where $\hat{p}_\pm$ is the pion direction of flight in the τ rest frame. These formulas can be expressed in terms of the laboratory momenta $(E_\pm^l, \vec{p}_\pm^{\,l})$:

$$\hat{r}_1 \cdot \vec{h}^\pm = \mp\frac{B}{s} det(\hat{p}_\pm^l, \hat{e}, \hat{\tau}) \quad , \quad \hat{r}_2 \cdot \vec{h}^\pm = \mp\frac{B}{s}\left((\hat{e}\cdot\hat{p}_\pm^l) - (\hat{e}\cdot\hat{\tau})(\hat{\tau}\cdot\hat{p}_\pm^l)\right) \tag{29}$$

with

$$(\hat{\tau}\cdot\hat{p}_\pm^l) = \pm\frac{2E_\tau E_\pm^l - m_\tau^2 - m_\pi^2}{2p_\tau p_\pm^l} \quad , \quad B = \frac{p_\pm^l}{p_\pm} \quad , \quad p_\pm = \frac{m_\tau^2 - m_\pi^2}{2m_\tau} \quad . \tag{30}$$

The formula above show that the normal polarisation is given by a volume element spanned by the beam, τ and pion whereas the transverse polarisation is basically the difference between two almost equal numbers, the cosine of the polar angle of the τ and the pion times the cosine of the angle between τ and pion. Therefore the normal polarisation should be sensitive to systematic errors like a twist inside the detector whereas the transverse polarisation depends more on the statistical error of the energy and angular measurement, resulting in a correlation between the three cosines. A systematic error on the angle between the two pions projected on the plane transverse to the beam axis can affect the normal polarisation as well the transverse polarisation because the polar angle of the reconstructed τ axis depends on the angle between the pions.

[3]Some detectors have a special high resolution tracking device, called a vertex detector, with a geometrical acceptance of around $c_0 = 0.6$. The cut in the polar angle has to be applied on the τ axis and not on the reconstructed tracks to avoid a bias especially on the transverse polarisation which can be seen by inspecting the term r_{20}, r_{02} and r_{22}. Therefore, the results are also given for $c_0 = 0.5$.

The systematic error is even more complicated due to the following effect: take an event and imagine a path from the true track position to the reconstructed position. It can be shown that the formula for calculating the vector, $\vec{v}$, which connects the two decay points of the τ^+ and the τ^- decay point is a continuous function of the measured quantities. Therefore the path from the true track position to the reconstructed position results in a continuous path from the true $\vec{v}$ to the reconstructed $\vec{v}$. There are two ways to obtain such a path, first by rotation, or by decreasing the length of $\vec{v}$ to zero and increasing it again but with a different direction, which is the case here. This means for a given event there is a certain probability to obtain a rather different τ axis. So one has again two possible τ axis if one takes the finite resolution into account. In principal, it is possible to calculate on an event by event basis the two possible directions of $\vec{v}$ and their probabilities. For each event one obtains two sets of $\vec{h}^{\pm}$ with their probabilities to be true and the spin density can be calculated by a weighted mean. There is a warning here: such a method is a challenge and needs an excellent knowledge of the detector. It is still not clear if the method proposed here and in [6, 8] will work in the end.

6 Test of the Generator

In the following the Monte Carlo generator is tested by performing the application described in section 5. The test is up to a certain order in ε but the generator itself dose not use these approximations. The spin density, r_{ab}, is extracted using the formulae described in the preceding section. A full maximum likelihood fit, as well as the approximations, are used. The extracted spin density is then compared with the input variables using the formulae above. 10^6 τ pairs are generated for each different set of input parameters and pairs of τ decay modes. The decay modes which are considered here are the leptonic decay modes and the π, ρ and a_1 decay modes.

All the various polarisations and correlations are correct in magnitude and sign up to the order of $\mathcal{O}(\varepsilon)$[4]. Since parts of the spin density change sign between forward and backward scattering (see the factor 'sc' in eq. (10)) the polarisation and correlation were tested in the forward and backward regions independently.

Bremsstrahlung was tested by a comparison with KORALZ. Up to a polar angle of $|\cos\theta_\tau| < 0.9$ no difference was observed for energies larger than 2GeV. In the range between 2GeV and one per mille of the beam energy, the number of photons generated was 20% less due to the lack of interference between final and initial state radiation.

[4]At first the six correlations r_{ab}, $0 \neq a \neq b \neq 0$ had a wrong sign. This sign can be flipped without changing all the other signs by taking the complex conjugate of the amplitudes or by modifying routine TRAL04 from KORALZ which is responsible for the boost from the τ rest frame to the laboratory system. (The link between SCOT and TAUOLA was done by copying some of the routines of KORALZ.) It turned out that TRAL04 had to be modified in a similar way as for KORALB [19]. I want to point out that this modification dos not affect any of the results of KORALZ because transverse spin correlation is not foreseen in KORALZ.

7 Use of the Generator

The main interface routine is `SCOT(IMODE,RPAPRO,RPADEC)`. The program has to be initialised by calling `SCOT` with `IMODE=-1`. An event can be generated by calling `SCOT` with `IMODE=0` whereas `IMODE=1` prints out some useful statistical information. `RPAPRO` and `RPADEC` are the input parameters. They are arrays of dimension 35 and should be defined by:

```
REAL RPAPRO(35),RPADEC(35)
INTEGER NPAPRO(35),NPADEC(35)
EQUIVALENCE (RPAPRO,NPAPRO)
EQUIVALENCE (RPADEC,NPADEC)
```

and the arrays are used in real and integer format. The parameters are listed in Table 1 and 2 were also the position of a particular parameter inside the array and the formats are shown. An example of how several events can be generated is shown in the FORTRAN code (routine `SCOTES`), where more technical information is also available. An example of how to extract the spin density is given in routine `SCOTAN`.

Table 1 and 2 shows the input parameters with their values used in the example. If `IFWEAK` is set to one then weak radiative corrections are implemented by use of the effective coupling constants `RHOWEA`, `SINWEA` and `ALPHA`. In this case, the vector and axial vector coupling constants are recalculated using the equation (5). With the parameter `COSTO` the polar angle range can be preselected.

The final output prints the input parameters of SCOT and TAUOLA. Furthermore the following statistical information is printed: number of generated events (`NRLOOP`), number of over-weighted events (`NRWTHI`), number of events with negative weights (`NRWTLO`, should be zero), number of rejected events according to section 4.3 (`NRWTRE`), number of accepted events (`NRWTAC`) and the $\tau\tau$ cross section within the polar angle range.

The output of the analysis routine `SCOTAN` shows the expected and measured asymmetries as defined in equation (23).

8 Conclusion

The Monte Carlo generator, SCOT, can be used to simulate the reaction $e^+e^- \rightarrow \tau^+\tau^-$ at energies around the Z peak, using the full Lorentz structure of the Z. The program takes into account all spin effects and allows e.g. the measurement of the real and imaginary part of the weak electric and magnetic dipole moments of the τ. Final state radiation is included using the leading log approximation of PHOTOS. The τ decays are performed by TAUOLA. The SCOT generator is written using spin amplitudes which allows easy modification of the Lorentz structure.

Def.	Pos.	Parameter	Meaning	example
R	1	CMSENE	$\sqrt{s}$, CMS-energy	91.18GeV
R	2	AMZ	M_Z , Z-mass	91.18GeV
R	3	GAMMAZ	Γ_Z , Z-width	2.484GeV
I	4	IFEXZ	Z-exchange on(=1)/off(=0)	1
I	5	IFEXPH	photon exchange on(=1)/off(=0)	1
R	6	EVRE	$Re(v_e)$ (cf. eq. (3))	-0.0475
R	7	EARE	$Re(a_e)$	-0.5941
R	8	EMRE	$Re(\mu_e)$	0.0 e/M_Z
R	9	EDRE	$Re(d_e)$	0.0 e/M_Z
R	10	ESRE	$Re(s_e)$	0.0 e/M_Z
R	11	EPRE	$Re(p_e)$	0.0 e/M_Z
R	12	EVIM	$Im(v_e)$	0.0 e/M_Z
R	13	EAIM	$Im(a_e)$	0.0 e/M_Z
R	14	EMIM	$Im(\mu_e)$	0.0 e/M_Z
R	15	EDIM	$Im(d_e)$	0.0 e/M_Z
R	16	ESIM	$Im(s_e)$	0.0 e/M_Z
R	17	EPIM	$Im(p_e)$	0.0 e/M_Z
R	18	TVRE	$Re(v_\tau)$	-0.0475
R	19	TARE	$Re(a_\tau)$	-0.5941
R	20	TMRE	$Re(\mu_\tau)$	0.05 e/M_Z
R	21	TDRE	$Re(d_\tau)$	0.05 e/M_Z
R	22	TSRE	$Re(s_\tau)$	0.0 e/M_Z
R	23	TPRE	$Re(p_\tau)$	0.0 e/M_Z
R	24	TVIM	$Im(v_\tau)$	0.0 e/M_Z
R	25	TAIM	$Im(a_\tau)$	0.0 e/M_Z
R	26	TMIM	$Im(\mu_\tau)$	0.05 e/M_Z
R	27	TDIM	$Im(d_\tau)$	0.05 e/M_Z
R	28	TSIM	$Im(s_\tau)$	0.0 e/M_Z
R	29	TPIM	$Im(p_\tau)$	0.0 e/M_Z
R	30	ALPHA	$\alpha(q^2 = M_Z^2)$	1/129.4
I	31	IPHOTO	final state radiation on(=1)/off(=0)	0
I	32	IFWEAK	weak corrections on(=1)/off(=0)	0
R	33	RHOWEA	$\rho = \rho_e = \rho_\tau$	1.0
R	34	SINWEA	$\sin^2\theta_W(q^2 = M_Z^2) = s_{e,eff} = s_{\tau,eff}$	0.233
R	35	COSTO	c_0 , polar angle range	0.5

Table 1. The input parameter RPAPRO of SCOT. The first column contains the definition (REAL or INTEGER), the second column shows the position in the array RPAPRO. The name of the parameters in the example is shown in the third column. The last two columns give the meaning of the parameter and the values in the example including the units.

Def.	Pos.	Parameter	Meaning	example
I	1	JAK1	decay mode (TAUOLA)	3
I	2	JAK2	decay mode (TAUOLA)	3
I	3	ITDKRC	radiation (TAUOLA)	0
R	4	RKODEC	k_0 (TAUOLA)	0.001
I	5	IDFF	PDG. id of τ (TAUOLA)	-15

Table 2. The input parameter RPADEC of SCOT. The first column contains the definition (REAL or INTEGER), the second column shows the position in the array RPADEC. The name of the parameters in the example is shown in the third column. The last two columns give the meaning of the parameter and the values in the example including the units.

A Appendix: Transformation from Helicity Basis to Cartesian Basis

The spin amplitude can be decomposed according to:

$$M_{\lambda_1\lambda_2\alpha_1\alpha_2} = \sum_{i=1,4} \lambda_{(i)}(\alpha_+ A_{(i)} + |\alpha_+| B_{(i)} + \alpha_- C_{(i)} + |\alpha_-| D_{(i)}) \tag{31}$$

with $\lambda_{(1)} = \lambda_+$, $\lambda_{(2)} = |\lambda_+|$, $\lambda_{(3)} = \lambda_-$, $\lambda_{(4)} = |\lambda_-|$ and $\lambda_\pm = \lambda_1 \pm \lambda_2$, $\alpha_\pm = \alpha_1 \pm \alpha_2$.

The spin density reads:

$$\begin{aligned} \rho_{\alpha_1\bar{\alpha}_1\alpha_2\bar{\alpha}_2} &= \tfrac{1}{4}\textstyle\sum_{\lambda_1\lambda_2} \quad M_{\lambda_1\lambda_2\alpha_1\alpha_2}\tilde{M}^*_{\lambda_1\lambda_2\bar{\alpha}_1\bar{\alpha}_2} \\ &= \tfrac{1}{2}\textstyle\sum_{(i)} \quad (\alpha_+ A_{(i)} + |\alpha_+| B_{(i)} + \alpha_- C_{(i)} + |\alpha_-| D_{(i)}) \\ &\qquad (\bar{\alpha}_+ \tilde{A}^*_{(i)} + |\bar{\alpha}_+| \tilde{B}_{(i)} +^* \bar{\alpha}_- \tilde{C}^*_{(i)} + |\bar{\alpha}_-| \tilde{D}^*_{(i)}) \end{aligned} \tag{32}$$

A... and $\tilde{A}$... may be different if ρ expresses only the interference between two interactions. We call them helicity factors. If $A = \tilde{A}$ then:

$$\begin{aligned} R^{(i)}_{00} &= 2 \quad (+|A_{(i)}|^2 + |B_{(i)}|^2 + |C_{(i)}|^2 + |D_{(i)}|^2) \\ R^{(i)}_{11} &= 2 \quad (-|A_{(i)}|^2 + |B_{(i)}|^2 - |C_{(i)}|^2 + |D_{(i)}|^2) \\ R^{(i)}_{22} &= 2 \quad (+|A_{(i)}|^2 - |B_{(i)}|^2 - |C_{(i)}|^2 + |D_{(i)}|^2) \\ R^{(i)}_{33} &= 2 \quad (+|A_{(i)}|^2 + |B_{(i)}|^2 - |C_{(i)}|^2 - |D_{(i)}|^2) \\ R^{(i)}_{01} &= 4 \quad (Re(B_{(i)}D^*_{(i)}) \pm Re(A_{(i)}C^*_{(i)})) \\ R^{(i)}_{32} &= -4 \quad (Im(B_{(i)}D^*_{(i)}) \pm Im(A_{(i)}C^*_{(i)})) \\ R^{(i)}_{02} &= -4 \quad (Im(A_{(i)}D^*_{(i)}) \pm Im(B_{(i)}C^*_{(i)})) \\ R^{(i)}_{31} &= 4 \quad (Re(A_{(i)}D^*_{(i)}) \pm Re(B_{(i)}C^*_{(i)})) \\ R^{(i)}_{03} &= 4 \quad (Re(A_{(i)}B^*_{(i)}) \mp Re(C_{(i)}D^*_{(i)})) \\ R^{(i)}_{12} &= -4 \quad (Im(A_{(i)}B^*_{(i)}) \mp Im(C_{(i)}D^*_{(i)})) \end{aligned} \tag{33}$$

The final R_{ab} then is the sum over (i). The $\pm$ sign refers to $R_{ab} \to R_{ba}$.

If $A_{(i)}$ is different from $\tilde{A}_{(i)}$ then the following replacement should be made:

$$|A_{(i)}|^2 \rightarrow 2Re(A_{(i)}\tilde{A}^*_{(i)}) \qquad A_{(i)}B^*_{(i)} \rightarrow A_{(i)}\tilde{B}^*_{(i)} + \tilde{A}_{(i)}B^*_{(i)} \tag{34}$$

B Appendix: The Helicity Factors

B.1 Z Boson Contribution

$$\begin{aligned}
A_{(1)} &= -(v_e\delta_1 + c\beta a_e a_\tau) \\
B_{(1)} &= -(\beta v_e a_\tau + c a_e \delta_1) \\
C_{(1)} &= s\beta a_e d_\tau \\
D_{(1)} &= -isa_e\delta_3 \\
A_{(2)} &= -(a_e\delta_1 + c\beta v_e a_\tau) \\
B_{(2)} &= -(\beta a_e a_\tau + c v_e \delta_1) \\
C_{(2)} &= s\beta v_e d_\tau \\
D_{(2)} &= -isv_e\delta_3 \\
A_{(3)} &= s\beta d_e a_\tau \\
B_{(3)} &= sd_e\delta_1 \\
C_{(3)} &= -p_e\delta_0 + c\beta d_e d_\tau \\
D_{(3)} &= -i(\beta p_e s_\tau + c d_e \delta_3) \\
A_{(4)} &= is\beta\mu_e a_\tau \\
B_{(4)} &= is\mu_e\delta_1 \\
C_{(4)} &= i(s_e\delta_0 + c\beta\mu_e d_\tau) \\
D_{(4)} &= -(\beta s_e s_\tau - c\mu_e\delta_3)
\end{aligned} \tag{35}$$

with:

$$\delta_0 = p_\tau - m a_\tau \qquad \delta_1 = v_\tau + m\mu_\tau \qquad \delta_3 = m v_\tau + \mu_\tau$$

and an overall normalisation factor for the amplitude of:

$$\frac{i4e^2}{q^2 - M_Z^2 + i\Gamma q^2/M_Z} \quad .$$

The longitudinal polarisation of the Z leads to additional terms:

$$\begin{aligned}
C_{(3)} &= -p_e\delta_0 \\
D_{(3)} &= -i\beta p_e s_\tau \\
C_{(4)} &= +is_e\delta_0 \\
D_{(4)} &= -\beta s_e s_\tau
\end{aligned} \tag{36}$$

with the same overall factor as above multiplied by $-q^2/M_Z^2$. But these terms are gauge dependent and can be gauged away at the Z resonance. Therefor to measure these contributions is a direct test of the gauge invariance.

B.2 The Photon Contribution

The photon leads to:

$$\begin{aligned} A_{(1)} &= -1 \\ B_{(2)} &= -c \\ D_{(2)} &= -ism \end{aligned} \tag{37}$$

with an overall factor for the amplitude of $4ie^2Q_eQ_f/q^2$.

The final density for a given (i) can be calculated by summing over the various contributions, e.g.

$$A_{(i)} = A_{(i)}(Z) + A_{(i)}(photon) \tag{38}$$

to calculate the various $R_{ab}^{(i)}$. The final density is then

$$R_{ab} = \sum_{i=1,4} R_{ab}^{(i)} \quad . \tag{39}$$

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```
      OUTPUT FROM SCOTES:
 NEVTES        10000
 CMSENE    91.179993
 NEVTES         JAK1          JAK2        ITDKRC
  10000            3             3             0
 RKODEC     0.001000
   IDFF          -15
              SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
              S                                      S
              S            SCOT GENERATOR            S
              S           ULRICH  STIEGLER           S
              S              JULY  1993              S
              S  CENTER OF MASS ENERGY  91.180 GEV   S
              S                                      S
              SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
```

(some output from TAUOLA which is deleted here)

```
***************************************************************************

 RANMAR INITIALIZED:    54217137           0          0

***************************************************************************

SCOT: GET WTNORM
SCOT:  WTNORM CALLCULATED
```

(some event listing which is deleted here)

```
              SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
              S                                      S
              S            SCOT GENERATOR            S
              S           ULRICH  STIEGLER           S
              S              JULY  1993              S
              S  CENTER OF MASS ENERGY  91.180 GEV   S
              S                                      S
              SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS


        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
        S      CMS ENERGY                 91.180           S
        S           POLAR ANGLE RANGE   COSTO       0.500  S
        S      Z MASS     Z WIDTH         91.180    2.484  S
        S      1/ALPHA    Q**2=MZ**2     129.400           S
        S      Z EXCHANGE       IFEXZ          1           S
        S      GAMMA            IFEXPH         1           S
        S      RADIATION        IPHOTO         0           S
        S      WEAK COR.        IFWEAK         0           S
        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS


        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
        S               COUPLING CONSTANTS FOR ELECTRON:   S
        S                                   REAL   IMAGINAR    S
        S      VECTOR      EVRE EVIM     -0.0475   0.0000  S
        S      AXIAL       EARE EAIM     -0.5941   0.0000  S
        S      ELECTRIC    EDRE EDIM      0.0000   0.0000  S
        S      MAGNETIC    EMRE EMIM      0.0000   0.0000  S
        S      SCALAR      ESRE ESIM      0.0000   0.0000  S
        S      PSYDO SCA.  EPRE EPIM      0.0000   0.0000  S
        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS


        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
        S                    COUPLING CONSTANTS FOR TAU:   S
        S                                   REAL   IMAGINAR    S
        S      VECTOR      TVRE TVIM     -0.0475   0.0000  S
        S      AXIAL       TARE TAIM     -0.5941   0.0000  S
        S      ELECTRIC    TDRE TDIM      0.0500   0.0500  S
        S      MAGNETIC    TMRE TMIM      0.0500   0.0500  S
        S      SCALAR      TSRE TSIM      0.0000   0.0000  S
        S      PSYDO SCA.  TPRE TPIM      0.0000   0.0000  S
        SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
```

```
SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
S                    STATISTIC:                        S
S          NRLOOP                   26393              S
S          NRWTHI     NRWTLO           12          0   S
S          NRWTRE     NRWTAC        16393      10000   S
S                                                      S
S       ACCEPTED      EVENTS        10000              S
S          CROSS SECTION nb                  0.83179   S
S                ERROR    nb               0.268E-02   S
S    THE CROSS SECTION IS WITHOUT BREMSSTRAHLUNG       S
SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS

SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
S                      TAUOLA PARAMETER                S
S             JAK1         JAK2          3          3  S
S             IDFF                     -15             S
S           ITDKRC      XK0DEC           0    0.00100  S
SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
DENSI ======DENSI output============
nrcalls                26493
V_E A_E     -0.0475     0.0000     -0.5941     0.0000
M_E D_E      0.0000     0.0000      0.0000     0.0000
S_E P_E      0.0000     0.0000      0.0000     0.0000
L_E          1.0000     0.0000
V_T A_T     -0.0475     0.0000     -0.5941     0.0000
M_T D_T      0.0500     0.0500      0.0500     0.0500
S_T P_T      0.0000     0.0000      0.0000     0.0000
L_T          1.0000     0.0000
IFEXPH IFEXZ      1       1
```

some output from TAUOLA which is deleted here

```
       OUTPUT FROM SCOTAN
              EXPECTED       MEASURED         ERROR
       P_T  -0.3494E-01  -0.3863E-01     0.1240E-01
       P_N  -0.3364E-01  -0.2122E-01     0.1220E-01
       P_L   0.1540E+00   0.1687E+00     0.1250E-01
      C_TT   0.8096E+00   0.8171E+00     0.3037E-01
      C_NN  -0.8096E+00  -0.8128E+00     0.2982E-01
      C_LL   0.1000E+01   0.9533E+00     0.3039E-01
      C_TN   0.0000E+00   0.3170E-03     0.2156E-01
      C_TL   0.3494E-01   0.4520E-01     0.2139E-01
      C_NL  -0.3494E-01  -0.2535E-01     0.2118E-01
```