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Study of the Weak Dipole Moment in $Z \rightarrow \tau\tau$ using the rho decay mode

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Abstract

A direct test of CP-invariance of the neutral current can be done by studying CP-odd spin correlation in the reaction $e^+e^- \rightarrow \tau^+\tau^-$. In this note we concentrate on the rho decay mode.

1 Introduction

It has been pointed out [?, ?] that various decay modes of the Z boson can be used to search for new CP-violating effects beyond the Kobayashi-Maskawa mechanism by measuring appropriate CP-odd correlations. These CP-odd correlations provide direct information on CP-odd form factors [?]. For $Z \to \tau^+\tau^-$ the corresponding on-shell amplitude contains only one CP-odd form factor, the weak dipole moment $d_\tau(m_Z)$.

An indirect measurement of the dipole moment can be obtained from the partial widths of Z decays. In the case of τ the partial width Γ_τ has a quadratic dependence on the dipole moment and one obtains an additional contribution $\Delta\Gamma_\tau$ to the width given by $\Delta\Gamma_\tau \approx |d_\tau(m_Z)|^2 m_Z^3/(24\pi)$ [?]. Using the data accumulated at LEP from 1989 to 1991 with the ALEPH detector the partial width was measured to be $\Gamma_\tau = (84.54 \pm 1.20) MeV$ [?]. This can be compared with the theoretical prediction $\Gamma_\tau^{SM} = (83.76 \pm 0.8) MeV$ given by the Standard Model [?], or with the measured partial width $\Gamma_{e,\mu} = (83.98 \pm 0.60) MeV$ [?] of e^+e^- and $\mu^+\mu^-$ production assuming the weak dipole moment of the light leptons to be zero. Taking the correlation between the data into account one obtains $\Delta\Gamma_\tau^{SM} = (0.75 \pm 1.44) MeV$ and $\Delta\Gamma_\tau^{e,\mu} = (0.57 \pm 1.47) MeV$ respectively, which gives an indirect limit on the weak dipole moment of $|d_\tau(m_Z)| < 1.1 \cdot 10^{-17} e \cdot cm$ at 95% confidence level for both cases.

A direct way to measure the weak and the electric dipole moment is to look for CP-odd quantities [?, ?]. The non-zeroness of such a quantity would tell us that the dipole moment is different from zero because for the $Z \to l\bar{l}$ vertex one can only construct a CP violating form factor by assuming a dipole moment [?].

CP-odd quantities for the leptonic Z boson decays $Z \to l\bar{l}$ are only accessible by measuring the correlation between the spins of the leptons. Due to the parity violation in τ decays the energy spectra and the angular distributions of the τ decay products depend on the polarization of the τ, which means we look at the correlation of the τ^+ and τ^- decay products. Various correlation functions have been proposed [?, ?]. One basic difference between them is their behaviour under time reversal [?, ?]. Even if one requires the assumptions yielding the CPT-theorem, absorbative parts in the amplitude contribute to T-odd correlations, because the T-transformation of an amplitude is given by its complex conjugate evaluated at negative time, so the imaginary part of the hamiltonian changes its sign. This means a CP-odd and CPT-odd quantity can only be nonzero if there is an absorbative term present if one assumes the CPT-theorem to hold. Therefore, a CP-odd but CPT-even quantity is required. As was shown in [?, ?] the expectation values for the cross products between the momenta of the τ^+ and τ^- decay products give CP-odd correlation functions. The final CP-odd variable has to be modified, having a CPT-even expectation value. This leads to the quantities [?]

$$T_{A\bar{B}\ i,j} = (\vec{P}_A - \vec{P}_{\bar{B}})_i \cdot (\vec{P}_A \times \vec{P}_{\bar{B}})_j \quad + \quad (i \leftrightarrow j) \tag{1}$$

$$\hat{T}_{A\bar{B}\ i,j} = (\hat{P}_A - \hat{P}_{\bar{B}})_i \cdot \frac{(\hat{P}_A \times \hat{P}_{\bar{B}})_j}{|\hat{P}_A \times \hat{P}_{\bar{B}}|} \quad + \quad (i \leftrightarrow j)\ , \tag{2}$$

where $\vec{P}_A(\vec{P}_{\bar{B}})$ is the momentum of one of the charged decay products of the negative (positive) τ-lepton and $\hat{P}_A(\hat{P}_{\bar{B}})$ their direction cosines. The expectation values $\langle T_{A\bar{B}\ i,j}\rangle$ and $\langle \hat{T}_{A\bar{B}\ i,j}\rangle$ change sign under CP transformation, but not under CPT. It is clear that only the expectation value but not the quantity $\overset{(\wedge)}{T}_{A\bar{B}\ i,j}$ itself (on the event by event

mode	C_{AB}	$\Delta d_\tau(m_Z)[e/m_Z]$	$\hat{C}_{AB}$	$\Delta d_\tau(m_Z)[e/m_Z]$
ll	250	0.068	0.62	0.078
$l\pi$	−240	0.133	−0.62	0.101
$l\rho$	64	0.357	0.093	0.464
$\pi\pi$	−1390	0.083	−1.82	0.083
$\pi\rho$	−1162	0.050	−1.54	0.049
$\rho\rho$	−694	0.081	−0.91	0.082

Table 1. The proportional constants C_{AB} and $\hat{C}_{AB}$ and the expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency.

basis) changes sign under CP transformation, which is exactly what we mean by the discrete symmetries P, C and T.

For unpolarized beams no CP-odd form factors in the production $e^+e^- \longrightarrow Z$ are observable which finally means that the mean value of $\overset{(\wedge)}{T}_{A\bar{B}\ i,j}$ is directly related to the τ-dipole moment [?]

$$\frac{\langle \overset{(\wedge)}{T}_{A\bar{B}\ i,j}\rangle + \langle \overset{(\wedge)}{T}_{B\bar{A}\ i,j}\rangle}{2} = \frac{m_Z}{e} d_\tau(m_Z) \cdot \overset{(\wedge)}{C}_{AB} \cdot diag(-1/6, -1/6, 1/3)\ . \qquad (3)$$

By the term $diag$ we mean a diagonal matrix with diagonal elements as given above. The linear dependence of the expectation value $\langle \overset{(\wedge)}{T}_{A\bar{B}\ i,j}\rangle + \langle \overset{(\wedge)}{T}_{B\bar{A}\ i,j}\rangle$ is due to the interference between the CP-odd form factor and the Standard Model contribution. In addition there is a quadratic term in $d_\tau(m_Z)$ which can be neglected for $d_\tau(m_Z) \ll e/m_Z$.

To obtain the above formulae $V-A$ interaction was assumed in τ decay.

The proportional constant $\overset{(\wedge)}{C}_{AB}$ depends on the τ decay mode. The integration over the phase space for calculating these constants was done by a Monte Carlo program written by the authors of reference [?], where also several values of $\overset{(\wedge)}{C}_{AB}$ are listed. The program was modified to have the ρ decay mode $\tau^\pm \longrightarrow \rho\nu$ with the final state $\pi^\pm\pi^0$ available (see next chapter). Table 1 shows the calculated proportionality constants for the various decay modes using $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. The expected error on the weak dipole moment is also shown. It was assumed, that the e, μ, π and also the ρ momentum is well reconstructed. The errors on momentum and angle are neglected.

Comparing the errors, one can see that there is no difference in the statistical error whether $T_{A\bar{B}\ i,j}$ or $\hat{T}_{A\bar{B}\ i,j}$ is used. According to a Monte Carlo study one gains up to a factor of 20 in resolution for the correlations containing the ρ channel if one reconstructs the ρ momentum as it was assumed in table 1. However for the $e\rho$ and $\mu\rho$ correlation one obtains the lowest error on the dipole moment if $\hat{T}_{A\bar{B}\ i,j}$ is used but defined by the $\pi^\pm$ or π^0 momentum and not by its sum, the $\rho^\pm$ momentum. For this case one finds $\hat{C}_{AB} = 0.34$ and $d_\tau(m_Z) = 0.124e/m_Z$. This surprising result is well tested but not understood (see next chapter).

The ρ particle has spin one and the angular distribution in the ρ rest frame of the decay products is given by spherical harmonics. Thus the z-component of the ρ spin can be measured by looking at the angular distribution of the pions in the ρ rest frame. This should in principal improve the analysing power [?] as was done in [?] for measuring the τ polarisation. Various weights on $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$ were tried but no improvement achieved.

2 Modification of the Monte Carlo Program

In this chapter I want to describe how the program used in [?] was modified.

The expectation value of $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$ can be calculated using the product of three spin density matrizes, one for the τ-pair production and two for the τ-decay. The density for τ-pair production is given in [?]. The densities for τ-decays are for instance for the pion decay mode (notation as in [?])

$$\mathcal{D} = \delta(E^*_\mp - E_0)(1 \pm \vec{\sigma} \cdot \hat{q}^*_\mp)$$

$$E_0 = m_\tau(1 + m_\pi^2/m_\tau^2)/2$$

where E^* is the pion energy in the τ rest frame and $\vec{\sigma}$ are the Pauli matrices [?].

A general formula for the decay spin density can be calculated as follows: The amplitude of the τ^- decay is given by $\bar{\nu}\gamma^\mu(1-\gamma_5)\tau J_\mu$ where J_μ is the hadronic current. The spin density is then

$$\mathcal{D}_{\alpha\beta} = \bar{\nu}(1-\gamma_5)\tau_\beta\bar{\tau}_\alpha\gamma^\nu(1-\gamma_5)\nu J_\mu J^*_\nu \quad . \tag{4}$$

In the Dirac representation the left handed neutrino is described by [?]

$$\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi \\ -\phi \end{pmatrix} \tag{5}$$

$$\phi = \begin{pmatrix} -\sin\theta/2e^{-i\phi} \\ \cos\theta/2 \end{pmatrix} \tag{6}$$

where the direction of the neutrino is defined by θ and ϕ. So the density can be calculated to be

$$\mathcal{D} = 2\sigma_L^\mu \rho_0 \sigma_L^\nu J_\mu J^*_\nu \tag{7}$$

with

$$\rho_0 = 1/2(1 - \vec{\sigma}\vec{n}) \qquad \sigma_L^\mu = (1, -\vec{\sigma})$$

and $\vec{n}$ is the direction of the neutrino. For $J = J^*$ one obtains

$$\mathcal{D} = 2(\sigma_L \cdot J)(n \cdot J) - J^2(\sigma_L \cdot n) \qquad n^\mu = (1, \vec{n}) \quad . \tag{8}$$

For the rho decay mode $\rho^-(q) \to \pi^-(q_1)\pi^0(q_2)$ the hadronic current is [?]

$$J^\mu = (q_1^\mu - q_2^\mu)/B(q^2) \tag{9}$$

where $B(q^2)$ is given by a Breit– Wigner resonance, or to be more precise, by the pion form factor via CVC. If one substitutes the hadronic current in the density above one obtains

$$\mathcal{D} = \frac{1}{k_0|B(q^2)|^2}\{2m_\tau(E_1-E_2)^2+(m_\rho^2-4m_\pi^2)k_0+\vec{\sigma}[2m_\tau(E_1-E_2)(\vec{q}_1-\vec{q}_2)+(m_\rho^2-4m_\pi^2)\vec{k}]\} \tag{10}$$

This is the density which was used in the Monte Carlo program. The form factor $B(q^2)$ and the phase space routines were taken from the decay routines of KORALZ.

If one integrates the density above one obtains

$$\mathcal{D} = 1 - \frac{m_\tau^2 - 2m_\rho^2}{m_\tau^2 + 2m_\rho^2}\vec{\sigma}\vec{q} \quad , \tag{11}$$

which is the same density as for the pion decay mode except for an additional factor $(m_\tau^2 - 2m_\rho^2)/(m_\tau^2 + 2m_\rho^2)$.

The program was tested in two different ways. First, if one uses the rho momentum in the definition of $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$ then one should get the same ressult as for the pion decay mode if the density for the pion decay mode is modified by the factor $(m_\tau^2 - 2m_\rho^2)/(m_\tau^2 + 2m_\rho^2)$. Second, in [?] the ρ analysis is done by using only charged pion coming from the ρ decay, so we can compare the results in [?] with our program by using only the charged pion momentum in the definition of $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$. Due to isospin conservation we can olso use only the neutral pion momentum.

All tests were in good agreement with the program used in [?] so we are now able to study various weighting functions to improve the analysing power.

3 Weighting Functions for the Rho Decay Mode

Since we do not really know how to calculate the best weights we are going to guess several functions. The quantity $T_{A\bar{B}\ i,j}$ was modified by

$$T_{A\bar{B}\ i,j} \rightarrow T_{A\bar{B}\ i,j} \cdot \omega_A \omega_{\bar{B}}$$

where ω is a CP and T even function. The momentum used in $T_{A\bar{B}\ i,j}$ is the ρ momentum.

The only quantity which is then left is the difference between the charged and neutral pion energy so we guess for the rho decay mode

$$\omega_1 = |E_+ - E_0|$$

and

$$\omega_2 = |\frac{m_\rho}{\sqrt{m_\rho^2 - 4m_\pi^2}} \frac{E_+ - E_0}{|\vec{q}|}|$$

to be good functions for the weights. ω_2 ist just the ρ decay angle.

From table 2 one can see that there is no gain in the analysing power if one uses this weights. A similar result was obtained with the normalized tensor $\hat{T}_{A\bar{B}\ i,j}$. Several other functions were tried without success.

mode	$\Delta d_0[e/m_Z]$	$\Delta d_1[e/m_Z]$	$\Delta d_2[e/m_Z]$
$l\rho$	0.357	0.454	0.380
$\pi\rho$	0.050	0.057	0.061
$\rho\rho$	0.081	0.075	0.089

Table 2. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses no weight function (Δd_0) and if one uses the functions ω_1 and ω_2 ($\Delta d_1, \Delta d_2$).

mode	$\Delta d_\tau(m_Z)(T_{A\bar{B}\ 3,3})[e/m_Z]$	$\Delta d_\tau(m_Z)(\hat{T}_{A\bar{B}\ 3,3})[e/m_Z]$
$l\rho$	0.357	0.464
$\pi\rho$	0.050	0.049
$\rho\rho$	0.081	0.082

Table 3. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses the reconstructed rho momentum.

The situation is even more difficult if one ask the question, what we gain by reconstructing the neutral pion. Table 3 shows the error on the dipole moment if one uses $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$ defined by the rho momentum whereas table 4 lists the error if $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$ are defined by the charged pion momentum. No weights are aplied.

One can see that one gains a large factor for the $\rho\rho$ correlation if the rho momentum is reconstructed. For the $l\rho$ correlation one gets the lowest error if $\hat{T}_{A\bar{B}\ 3,3}$ defined by the charged pion momentum is used. We have no explanation for this result.

Let us summarise the tables. Table 3, where the reconstructed ρ momentum was used shows that there is no difference between $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. If only the charged π momentum is used (table 4) one obtains differences of up to a factor of 20 between $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. Table 2 shows that one does not gain analyzing power by using weights. Since the best analysing power for the lepton-rho correlation was obtained by using $\hat{T}_{A\bar{B}\ 3,3}$ defined by the charged pion momentum only, the author thinks that the best CP-odd and CPT-event quantity is much more complicated than just the cross product times a CPT-odd function $(\omega_A\omega_{\bar{B}}(\vec{P}_A - \vec{P}_{\bar{B}})_3)$.

mode	$\Delta d_\tau(m_Z)(T_{A\bar{B}\ 3,3})[e/m_Z]$	$\Delta d_\tau(m_Z)(\hat{T}_{A\bar{B}\ 3,3})[e/m_Z]$
$l\rho$	2.000	0.124
$\pi\rho$	0.068	0.078
$\rho\rho$	0.130	2.070

Table 4. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses only the charged pion momentum.

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