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Search for CP violation in $Z \to \tau\tau$

T. Fischer, C. Gotzhein, U. Stiegler

Abstract

Using the data accumulated at LEP in 1990 and 1991 with the ALEPH detector, a direct test of CP-invariance of the neutral current is performed. The method is based on CP-odd correlations of the τ decay products in the reaction $e^+e^- \to \tau^+\tau^-$. No evidence for CP-violation is observed. The weak dipole moment has been measured $d_\tau(m_Z) = (17.5 \pm 14.3 \pm 8.5) \times 10^{-18} e \cdot cm$ which results in an upper limit on the weak dipole moment of $|d_\tau(m_Z)| \leq 4.4 \times 10^{-17} e \cdot cm$ with 95% confidence level.

1 Introduction

It has been pointed out [1,2,3] that various decay modes of the Z boson can be used to search for new CP-violating effects beyond the Kobayashi-Maskawa mechanism by measuring appropriate CP-odd correlations. These CP-odd correlations provide direct information on CP-odd form factors [3]. For $Z \to \tau^+\tau^-$ the corresponding on-shell amplitude contains only one CP-odd form factor, the weak dipole moment $d_\tau(m_Z)$. Therefore the search for CP violation is related to the measurement of the weak dipole moment.

An indirect measurement of the dipole moment can be obtained from the partial widths of Z decays. In the case of the τ the partial width Γ_τ has a quadratic dependence on the dipole moment and one obtains an additional contribution $\Delta\Gamma_\tau$ to the width given by $\Delta\Gamma_\tau \approx |d_\tau(m_Z)|^2 m_Z^3/(24\pi)$ [3]. Using the data accumulated at LEP from 1989 to 1991 with the ALEPH detector the partial width was measured to be $\Gamma_\tau = (84.54 \pm 1.20)MeV$ [4]. This can be compared with the theoretical prediction $\Gamma_\tau^{SM} = (83.7 \pm 0.4)MeV$ given by the Standard Model [5], or with the measured partial width $\Gamma_{e,\mu} = (83.98 \pm 0.60)MeV$ [4] of e^+e^- and $\mu^+\mu^-$ production assuming the weak dipole moment of the light leptons to be zero. Taking the correlation between the data into account one obtains $\Delta\Gamma_\tau^{SM} = (0.84 \pm 1.26)MeV$ and $\Delta\Gamma_\tau^{e,\mu} = (0.56 \pm 1.47)MeV$ respectively, which corresponds for both cases to an indirect limit on the weak dipole moment of $|d_\tau(m_Z)| < 1.1 \cdot 10^{-17} e \cdot cm$ at 95% confidence level.

The on-shell $Z \to ll$ vertex contains only one CP-violating form factor, namely the weak dipole moment $d_\tau(m_Z)$. A direct way to measure this weak dipole moment is to look for CP-odd quantities [1,2,3] as it was recently done by the OPAL collaboration [6]. CP-odd quantities for the leptonic Z boson decays $Z \to l\bar{l}$ are, in the absence of radiation, only accessible by measuring the correlation between the spins of the leptons. Due to the parity violation in τ decays the energy spectra and the angular distributions of the τ decay products depend on the polarization of the τ, which means we have to look at the correlation of the τ^+ and τ^- decay products. Various correlation functions have been proposed [1,2,3]. One basic difference between them is their behaviour under time reversal [7,8]. A quantity which is T-odd but cannot be classified with respect to CP can also receive contributions from absorptive parts in the amplitude generated by CP-conserving interactions. Therefore it is advantageous to use observables which classify as being CP-odd. They cannot be faked by CP-conserving interactions. The simplest possibility are CP-odd but CPT-even quantities which require CP violation but no additional absorptive parts in order to be non-zero. As was shown in [3,7] the expectation values for the cross products between the momenta of the τ^+ and τ^- decay products are CP-odd correlation functions. This leads for instance to the following quantities [7]

$$T_{A\bar{B}\ i,j} = (\vec{P}_A - \vec{P}_{\bar{B}})_i \cdot (\vec{P}_A \times \vec{P}_{\bar{B}})_j \quad + \quad (i \leftrightarrow j) \tag{1}$$

$$\hat{T}_{A\bar{B}\ i,j} = (\hat{P}_A - \hat{P}_{\bar{B}})_i \cdot \frac{(\hat{P}_A \times \hat{P}_{\bar{B}})_j}{|\hat{P}_A \times \hat{P}_{\bar{B}}|} \quad + \quad (i \leftrightarrow j)\ , \tag{2}$$

where $\vec{P}_A(\vec{P}_{\bar{B}})$ is the momentum of the charged decay products of the negative (positive)

mode	C_{AB}	$\Delta d_\tau(m_Z)[e/m_Z]$	$\hat{C}_{AB}$	$\Delta d_\tau(m_Z)[e/m_Z]$
ll	250	0.130	0.62	0.143
$l\pi$	−240	0.173	−0.62	0.138
$l\rho$	64	0.516	0.093	0.625
$\pi\pi$	−1390	0.083	−1.82	0.083
$\pi\rho$	−1162	0.050	−1.54	0.049
$\rho\rho$	−694	0.081	−0.91	0.082

Table 1. The proportionality constants C_{AB} and $\hat{C}_{AB}$ and the expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency. It was assumed that the ρ momentum can be reconstructed.

τ-lepton and $\hat{P}_A(\hat{P}_{\bar{B}})$ its momentum direction. The expectation values $\langle T_{A\bar{B}\ i,j}\rangle$ and $\langle \hat{T}_{A\bar{B}\ i,j}\rangle$ change sign under CP transformation, but not under CPT. It is clear that only the expectation value but not the quantity $\hat{T}_{A\bar{B}\ i,j}$ itself (on the event by event basis) changes sign under CP transformation, which is exactly the meaning of the discrete symmetries P, C and T.

For unpolarized beams no CP-odd form factors in the production $e^+e^- \rightarrow Z$ are observable which finally means that the mean value of $\hat{T}_{A\bar{B}\ i,j}$ is directly related to the dipole moment of the τ [7][1]

$$\frac{\langle \hat{T}_{A\bar{B}\ i,j}\rangle + \langle \hat{T}_{B\bar{A}\ i,j}\rangle}{2} = \frac{m_Z}{e} d_\tau(m_Z) \cdot \hat{C}_{AB} \cdot diag(-1/6, -1/6, 1/3) \ . \tag{3}$$

By the term *diag* we mean a diagonal matrix with diagonal elements as given above. The linear dependence of the expectation value $\langle \hat{T}_{A\bar{B}\ i,j}\rangle + \langle \hat{T}_{B\bar{A}\ i,j}\rangle$ on $d_\tau(m_Z)$ is due to the interference between the CP-odd form factor and the Standard Model contribution. In addition there are terms of quadratic order in $d_\tau(m_Z)$, resulting from the normalization of $\langle \hat{T}_{A\bar{B}\ i,j}\rangle$, which can be neglected for $d_\tau(m_Z) \ll e/m_Z$.

To obtain the above formulas $V - A$ interaction was assumed in τ decay. Actually one can show [9] that to lowest order in the Standard Model $\langle \hat{T}_{A\bar{B}\ i,j}\rangle$ does not receive contributions from possible CP-violating effects in the τ decay amplitudes.

The proportionality constant $\hat{C}_{AB}$ depends on the τ decay mode. The integration over the phase space for calculating these constants was done by a Monte Carlo program written by the authors of references [7,10], where also the values of $\hat{C}_{AB}$ are listed. Table 1 shows the calculated proportionality constants for the various decay modes. The expected error on the weak dipole moment is also shown using $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. It was assumed that the e, μ, π and also the ρ momentum is well reconstructed.

Comparing the errors, one can see that there is no difference in the statistical error whether $T_{A\bar{B}\ i,j}$ or $\hat{T}_{A\bar{B}\ i,j}$ is used. For the ρ decay mode there are in principle three different definitions of $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$; one can use the momentum of the charged

[1]If an imaginary part of $d_\tau(m_Z)$ is taken into account, $d_\tau(m_Z)$ has to be replaced by $Re\ d_\tau(m_Z)$.

pion only, the neutral pion only, or the sum of both. According to a Monte Carlo study one gains up to a factor of 20 in resolution for the correlations containing the ρ channel if one reconstructs the ρ momentum and not only the charged pion momentum [10]. However for the $e\rho$ and $\mu\rho$ correlation one obtains the lowest error on the dipole moment if $\hat{T}_{A\bar{B}\ i,j}$ is used but defined by the $\pi^{\pm}$ or π^0 momentum and not by their sum, the $\rho^{\pm}$ momentum. For this case one finds $\hat{C}_{AB} = 0.34$ and $\Delta d_\tau(m_Z) = 0.18e/m_Z$ [10].

The ρ particle has spin one and the angular distribution of the decay products is given by spherical harmonics in the ρ rest frame. Thus the z component of the ρ spin can be measured by looking at the angular distribution of the pions in the ρ rest frame. This should in principle improve the analyzing power [11] as was done in [12] for measuring the τ polarisation. Various weights on $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$ were tried but no improvement achieved (cf. appendix A).

The $\hat{T}_{A\bar{B}\ i,j}$ are highly correlated for the various $i = j$ so we will concentrate on the element with $i = j = 3$, which has the highest analyzing power.

2 Event Selection

The techniques used to select the τ pairs and to identify the τ decay modes have been described in detail elsewhere [12,13]. In this analysis we will use the τ decay modes $\tau \to e\nu\nu$, $\tau \to \mu\nu\nu$, $\tau \to \pi\nu$ and $\tau \to \rho\nu$. The data for this analysis were accumulated at LEP in 1990 and 1991 with the ALEPH detector. The integrated luminosity was $18.8pb^{-1}$ distributed in $\sqrt{S}$ around the Z mass [14] and one would expect about 21600 $\tau\tau$ events [14]. The τ pair selection is only used for studying the background.

The momentum of each particle should be greater than $3GeV$. The scattering angle was restricted to be $|\cos\theta| < 0.9$ and the cosine of the acolinearity to be less than -0.95.

For the muon identification we used the program of ref. [15]. The pion selection is done with the program of ref. [16] using the new "transverse analysis". The rho identification is described in ref. [17]. Electrons are selected in a similar way as in [12,18].

To get a further reduction of the μ pair and Bhabha background the following cuts are applied: If a $\mu\mu$ event was identified we require that the highest particle momentum should be less than 85% of the beam energy and the lowest momentum has to be less than 60% of the beam energy. Furthermore there should be no photon found with GAMPEC with an energy above $1GeV$. For $\mu\pi$ events the muon momentum has to be lower than 90% of the beam energy. In case of $e\pi$ ($e\rho$) pairs we require the wire energy to be lower than $0.65(0.8)\sqrt{S}$. Bhabha background causes a non-zero expectation value of $\hat{T}_{A\bar{B}\ 3,3}$ due to bremsstrahlung (cf. chapter 3). Therefore electron pairs are rejected.

Table 2 shows the number of selected pairs, the efficiency and the background. The high background for $\mu\mu$ events is due to the two-photon background. The background for the other modes is dominated by particle misidentification. From the number of selected pairs one can calculate the branching ratio for $\tau^+\tau^- \to X^+Y^-\nu\bar{\nu}$ as shown in table 2. The overall normalization was taken from [14], where the same data sample was used. The branching ratios are in good agreement with ref. [19] so we expect no

mode	no. of events	efficiency	background	br. ratio
$e\mu$	601	$(51.4 \pm 1.4)\%$	$(2.6 \pm 0.4)\%$	$(5.27 \pm 0.27)\%$
$e\pi$	414	$(37.4 \pm 1.7)\%$	$(10.0 \pm 1.2)\%$	$(4.61 \pm 0.32)\%$
$e\rho$	486	$(28.0 \pm 1.2)\%$	$(3.0 \pm 0.6)\%$	$(7.79 \pm 0.50)\%$
$\mu\mu$	481	$(63.9 \pm 2.0)\%$	$(13.2 \pm 1.2)\%$	$(3.02 \pm 0.18)\%$
$\mu\pi$	417	$(45.4 \pm 1.7)\%$	$(10.1 \pm 1.1)\%$	$(3.82 \pm 0.25)\%$
$\mu\rho$	663	$(35.5 \pm 1.2)\%$	$(8.2 \pm 0.8)\%$	$(7.93 \pm 0.43)\%$
$\pi\pi$	128	$(36.2 \pm 2.8)\%$	$(11.0 \pm 2.0)\%$	$(1.46 \pm 0.18)\%$
$\pi\rho$	365	$(26.2 \pm 1.5)\%$	$(12.4 \pm 1.4)\%$	$(5.65 \pm 0.46)\%$
$\rho\rho$	290	$(21.7 \pm 1.6)\%$	$(11.4 \pm 1.6)\%$	$(5.48 \pm 0.54)\%$
sum	3845			

Table 2. The number of selected decay modes, their efficiencies, backgrounds, and the measured branching ratios.

large systematic errors on the background and efficiency.

The momenta of e, μ and π are measured by using the TPC and ITC. The ρ momentum is calculated by adding the charged pion momentum and the reconstructed photon momenta using the ECAL storey information. If only one photon was reconstructed then we take its momentum as being the π^0 momentum [17].

3 Systematic Errors

As we will see the systematic errors on the expectation value of $\hat{T}_{A\bar{B}\ 3,3}$ are better to handle than the ones on $T_{A\bar{B}\ 3,3}$ so we will concentrate on $\hat{T}_{A\bar{B}\ 3,3}$ in the following. There are two kinds of systematic errors: The first is an error on the proportionality constant $\hat{C}_{AB}$ and the second one is an error on $\hat{T}_{A\bar{B}\ i,j}$ faking a CP-odd form factor. Errors in estimating the background and efficiency belong to the first group. The second kind of error is caused by internal effects of the detector like a twist of the TPC endcaps. The error on $\hat{C}_{AB}$ contributes in two ways to the error on the dipole moment: as a relative error on the measured dipole moment and as a relative error on the statistical error.

Since the constant $\hat{C}_{AB}$ depends on the decay mode it will be modified due to the background. The relative change of $\hat{C}_{AB}$ is shown in the first column of table 3. There are two sources of errors on the relative change: On the one hand, there are experimental uncertainties on the background (first error in first column of table 3) and on the other hand the proportionality constant $\hat{C}_{AB}$ is not known for all types of backgrounds (second error in first colum of table 3). The background is known to better than 50% so the first error was estimated by varying the background by a relative amount of 50%. The second error is reasonably well estimated by varying $\hat{C}_{AB}$ from -1 to 1 if $\hat{C}_{AB}$ is unknown. Cuts like acolinearity, lowest allowed momentum of the track and geometrical acceptance also lead to a change of $\hat{C}_{AB}$ (see table 3). Varying the cuts in a wide range leads to no significant change of $\hat{C}_{AB}$ within the statistical

mode	backg.	cuts	$\hat{C}_{AB}$
$e\mu$	$0.984 \pm 0.008 \pm 0.005$	1.168	$+0.771 \pm 0.014$
$e\pi$	$0.876 \pm 0.062 \pm 0.058$	1.132	-0.614 ± 0.053
$e\rho$	$0.956 \pm 0.022 \pm 0.107$	1.038	$+0.338 \pm 0.038$
$\mu\mu$	$0.961 \pm 0.020 \pm 0.012$	1.147	$+0.683 \pm 0.019$
$\mu\pi$	$0.868 \pm 0.066 \pm 0.034$	1.111	-0.597 ± 0.045
$\mu\rho$	$0.962 \pm 0.019 \pm 0.061$	1.038	$+0.340 \pm 0.024$
$\pi\pi$	$0.942 \pm 0.029 \pm 0.005$	1.092	-1.869 ± 0.056
$\pi\rho$	$0.900 \pm 0.050 \pm 0.050$	1.054	-1.455 ± 0.103
$\rho\rho$	$0.886 \pm 0.057 \pm 0.134$	1.009	-0.816 ± 0.119

Table 3. The relative change of $\hat{C}_{AB}$ due to background (first colum), due to several cuts (second colum) and the final proportionality constant $\hat{C}_{AB}$. The errors in the first colum are for the experimental and theoretical uncertainties on the background. The error on $\hat{C}_{AB}$ counts for the error due to background and the statistical error of the Monte Carlo data.

error of the Monte Carlo data. The finite momentum and angular resolution have no significant influence on the proportionality constant according to the Monte Carlo data. An error caused by a wrong definition of the z-axis also belongs to the first group of errors, because the measured $\hat{T}_{A\bar{B}\ 3,3}$ is then a linear combination of the various $\hat{T}_{A\bar{B}\ i,j}$. The error is negligibly small because the geometry is known up to $1 mrad$.

Bremsstrahlung causes a systematic shift in the acoplanarity measured in the plane perpendicular to the beam axis. This leads to a shift in $\hat{T}_{A\bar{B}\ 3,3}$ equal in magnitude but of opposite sign for forward and backward scattering. Since the forward-backward asymmetry in τ pair production is low we can neglect this effect. Of course a high Bhabha background causes a non-zero expectation value. Therefore the ee correlation was not used.

A twist of the TPC endcaps can be measured by analyzing direct μ pair events because a CP violating form factor in μ pair production does not contribute to $\langle \hat{T}_{A\bar{B}\ i,j} \rangle$ if there is no hard radiation [3]. From the μ pairs recorded in 1990 and 1991 we obtained $\langle \hat{T}_{3,3} \rangle = (1.22 \pm 0.82) \cdot 10^{-2}$. The relation between $\langle \hat{T}_{3,3} \rangle$ and the twist angle ω, defined as the difference in ϕ between the two endcaps, depends on the acolinearity distribution of the untwisted case. To take this into account, we have propagated the error on $\langle \hat{T}_{3,3} \rangle$ measured above to an error on $\langle \hat{T}_{3,3} \rangle$ for τ pairs. This was done by generating μ and τ pair Monte Carlo events with twisted endcaps. We obtained $\omega = (0.19 \pm 0.13) mrad$, which translates to $|\langle \hat{T}_{3,3} \rangle| < 4 \cdot 10^{-3}$ for the various τ decay modes at a confidence level of 68%. This contribution results in an error on the dipole moment which is around a factor ten lower than the statistical error. (cf. chapter 4). The same systematic error could also be caused by inhomogeneities in the electric and magnetic fields of the TPC, which we would also see with μ pair events.

If one of the end caps is rotated around an axis perpendicular to the beam axis one would also obtain a systematic shift in $\langle \hat{T}_{A\bar{B}\ 3,3} \rangle$. Since the geometry is known better than $1 mrad$ we expect a contribution to $\langle \hat{T}_{A\bar{B}\ 3,3} \rangle$ of less than $1 \cdot 10^{-3}$.

A more general way to measure the systematic errors caused by detector effects is to inspect the two-dimensional distribution of $\hat{T}_{A\bar{B}\ 3,3}$ versus $\cos\theta$ for nonradiating μ pairs [20]. A non-zero expectation value of $\hat{T}_{A\bar{B}\ 3,3}$ for nonradiating μ pairs can only be caused by detector effects and would result in different numbers of entries in bins opposite in sign of $\hat{T}_{A\bar{B}\ 3,3}$. Using the nonradiating μ pairs recorded in 1990 and 1991 we divide the two-dimensional distribution (8 bins in $\cos\theta$ and 2×5 bins in $\hat{T}_{A\bar{B}\ 3,3}$, see fig. 1) for $\hat{T}_{A\bar{B}\ 3,3} > 0$ by the distribution for $\hat{T}_{A\bar{B}\ 3,3} < 0$. These ratios, normalized by their errors, are not really gaussian distributed (see fig. 2).[2] Therefore there might be a systematic effect. This possible systematic effect, propagated from μ pair events to τ pair events as described in the following paragraph, is added as an error on the dipole moment. We do not correct for it.

We expect that the systematic error on each bin is less than the measured ratios plus the statistical error of that particular bin. This systematic error on the ratio for the various $\hat{T}_{A\bar{B}\ 3,3}$ and $\cos\theta$ bins is propagated to the τ pair distribution: At first the measured ratios from μ pairs are smeared bin by bin with their errors. The number of entries of the two-dimensional distribution of 50,000 Monte Carlo τ pairs for $\hat{T}_{A\bar{B}\ 3,3} > 0$ was multiplied for each bin with the smeared ratio (of μ pairs) of that particular bin. The shift $\Delta\hat{T}$ of the mean value of $\hat{T}_{A\bar{B}\ 3,3}$ due to this correction was calculated. If one repeats this procedure several times with different initializations of the random number generator that was used for the smearing one obtains a gaussian distribution of the shifts $\Delta\hat{T}$. The mean values and the widths of these distributions are listed in table 4 for the various τ decay modes. The resulting errors on the limit on the dipole moment is also shown in table 4 with a confidence level of 68% if one does not correct by the mean values. These limits are of the order of the statistical error (cf. chapter 4).

This method reflects mainly the errors on the angular measurement so it is only relyable for $\hat{T}_{A\bar{B}\ 3,3}$ but not for $T_{A\bar{B}\ 3,3}$.

We want to point out that the final systematic error on the dipole moment due to detector effects is rather low because of the different sign of the proportionality constant $\hat{C}_{AB}$ for the various decay modes.

A constant shift $\Delta\hat{T}$ for all decay modes can be measured by inspecting the dipole moments d_+ and d_- calculated from the decay modes with positive or negative $\hat{C}_{AB}$ respectively. The difference between these two moments depends only on the shift but not on a physical dipole moment. An appropriately weighted sum of these moments depends on the physical dipole moment alone. The formulas are calculated in appendix B.

[2] This is a pity or a chance to improve our nice ALEPH detector or just a statistical accident.

mode	$\Delta\hat{T}$	r.m.s.	$\Delta d_\tau(m_Z)$ $[e/m_Z]$
$e\mu$	-0.009	0.021	-0.117
$e\pi$	-0.031	0.021	0.254
$e\rho$	-0.031	0.026	-0.506
$\mu\mu$	-0.009	0.021	-0.132
$\mu\pi$	-0.031	0.021	0.261
$\mu\rho$	-0.031	0.026	-0.503
$\pi\pi$	-0.030	0.020	0.080
$\pi\rho$	-0.026	0.022	0.099
$\rho\rho$	-0.038	0.007	0.165

Table 4. The expected shift $\Delta\hat{T}$ on $\hat{T}_{A\bar{B}\ 3,3}$ measured with μ pairs, the r.m.s. of the $\Delta\hat{T}$ distribution and the systematic error on the limit on $d_\tau(m_Z)$ at 68% confidence level. The sign, which is important for the final error, is given by the product of $\Delta\hat{T}$ and $\hat{C}_{AB}$.

4 Data Analysis and Result

Table 5 shows the measured dipole moment in units of $\frac{e}{m_Z}$ using $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. In case of the $e\rho$ and $\mu\rho$ correlation one obtains the highest analyzing power with $\hat{T}_{A\bar{B}\ 3,3}$ given by the lepton and charged pion momentum as discussed above, so we use this definition of T and $\hat{T}$ for the lepton-ρ correlation. Adding the quantities $T_{1,1}$ and $T_{2,2}$ does not improve the result substantially because the analyzing power for these two quantities is lower by a factor of 2 and the $T_{i,i}$ are highly correlated ($\hat{T}_{1,1} + \hat{T}_{2,2} + \hat{T}_{3,3} = 0$).

In table 5, one can see the agreement between $d_\tau(m_Z)$ using $T_{A\bar{B}\ 3,3}$ or $\hat{T}_{A\bar{B}\ 3,3}$. The result is consistent with zero dipole moment.

Since the systematic errors on $\hat{T}_{A\bar{B}\ 3,3}$ are better to handle than the ones on $T_{A\bar{B}\ 3,3}$ we will use $\hat{T}_{A\bar{B}\ 3,3}$ for calculating the final limit. The systematic error on the dipole moment due to detector effects is given by the sum of the values listed in table 4 but weighted as in table 5. One obtains $\delta d_\tau(m_Z) = 0.039e/m_Z$. The mean value of $d_\tau(m_Z)$ as well as the statistical error on $d_\tau(m_Z)$ receive an additional error due to the error on $\hat{C}_{AB}$. Propagation of the errors assuming them to be gaussian results in a relative error on the mean value of $d_\tau(m_Z)$ of 5.6% and on the statistical error of 2.9% (cf. appendix B). Therefore the measured dipole moment is

$$d_\tau(m_Z) = 0.081(\pm 5.6\%) \pm 0.064(\pm 2.9\%) \pm 0.039 \quad \left[\frac{e}{m_Z}\right] \quad . \tag{4}$$

The first and second number shows the mean value and the statistical error with their relative error and the last number counts for the systematic error due to detector effects. The formulas for the propagation of the errors are shown in appendix B.

mode	nr. of events	$d_\tau(m_Z)$ (T)	$d_\tau(m_Z)$ $(\hat{T})$
$e\mu$	601	0.333 ± 0.174	0.378 ± 0.190
$e\pi$	414	0.419 ± 0.350	0.262 ± 0.249
$e\rho$	486	7.564 ± 6.315	-0.056 ± 0.414
$\mu\mu$	481	-0.276 ± 0.208	-0.228 ± 0.202
$\mu\pi$	417	0.516 ± 0.342	0.375 ± 0.255
$\mu\rho$	663	-9.664 ± 7.597	-0.450 ± 0.362
$\pi\pi$	128	0.265 ± 0.205	0.212 ± 0.146
$\pi\rho$	365	0.201 ± 0.125	0.071 ± 0.107
$\rho\rho$	290	-0.262 ± 0.204	-0.289 ± 0.238
sum	3845	0.137 ± 0.074	0.081 ± 0.064

Table 5. The number of selected decay modes and the measured dipole moment using $T_{A\bar{B}\ i,j}$ and $\hat{T}_{A\bar{B}\ i,j}$. The dipole moment is given in units of e/m_Z. The error contains only the statistical one. For the $e\rho$ and $\mu\rho$ correlation we used only the charged pion momentum and not the reconstructed ρ momentum. The last row shows the mean of the dipole moments weighted with their statistical errors.

The weighted mean value of the dipole moments d_+ and d_- for positive and negative $\hat{C}_{AB}$ are

$$d_+ = -0.005 \pm 0.123 \qquad d_- = 0.112 \pm 0.074 \qquad \left[\frac{e}{m_Z}\right] \quad .$$

These moments differ from each other if a significant shift on $\hat{T}_{A\bar{B}\ 3,3}$ in the same direction is present for the various decay modes. The shift $\Delta\hat{T}$ in the expectation value of $\hat{T}_{A\bar{B}\ 3,3}$ is calculated to be (cf. appendix B)

$$\Delta\hat{T} = -0.0154 \pm 0.0189 \quad .$$

An appropriately weighted sum of d_+ and d_- results in a weak dipole moment which is independent of a constant shift of $\hat{T}_{A\bar{B}\ 3,3}$. One obtains

$$d_\tau(m_Z) = 0.073(\pm 5.6\%) \pm 0.064(\pm 2.9\%) \pm 0.003 \qquad \left[\frac{e}{m_Z}\right] \quad . \tag{5}$$

The last error in equation (5) is caused by the product of the $\Delta\hat{T}$ and the relative error on $\hat{C}_{AB}$ (cf. appendix B). The measured dipole moments using equation (4) or (5) agree within their systematic errors. The statistical errors happen to be the same.

If we add the relative error on the mean value and the error due to detector effects in quadrature and apply the correction to the statistical error we obtain

$$d_\tau(m_Z) = 0.081 \pm 0.066 \pm 0.039 \quad \left[\frac{e}{m_Z}\right] \tag{4}$$

$$d_\tau(m_Z) = 0.073 \pm 0.066 \pm 0.005 \quad \left[\frac{e}{m_Z}\right] \qquad (5)$$

where the first error counts for the corrected statistical error and the second one for the systematic error. In units of $e \cdot cm$ we obtain

$$d_\tau(m_Z) = (17.5 \pm 14.3 \pm 8.5) \cdot 10^{-18} e \cdot cm \qquad (4)$$

$$d_\tau(m_Z) = (15.8 \pm 14.3 \pm 1.1) \cdot 10^{-18} e \cdot cm \qquad (5)$$

Using equation (4), this null result yields a limit on the weak dipole moment of

$$|d_\tau(m_Z)| < 4.4 \cdot 10^{-17} e \cdot cm$$

at a confidence level of 95%. The systematic and statistical errors are added in quadrature for calculating the limit above. From equation 5 one obtains

$$|d_\tau(m_Z)| < 3.9 \cdot 10^{-17} e \cdot cm \qquad .$$

The difference between the results from equation (4) and (5) is rather small. Being conservative we prefer the result of equation (4).

5 Appendix A: The Rho Chanel

Several numbers given in [21] where wrong by a factor of $\sqrt{2}$ so this appendix is an erratum to [21]. Furthermore we now have a better understanding of the result.

The program written by the authors of [7] allows to use the ρ momentum or the single pion momentum in the definition of $\hat{T}_{A\bar{B}\ i,j}$ if the τ decays into $\rho\nu$. Acording to [11] one would expect an improvement of the analyzing power if the angle between one pion and the rho direction of flight is also used. To do so the program of [7] was modified as described in [21] to have the momenta of the charged and neutral pion available. The idea is to weight $\hat{T}_{A\bar{B}\ i,j}$ with a function which depends on the rho decay angle.

Since we do not really know how to calculate the best weights we are going to guess several functions. The quantity $T_{A\bar{B}\ i,j}$ was modified by

$$T_{A\bar{B}\ i,j} \rightarrow T_{A\bar{B}\ i,j} \cdot \omega_A \omega_{\bar{B}}$$

where ω is a CP and T even function. The momentum used in $T_{A\bar{B}\ i,j}$ is the ρ momentum.

The only quantity which is then left is the difference between the charged and neutral pion energies so we guess for the rho decay mode

$$\omega_1 = |E_+ - E_0|$$

and

$$\omega_2 = \left|\frac{m_\rho}{\sqrt{m_\rho^2 - 4m_\pi^2}} \frac{E_+ - E_0}{|\vec{q}|}\right|$$

mode	$\Delta d_0[e/m_Z]$	$\Delta d_1[e/m_Z]$	$\Delta d_2[e/m_Z]$
$l\rho$	0.516	0.454	0.380
$\pi\rho$	0.050	0.057	0.061
$\rho\rho$	0.081	0.106	0.126

Table A1. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses no weight function (Δd_0) and if one uses the functions ω_1 and ω_2 ($\Delta d_1, \Delta d_2$).

mode	$\Delta d_\tau(m_Z)(T_{A\bar{B}\ 3,3})[e/m_Z]$	$\Delta d_\tau(m_Z)(\hat{T}_{A\bar{B}\ 3,3})[e/m_Z]$
$l\rho$	0.516	0.625
$\pi\rho$	0.050	0.049
$\rho\rho$	0.081	0.082

Table A2. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses the reconstructed rho momentum.

to be good functions for the weights. ω_2 ist just the ρ decay angle.

From table A1 one can see that there is no gain in the analyzing power if one uses these weights. A similar result was obtained with the normalized tensor $\hat{T}_{A\bar{B}\ i,j}$. Several other functions were tried without success.

What can be gained by reconstructing the neutral pion? Table A2 shows the error on the dipole moment if one uses $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$ defined by the rho momentum whereas table A3 lists the error if $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$ are defined by the charged pion momentum. No weights are applied.

One can see that one gains a large factor for the $\rho\rho$ correlation if the rho momentum is reconstructed. For the $l\rho$ correlation one gets the lowest error if $\hat{T}_{A\bar{B}\ 3,3}$ defined by the charged pion momentum is used. The explanation for this result is as follows [9,10]: The constant $\hat{C}_{AB}$ can approximately be written as the sum of two constants $\hat{C}_A$ and $\hat{C}_{\bar{B}}$ where $\hat{C}_A$ ($\hat{C}_{\bar{B}}$) depends only on the τ^+ (τ^-) decay mode. Inspecting the numbers one finds that $\hat{C}_l \approx -\hat{C}_\rho$ if the rho momentum is used. This leads to a cancellation of the analyzing power. If only the pion momentum is used one has $\hat{C}_\rho \approx 0$ so the

mode	$\Delta d_\tau(m_Z)(T_{A\bar{B}\ 3,3})[e/m_Z]$	$\Delta d_\tau(m_Z)(\hat{T}_{A\bar{B}\ 3,3})[e/m_Z]$
$l\rho$	2.828	0.175
$\pi\rho$	0.068	0.078
$\rho\rho$	0.130	2.070

Table A3. The expected statistical error on the dipole moment in units of e/m_Z for 10^6 Z assuming 100% efficiency if one uses only the charged pion momentum.

analyzing power is given by $\hat{C}_l$. The approximation used is only true for $\hat{T}_{A\bar{B}\ i,j}$ but not for $T_{A\bar{B}\ i,j}$.

Let us summarize the tables. Table A2, where the reconstructed ρ momentum was used, shows that there is no difference between $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. If only the charged π momentum is used (table A3) one obtains differences of up to a factor of 20 between $T_{A\bar{B}\ 3,3}$ and $\hat{T}_{A\bar{B}\ 3,3}$. Table A1 shows that one does not gain analyzing power by using weights.

6 Appendix B: Propagation of Errors

In this appendix we describe the formulas used for the propagation of the errors.

Let us assume that the various dipole moments are measured to be d_i with statistical errors σ_i. The index counts for the different decay modes. The weighted mean and its error are given by:

$$d = \frac{\sum d_i/\sigma_i^2}{\sum 1/\sigma_i^2} \qquad \sigma^2 = \frac{1}{\sum 1/\sigma_i^2} \quad .$$

The systematic error Δd due to detector effects is then given by substituting in the formula above d_i by the individual error Δd_i.

The propagation of the relative errors $\delta c_i/c_i$ on $\hat{C}_{AB}$ to an error on the dipole moment is done by

$$(\delta d)^2 = \sum \left(\frac{\partial d}{\partial d_i}\right)^2 \delta d_i^2 = \sigma^4 \sum \left(\frac{d_i}{\sigma_i^2} \cdot \frac{\delta c_i}{c_i}\right)^2$$

The propagated error on the statistical error is calculated in the same way:

$$(\delta \sigma)^2 = \sum \left(\frac{\partial \sigma}{\partial \sigma_i}\right)^2 \delta \sigma_i^2 = \sigma^6 \sum \left(\frac{1}{\sigma_i^2} \cdot \frac{\delta c_i}{c_i}\right)^2$$

The formulas are valid if the errors on $\hat{C}_{AB}$ are gaussian distributed and not correlated between the various decay modes.

Assume one measures the dipole moments d_+ and d_- for positive and negative $\hat{C}_{AB}$ given by

$$d_\pm = \frac{\sum_\pm d_i/\sigma_i^2}{\sum_\pm 1/\sigma_i^2} \qquad \sigma_\pm^2 = \frac{1}{\sum_\pm 1/\sigma_i^2} \quad ,$$

where the sum runs over the set of moments with positive (negative) $\hat{C}_{AB}$. If these dipole moments are caused by a theoretical moment d^{th} and by a shift $\Delta\hat{T}$ we expect

$$d_\pm = d^{th} \pm 3\Delta\hat{T}|\langle 1/c_\pm\rangle| \qquad \langle 1/c_\pm\rangle = \sigma_\pm^2 \sum_\pm \frac{1}{\sigma_i^2 c_i} \quad .$$

Therefore from the sum of d_+ and d_- weighted with $\langle 1/c_\pm\rangle$ one obtains the dipole moment

$$d^{th} = \left(\frac{d_+}{|\langle 1/c_+\rangle|} + \frac{d_-}{|\langle 1/c_-\rangle|}\right)\left(\frac{1}{|\langle 1/c_+\rangle|} + \frac{1}{|\langle 1/c_-\rangle|}\right)^{-1}$$

and the shift $\Delta\hat{T}$

$$\Delta\hat{T} = \frac{d_+ - d_-}{3}\left(\frac{1}{|\langle 1/c_+\rangle|} + \frac{1}{|\langle 1/c_-\rangle|}\right)^{-1}$$

As above errors on the proportionality constants result in relative errors on the measured dipole moment and on the statistical error. In addition one obtains an error Δd depending on the product of $\Delta\hat{T}$ and δc_i

$$\Delta d = 3(|\Delta\hat{T}| + \delta\Delta\hat{T})\sqrt{(r_+^2 + r_-^2)} \cdot \left(\frac{1}{|\langle 1/c_+\rangle|} + \frac{1}{|\langle 1/c_-\rangle|}\right)^{-1} \quad ,$$

where $r_\pm$ are the relative errors on $|\langle 1/c_\pm\rangle|$

$$\left(r_\pm \cdot |\langle 1/c_\pm\rangle|\right)^2 = \sum_\pm \left(\frac{\delta c_i}{\sigma_i^2 c_i^2}\right)^2 \quad .$$

The statistical error $\delta\Delta\hat{T}$ is added to $\Delta\hat{T}$ so the result corresponds to a limit on the dipole moment with 68% confidence level.

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Figure Captions

Fig. 1 The two-dimensional distribution of $\hat{T}_{A\bar{B}\ 3,3}$ versus $\cos\theta$ for nonradiated μ pairs

Fig. 2 The ratios between the bins with $\hat{T}_{A\bar{B}\ 3,3} > 0$ and $\hat{T}_{A\bar{B}\ 3,3} < 0$ minus one, divided by their errors.