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On the Interpolation of the Bhabha Cross Section used in JULIA

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Abstract

This note proposes a way to interpolate the predicted Bhabha cross section to other values of the beam energy, resonance mass or width than used in the original estimate and to use this interpolation in JULIA whenever needed.

1 Introduction

The Monte Carlo calculation of the Bhabha cross sections to be used in the various luminosity estimates in JULIA is very time consuming. It can thus not be done for every beam energy before the start of JULIA. The cross sections depend in addition on the values of the mass and width of the Z^0 which will not be well known before the start of LEP.

It can therefore not be avoided that some interpolation must be done.

The Bhabha cross section, σ, is to lowest order given by:

$$\sigma(E,E_Z,\Gamma) = \frac{A}{E^2}\cdot(1+\frac{B\cdot(E^2-E_Z^2)\cdot\Gamma^e + C\cdot\Gamma^{e2}}{(E^2-E_Z^2)^2+(E_Z\cdot\Gamma)^2})$$

where

E is the center of mass energy

E_Z is the Z^0 mass

Γ^e is the electron width of the Z^0 at the energy E

Γ is the Z^0 width at the energy E

Γ^e_Z is the electron width of the Z^0 at the Z^0 mass

Γ_Z is the Z^0 width at the Z^0 mass

The 'constants' A, B and C depend on the cuts. Higher order terms, i.e. radiative corrections, would also give a weak dependence on E. It is this later dependence which gives the main uncertainty if the expression is used as an approximation to all orders and makes it necessary to recalculate A, B and C when going to larger energies.

Assuming

$$\Gamma = \Gamma_Z \frac{E + E_Z}{2 \cdot E_Z}$$

$$\Gamma^e = \Gamma^e_Z \frac{E + E_Z}{2 \cdot E_Z}$$

and redefining the constants B and C gives

$$\sigma(E, E_Z, \Gamma_Z) = \frac{A}{E^2} \cdot (1 + \frac{B \cdot (E - E_Z) + C}{(E - E_Z)^2 + (\frac{\Gamma_Z}{2})^2})$$

It is proposed to use for the interpolated cross section the following:

$$\sigma(E, E_Z, \Gamma_Z) = \sigma(E_{ref}, E_{Z,ref}, \Gamma_{Z,ref}) \cdot (\frac{E_{ref}}{E})^2 \cdot \frac{1 + \frac{B \cdot (E - E_Z) + C}{(E - E_Z)^2 + (\frac{\Gamma_Z}{2})^2}}{1 + \frac{B \cdot (E_{ref} - E_{ref,Z}) + C}{(E_{ref} - E_{ref,Z})^2 + (\frac{\Gamma_{ref,Z}}{2})^2}}$$

Typical values of 4*B/Γ_Z^2 and 4*C/Γ_Z^2 are -1.3%GeV^{-1} and 0.1%, respectively. This means that the contribution from C is below the precision with which the reference cross section can be calculated. It is kept here for completeness.

A simple way to find σ_{ref}, B and C is to calculate the cross section for E equal to $E_Z + \Gamma_Z/2, E_Z and E_Z + \Gamma_Z/2$, respectively. This will cover the region of interest in the beginning. Obviously the calculation should be repeated with better values when needed.

The values of B, C, E_Z and Γ_Z for each luminosity estimate should be given in the JULIA luminosity end of run bank to allow for a simple recalculation of the luminosity when new values of E_Z and Γ_Z are available.