ALEPH 89-186
Physic 89-73
F. Bird et al.
19.12.89

Luminosity Errors

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December 18, 1989

Abstract

We discuss in this note the contributions to the luminosity errors in our first paper, [1].

1 Introduction

Bhabha events are selected on the basis of the energy deposited in LCAL towers. Neighboring towers containing more than 50 MeV are joined into clusters, giving energy and positions of the clusters. Events are required to have a cluster reconstructed on each side of the interaction point.

An asymmetric selection is performed involving four cuts:

1. An asymmetry A_x is defined as:

$$A_x = \frac{E_{in} - E_{out}}{E_{in} + E_{out}}$$

   where the E_{in} is the energy sum in three neighboring towers along y centered around the tower with the maximum energy. Only energies in the first storey are used. E_{out} is the similar sum of energies in the two adjacent sets of pads. This gives two values of A_x. A_y is defined using towers along the x-axis.

   For events with an odd event number, showers on the e^+ side are required to have the tower with the maximum energy in some fiducial region defined along pad boundaries. The boundary is at least half a tower away from the detector edge as shown in figure 3 of [1]. Events where the tower with the maximum energy is next to the boundary are further required to have the relevant A positive when the combination with the triplet outside the boundary is used for E_{out}.

2. The total energy deposited on the side of the acceptance cut is required to be larger than 55% of the beam energy; on the other side only a total energy deposition of 44% is required, see figure 4a of [1].

   The respective role of the sides are interchanged for events with an even event number.

3. The difference in azimuth, $\Delta\phi$, between the e^+ and e^- cluster is required to be larger than 170^0, figure 4b of [1].

4. The cluster centroids are required to be more than 1 cm in x from the edge of the aluminum extrusions.

This selection was used in [1] and is known as method 5. The last cut does not affect the selection on the fiducial side, but the cut removes 1% of the clusters on the non-fiducial side when applied as the last cut.

Method 6 is similar but here it is required that the boundary is at least one full pad from the detector edge, except along the vertical cracks where the distance from the edge is 0.9 pads. The effect of the last cut is very small for method 6 as Bhabha events are rather collinear.

Method 4 does not use cut 1.

The contribution from table 1 of our paper to each of the cuts will be described in the following. The contribution to cut 4 will be described as part the contribution to cut 1.

2 Transverse and longitudinal shower profile

The error in the luminosity from these sources has two parts:

$$\frac{\Delta L}{L} = \frac{\Delta L_{loss}}{L} + \frac{\Delta L_{acc}}{L}$$

coming from the energy and acceptance cuts, respectively.

The first term comes as an increase of the shower size, both longitudinal and transverse leads to a loss of energy and therefore a loss of events when the energy falls below the threshold. The effect of the longitudinal shape is small. Less than 1% of the energy from a 45 GeV electron is leaking out of the 24.5 radiations lengths deep calorimeter.

The second term comes as the effective boundary of the acceptance region will expand with increasing transverse shower size if the pads used to calculate E_{in} are larger than the one used when calculating E_{out}. The shower size is measured as the rms of the transverse energy distribution in a cluster, figure 1.

There are two parameters in the Monte Carlo that controls the longitudinal profile. The transverse size is assumed to be equal to the shower depth times the transverse parameter times the ratio of the longitudinal parameters. The result is that all three parameters influence the shower size.

Another contribution to this uncertainty comes from the fact that the pads change size in steps of $\frac{1}{40}$" or 0.063 cm. Thus storey 1 is composed of 6 layers with pad size 2.8575 cm followed by 3 layers with pad size 2.921 cm.

The argument for using storey 1, as it is done in the paper rather than the sum of storey 1 and storey 2 is that the ΔL would be bigger for the latter as the shower grows with depth. Contrary, the argument for using both storeys, is that the influence from the padsize changes is smaller when more changes are involved.

To study this, events selected using only storey 1 and only storey 2 were compared. The number of events accepted by storey 1 were 4169 and 4171 in the data [2] and in the Monte Carlo 9276 and 9296 respectively. This can be used as a measure of the uncertainty in ΔL when data and Monte Carlo is compared. The result indicates that the size of the shower in the two stories is indeed much smaller than the pad size, see also section 8.

An additional argument for using the sum of storey 1 and 2 is that the error introduced by calibration errors e.g. a dead wire, see section 4, is smaller and that the fluctuations are smaller in this case.

An increase in the shower size leads to an increased number of events especially for method 5 where the outside pads are smaller than the inside pads in contrasts to method 6 where all important pads are equal.

Monte Carlo studies show that:

$$\frac{\Delta L}{L} = (-0.019 \pm 0.003) \cdot \Delta r_{show}, \qquad \text{method 4}$$

where Δr_{show} is the uncertainty in the transverse shower size in cm. The $\pm$ reflects the variation found when the shower is given a certain size using the different simulations. The dominant term is the loss of event due to the energy cut, especially since the measured position is rather independent of the actual position of the cluster when it is near the edge.

$$\frac{\Delta L}{L} = (0.024 \pm 0.008) \cdot \Delta r_{show}, \qquad \text{method 6}$$

The dominant term is the shift in the boundary as we expect no significant loss of events due to the energy cut since this boundary is far from the edge.

$$\frac{\Delta L}{L} = (0.054 \pm 0.006) \cdot \Delta r_{show}, \qquad \text{method 5}$$

Both terms contribute in this case.

Δr_{show} is estimated from the agreement of the average value between the data and the Monte Carlo using the rms. of the shower profile and the number of towers contributing to the cluster.

$$\Delta r_{show} = 0.006 cm$$

Which gives an error of 0.0003.
Figure 1 shows good agreement with the average values but the width is larger in the data than in the Monte Carlo.

The effect of the error can be estimated as follows. For later use we express the uncertainty in the expansion of the acceptance region, Δx_{show}.

$$\frac{\Delta L_{acc}}{L} = 2 \cdot \frac{< \Delta x_{show} >}{r_{min}} \cdot \frac{9}{8}$$

$$r_{min} = 16 cm. \qquad \text{method 5}$$

$$r_{min} = 18 cm. \qquad \text{method 6}$$

The last factor comes from assuming that r_{max} is equal to 3 times r_{min}. This assumption will be used throughout the note. The increase in r_{min} from method 5 to method 6 is less than one pad as the boundary is not changed everywhere.

Assuming that the transverse loss of energy for method 6 is zero gives:

$$< \Delta x_{show} > \in [0.43 \cdot \Delta r_{show}, 0.58 \cdot \Delta r_{show}] \qquad \text{method 5}$$

$$< \Delta x_{show} > = 0.19 \cdot \Delta r_{show} \qquad \text{method 6}$$

A position resolution of σ_x gives an increase of the cross section [3] of:

$$\frac{\Delta L_{increase}}{L} \simeq 3 \cdot \sigma_x^2 \cdot \left(\frac{1}{r_{min}^2} + \frac{1}{r_{max}^2}\right) = 3 \cdot \frac{\sigma^2}{r_{min}^2} \cdot \frac{10}{9}$$

This effect is included in the Monte Carlo but it has an error of:

$$\frac{\Delta L}{L} = 3 \cdot \frac{\Delta\sigma_x^2}{r_{min}^2} \cdot \frac{10}{9}$$

Assuming the maximum change of x_{show} with r_{show} gives from figure 1.

$$\Delta r_{show}^2 = 0.052cm^2$$

$$\Delta\sigma_x^2 = 0.19^2 \cdot \Delta r_{show}^2$$

$$\frac{\Delta L}{L} = 0.0007$$

which combined with the above gives 0.001. It is 0.005 in the paper.

3 Energy scale

The uncertainty in the energy scale $\frac{\Delta E}{E}$ contributes to the error through:

$$\frac{\Delta L}{L} = (\frac{E}{dE} \cdot \frac{dN}{N}) \cdot \frac{\Delta E}{E}$$

where E is the energy at the energy threshold and N is the number of events. Varying the energy cut in the range 25 GeV to 35 GeV gives:

$$\frac{E}{dE} \cdot \frac{dN}{N} = 0.15$$

The same number can to a good approximation be obtained from the energy plot in figure 4a of [1].

The relative error in the energy can be determined from the agreement between the Monte Carlo energy spectrum and the above experimental plot taking into account the variation between the modules. It is found that:

$$\frac{\Delta E}{E} \simeq 0.015$$

which gives an error of 0.002. It is 0.002 in the paper.

4 Energy resolution and cell-to-cell calibration

The test beam has shown that the energy resolution at 45 GeV/c is 3% in well behaved regions. A single calibration constant is used for each module. This results in a wider energy spectrum than if each pad had its own constant.

The energy threshold is very low, with very few events near it. Since it is only the net loss across the threshold that counts, the energy resolution has a small effect on cut 2.

A systematic difference in the cell-to-cell calibration might affect cut 1 through a change in the boundary position. A change in the energy of the triplet energies will change the average boundary position x with:

$$\Delta x_{cal} = \frac{E \cdot dA}{dE} \cdot \frac{dx}{dA} \cdot \sqrt{\frac{1}{N_{eff}}} \cdot \frac{\Delta E}{E} \cdot \frac{1}{2}$$

where E is the energy on one side of the boundary and A is the asymmetry as defined in cut 1.

Assuming that the two triplets vary independently then

$$\frac{E \cdot dA}{dE} = \sqrt{\frac{1}{2}}.$$

From figure 4 it is found that:

$$\frac{dx}{dA} = 0.5cm.$$

N_{eff} is the effective number of independent triplets in a module. It takes into account that not all triplets vary coherently across the boundary. It is set to:

$$N_{eff} = 4.$$

$\Delta E/E$ is estimated from the uniformity of the wires and precision with which the wires are placed, the fluctuations in the amplifiers pedestals and gains and the effect of possible dead wires.

The wires are uniform to $\leq$ 0.0001 cm and they are placed to $\pm$ 0.001 cm in a position where the gain is insensitive to the exact position. This means that such uncertainties are small.

The pedestal fluctuations per pad are less than 10 MeV and the gain fluctuations are less than 1% as measured by the amplifiers alone. The energy in the first storey is about 27% of the shower energy or 12 GeV. If this energy is distributed evenly between the two sides and the three pads are assumed to fluctuate coherently in gain then this fluctuation is 60 MeV.

There are no dead wires in the critical regions at the moment. If it happens then the signal will be reduced with $\frac{1}{36}$ for storey 1 and $\frac{1}{96}$ for storey 1 plus 2. For storey 1 this corresponds to 166 MeV, which together with the pedestal and gain fluctuations gives:

$$\frac{\Delta E}{E} = 0.03$$

.

The last factor in the expression for the uncertainty in Δx_{cal} comes from the assumption that the four modules are independent.

Inserting gives for the average over the four modules:

$$\Delta x_{cal} = 0.003cm.$$

The error in the increase of the luminosity is small, since the increase itself is small and only the uncertainty in Δx_{cal} matters.

If however Δx_{cal} is taken as the systematic error then:

$$\frac{\delta L}{L} \leq 2 \cdot \frac{\Delta x_{cal}}{r_{min}} \cdot \frac{9}{8} = 0.0004$$

Monte Carlo studies indicate luminosity changes of 0.5% for a systematic shift of 5% of each summing amplifier, which is a very large shift. But the observed effect is within the expected fluctuations from Monte Carlo statistics. It does not contradict the estimate above.

It is 0.007 in the paper.

5 External alignment and beam parameters

By external alignment and beam parameters is meant the alignment precision with which we know the absolute positions of the center of a set of modules with respect to the inside of the TPC. It is known from construction and survey.

There are several contributions:

$$\Delta x^2_{ext} = \Delta x^2_{TPC} + \Delta x^2_{survey} + 0.5 \cdot (\Delta x^2_{opening} + \Delta x^2_{dowel} + \Delta x^2_{print})$$

$$\Delta x_{TPC} = 0.005cm$$

This is the error between the inside of the TPC and the outside fiducials.

$$\Delta x_{survey} = 0.021cm$$

Survey error of the LCAL front Al plate. 8 points on each of the two detector halves were measured with a precision of .080 cm in each direction. A model with 7 parameters describes the measurements well with the above measurement errors. The model uses 3 coordinates, 3 Euler angles and one rotation of one module with respect to the other around a vertical axis.

$$\Delta x_{opening} = 0.009cm.$$

This survey error of one module comes from the opening of the two detector halves around a vertical axis.

$$\Delta x_{dowel} = 0.011cm.$$

Error when going from front of Al plate to dowel surface. The main contribution is from the uncertainty in how perpendicular the dowel is to the Al-plate. In principle it depends on z.

$$\Delta x_{print} = 0.012cm.$$

Error from dowel surface to point on print. Here it is assumed that all prints in a module have systematically the same error. If all four modules were identical this error would be absent in the formula above. The prints are pinned to the lead. The main contribution comes from this.

Inserting gives:

$$\Delta x_{ext} = .025cm.$$

which leads to an increase of the luminosity of:

$$\frac{\Delta L_{acc}}{L} = 3 \cdot (\frac{\Delta x_{ext}}{r_{min}})^2 \cdot \frac{10}{9} = 0.00001$$

This error can be checked by comparing the vertex x and y as observed by the TPC and LCAL.

y_{LCAL} is found as the average value of:

$$y_{LCAL} = \frac{y_A + y_B}{2} - \frac{0.3 \cdot B \cdot z}{2\dot{p}} \cdot x_B$$

and

$$x_{LCAL} = \frac{x_A + x_B}{2} - \frac{0.3 \cdot B \cdot z}{2\dot{p}} \cdot y_B$$

Figure 2 and 3 show these distributions.

Comparing the TPC and the LCAL gives:

$$(< x_{TPC} > - < x_{LCAL} >)^2 \simeq \frac{rms_{TPC}}{N_{TPC}} + \frac{rms_{LCAL}}{N_{LCAL}} + \frac{\Delta x_{ext}^2}{2}$$

The average value for LCAL is for all LEP settings 0.020 cm closer to the nominal value than the TPC both in x and y in agreement with the above.

The spreads of 0.120 cm and 0.135 cm depend in addition to statistics on the flutuations of the beam position, lack of colinearity of the clusters due to photon emission and on the measurement accuracy, which was $0.15/\sqrt{2}$ cm in the test beam. The spreads give upper limits of 0.17 cm and 0.19 cm respectively.

The external alignment error also contributes to the $\Delta\phi$ cut error. Averaging over the polar angle gives:

$$\frac{\delta L}{L} \leq 2 \cdot \frac{\Delta x_{ext}}{r_{min}} \cdot \frac{dN}{N \cdot d\phi}$$

$$\frac{dN}{N \cdot d\phi} = 0.12 rad^{-1}$$

is the relative change of the number of events when changing the coplanarity angle. This can be seen from figure 4b in [1]. Inserting gives:

$$\frac{\Delta L}{L} \leq .0002$$

It is 0.002 in the paper.

6 Internal alignment and inner radius

By internal alignment is meant the aligment precision with which we know the distance between the two boundaries which define the acceptance region on one side of the interaction point. Again it is known from construction and survey.

Assuming now that the error Δx_{print} is fully correlated to get the maximum error, then one can write the error in the half distance between the modules as:

$$\Delta x^2_{int} = \frac{1}{4} \cdot (\Delta x^2_{fix} + \Delta x^2_{opening} + \Delta x^2_{dowel}) + \Delta x^2_{print}$$

The only new error is:

$$\Delta x_{fix} = 0.0025cm.$$

which is the error in the relative fixation of the two modules. It is composed of 3 gaps of 0.005 cm each and given by construction.

Inserting numbers gives:

$$\Delta x_{int} = 0.014cm.$$

Which with

$$\frac{\Delta L}{L} = 2 \cdot \frac{\Delta x_{int}}{r_{min}} \cdot \frac{9}{8}$$

gives an error of 0.002. It is 0.010 in the paper.

7 Description of material

The error comes from an incomplete Monte Carlo description of the detector. The cables from LCAL and SATR, the electronic boxes of the SATR, the internal wiresupports of LCAL and the support structure of the TPC were not in the Monte Carlo simulation.

The result of these imperfections is that the Monte Carlo does not describe the data. For example is the distribution of cluster width broader than predicted, see figure 1. Also the energy spectrum is not correctly described as is apparent in the energy plot of [1].

Part of the broadening comes from the fact that the Monte Carlo spectrum is only valid for the central energy in the energy scan while the data is the sum over all energies.

The deduced luminosity depends, even when this is taken into account, on the energy threshold which anyway is somewhat arbitrary. Its value is set to be larger than the trigger threshold to ensure that the trigger is efficient. The trigger threshold is in turn set high enough to keep random background at a sufficiently low level.

The excess after correcting for the different energies of the Monte Carlo over the data is 3.2$\pm$ 0.5% in the range from 25 GeV to 35 GeV as deduced from the energy plot in [1].

Using data taken in the beginning of the run with a threshold at 11 GeV gives an excess of 0.8$\pm$ 0.5% in the range from 15 GeV to 25 GeV.

Under the assumption that the excess is a linear function of the energy then the two measurements suggest that 0.8$\pm$ 0.5% is equal to the total increase of the luminosity which one would get by lowering the threshold.

The error is set equal to this value e.g. 0.008. Note that an increased threshold will increase this error. This error is 0.005 in the paper.

8 Monte Carlo simulation of detector response

The detector response is assumed to be uniform in active regions of the detector. However the charge distribution on the graphite is systematically larger near the wire giving a systematic shift Δx_{resp}, which was not considered in [1]. This effect can be studied by comparing selections with storey 1 or 2, both for data and Monte Carlo. The boundaries are differently positioned with respect to the wires in the two storeyes. The result is that the number of events only changes very little as already mentioned in section 2.

9 Monte Carlo statistics

The Monte Carlo cross section which is used in the luminosity determination is calculated by integrating an approximate matrix element over a region of phasespace corrected by a Monte Carlo generation of events with weight, w_i such that:

$$\sigma_T =< w > \cdot N_T$$

and

$$\Delta\sigma_T = \sqrt{(< w_i - < w >>^2 \cdot N_T)} \equiv a \cdot < w > \cdot \sqrt{N_T}$$

where σ_T is the cross section in the full phasespace region and N_T is the total number of events generated.

The cross section in a restricted region, e.g. method 5, is given by:

$$\sigma = \frac{N}{N_T} \cdot \sigma_T = \alpha \cdot \sigma_T$$

$$\frac{\Delta\sigma}{\sigma} = \sqrt{\frac{\frac{1-\alpha}{\alpha} + a}{N_T}}$$

The term $1 - \alpha$ comes from the fact that it is the fixed number N_T which gives the cross section σ_T.

Inserting a = 0.23 , $\alpha_5 = 0.36$ and $N_T = 50000$ gives an error of .006. It is 0.009 in the paper as implied by the 2% overall error.

10 Extra checks

The first check is the check that all methods give the same luminosity within errors. Here we shall discuss the error contribution from cut 1 to the difference between method 5 and 6.

$$\begin{aligned}\Delta(\tfrac{\Delta L}{L})^2 = \; & (\tfrac{1}{\alpha_6} - \tfrac{1}{\alpha_5}) \cdot \tfrac{1}{N_T} + \tfrac{1}{N_6} - \tfrac{1}{N_5} + \\ & (2 \cdot (\tfrac{1}{rmin_5} - \tfrac{1}{rmin_6})\tfrac{9}{8})^2 \cdot (\Delta x^2_{cal} + \Delta x^2_{int} + \Delta x^2_{resp}) + \\ & ((0.58 - 0.19) \cdot \Delta r_{show} \cdot 2 \cdot (\tfrac{1}{rmin_5} - \tfrac{1}{rmin_6})\tfrac{9}{8})^2 + \\ & ((0.58 - 0.19)^2 \cdot \Delta\sigma^2_{show} \cdot 3 \cdot (\tfrac{1}{rmin_5^2} - \tfrac{1}{rmin_6^2})\tfrac{10}{9})^2\end{aligned}$$

N_5 and N_6 are the number of events in method 5 and 6, respectively, after background subtraction . This expression is calculated for a fixed number of Monte Carlo events in method 5.

Inserting the above numbers with $\alpha_6 = 0.26$, $N_5 = 4628$ and $N_6 = 3343$ gives:

$$\Delta(\frac{\Delta L}{L}) = 0.01$$

which is larger than the observed difference.

Plotting the asymmetry A as function of the x of the LCAL cluster for small values of y, figure 4, puts limits on the $\Delta x's$ through:

$$\Delta x^2 = \Delta x^2_{show} + \Delta x^2_{cal} + \Delta x^2_{int} + \Delta x^2_{resp}.$$

Inserting the above with $x_{resp} = 0$ predicts that:

$$\Delta x = 0.015 cm.$$

Δx is dominated by the internal alignment error. The agreement on the two boundaries is within this error. It should however not be forgotten that the asymmetry and the cluster position is related as 27% of the energy is used for the asymmetry measurement and all the energy is used for the cluster position.

In [1] beamchambers from the testbeam were used instead of the cluster to get an estimate of a possible error.

In any case what matters is not the change in the apparent boundary position, but the precision by which the Monte Carlo describes the data.

An alternative plot is the efficiency across the boundary, figure 5 as function of the LCAL cluster. The efficiency near the boundary x_0 can be written as:

$$eff \simeq \frac{1}{2} - \left(\frac{\Delta x}{2 \cdot \sqrt{3} \cdot \sigma} + \frac{3 \cdot \sqrt{3} \cdot \sigma}{2 \cdot x_0}\right) + \frac{x - x_0}{2 \cdot \sqrt{3} \cdot \sigma}$$

where σ is the resolution near x_0. This expression predicts a resolution of 0.06 cm. The steepness of the spectrum is partly coming from the correlation in the data. There is good agreement between the data and the Monte Carlo.

Similar plots will be shown in [4] where the SATR track is used instead of the cluster. The correlation is avoided this way.

In figure 6 is plotted the polar angle of the LCAL cluster minus polar angle of the SATR track for for events selected with method 5. The Monte Carlo predicts a mean value of 0.17 mrad which comes as clusters near the boundary systematically gets a too large polar angle. The mean value in the data is 0.02 mrad. The shift corresponds to 0.04 cm at the LCAL position e.g. $\Delta x \leq 0.04$. This shift gives an upper limit of the error from cut 1 of 0.005.

The predicted width from the Monte Carlo is 0.31 mrad and it is 0.56 mrad in the data which corresponds to 0.087 cm and 0.157 cm at the position of LCAL. It is a combination of the LCAL and SATR resolution and is for the data consistent with the value found in section 5.

11 Higher order radiative effects

This error is rather arbitarily set to:

$$\frac{\Delta L}{L} = \frac{\delta^2}{2}$$

where

$$\delta = \frac{\sigma(\alpha^3)}{\sigma(\alpha^2)} - 1 = -0.02...0.03$$

The exact value depends on the selection criteria [5]. It is roughly the same for all runs and energies and it should be the same for all experiments.

This value of δ gives an error of less than 0.001.

An additional contribution is the uncertainty in α due to QCD effects.

This error is set to 0.010 in the paper.

12 Conclusion

The new error estimates are given in the table with the old values in parenthesis.

transverse and longitudinal shower profile	0.001 (0.005)
energy scale	0.002 (0.002)
energy resolution and cell-to-cell calibration	0.001 (0.007)
external alignment and beam parameters	0.001 (0.002)
internal alignment and inner radius	0.002 (0.010)
description of material	0.008 (0.005)
trigger efficiency	0.002 (0.002)
Monte Carlo statistics	0.006 (0.009)
total experimental systematic error	0.011 (0.017)
higher order radiative effects	$\leq$ 0.010 (0.010)

The conclusion is that the overall error in our paper is high but reasonable.

The discussion of the errors often centers on the error in the three cuts rather than on the individual error contributions.

Cut on asymmetry and boundary	0.003 (0.014)
Cut on energy	0.008 (0.005)
Cut on $\Delta\phi$	0.001 (0.002)
trigger efficiency	0.002 (0.002)
Monte Carlo statistics	0.006 (0.010)
total experimental systematic error	0.011 (0.017)
higher order radiative effects	$\leq$ 0.010 (0.010)

Relating the error on the asymmetry cut to an error in the inner radius through:

$$\frac{\Delta L}{L} = 2 \cdot \frac{\Delta x_{cut}}{r_{min}} \cdot \frac{9}{8}$$

gives an uncertainty of 0.02 cm in the inner radius averaged over the four modules, which is about half of the upper limit found using SATR.

The error from the incomplete Monte Carlo description can in future publications be reduced by improving the Monte Carlo description. To the extent that this is not enough then extra cuts which leave out regions which are difficult to describe can be applied. This will reduce the error from the energy cut and slightly reduce the error from the asymmetry cut. This however increases the geometrical and statistical errors.

It is obviously possible to reduce the error from the Monte Carlo statistics.

To the systematic error must be added the statistical error which is given by:

$$\frac{\Delta L_{stat}}{L} = \sqrt{\frac{1 + 2 \cdot \frac{N_{Back}}{N_5}}{N_5}}$$

where N_{Back} is the number of estimated remaining background events.

References

[1] D.Decamp et al.: Determination of the Number of Light Neutrino Species.

[2] L.Garrido private communication.

[3] J.D.Hansen: ALEPH Note 88-30/ Physics 88-10

[4] Forthcoming note on SATR performance.

[5] H.Burkhardt: Notes of LUMI meeting 8-9-1989

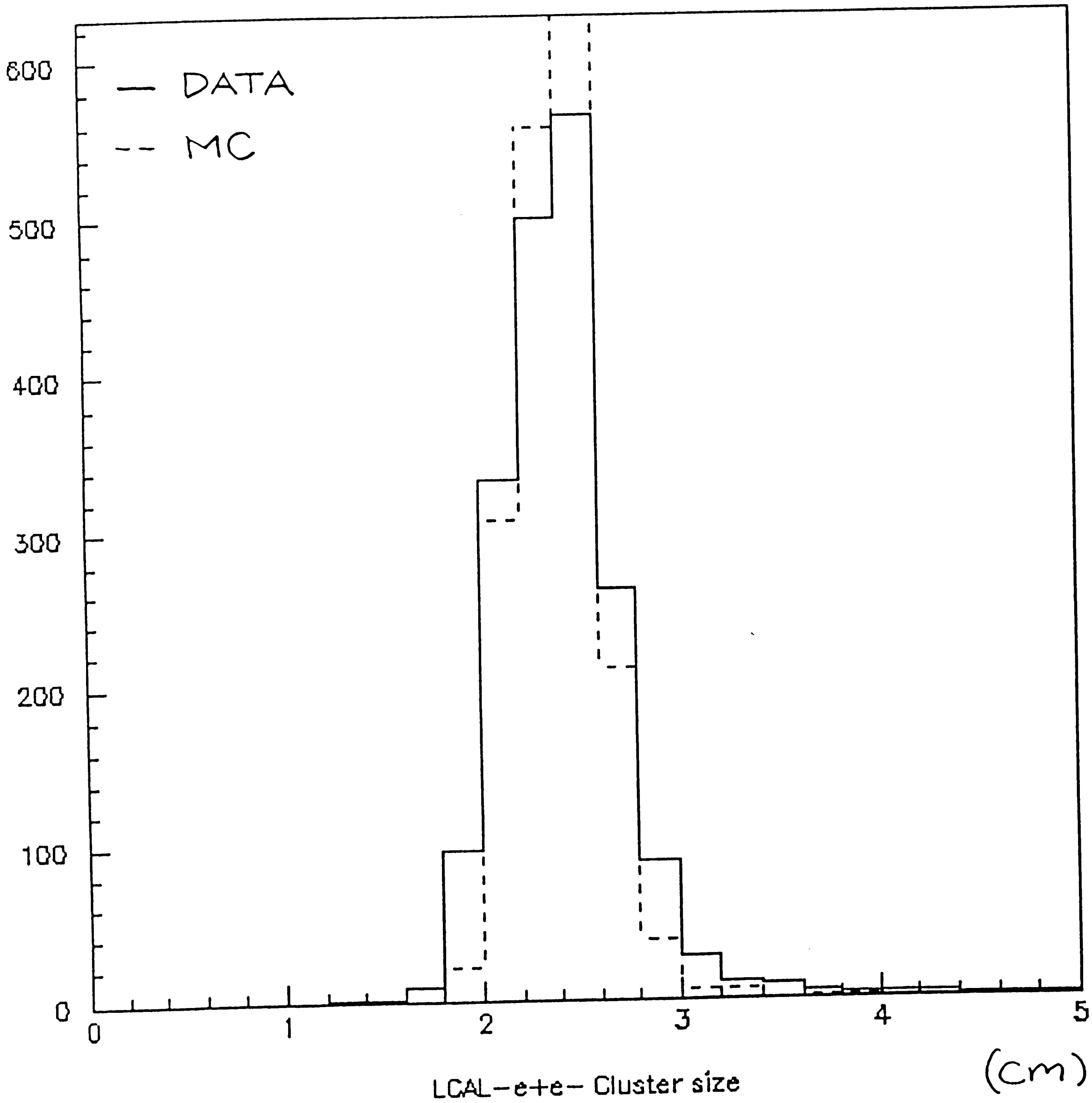


Figure 1: Shower width

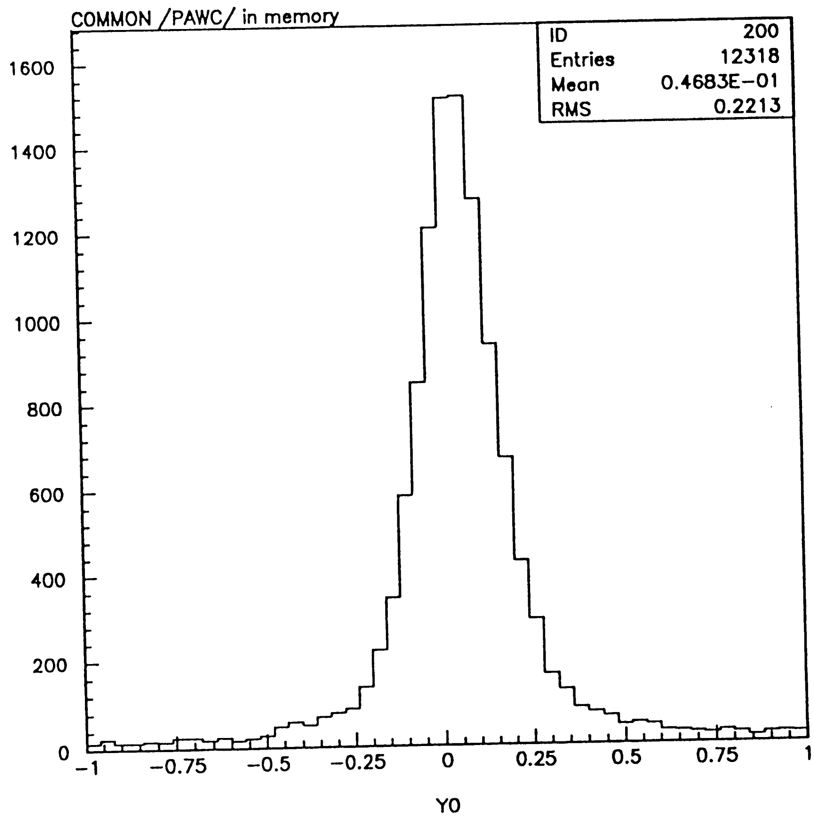


Figure 2: Measured vertex y

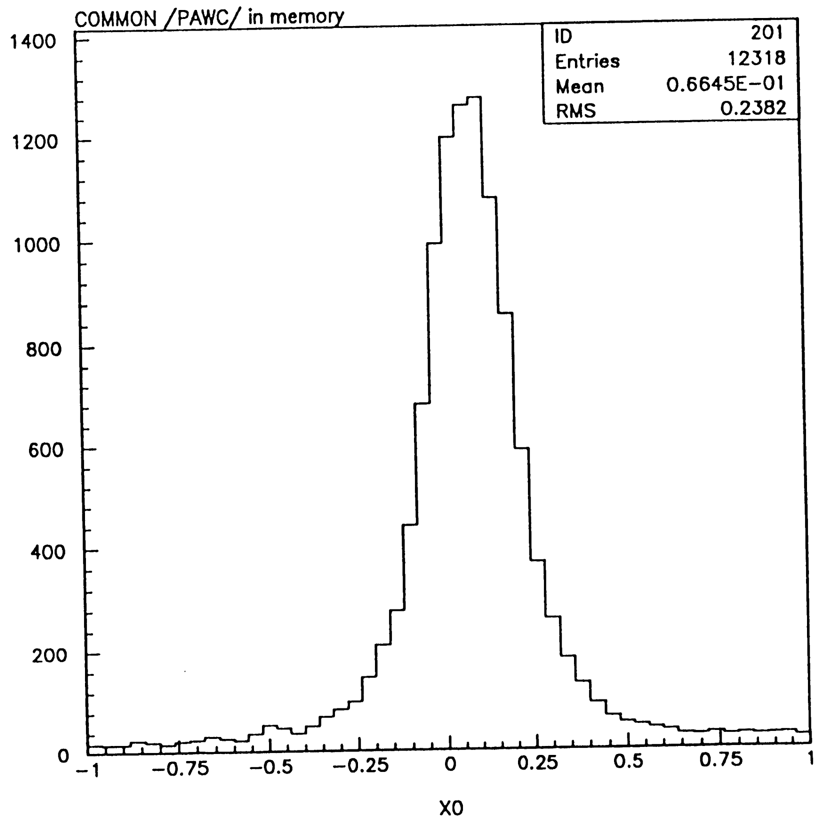
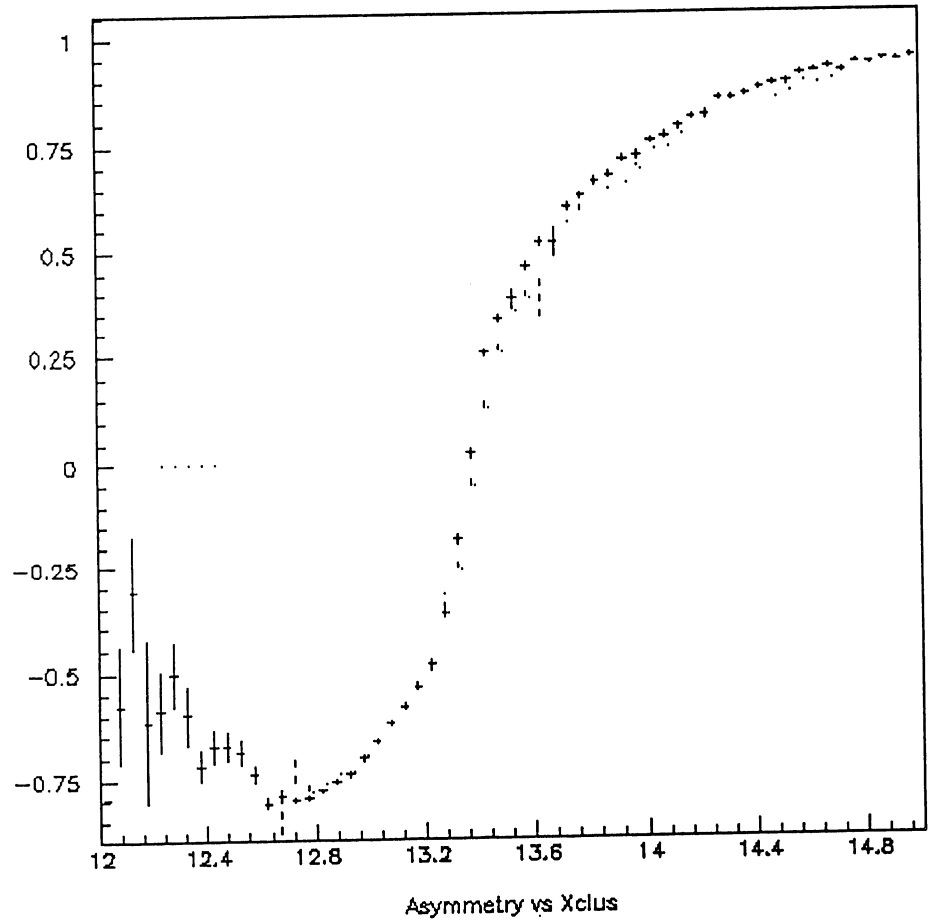


Figure 3: vertex x



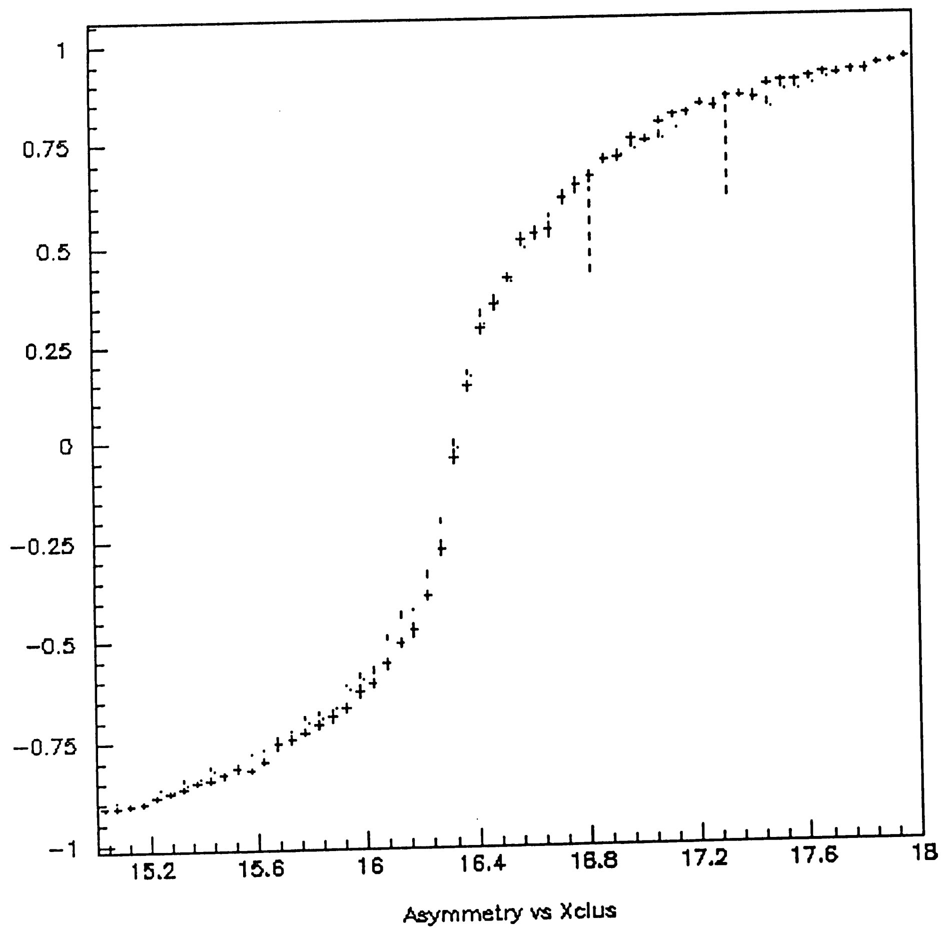


Figure 4: Asymmetry A as function of x of LCAL cluster for small y, a) method 5 boundary, b) method 6 boundary

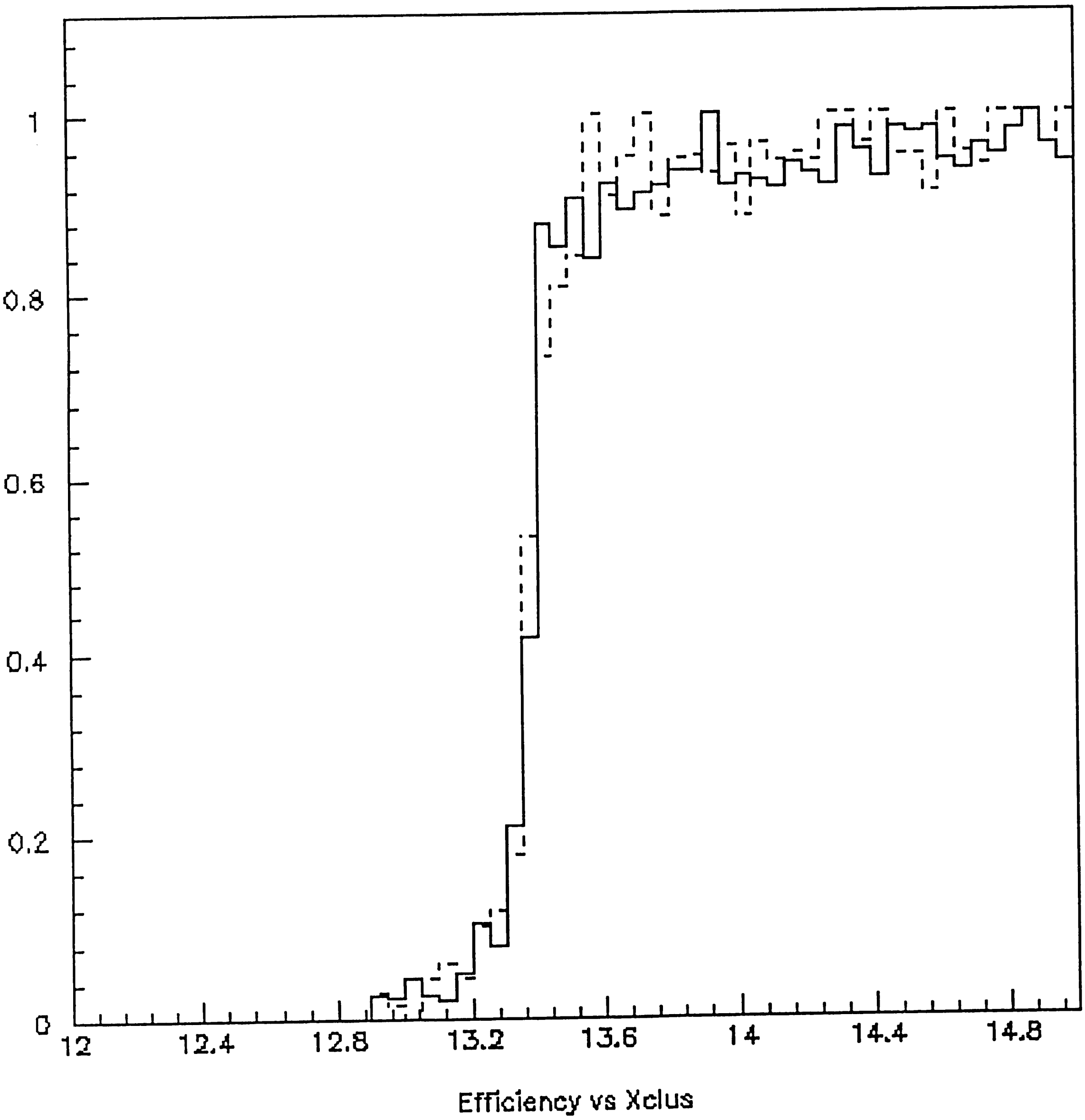


Figure 5: Efficiency as function of x of LCAL cluster for small y, method 5 boundary

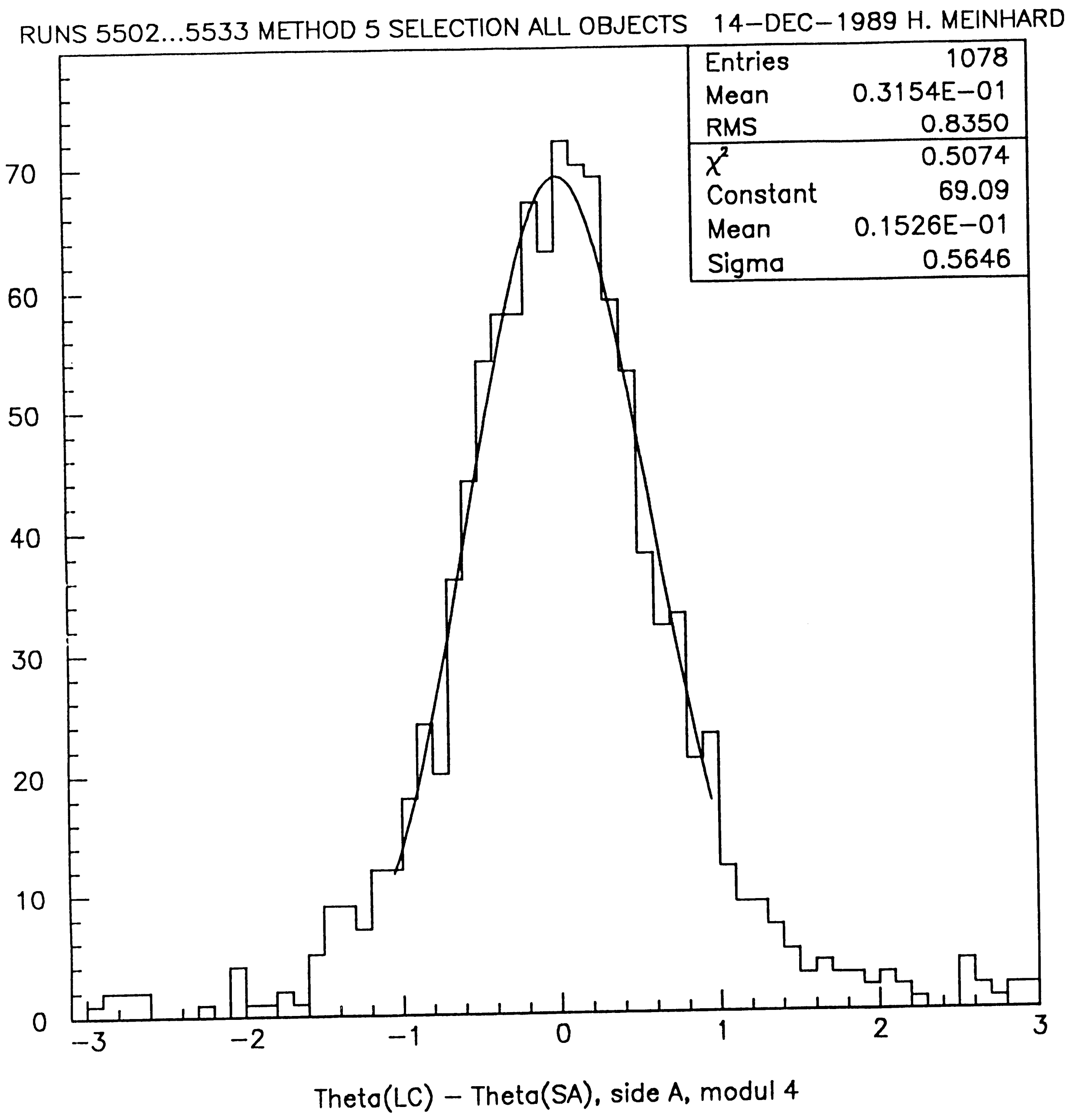


Figure 6: Polar angle of LCAL cluster - polar angle of SATR track