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A Bhabha event generator for luminosity studies

by

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ABSTRACT

The Monte Carlo event generator for Bhabha scattering MILLOR for LEP/SLC energies is described. It includes the $O(\alpha^3)$ calculations needed to make good luminosity studies when events with at least one electron and one positron detected are used. A comparison with BABAMC event generator [2] is performed. Moreover, differences from the lowest order Q.E.D. cross section at very small angles for the Small Angle Luminosity Monitor (SALM) for ALEPH are shown.

1. Introduction

Bhabha scattering is of fundamental importance for e^+e^- colliders since it will be used as a reference process to measure the luminosity. It also serves as a precision test of the G.W.S. theory. In order to compare with precise experimental data, radiative corrections have to be taken into account.

The standard way to include $e^+ e^- (\gamma)$ events is to define a cut for the relative emitted photon energy $k_0 \equiv \Delta E_\gamma / E_{beam}$ below which an event is considered to be 'soft' (that is, in practice the final state is only e^+e^-) and above which it will be 'hard'. We tipically take $k_0 = 0.01$. With this in mind we have constructed an event generator (MILLOR) optimised for the study of luminosity measurements using events with both charged particles detected, including M. Greco's analytical expressions [1] for the soft part and F.A. Berends, R. Kleiss & W. Hollik calculations [2] for the hard part. Modifications to [1] to make a more reliable generator will be discribed below.

As an example of the results obtained with MILLOR we show also the analysis of two used programs (MAG and MAGFIX) to study the acceptance for the Small Angle Luminosity Monitor (SALM) for ALEPH, showing that radiative corrections are essential to make a good study.

The outline of this paper is as follows. In section 2 we make a theoretical description of MILLOR describing which diagrams contains and which modifications have been made to [1]. Section 3 is a reference guide explaining the meaning of the parameters used and the range of validity of the generator. Section 4 contains some results studying the acceptance of the SALM and some relevant distributions. Finally, in section 5 we present the conclusions that can be drawn.

2. Description of MILLOR

As mentioned before, the generator has two parts: soft and hard. For the soft part we include complete Q.E.D. radiative corrections (all one loop diagrams) and soft photon effects to the complete electroweak lowest order calculation. This is based on ref. [1]. We have also included as pure and main weak correction the self energy of the Z^0 boson by substituting (M_0 is the Z^0 mass; see [2] for notation)

$$1/(q^2 - M_0^2) \longrightarrow 1/(q^2 - M_0^2 + \hat{\Sigma}^z(q^2)) \tag{1}$$

in the Z^0 propagator and in both channels s and t. On the other hand, for the hard part we include the exact matrix element squared to order α^3. Following ref. [2] the method of helicity amplitudes to evaluate the cross section neglecting fermion masses is used. The masses needed to describe properly the collinear photon peaks are included later in the so called 'mass effect factors'.

The main formulae used in the program have been obtained from reference [1]. However, we have made substantial modifications, namely:

(1) Solve a numerical problem ocuring when $s = M_0^2$ in $C_F^{(i)}$, $i = 7, 8, 9$.

(2) Use expansions in k_0 to not resum to all orders the soft effects.

(3) Use more accurate expressions in the cross section for the vacuum polarization of the photon.

All these changes are explained in the following.

The first point concerns the C_F array. When the energy of the process is just that of the Z^0 peak ($s = M_0^2$) then $R'(s) = 0$ and this causes problems in $C_F^{(i)}$ ($i = 7, 8, 9$). But this is a trivial problem and can be solved immediately observing that $d\sigma_0(i)$ ($i = 7, 8, 9$) also contain a factor $R'(s)$. So we have just redefined $\{C_F^{(i)},\ i = 7, 8, 9\}$ and $\{d\sigma_0^{(i)},\ i = 7, 8, 9\}$ to avoid $R'(s)$ in the denominators thus avoiding problems in numerical calculations.

The second point has to do with the resummation effects. Usual exponentiation [3,4] is used in ref. [1] in order to resum to all orders these effects. But to get a good agreement with the Bhabha generator from F.A. Berends & R. Kleiss at PETRA energies [5] we have to use the following expansions in C_{infra} ($\Delta \equiv k_0$) :

$$\begin{aligned} \Delta^{2\beta_e+2\beta_{int}} &\simeq 1 + 2 \cdot ln\Delta \ \ (\beta_e + \beta_{int}) \\ \Delta^{\beta_e+\beta_{int}} &\simeq 1 + ln\Delta \ \ (\beta_e + \beta_{int}) \\ \Delta^{\beta_e} &\simeq 1 + ln\Delta \ \ \beta_e \end{aligned} \tag{2}$$

So the soft effects are included in MILLOR only to order α^3. In fact this makes differences even of some few per cent in the total cross section. Thus it is not clear that not include the resummation is the best way to follow in a Monte Carlo treatment. Actually, this is a point that we are still studying.

The last change is a modification in the vacuum polarization of the photon. In ref. [1] simplified expressions for $\delta_\pi(s)$ and $\delta_\pi(t)$ are used, neglecting terms containing fermion masses. This can't be done for heavy fermions because for instance in the t channel the invariant t becomes of the order of m_f^2. As an example at the Z^0 peak for the charm quark we have that $2m_c^2/\mid t \mid \gg 1$ at angles $\sim 10mrad$. Instead we have used exact calculations based on ref. [2] (see formula (3.8) therein for details).

Clearly MILLOR doesn't contain all the possible weak corrections. For instance it includes neither weak vertex corrections nor weak box diagrams, but these contributions are negligible for angles of the luminosity monitors. This is specially clear for the SALM (see for instance figures 5.9 and 5.10 of ref. [2]).

3. How to use MILLOR

In this section we describe the parameters used as input by MILLOR. We also show the parameters needed by the MAG and MAGFIX programs, which use the MILLOR generator to calculate the acceptance for the SALM, the Small Angle Luminosity Monitor for ALEPH. These programs are also easy to use for any other luminosity monitor shapes.

3.1. MILLOR parameters

EB the beam energy (in GeV)

THMIN minimum scattering angle of e^-, e^+ (in rad.)

THMAX maximum scattering angle of e^-, e^+ (in rad.)

The acceptability criterion is the following:

$$cos\theta_{max} < cos\theta_{e^+} < cos\theta_{min}$$
$$cos\theta_{max} < -cos\theta_{e^-} < cos\theta_{min}$$

XKMIN minimum bremsstrahlung energy (in units of EB)

XKMAX maximum bremsstrahlung energy (in units of EB)

XK0 defines the separation between soft and hard bremsstrahlung energy (in units of EB). Default value: XK0=0.01

NEV number of events to be generated

WMAX estimated maximum weight. It has to be provided by the user, and chosen as small as possible. If the weight of an event is greater than WMAX a message will be printed

M0 the Z^0 mass (in GeV) (the Z^0 width is calculated in the program)

Allowed values:

XKMIN=0 or XKMAX=1 are allowed
THMIN=0 or THMAX=π are NOT allowed
If XKMIN < XK0 it is assumed to be XKMIN=0

The incoming e^+ goes in the $z > 0$ direction, but due to the ALEPH convention of axis after having generated the event the outgoing e^+ goes in the $z < 0$ direction.

3.2. MAG and MAGFIX parameters

The SALM is the monitor for ALEPH for very forward Bhabha events. Before a Bhabha event impacts the calorimeter it passes through a magnetic quadrupole located in the region $3.7m < \mid z \mid < 5.7m$. This is also simulated in both programs. For a complete description of the SALM see ref. [6].

MAG and MAGFIX allow any directions for the incoming e^+e^- to study the beam divergences problem. Then the particles are boosted in order to put them in the center of mass reference (in which go back to back) and finally rotated to put them along the z axis. After that an event is generated (MILLOR accepts only incoming e^+e^- back to back, along the z axis and with e^+ in the $z > 0$ direction). The backward rotation and backward boost have to be performed in this order for each generated event to get the original frame of reference. MAG is for gaussian distributed incoming directions around two given axis and MAGFIX is for fixed incoming directions.

MAG and MAGFIX also permit the study of the deplacement of the interaction point by simple generating three gaussian numbers for the vertex position.

Meaning of the parameters:

- X0 minimum distance in x-axis from the beam line of the calorimeter (in cm) (= 6.5 for the SALM monitor)
- XF maximum distance in x-axis from the beam line of the calorimeter (in cm) (= 8.5 for the SALM monitor)
- YDIM length of the calorimeter in y-axis (in cm). It is supposed to be YDIM/2 with $y > 0$ and the other half with $y < 0$ (= 5 for the SALM monitor)
- ZFINAL $\mid z \mid$ position of the monitors (in cm) (= 770 for the SALM monitor)
- ENCUT energy cut in a monitor below which an event is rejected (in GeV)
- XM1,XM2,XM3 mean vertex position (in cm)
- SD1,SD2,SD3 standard deviations for the vertex position (in cm)
  Default values: SD1=0.025, SD2=0.005, SD3=1

In fact the SALM detector consists of four identical calorimeters located symmetrically on each side of the beam pipe with respect to the horizontal plane, and on each side of the interaction point along the beam line. The four monitors are labeled 1, 2, 3 and 4. Monitors 1 and 2 have both $z > 0$, and $x > 0$ and $x < 0$ respectively, while monitors 3 and 4 have both $z < 0$, and $x > 0$ and $x < 0$ respectively.

JWR =1 will write in logical unit 7 the coincidence events between monitors 1 and 4

THPR divergence polar angle for incident e^+ (in rad.)

PHIPR divergence azimuthal angle for incident e^+ (in rad.)

THMR divergence polar angle for incident e^- (in rad.)

PHIMR divergence azimuthal angle for incident e^- (in rad.)

DIVSDR* standard deviation of polar divergence angle for incident e^+e^- (in rad.) In this case the azimuthal angle is generated uniformly around incident directions

(* Only in MAG program)

The output of the programs consists of many acceptance numbers NACC(K) and the corresponding cross sections SIGMA(K) obtained with

$$SIGMA(K) = SIGREF \cdot \frac{NACC(K)}{NEV} \tag{3}$$

where SIGREF is the reference cross section in a region that contains the monitors. The definition of these numbers depends strongly on the particular necessity of the user.

The various NACC(K) are defined in our programs as follows:

NACC(1) no. of e^- that impact in 1; when accepted IFLAG(1)=1

NACC(2) no. of e^- that impact in 2; when accepted IFLAG(2)=1

NACC(3) no. of e^+ that impact in 3; when accepted IFLAG(3)=1

NACC(4) no. of e^+ that impact in 4; when accepted IFLAG(4)=1

NACC(5) no. of γ that impact in 1; when accepted IFLAG(5)=1

NACC(6) no. of γ that impact in 2; when accepted IFLAG(6)=1

NACC(7) no. of γ that impact in 3; when accepted IFLAG(7)=1

NACC(8) no. of γ that impact in 4; when accepted IFLAG(8)=1

NACC(9) no. of times that at least one particle impacts in a monitor

NACC(10) no. of coincidences 1-4 with only $e^{\pm}$

NACC(11) no. of coincidences 2-3 with only $e^{\pm}$

NACC(12) no. of coincidences 1-4 with $e^{\pm}$ or γ

NACC(13) no. of coincidences 2-3 with $e^{\pm}$ or γ

NACC(14) no. of coincidences 1,2-3,4 with only $e^{\pm}$

NACC(15) no. of coincidences 1,2-3,4 with $e^{\pm}$ or γ

NACC(16) idem that NACC(9) but with energy cut

NACC(17) idem that NACC(10) but with energy cut

NACC(18) idem that NACC(11) but with energy cut

NACC(19) idem that NACC(12) but with energy cut

NACC(20) idem that NACC(13) but with energy cut

NACC(21) idem that NACC(14) but with energy cut

NACC(22) idem that NACC(15) but with energy cut

We are also interested in finding an inner region in monitor 1 for which we have coincidence with monitor 4. For this purpose we define EPSX (and EPSY) which are the reductions in x-direction (and y-direction) at both edges of the monitor (default values: EPSX=0.25cm, EPSY=0cm). So we can also define:

NACC(23) no. of e^- in 1 with reduction EPSX, EPSY; then IFLAG(9)=1

NACC(24) no. of coincidences 1-4 with only $e^{\pm}$ and with IFLAG(9)=1

NACC(25) idem with energy cut

NACC(27) no. of e^- in 1 with reduction $2 \cdot EPSX,\ EPSY$; then IFLAG(10)=1

NACC(28) no. of e^- in 1 with reduction $3 \cdot EPSX,\ EPSY$; then IFLAG(11)=1

NACC(31) idem that NACC(24) but with IFLAG(10)=1

NACC(32) idem that NACC(24) but with IFLAG(11)=1

NACC(26) idem that NACC(23) but with energy cut

NACC(29) idem that NACC(27) but with energy cut

NACC(30) idem that NACC(28) but with energy cut

NACC(33) idem that NACC(31) but with energy cut

NACC(34) idem that NACC(32) but with energy cut

NACC(35) idem that NACC(1) but with energy cut

If we want to know the probabilities that once we have an impact in an inner region in 1 we have another impact in the whole oposite monitor 4 (conditioned probabilities) we define:

$$\begin{aligned}
for\ EPSX &= 0\ \ , & PROB(1) &= NACC(17)/NACC(35) \\
for\ EPSX &= 0.25, & PROB(2) &= NACC(25)/NACC(26) \\
for\ EPSX &= 0.50, & PROB(3) &= NACC(33)/NACC(29) \\
for\ EPSX &= 0.75, & PROB(4) &= NACC(34)/NACC(30)
\end{aligned}$$

Note that $PROB(1) < PROB(2) < PROB(3) < PROB(4)$ taking enough statistics. This definition of the reduced inner regions in monitor 1 will help us to minimize the dependence on the vertex position and on the divergence angle in the counting rate by simply taking an inner region small enough so that the acceptance becomes less dependent of the mentioned effects, without decreasing too much that counting rate.

Furthermore, to study also the effect of the photons in the monitor we still define:

NACC(37) no. of γ in 1 with reduction EPSX, EPSY; then IFLAG(12)=1

NACC(38) no. of γ in 1 with reduction $2 \cdot EPSX,\ EPSY$; then IFLAG(13)=1

NACC(39) no. of γ in 1 with reduction $3 \cdot EPSX,\ EPSY$; then IFLAG(14)=1

IFLAG(15)=1 when IFLAG(1)=1 or IFLAG(5)=1

THCEN theoretical centroid of an event, calculated as

$$\frac{E_{e^-} \cdot x_{e^-} + E_\gamma \cdot x_\gamma}{E_{e^-} + E_\gamma} \tag{4}$$

if e^- and γ in 1

NACC(43) no. of theoretical centroids in 1 with reduction EPSX, EPSY; then IFLAG(16)=1

NACC(44) no. of theoretical centroids in 1 with reduction $2 \cdot EPSX,\ EPSY$; then IFLAG(17)=1

NACC(45) no. of theoretical centroids in 1 with reduction $3 \cdot EPSX,\ EPSY$; then IFLAG(18)=1

NACC(49) no. of coincidences 1-4 for th. cen. in inner region in 1 with reduction EPSX, EPSY

NACC(50) no. of coincidences 1-4 for th. cen. in inner region in 1 with reduction $2 \cdot EPSX,\ EPSY$

NACC(51) no. of coincidences 1-4 for th. cen. in inner region in 1 with reduction $3 \cdot EPSX,\ EPSY$

NACC(52) no. of centroids in 1

NACC(36) no. of γ in 1 with energy cut

NACC(40) idem that NACC(37) but with energy cut

NACC(41) idem that NACC(38) but with energy cut

NACC(42) idem that NACC(39) but with energy cut

NACC(46) idem that NACC(43) but with energy cut; $(E_{e^-} + E_\gamma > ENCUT)$

NACC(47) idem that NACC(44) but with energy cut; "

NACC(48) idem that NACC(45) but with energy cut; "

NACC(53) idem that NACC(52) but with energy cut; "

NACC(54) idem that NACC(49) but with energy cut; "

NACC(55) idem that NACC(50) but with energy cut; "

NACC(56) idem that NACC(51) but with energy cut; "

Now the conditioned probabilities are:

$$
\begin{aligned}
&for\ EPSX = 0\ \ , && PROB(5) = NACC(19)/NACC(53)\\
&for\ EPSX = 0.25, && PROB(6) = NACC(54)/NACC(46)\\
&for\ EPSX = 0.50, && PROB(7) = NACC(55)/NACC(47)\\
&for\ EPSX = 0.75, && PROB(8) = NACC(56)/NACC(48)
\end{aligned}
$$

Also $PROB(5) < PROB(6) < PROB(7) < PROB(8)$.

4. Results

We present here some results using the MILLOR generator. First of all we are interested in making a comparison with BABAMC [2], which includes complete weak corrections. These are shown in tables from 1 to 4. We can see a great agreement for the angular region of the luminosity monitors and at energies up to the Z^0 peak. Differences appear when $\theta_{min} > 30°$ but they become smaller at PETRA energies, showing good agreement with the corresponding Bhabha generator of ref. [5].

We have also obtained some results with the MAG program. To exhibit that radiative corrections are essential we have constructed another generator (LOWGROS) with the same parameters that MAG but simulating only the lowest order Q.E.D. cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2} \cdot \frac{(3 + cos^2\theta)^2}{(1 - cos\theta)^2} \tag{5}$$

The results are shown in figure 1. For instance, the cross section in the SALM monitor with this approximation (putting DIVSDR=0 and EPSX=0) is $0.67\mu barn$ while with radiative corrections is $0.57\mu barn$.

Finally we have investigated the dependence on the standard deviation of the polar divergence angle (DIVSDR) and on the x-coordinate of the mean vertex position (XM1) in the cross section for the SALM. The results are presented in figures 2 and 3 showing that σ_{SALM} (for coincidence events) decreases as DIVSDR or XM1 increases.

5. Conclusions

The Monte Carlo event generator for luminosity studies MILLOR at LEP/SLC energies has been constructed. Very good agreement with BABAMC generator has been demonstrated in that angular region. Moreover radiative corrections are shown to be essential to perform a correct analysis for the Small Angle Luminosity Monitor for ALEPH.

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TABLE CAPTIONS

1. Comparison between MILLOR and BABAMC for $\theta_{min} = 0.2°$, $\theta_{max} = 0.8°$ and $\sqrt{s} = M_0 = 93\ GeV$.

2. Idem at $\sqrt{s} = 40\ GeV$.

3. Idem with $\theta_{min} = 30°$, $\theta_{max} = 150°$ and $\sqrt{s} = M_0 = 93\ GeV$.

4. Idem with $\theta_{min} = 50°$, $\theta_{max} = 130°$ and $\sqrt{s} = 40\ GeV$.

FIGURE CAPTIONS

1. Cross section for the SALM monitor for $\sqrt{s} = 100\ GeV$ showing the importance of radiative corrections.

2. Idem as a function of DIVSDR for three different values of EPSX

3. Idem as a function of Δx ($\equiv XM1$) for three different values of EPSX

	$\sigma(nb)$	$\sigma_s(nb)$	$\sigma_h(nb)$
MILLOR	8048.7±11	4906.8±1.6	3141.9±10
BABAMC	8018.0±16	4904.8	3114.0±16

Table 1. *Comparison between MILLOR and BABAMC for* $\theta_{min} = 0.2^\circ,\ \theta_{max} = 0.8^\circ$ *and* $\sqrt{s} = M_0 = 93\ GeV$

	$\sigma(nb)$	$\sigma_s(nb)$	$\sigma_h(nb)$
MILLOR	49957±69	33563±9	16394±60
BABAMC	49959±96	33548	16411±96

Table 2. *Idem at* $\sqrt{s} = 40\ GeV$

	$\sigma(nb)$	$\sigma_s(nb)$	$\sigma_h(nb)$
MILLOR	0.303±0.002	0.0645±0.0005	0.239±0.001
BABAMC	0.313±0.002	0.0715	0.241±0.002

Table 3. *Idem for* $\theta_{min} = 30^\circ,\ \theta_{max} = 150^\circ$ *and* $\sqrt{s} = M_0 = 93\ GeV$

	$\sigma(nb)$	$\sigma_s(nb)$	$\sigma_h(nb)$
MILLOR	0.479±0.002	0.1568±0.0004	0.322±0.001
BABAMC	0.483±0.001	0.1538	0.329±0.001

Table 4. *Idem for* $\theta_{min} = 50^\circ,\ \theta_{max} = 130^\circ$ *and* $\sqrt{s} = 40\ GeV$

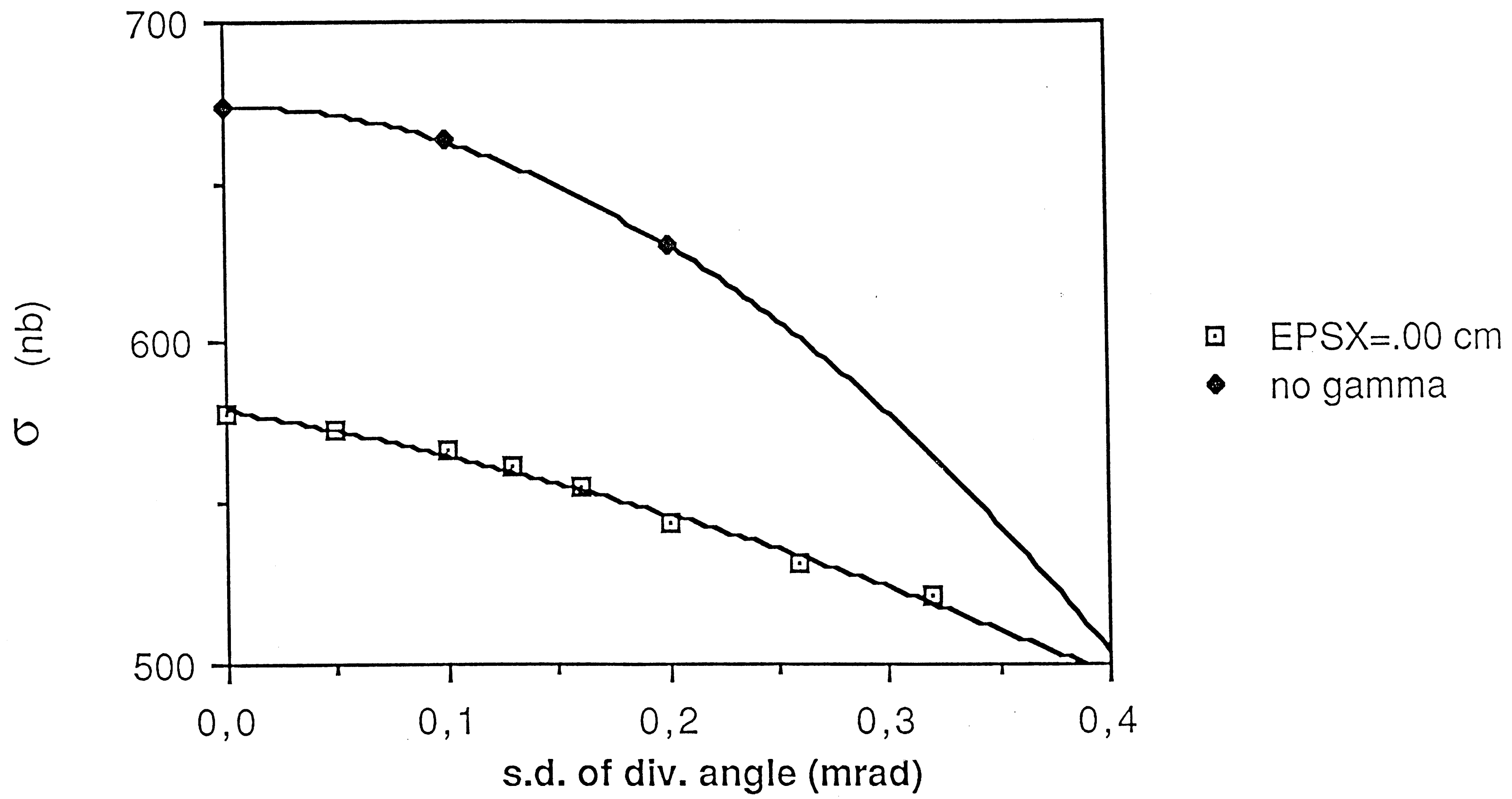

700
600
500
σ (nb)
0,0
0,1
0,2
0,3
0,4
s.d. of div. angle (mrad)
EPSX=.00 cm
no gamma

Fig. 1

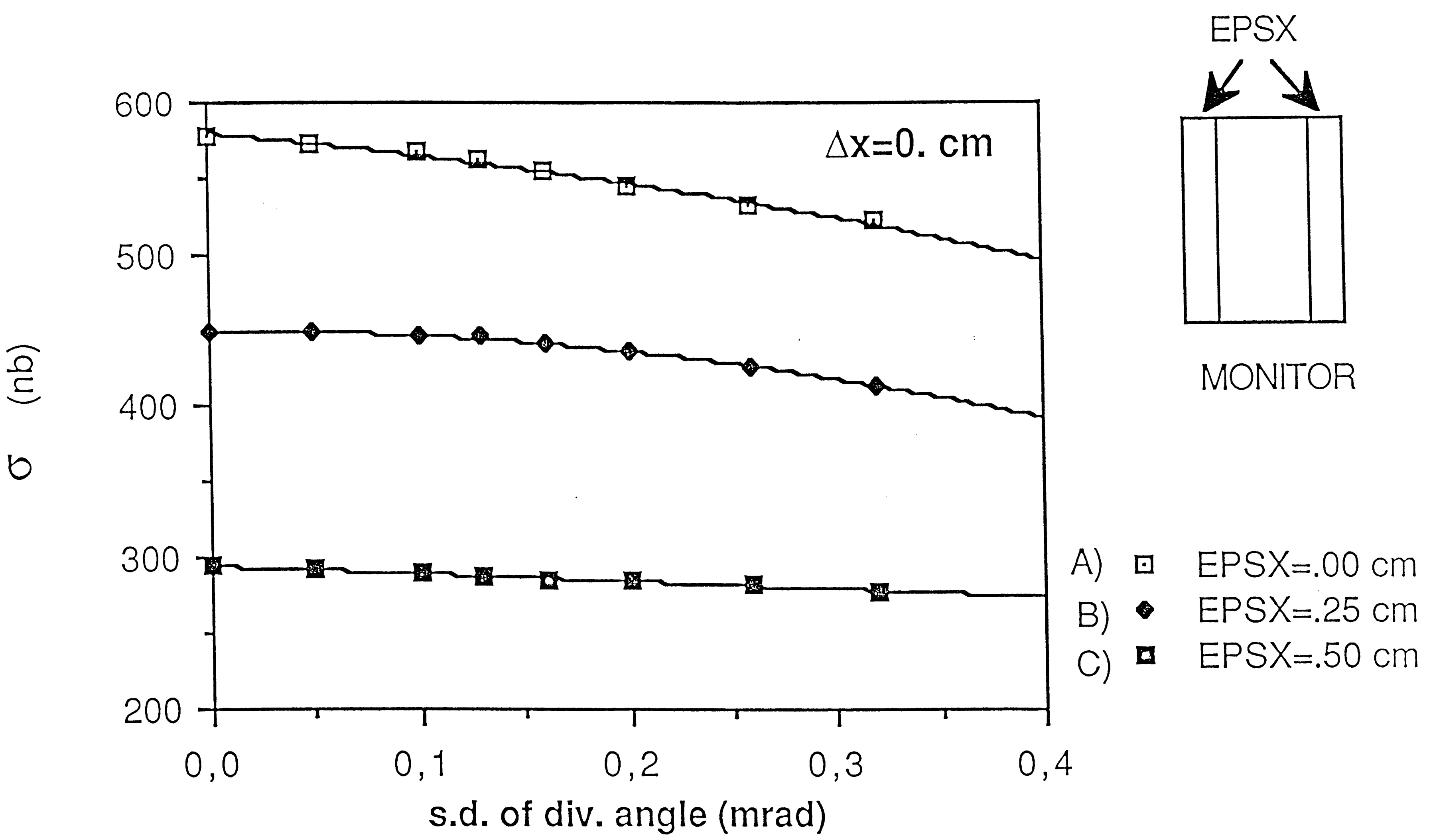
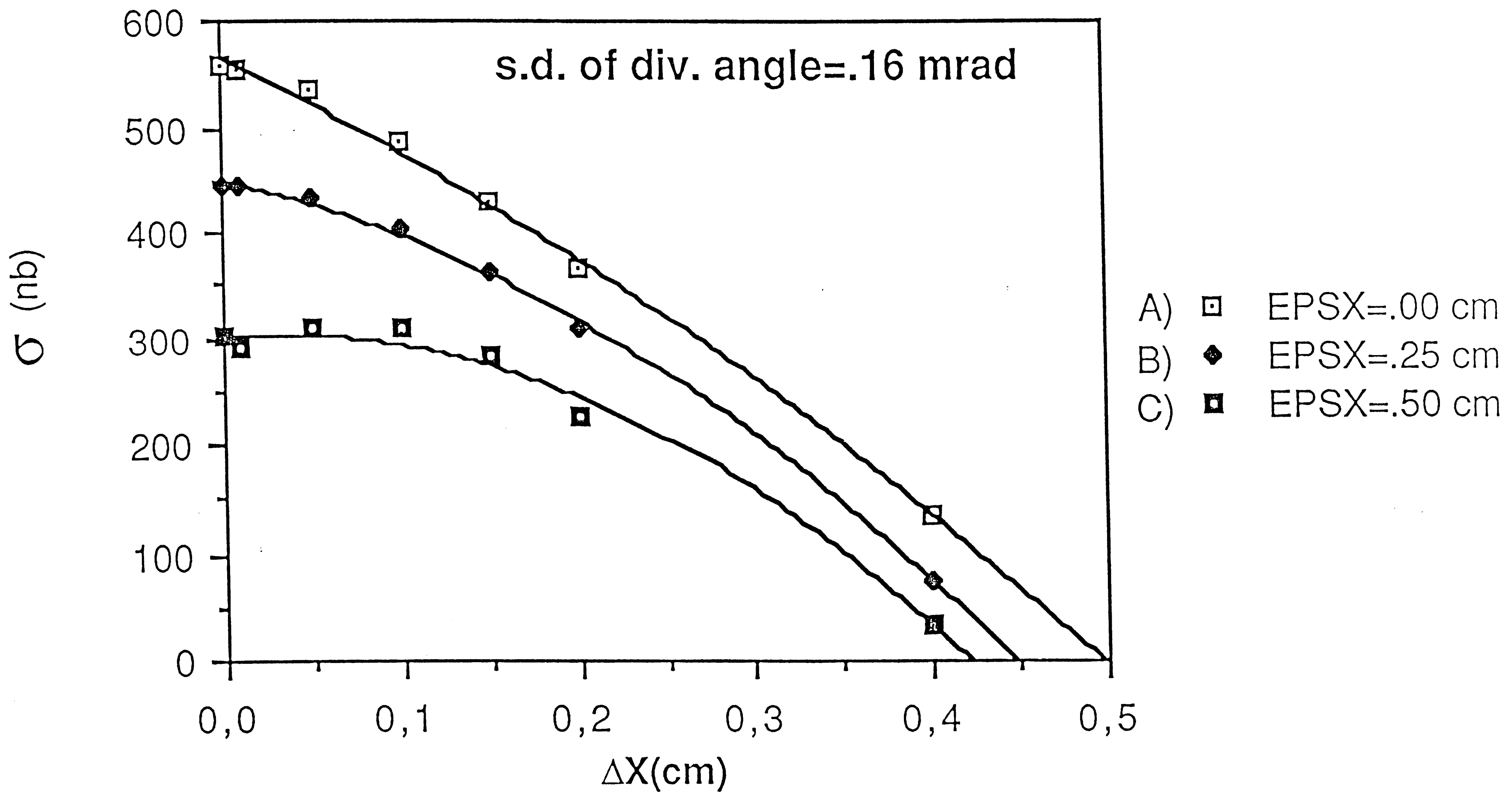

EPSX
MONITOR
600
500
400
300
200
σ (nb)
Δx=0. cm
0,0
0,1
0,2
0,3
0,4
s.d. of div. angle (mrad)
A) EPSX=.00 cm
B) EPSX=.25 cm
C) EPSX=.50 cm

Fig. 2



s.d. of div. angle=.16 mrad
600
500
400
300
200
100
0
σ (nb)
0,0
0,1
0,2
0,3
0,4
0,5
ΔX(cm)
A) EPSX=.00 cm
B) EPSX=.25 cm
C) EPSX=.50 cm

Fig. 3