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S. Dugeay, P. Henrard
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Sylvie Dugeay
Pierre Henrard

# HEAVY QUARK PRODUCTION WITH THE LUND GENERATOR COMPARISON WITH $e^+e^-$ DATA

After a brief introduction to fragmentation models and a summary of the status of the models used (section **A**), we shall concentrate on the heavy flavour sector (c and b quarks), as all the models give a fairly good description of the data in the light flavour sector. Section **B** presents the two main fragmentation functions used to describe the heavy quark fragmentation process. It is actually not straightforward to go from the experimental results to the theoretical expectations via Monte-Carlo predictions, due to the specific way each model performs the fragmentation and to the multiple definitions of the fragmentation variables used by the experiments. This has led in the past to confusion in comparing the experimental results and we will try to explain why. We then recall in section **C** some experimental results, mainly at PEP and PETRA energies. The main features of the Lund generator will be outlined in the following and last section **D**, with our recommendations for the adjustment of the value of some parameters, which seem to us not too "*folklorique*".

## A- FRAGMENTATION MODELS SEEN BY PEDESTRIANS

All the available data in high-energy $e^+e^-$ annihilation show that the way to go to hadronic final states can be described by two consecutive processes. First, the parton production $e^+e^- \to q\bar{q}(g)(g)$ which can be calculated in the framework of the electroweak theory plus perturbative QCD. Then, the hadronisation of the partons (see fig. 1).

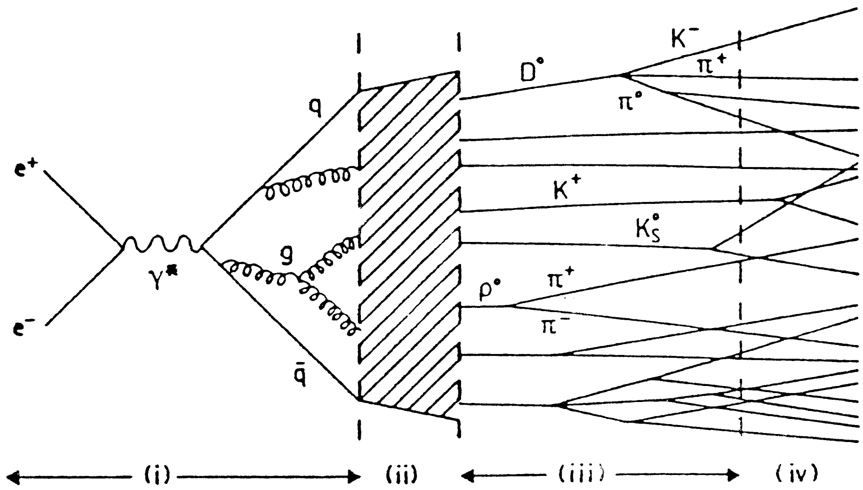


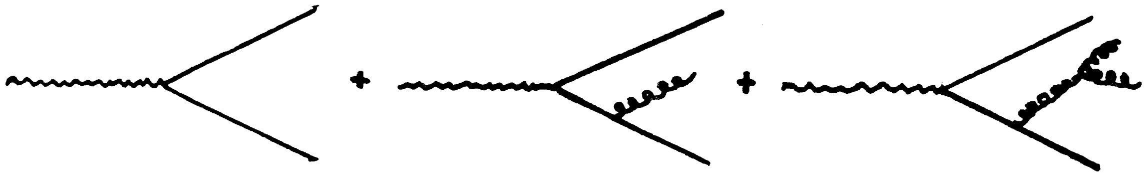

**fig. 1 :**
*Shematic illustration of an $e^+e^-$ annihilation event : (i) Perturbative phase. (ii) Fragmentation. (iii) particle decays and (iv) experimental observation.*

Fragmentation models are used to describe this transition from quarks and gluons to the observable final states of hadrons : due to the strong confinement of coloured objects, this is not calculable and only phenomenological approaches exist to describe it. The need of simulation programs to analyse the data has given birth to a certain number of Monte-Carlo generator, based on different models. Before going further in the review of one of the most popular, the LUND generator, let's try to give an idea of the way models treat the reaction $e^+e^- \rightarrow hadrons$.

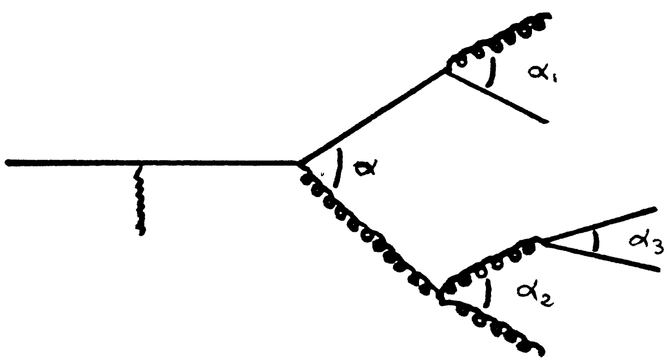
Restriting ourselves to the most commonly used models we have :

- For parton production, calculations according to

  - **the second order perturbative QCD matrix elements (MA).**



  - **leading-logarithmic parton-shower evolution (LLA).**




Parton shower evolution tries to implement all orders and is based on the iterative use of the splittings $q \rightarrow qg$, $g \rightarrow gg$, $g \rightarrow q\bar{q}$ as given by the leading-logarithmic QCD probabilities ( Altarelli-Parisi equations). The partons are allowed to radiate until their energy falls below a cut-off $Q_0$. Coherent effects (interferences of soft gluons) are taken into account by angular ordering ( $\alpha > \alpha_1, \ \alpha_2; \ \alpha_2 > \alpha_3$ ) leading to a suppression of soft gluon emission.

- For the subsequent hadronisation, fragmentation is performed according to

  - **the string scheme (SF).**

Coloured quarks and gluons are created at small distances. As two of them move apart, they stretch the colour line of force or string between them. They transfer their energy and momentum to the string and its breaking results into jets of colourless hadrons, with restricted transverse momentum with respect to the fragmentation direction, and a flavour dependent distribution of longitudinal momentum. The development of the longitudinal fragmentation process is usually parametrised by a scaling function $f(z)$,

where $$z = (E + p_{||})_{meson} / \ (E + p)_{quark} \ ,$$

is the fraction of energy and momentum component parallel to the fragmenting quark direction of the resulting meson. In addition, a transverse momentum component is given to the meson, parametrised by a gaussian distribution.

- **cluster fragmentation (CF).**

  This model is commonly used in combination with the LLA parton production. The neighbouring $q\overline{q}$ pairs along the colour flux lines are combined to form colourless clusters. These clusters are then decayed into one or two particles, according to the phase-space model. This approach does not require any parametrisation of fragmentation functions contrary to the previous model. Transverse momentum is given to hadrons by the cluster decays. In recent implementations of the CF approach, the case of high-mass clusters, which cannot be allowed to decay isotropically into 2 bodies is solved by using a string-like break-up of these clusters into lighter clusters. One problem however remains : one heavy cluster of given mass produces more particles than a primary gluon of the same mass. This means that an increase of the QCD scale results in more produced gluons but no increase in the number of final-state hadrons.

- **independent fragmentation (IF).**

  In the CM frame of the event, the fragmentation of the outgoing partons is considered completely independently of each others. An iterative scheme is assumed for the fragmentation of each jet, which is not Lorentz invariant. Flavour, momentum and energy conservation are restored at the end of the process and the procedure to achieve that has been implemented differentely in various models. This approach suffers mainly from two problems : it does not reproduced the so-called "string effect" which predicts that the region between the $q$ and $\overline{q}$ jets is less populated than the region between the $g$ and $q$ or $g$ and $\overline{q}$ jets; and the treatment of two colinear outgoing partons is not very well accounted, as the multiplicity from their fragmentation is larger than from a single parton with the same energy.

- Short discussion about some models.

| MODEL | MA | $LLA_{inc}$ | $LLA_{coh}$ | IF | SF | CF |
|---|---|---|---|---|---|---|
| Webber | | | ⊗ | | | ⊗ |
| Caltech II | | | ⊗ | | ⊗— | —⊗ |
| Lund 6.3 | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | |

The MARK II experiment has compared the results of their analysis of multihadronic events (all flavours mixed) at a centre of mass energy of 29 GeV with these models [1]. First they have tuned the parameters of the various Monte-Carlos in order to reproduce their data as well as possible. Then they have looked at some global shape observables (such as sphericity, thrust, aplanarity, jet invariant mass...) and inclusive particle distributions (such as $P_T$, rapidity, $x = 2P/\ E_{cm}$ ...) and have calculated the $\chi^2$ of the fit of these experimental distributions to each model. Their results are the following :

| model | sum of $\chi^2$ of all distributions |
|---|---|
| LUND LLA | 960 |
| LUND MA | 1230 |
| WEBBER | 2870 |
| CALTECH II | (for 450 bins) 6830 |

The Caltech II model (which deals for the fragmentation with a mixed scheme of string and cluster) seems to have problems at the parton shower level, and overestimates tails of distributions. The problems with the Webber model seem instead to arise from the hadronisation, leading mainly to an overestimation of the number of events with low multiplicities, low $P_T$, high thrust, while the Lund MA suffers from a lack of 3 or more parton events. Finally, the Lund LLA model gives the best description of the data, with discrepancies which are of the same order as the ones between Mark II and other experimental results.

Comparisons between these models at 93 $GeV$ have been also performed. The LUND MA model gives the predictions most different from the others, and the discrepancies between the various shower models are for most of the distributions, of the same order of magnitude at 93 $GeV$ as at 29 $GeV$.

## B- FRAGMENTATION FUNCTIONS FOR HEAVY QUARKS

Fragmentation functions have been introduced to account for the transformation from quarks to hadrons, which is not calculable, and defined, in the absence of QCD corrections as :

$$f_q^h(z) = \frac{1}{\sigma(\, e^+e^- \to q\bar{q}\,)} \frac{d\sigma(\, e^+e^- \to hX\,)}{dz} ,$$

which is scale invariant,

and where $z = (E + p||)_{meson} / \, (E + p)_{quark}$ .

For the light quarks, one of the first and most popular fragmentation functions is the "*Field - Feynman*" one,

$f^h(z) = 1 - a - 3a\,(1-z)^2$, for $u,\ d,\ s$.

(TASSO 1984 [**2**] : $a = 0.57 \pm 0.20$)

(LUND default : $a = 0.77$)

Kinematical arguments show that for a heavy quark, a larger part of its energy is carried by the daughter hadron than for a light one, leading to harder fragmentation functions peaked towards high values of $z$ for c- and b-hadrons. This has led to fragmentation functions dependent upon the mass of the created hadron.

The Lund group has proposed the following function :

- the **"Lund symmetric fragmentation function"** :

$$f(z) = \frac{1}{z}(1-z)^a\, e^{-bm_T{}^2/\,z},$$

as a result, for heavy quarks :

$$< z > \to 1 - \frac{1+a}{bm_T{}^2} .$$

where $m_T$ is the transverse mass of the produced hadron and the parameters $a$ and $b$ must be determined experimentally. But as we will see in the section **D** of this note, this function, for a heavy quark, is too peaked at high values of $z$ and does not reproduce very well the data. In fact, for analyses specific to heavy quark physics, the form earlier presented by Petersen *et al.* has been widely adopted :

- the so-called **"Petersen fragmentation function"** :

$$f(z) = \frac{1}{z(1 - \frac{1}{z} - \frac{\epsilon_Q}{(1-z)})^2} ,$$

where only one parameter $\epsilon_Q$, to be determined experimentally for each heavy quark, is needed.

The effect of the mass of the parent quark is put in $\epsilon_Q$ which is expected to be approximately, for *e.g.* a $Qq$ meson, :

$$\epsilon_Q \approx \frac{{M_q}^2}{{M_Q}^2} .$$

But even with the "quasi"-universal adoption of the Petersen fragmentation function for c- and b-quarks, there are some ambiguities in the interpretation and use of the fragmentation variable and the comparison of the results between the various experiments is not straightforward, due to its many different formulations. Actually, if the $z$ variable is a convenient one for theoretical studies, it is not at all the case from the experimental point of view : the $(E+p)_{quark}$ quantity is not directly accessible, as one must correct it for initial state radiation, and gluon (photon) bremsstrahlung in the final state (see fig. 2).

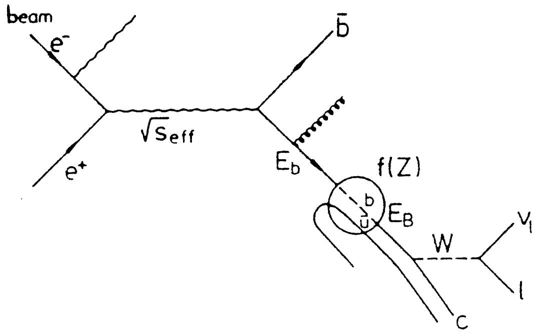



**fig. 2 :** $e^+e^- \rightarrow b\bar{b}$
*Effects of QED and QCD radiation*

No less than 5 definitions of the fragmentation variable have been used by PEP and PETRA experiments

$$z = \frac{(E + p||)_{meson}}{(E+p)_{quark}} ,$$

$$z_E = \frac{E_{had}}{E_q} ,$$

$$x_E = \frac{E_{had}}{E_{beam}} ,$$

$$x_p = \frac{P_{had}}{P_{max}}, \quad where \ P_{max} = \sqrt{{E_{beam}}^2 - {M_{had}}^2} ,$$

$$x_\gamma = \frac{2E_{had}}{\sqrt{s_{eff}}} .$$

Clearly, they are not the same as $z$, due to QED radiation in the initial and final state, and QCD radiation in the final state :

$$z \approx z_E > x_\gamma > x_E, \, x_p .$$

Figure 3, which reproduces the expected $x_E$ spectrum for $D^*$ mesons softened by QED and QCD effects, illustrates this effect [3]. Despite these significant differences between the variables, their distributions are often fitted directly to the Petersen function and the corresponding $\epsilon_Q$ calculated. Corrections therefore have to be applied to relate the measured distributions of $f(x)$ to the theoretical distribution of $f(z)$. And

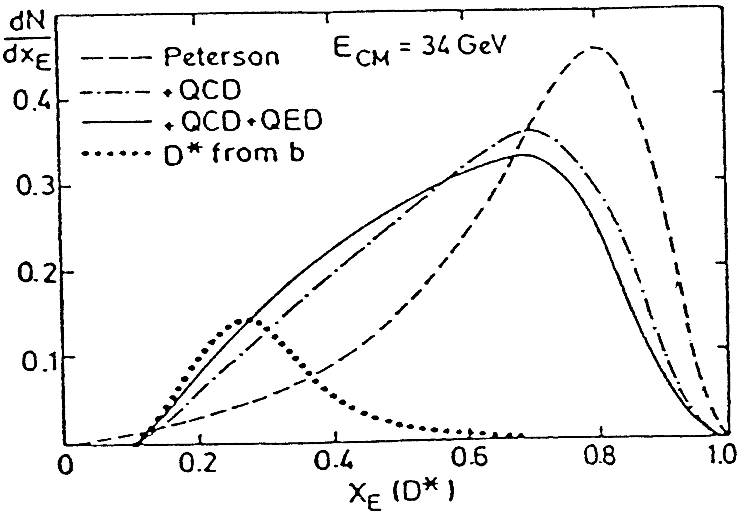



**fig. 3** *Softening of the energy-spectrum of $D^*$ due to QED initial state photon and QCD gluon radiation, calculated for the process $e^+e^- \to c\bar{c}$ at $\sqrt{s}$ = 34 GeV using the Lund model and a fragmentation function $f(z)$ of Petersen et al. Also indicated is the expected spectrum of $D^*$ from b-quark decays.*

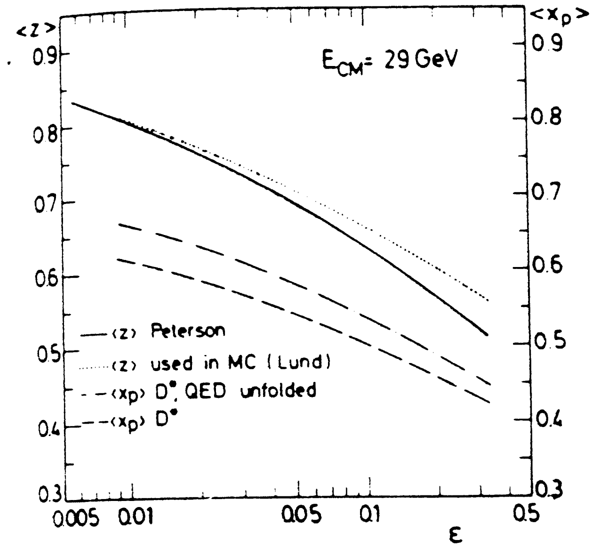



**fig. 4** *Relation between $< z_c >$ and $< x_p >$ of $D^*$ as a function of the parameter $\epsilon_c$ of the Petersen fragmentation function $f(z)$, calculated with the Lund model at $\sqrt{s}$ = 29 GeV including QED and QCD radiation effects.*

| $E_{cm}$ $(GeV)$ | Experiment | Observable | $< x >_{D^*}$ | $< z_c >$ |
|---|---|---|---|---|
| 10 | CLEO | $x_p > 0.35$ | 0.675 $\pm$ 0.015 | 0.73 $\pm$ 0.02 |
| 10 | ARGUS | $x_p > 0.10$ | 0.589 $\pm$ 0.017 | 0.695 $\pm$ 0.020 |
| 29 | DELCO | $x_p > 0.35$ | 0.585 $\pm$ 0.023 | 0.665 $\pm$ 0.035 |
| 29 | TPC | $x_E > 0.30$ | 0.585 $\pm$ 0.038 | 0.695 $\pm$ 0.050 |
| 29 | MARK II | $x_E > 0.40$ | 0.604 $\pm$ 0.040 | 0.66 $\pm$ 0.06 |
| 29 | HRS | $x_E > 0.20$ | 0.525 $\pm$ 0.017 | 0.700 $\pm$ 0.025 |
| 34 | JADE $D^{*+}$ | $x_E > 0.40$ | 0.635 $\pm$ 0.025 | 0.71 $\pm$ 0.03 |
| 34 | JADE $D^{*0}$ | $x_E > 0.50$ | 0.660 $\pm$ 0.025 | 0.68 $\pm$ 0.04 |
| 34 | TASSO | $x_E > 0.40$ | 0.647 $\pm$ 0.028 | 0.73 $\pm$ 0.04 |
| 10 | comb. 10 GeV | | | 0.713 $\pm$ 0.014 |
| 29 | comb. 29 GeV | | | 0.687 $\pm$ 0.018 |
| 34 | comb. 34 GeV | | | 0.707 $\pm$ 0.021 |
| | all comb. | | | 0.704 $\pm$ 0.010 |

**Table 1.**

*$< x >$ and reconstructed $< z_c >$ from $D^{*+}$ measurements. For each experiment the measured x-range and the resulting $< x >$ are presented. The values for $< z_c >$ are corrected for QED and QCD radiation, as well as for the expected background of b-quark decays and the acceptance in x.*

in fact, the Petersen function is well suited to parametrising the underlying $z$ spectrum but does not provide an adequate description of the measured $x_E, x_p$ ... spectra. And, the value of $\epsilon_Q$ founded by fitting directly the analytical formula to the data, with a fragmentation variable which is $x_E$ or $x_p$ is quite different from the one extracted via a MC simulation where QCD radiative corrections are taken into account.

For example, the CLEO collaboration find, from the $x_p$ spectrum of $D^*$ mesons :

- from a direct fit, where the fragmentation variable is $x_p$ :

$\epsilon_c \ = \ 0.156 \ \pm 0.015$ ( *with a* $\chi^2/DF$ *of* 40.0/10 ),

- from a fit via Lund MA, where the fragmentation vaiiable is $x_p$ unfolded for QCD radiation :

$\epsilon_c \ = \ 0.076 \ \pm 0.009$ ( *with a* $\chi^2/DF$ *of* 7.8/9 ), in better agreement with the theoretical expectation ($\epsilon_c \ \approx \ (m_{u,d}/\ m_c)^2$).

As a consequence of that, various functions have been developped [6], which give better results by direct fit to the data than the Petersen function, but are adapted to fragmentation variables which do not take into account radiative corrections. See for example ref [4,5] from the ARGUS collaboration, and [6] from the CLEO collaboration where they have fitted their $x_p$ spectrum obtained from $D^*$ mesons with the Petersen function and another one, the "*Kartvelishvili function*" :

$f(x) = x^a(1-x)$ ,

ARGUS has measured : $a = 1.5 \ \pm \ 0.2$,

and CLEO : $a = 1.40 \ \pm \ 0.18$ , ( *with a* $\chi^2/DF$ *of* 6.0/10 ),

and have found for this function a better agreement with the data.

But in a Monte-Carlo model, as the QCD corrrections are accounted before the fragmentation step, the fragmentation variable used is $z$ and it seems that it is better suited to use fragmentation functions which are specific to this variable. Furthermore, there is no such results, to our knowledge, available for the $b$-quark.

There are other complications due to the use of various fragmentation variables : whatever the fragmentation model used, the value of $z$ or [$z(prim.)$] of the primary hadron given by the primordial fragmentation function is not identical to the one, [$z(rec.)$], which is reconstructed from the final state hadron momenta and energies. In fact, except in the string model, energy-momentum is not conserved during the hadronisation process, but restored later by a rescaling of the particle momenta and energies, leading to an inequality between $z(prim.)$ and $z(rec.)$. In string models, the energy-momentum is conserved locally, at each step of the hadron generation process, but with constraints on the energy-momentum of the produced hadron. Furthermore, in the LUND model in particular, the energy and momentum available for the formation of the primary hadron is not interpreted as a fraction of the one of the primary quark but as a component of the energy-momentum of the remaining and unfragmented system. The consequence is that in some cases of hard gluon emission, the produced hadron can have *a larger energy-momentum than the primary quark*, and so a value of $z(rec.)$ greater than unity (!) (see fig. 7.a and 7.b). Anyway, the value of $z(rec.)$ is generally larger than $z(prim.)$, with the magnitude of the difference dependent upon the strength of the gluon coupling constant and the definition and value of a cut-off parameter defining the end of the parton splitting process [7] (see fig. 4).

## C- PEP and PETRA RESULTS

As $c\bar{c}$ and $b\bar{b}$ production is expected to be suppressed in the fragmentation chain due to their high masses, the energy and momentum spectrum of the hadron containing the primary heavy quark, or its weak decay products, gives direct access to information about fragmentation.

The ratio of the $c\bar{c}$ cross section over the total hadronic $q\bar{q}$ cross section is about 4/ 11 but only 1/ 11 for the $b\bar{b}$ case. Furthermore the mean multiplicity of particles from the decay of two charmed hadrons ($5.11 \pm 0.21 \pm 0.20$, considering only $D^*$ and $D$ production ) is roughly half of the one from two beauty hadrons ($10.99 \pm 0.06 \pm 0.29$ , with only $B_u$, $B_d$ production ). As a result of that, the fragmentation function of the c-quark is better known and has been measured with 2 methods : complete reconstruction of $D^*$ mesons or studies of inclusive leptons from semileptonic decay spectra. For the b, only the latter method has been performed. The lepton momentum, dependent upon the momentum of the parent hadron, contains information on the fragmentation of the heavy quark while its transverse momentum relative to the jet axis facilitates separation between the quark flavours. In a typical analysis, the $P, P_T$ spectra are simulated according to the Petersen fragmentation function and the semileptonic decay models of charmed and beauty hadrons, for various values of $\epsilon_Q$, and fitted to the corresponding spectra from the data. This method is less precise, because the contamination of the c (b) enriched sample by b (c) events and other kinds of background is not very well controlled.

In the two cases, if the mean value of the fragmentation function is more or less well measured, the determination of its precise shape suffers from a lack of statistics. As a matter of fact, only the measured mean value of the fragmentation variable is usually given.

**- c fragmentation (world averages) [7,8]**

- from $D^*$ reconstruction

$< z(rec.) >= 0.70 \pm 0.01 \pm 0.03$ , in the C.M. energy range from 10 GeV to 34 GeV.

The last (systematic) error of $\pm 0.03$ is added to take into account uncertainties on $\Lambda_{QCD}$, the QCD cut-off for gluon radiation and the Monte-Carlo technicalities for the correction procedure. Table 1 shows the results of the various experiments. The measured quantities are $x_E$ or $x_p$ and they are unfolded for QCD and QED effects in order to extract $z$. We can note the scaling behaviour of $z$, which is to be expected since QCD effects have been unfolded.

- from the lepton spectrum

$< z(rec.) >= 0.67 \pm 0.02 \pm 0.02$ ,

resulting in a value of $\epsilon_c = 0.06 \, ^{+0.02}_{-0.01} \, ^{+0.02}_{-0.01}$ .

We can note that this value of $< z(rec.) >$ is expected to be smaller than the previous one as it comes from an average of an unknown mixture of primary charmed mesons and baryons : a lepton coming from the decay chain $c \to D^* \to D \to l$ will give a softer $z$ than a lepton from $c \to D \to l$ ; it is the same for the decay $c \to F \to l$, as the $\epsilon$ parameter for the $F$ meson is expected (and measured $< z(rec.) >_{F^+} = 0.60 \pm 0.02$ **[5,6,8]** ) to be higher than for $D$ mesons, which then has a softer fragmentation (the $c$-quark of the $F$ meson is more "slown" by the $s$-quark, heavier than $u$ or $d$-quark).

**- b fragmentation (world averages) [7,8,9]**

- from the lepton spectrum

$< z(rec.) >= 0.83 \pm 0.01 \pm 0.02$ ,

resulting in a value of $\epsilon_b = 0.006 \, ^{+0.001}_{-0.001} \, ^{+0.002}_{-0.002}$ .

Note that the subsequent ratio

$$\frac{\epsilon_c}{\epsilon_b} = \frac{{M_b}^2}{{M_c}^2} = 10 \, ^{+4}_{-2} \, ^{+5}_{-4}$$

is in good agreement with an expectation of about 10.

Table 2 shows the results of the various experiments.

| Exp | $l$ | MC Model | $E_{cm}$ GeV | $\epsilon_b(x_E)$ | $\epsilon_b(z(rec.))$ | $< x_E >_b$ (%) | $< z(rec.) >_b$ (%) |
|---|---|---|---|---|---|---|---|
| TASSO | $\mu$ | Lund | 34.5 | | $.0025\,^{-.0025}_{+.029}\,^{-.0013}_{+.011}$ | | $87\,^{+13}_{-15}\,^{+2}_{-9}$ |
| TASSO | $e$ | FF + Ali | 34.6 | | $.005\,^{-.005}_{+.022}\,^{-.005}_{+.020}$ | | $84\,^{+15}_{-10}\,^{+15}_{-11}$ |
| JADE | $\mu$ | Lund | 34.6 | | $.0035\,^{-.002}_{+.004}\,^{-.0025}_{+.005}$ | | $86 \pm 4 \pm 5$ |
| MAC | $\mu$ | FF + Ali | 29 | | $.008\,^{-.008}_{+.037}$ | | $80 \pm 10$ |
| DELCO | $e$ | Lund | 29 | $.033\,^{-.017}_{+.032}$ | | $72 \pm 5$ | $83 \pm 5$ ( $\pm 3$) |
| TPC | $\mu$ | Lund | 29 | $.011\,^{-.007}_{+.015}\,^{-.007}_{+.011}$ | | $80 \pm 5 \pm 5$ | $93 \pm 5 \pm 5$ ( $\pm 3$) |
| TPC | $e$ | Lund | 29 | $.033\,^{-.019}_{+.037}\,^{-.012}_{+.019}$ | | $74 \pm 5 \pm 3$ | $83 \pm 5 \pm 3$ ( $\pm 3$) |
| MARK II | $\mu$ | FF + Ali | 29 | | $.042\,^{-.041}_{+.218}\,^{-.035}_{+.120}$ | | $73 \pm 15 \pm 10$ |
| MARK II | $e$ | FF + Ali | 29 | | $.015\,^{-.011}_{+.022}\,^{-.011}_{+.022}$ | | $79 \pm 6 \pm 6$ |

**Table 2.**

*A compilation of the latest results on bottom quark fragmentation from inclusive lepton studies. The additional systematic error given in brackets in the final column refers to the uncertainty in extracting* $< z(rec.) >$ *from* $< x_E >$.

### - c and b fragmentation from multiplicity measurements [7,9]

There is another way to obtain information about fragmentation, through the mean charged multiplicity measurement $< n >$. By distinguishing between the contributions to $< n >$ from the decay of the primary hadrons and from the remainder of the fragmentation process ($< n_{nl} >$ the non-leading multiplicity), an average non-leading energy $< E_{nl} >$ can be extracted by using the measured variation of $< n >$ as a function of $E_{cm}$. This gives access to $< x_{nl} >= 1- < E_{nl} > / E_{cm}$. The results are reported in table 3 , where the leading multiplicity for c and b hadrons used is the one measured by other experiments. The $< z(rec.) >_c$ is around 0.60 and $< z(rec.) >_b$ around 0.94, in apparent disagreement with the determinations reported above. The multiplicity results however suffer from much larger systematic uncertainties : the flavour of the events is tagged by a lepton of high $P_T$ as in the lepton spectrum method. A fit to the $P$, $P_T$ is performed in terms of semileptonic branching ratios and fragmentation functions of heavy quarks to extract the b (resp. c ) contribution to the sample. But it has been proven (see for example [9]) that in this kind of procedure, the results on $BR_{SL}$ are highly affected by systematic errors in the fragmentation process and QCD calculations. This leads to an overestimation of the $BR_{SL}$ of B mesons ( some 20% higher than other measurements ) and then of the b content of the sample ( the opposite happens for the c sample ). Consequently, the correction for the background is underestimated, and the same is true for the multiplicity of b jets. As a result, the fragmentation variable is found too large in the b case, too small in the c case.

| Exp. | Meth. | $E_{cm}$ GeV | $< n_{nl} >_c$ | $< E_{nl} >_c$ GeV | $< x_E >_c$ (%) | $< z(rec.) >_c$ (%) |
|---|---|---|---|---|---|---|
| TASSO | $D^*$ | 34.4 | 9.9 $\pm 1.0$ $\pm 0.6$ | $20\ ^{-3}_{+4}\ ^{-2}_{+3}$ | $42\ ^{+9}_{-12}\ ^{+6}_{-9}$ | $54\ ^{+9}_{-12}\ ^{+6}_{-9}$ ($\pm3$) |
| HRS | $D^*$ | 27.3 | 8.1 $\pm 0.4$ $\pm 0.5$ | $14.4\ ^{-1.6}_{+1.0}\ ^{-2.0}_{+1.3}$ | $47\ ^{+6}_{-3}\ ^{+8}_{-5}$ | $59\ ^{+6}_{-3}\ ^{+8}_{-5}$ ($\pm3$) |
| MARK II | $l$ | 29.0 | 8.1 $\pm 0.5$ $\pm 0.9$ | $14.4\ ^{-2.0}_{+1.3}\ ^{-3.2}_{+3.0}$ | $50\ ^{+7}_{-4}\ ^{+11}_{-10}$ | $64\ ^{+7}_{-4}\ ^{+10}_{-11}$ ($\pm3$) |
| TPC | $l$ | 29.0 | 8.4 $\pm 0.9$ $\pm 0.9$ | $15.0\ ^{-3.0}_{+3.2}\ ^{-3.0}_{+3.2}$ | $48\ ^{+11}_{-11}\ ^{+11}_{-11}$ | $61\ ^{+11}_{-11}\ ^{+11}_{-11}$ ($\pm3$) |

(a)

| Exp. | Meth. | $E_{cm}$ GeV | $< n_{nl} >_b$ | $< E_{nl} >_b$ GeV | $< x_E >_b$ (%) | $< z(rec.) >_b$ (%) |
|---|---|---|---|---|---|---|
| MARK II | $l$ | 29. | 5.1 $\pm 0.5$ $\pm 1.0$ | $5.7\ ^{-0.8}_{+0.9}\ ^{-1.8}_{+1.8}$ | $80\ ^{+3}_{-3}\ ^{+6}_{-6}$ | $93\ ^{+3}_{-3}\ ^{+6}_{-6}$ ($\pm3$) |
| TPC | $l$ | 29. | 5.7 $\pm 1.0$ $\pm 1.0$ | $6.7\ ^{-1.7}_{+2.2}\ ^{-1.7}_{+2.2}$ | $77\ ^{+6}_{-8}\ ^{+6}_{-8}$ | $90\ ^{+6}_{-8}\ ^{+6}_{-8}$ ($\pm3$) |
| DELCO | $l$ | 29. | 4.2 $\pm 0.9$ $\pm 1.0$ | $4.0\ ^{-1.7}_{+1.7}\ ^{-1.9}_{+1.8}$ | $86\ ^{+6}_{-6}\ ^{+7}_{-6}$ | $100\ ^{+6}_{-6}\ ^{+7}_{-6}$ ($\pm3$) |

(b)

**Table 3.**

*Results on (a) charm and (b) bottom fragmentation from charged multiplicity measurements. The systematic error given in brackets in the final column refers to the uncertainty in extracting $< z(rec.) >$ from $< x_E >$.*

## D- THE LUND GENERATOR (version JETSET 6.3)

The use of this generator allows a certain number of options :

- **MA** and **LLA** approaches for the parton production step,
- **IF** (independent fragmentation, which is now obsolete so we can forget about it "*on the spot*") and **SF** for the hadronisation.
- Further, many fragmentation functions are available, such as Lund symmetric, Petersen, Field-Feynman, ...

The more important parameters to play with are the following :

1- the cut-off used to resolve between partons,
2- the $QCD$ scale, for the *parton generation.*
3- the choice of the fragmentation function and its parameters,
4- the choice of the width of the gaussian which gives a transverse momentum to the hadrons with respect to the primary quark direction, for the *subsequent hadronisation process.*

### a) Matrix Element approach

If $O(\alpha_s^2)$ seems to be adequate at low $\sqrt{s}$, there is now accumulating evidence that this is not enough to provide a fully satisfactory description of present data at *PEP-PETRA* energies and even larger discrepancies can be expected at *TRISTAN-SLC-LEP* energies : data demand higher parton multiplicities than this method can provide and second order matrix element calculations are probably not sufficient.

It suffers also from another problem which is due to the specific technique used in Lund : the control of the production of 2-3-4 parton final states is determined by the value of $\Lambda_{QCD}$ and a cut-off, $y_{min} = m_{ij}^2/\ s$ which is the scaled minimum invariant mass between partons $i$ and $j$ for the two partons not to merge into one. Actually, comparison with data shows that the $y_{min}$ value should be made as small as possible and its best value is more or less the kinematic limit in the generator. In order to use the same $y_{min}$ value at different centre of mass energies, as it is done by all the experiments, one must then compensate for the fact that $y_{min}$ does not scale with $s$. In other words, this means that the fragmentation scheme has to be $Q^2$-dependent, and this is not the case in all fragmentation models. So one is forced to play with the value(s) of the parameter(s) of the fragmentation function in order to reproduce the data. For example, using the Petersen form for heavy quark generation, the optimum values of the $\epsilon_Q$ parameter in Lund are [**4**] :

- at $E_{cm} = 29GeV$ : $\epsilon_c(z(prim.)) = 0.10$, $\epsilon_b(z(prim.)) = 0.015$,
- at $E_{cm} = 35GeV$ : $\epsilon_c(z(prim.)) = 0.09$, $\epsilon_b(z(prim.)) = 0.012$,
- at $E_{cm} = 45GeV$ : $\epsilon_c(z(prim.)) = 0.08$, $\epsilon_b(z(prim.)) = 0.010$,
- at $E_{cm} = M(Z^0)$ : $\epsilon_c(z(prim.)) =$ ? , $\epsilon_b(z(prim.)) =$ ? .

They are not directly comparable with the ones measured, which are $E_{cm}$ independent, but ensure the compensation for the $y_{min}$ cut-off. Then, $< z(rec.) >$ remains constant with respect to the centre of mass energy while $< z(prim.) >$ increases slightly with it, as $\epsilon_Q$ decreases.

Table 4 and fig. 5.a (for $b\bar{b}$ events) illustrate the disastrous effect of this fixed cut-off ($y_{min} = 0.015$) : at 30 GeV, the $M_{ij}$ invariant mass above which two partons are resolved is 3.5 GeV, while it goes to 11.5 GeV at 94 GeV ( we recall that this cut-off is 1 Gev in the LLA model, for all $\sqrt{s}$ values). This leads to the contradictory situation that the mean number of radiated gluons falls from 0.9 to 0.8 ( grows 1.7 to 5.7 for LLA), while $\alpha_s$ decreases by about 20 % in the meantime. Then, $< z(rec.) >$ also decreases, due to the specific definition of $z$ in Lund (cf. **B**), as we can see in tables 5. The first part ( with the Lund fragmentation function ) of these tables shows the decrease of $< z(rec.) >$ for the MA option from 30 to 94 GeV, while it stays roughly constant for the LLA option. In the second part of these tables, we use the Petersen fragmentation function to try to compensate for that : for the $b\bar{b}$ events, with the recommended value of 0.015 at 30 GeV for $\epsilon_b$, the $< z(rec.) >$ goes from 0.84 to 0.79 at 94 GeV. A value for $\epsilon_b$ of 0.07 restores the initial value. Anyway, the best parameters for this MA option to fit the existing data, as given by the experiments, are [**1**] :

1- $\Lambda_{QCD} = 0.5\ GeV$,
2- $y_{min} = 0.015$ (fixed),

⋄

| MODEL | $E_{cm}$ $(GeV)$ | $< n_{nl} >$ | $< n_{tot} >$ | $< E_{nl} >$ $(GeV)$ | $< x_E >$ | $< z(rec.) >$ |
|---|---|---|---|---|---|---|
| LLA | 29 | 2.8 | 13.8 | 3.5 | 0.89 | 0.97 |
| LLA | 35 | 3.8 | 14.8 | 4.9 | 0.86 | 0.97 |
| LLA | 94 | 10.8 | 21.8 | 22.7 | 0.76 | 0.98 |
| MA | 29 | 4.1 | 15.1 | 4.8 | 0.84 | 0.95 |
| MA | 35 | 4.7 | 15.7 | 6.2 | 0.83 | 0.94 |
| MA | 94 | 8.8 | 19.8 | 18.1 | 0.81 | 0.93 |
| MA + Peters. | | | | | | |
| $\epsilon_b$=0.015 | 29 | 5.8 | 16.8 | 7.3 | 0.75 | 0.84 |
| $\epsilon_b$=0.015 | 35 | 6.8 | 17.8 | 9.7 | 0.72 | 0.83 |
| $\epsilon_b$=0.015 | 45 | 8.2 | 19.2 | 14.8 | 0.71 | 0.82 |
| $\epsilon_b$=0.015 | 94 | 11.8 | 22.8 | 28.5 | 0.69 | 0.79 |
| $\epsilon_b$=0.012 | 35 | 6.7 | 17.7 | 9.1 | 0.74 | 0.85 |
| $\epsilon_b$=0.010 | 45 | 7.8 | 18.8 | 12.0 | 0.74 | 0.84 |
| $\epsilon_b$=0.007 | 94 | 11.1 | 22.1 | 26.1 | 0.72 | 0.83 |

**Table 4.** $e^+e^- \rightarrow b\bar{b}$

*Dependence of $< z(rec.) >$ on $E_{cm}$ for the MA option. Play with the parameter $\epsilon_b$ of the Petersen fragmentation function to keep it constant.*

| MODEL | $E_{cm}$ $(GeV)$ | $< n >_{gluon}$ | $< E/gluon >$ $(GeV)$ | $< E >_g$ $(GeV)$ | $< E >_q$ $(GeV)$ |
|---|---|---|---|---|---|
| LLA | 30 | 1.7 | 1.5 | 2.7 | 27.3 |
| MA | 30 | 0.9 | 3.8 | 3.7 | 25.3 |
| LLA | 94 | 5.7 | 3.1 | 17.4 | 76.6 |
| MA | 94 | 0.8 | 14.2 | 11.4 | 82.6 |

**Table 5.** $e^+e^- \rightarrow b\bar{b}$

*Comparison between the LLA and the MA options.*

3- the Lund fragmentation-function parameters : $a = 0.9$, $b = 0.7$,
4- the $\sigma_q$ parameter of the gaussian $= 0.265\ GeV/c$.

More practically, this means to change the default value (D) of the following parameters :

| | |
|---|---|
| - MSTE(1) = 2 | to get the MA treatment (D=3, coherent parton shower). |
| - PARE(8) = 0.015 | is the $y_{min}$ cut-off (D=0.020). |
| - PAR(31) = 0.9 , PAR(32) = 0.7 | are the $a$ and $b$ parameters for the Lund fragmentation function, well suited for the light $u$, $d$, $s$.-quarks. ( D=0.5-0.9, optimised for the LLA treatment). |
| - PAR(12) = 0.375 | is the $\sqrt{2}\sigma_q$ parameter (D=0.350 GeV). |

**b) Coherent parton shower approach**

This is the default option in LUND, and of all the models the one which reproduce best the data. ( note however that since this option in its present version is available since 1986 only, most of the experiments, except Mark II to our knowledge, which have used Lund to compare predictions and data to extract their results have employed the MA approach ). The problem of the scaled cut-off of the MA scheme is no more present here, as a fixed cut-off $Q_0$ is used to stop the radiation of the partons, with a value of about 1 $GeV$.

Table 6 reports the mean multiplicity, energy, ... of quarks and gluons for the different types of quarks $d$, $s$, $c$, $b$ , and figures 5.a and 5.b, 6.a and 6.b ( for b events and d, s, c events respectively ) show the gluon multiplicity and quark energy, this for two different $E_{cm}$ energies, 30 and 94 GeV.

| QUARK TYPE | $E_{cm}$ $(GeV)$ | $< n >_{gluon}$ | $< E/gluon >$ $(GeV)$ | $< E >_g$ $(GeV)$ | $< E >_q$ $(GeV)$ |
|---|---|---|---|---|---|
| d | 30 | 5.7 | 2.3 | 13.1 | 16.9 |
| s | 30 | 5.6 | 2.2 | 12.4 | 17.6 |
| c | 30 | 4.2 | 1.8 | 7.7 | 22.3 |
| b | 30 | 1.7 | 1.5 | 2.7 | 27.3 |
| | | | | | |
| d | 94 | 9.6 | 4.6 | 46.2 | 47.8 |
| s | 94 | 9.5 | 4.4 | 42.9 | 51.1 |
| c | 94 | 8.3 | 3.7 | 31.9 | 62.1 |
| b | 94 | 5.7 | 3.1 | 17.4 | 76.6 |

**Table 6.**
*LLA option. Comparison between the different type of quark.*

The best parameters for this LLA option to fit the existing data as given by MARK II are [1] :

1- $\Lambda_{QCD}$ $= 0.4\ GeV$ (cannot be directly correlated to the usual one)
2- the cut-off $Q_0$ $= 1.0\ GeV$ (cut-off to end parton showering)
3- the Lund fragmentation-function parameters $a = 0.45$, $b = 0.9$
4- the $\sigma_q$ parameter of the gaussian $= 0.230\ GeV/c$

| FRAGM. FC. | $E_{cm}$ $(GeV)$ | $< n_{nl} >$ | $< n_{tot} >$ | $< E_{nl} >$ $(GeV)$ | $< x_E >$ | $< z(rec.) >$ |
|---|---|---|---|---|---|---|
| Lund fc. | 29 | 2.8 | 13.8 | 3.5 | 0.89 | 0.97 |
| Peters. fc. | 29 | 5.1 | 16.1 | 6.7 | 0.77 | 0.84 |

(a)

| FRAGM. FC. | $E_{cm}$ $(GeV)$ | $< n_{nl} >$ | $< n_{tot} >$ | $< E_{nl} >$ $(GeV)$ | $< x_E >$ | $< z(rec.) >$ |
|---|---|---|---|---|---|---|
| Lund fc. | 29 | 7.3 | 12.8 | 10.5 | 0.64 | 0.87 |
| Peters. fc. | 29 | 8.3 | 13.8 | 13.3 | 0.54 | 0.75 |

(b)

**Table 7.** $e^+e^- \rightarrow b\bar{b}$ **(a)**, $c\bar{c}$ **(b)**
*With the LLA option. Comparison between the Lund and the Petersen fragmentation function* $(\epsilon_b = 0.015,\ \epsilon_c = 0.10)$.

| $\epsilon_b(z(prim.) =$ | $E_{cm}$ $(GeV)$ | $< n_{nl} >$ | $< n_{tot} >$ | $< E_{nl} >$ $(GeV)$ | $< x_E >$ | $< z(rec.) >$ |
|---|---|---|---|---|---|---|
| 0.015 | 29 | 5.1 | 16.1 | 6.7 | 0.77 | 0.84 |
| 0.015 | 35 | 6.3 | 17.3 | 8.9 | 0.75 | 0.85 |
| 0.015 | 94 | 13.1 | 24.1 | 31.7 | 0.66 | 0.84 |

(a)

| $\epsilon_c(z(prim.) =$ | $E_{cm}$ $(GeV)$ | $< n_{nl} >$ | $< n_{tot} >$ | $< E_{nl} >$ $(GeV)$ | $< x_E >$ | $< z(rec.) >$ |
|---|---|---|---|---|---|---|
| 0.10 | 29 | 8.3 | 13.8 | 13.3 | 0.54 | 0.75 |
| 0.10 | 94 | 16.5 | 22.0 | 49.7 | 0.46 | 0.75 |
| 0.15 | 29 | 8.5 | 14.0 | 13.9 | 0.52 | 0.72 |
| 0.20 | 29 | 8.7 | 14.2 | 14.4 | 0.50 | 0.70 |
| 0.23 | 29 | 8.7 | 14.2 | 14.6 | 0.49 | 0.69 |
| 0.23 | 94 | 16.9 | 22.4 | 54.3 | 0.42 | 0.69 |

(b)

**Table 8.** $e^+e^- \rightarrow b\bar{b}\ (a),\ c\bar{c}\ (b)$
*LLA option with Petersen fragmentation function. Choice of* $\epsilon_Q$ *to reproduce the measured* $< z(rec.) >$.

Here, you have to change the following default parameters :

| | |
|---|---|
| - PAR(31) = 0.45 | is the $a$ parameter for the Lund fragmentation function. |
| - PAR(12) = 0.325 | is $\sqrt{2}\sigma_q$ in $GeV$. |

**c) Petersen fragmentation function for heavy quarks**

The Lund function seems not well suited for c and b quarks and the Petersen seems today the better and is currently used. There is the possibility in Lund to use a mixed scheme for fragmentation functions:

| | |
|---|---|
| - MST(4) = 3 | to get a mixed scheme for the fragmentation functions : the Lund one for $u$, $d$, $s$ quarks and the Petersen one for $c$, $b$. (D=1, Lund fragm. fc.) |
| - PAR(44) | |
| - PAR(45) : | are the opposites of the $\epsilon_c$ and $\epsilon_b$ parameters for the Petersen fragmentation fc. |

The differences between the Lund and Petersen fragmentation functions are shown in figures 7.a, 7.b for the $z$ variable (note however that $\approx$ 13 % of the events have a $z$ greater than 1 in both cases ); 8.a, 8.b for the charged multiplicity from fragmentation; 9.a and 9.b for the energy from fragmentation and the momentum of the primary hadron ( don't forget that fragmentation means always particles from fragmentation but also from QCD radiation), at a centre of mass energy of 29 GeV, and for $\epsilon_b$ =0.015 and $\epsilon_c$ =0.1. Tables 7.a and 7.b compare again the two functions, for some quantities. The $< x_E >$ and $< z(rec.) >$ decrease by $\approx$ 13 % from the Lund function to the Petersen one, while the $< n_{frag.\ charg.} >$ and $< E_{non-leading} >$ grow by $\approx$ 14 % for $c\bar{c}$ events and 80 % for $b\bar{b}$ events for which the Lund function is definitely too hard.

Finally the tables 8.a, 8.b show our preferred choice for the $\epsilon_Q$ parameters. With $\epsilon_b$ = 0.015 and $\epsilon_c$ = 0.23, the $< z(rec.) >$, $< x_E >$, $< n_{frag.\ charg.} >$ and $< E_{non-leading} >$ found agree quite well with the data (tables 1, 2, 3).

So, finally we choose :

| |
|---|
| - PAR(44) = - 0.230 |
| - PAR(45) = - 0.015 |

**d) Remark**

The value of $< z(rec.) >$ depends upon the relative rates of vector (V), pseudo-scalar (P) mesons and baryons.

For light quarks, TASSO (1984) has measured [2] :

$$\frac{P}{P + V} = 0.56 \pm 0.15 .$$

The default in Lund for the probability of having a spin 1 hadron is 0.5 ( PAR(8) ).

For charmed mesons, CLEO has measured the ratio :

$$\frac{D_{dir.}}{D_{dir.} + D^*} = 0.28 \pm 0.15 .$$

And the default in Lund for the probability of having a spin 1 is 0.75 ( PAR(10) ), for heavy charmed and bottom hadrons.

Baryonic production is suppressed by a factor of 0.1 in Lund ( PAR(1) = $\frac{P(qq)}{P(q)}$ suppression of diquark compared to quark production ).

These default values seem reasonable, within the limits of experimental results.

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**4.** " Production and Decay of the Charged $D^*$ Meson in $e^+e^-$ Annihilation at 10 GeV Centre of Mass Energy ". ARGUS Collaboration.
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**6.** " Charm Production in non Resonant $e^+e^-$ Annihilation at $\sqrt{s} = 10.55\ GeV$ ".
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**7.** " Upon the Determination of Heavy Quark Fragmentation Functions in $e^+e^-$ Annihilation ".
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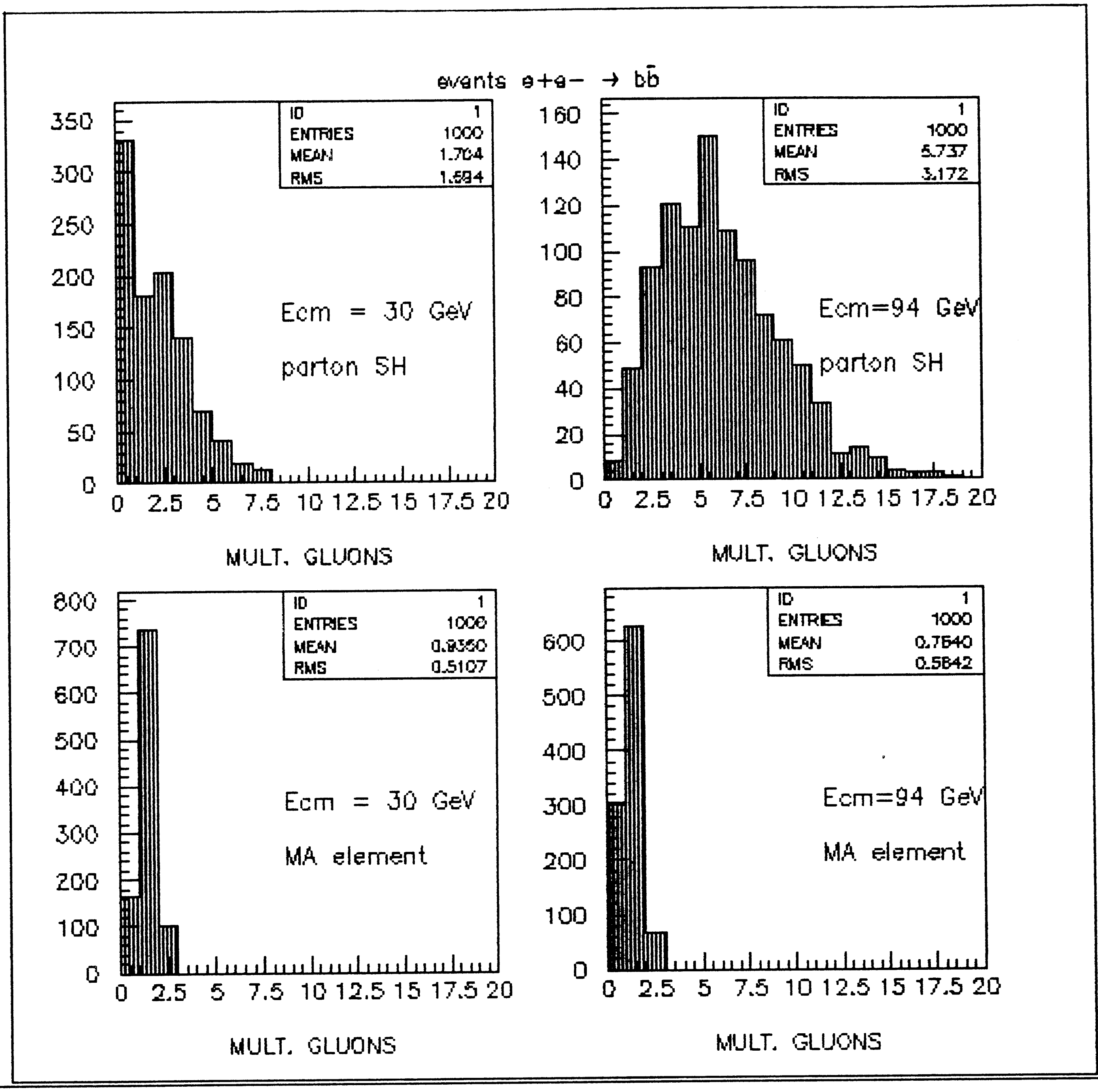


**figure 5.a :**
*Comparison between the MA and LLA option.*
*events* $e^+e^- \to b\bar{b}$
*Gluon multiplicity at* $\sqrt{s}$ = 30 *and* 94 *GeV*.

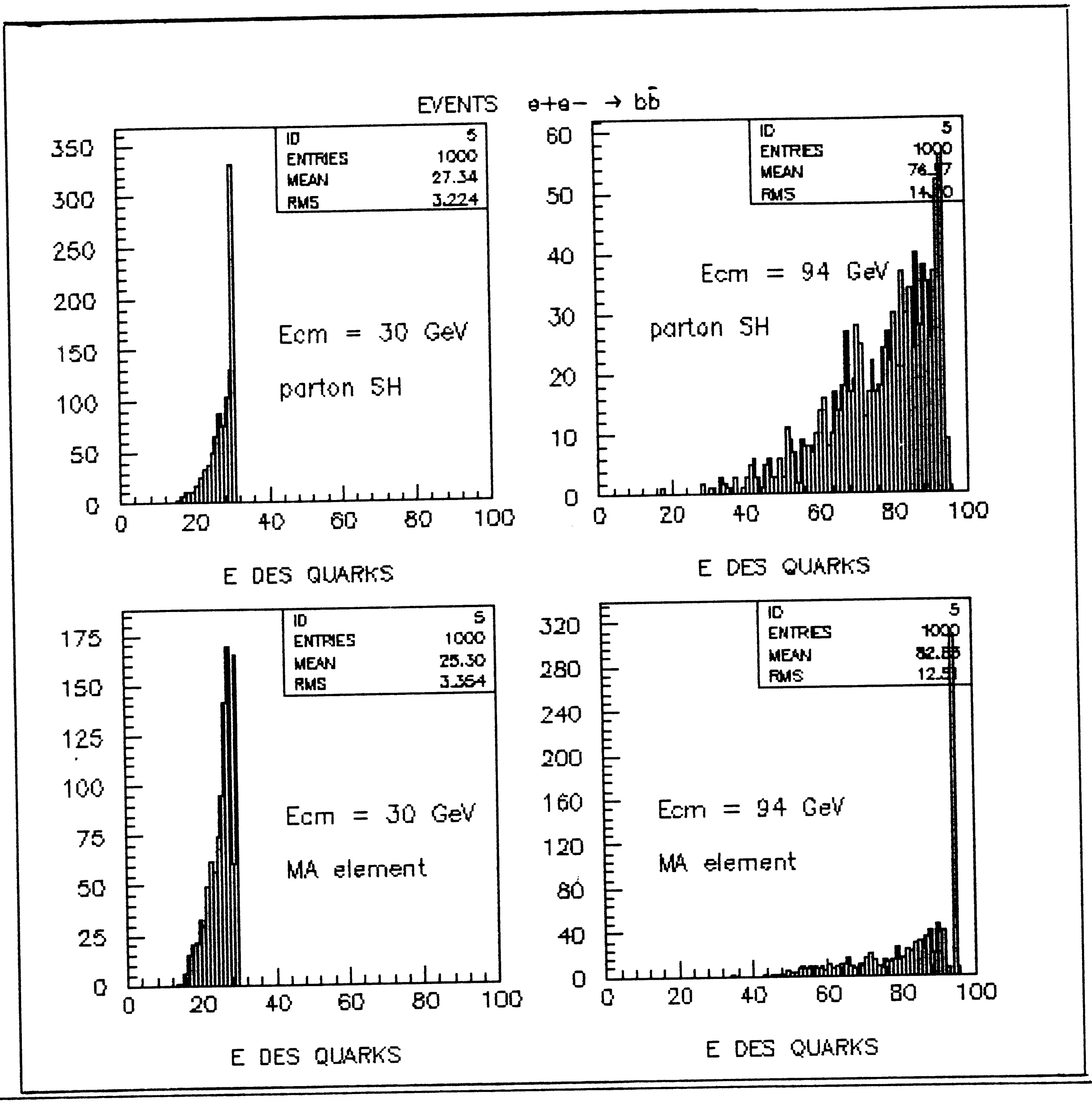


**figure 5.b :**
*Comparison between the MA and LLA option.*
*events* $e^+e^- \to b\bar{b}$
*Total energy from quarks in the event, at* $\sqrt{s} = 30$ *and* 94 *GeV*.

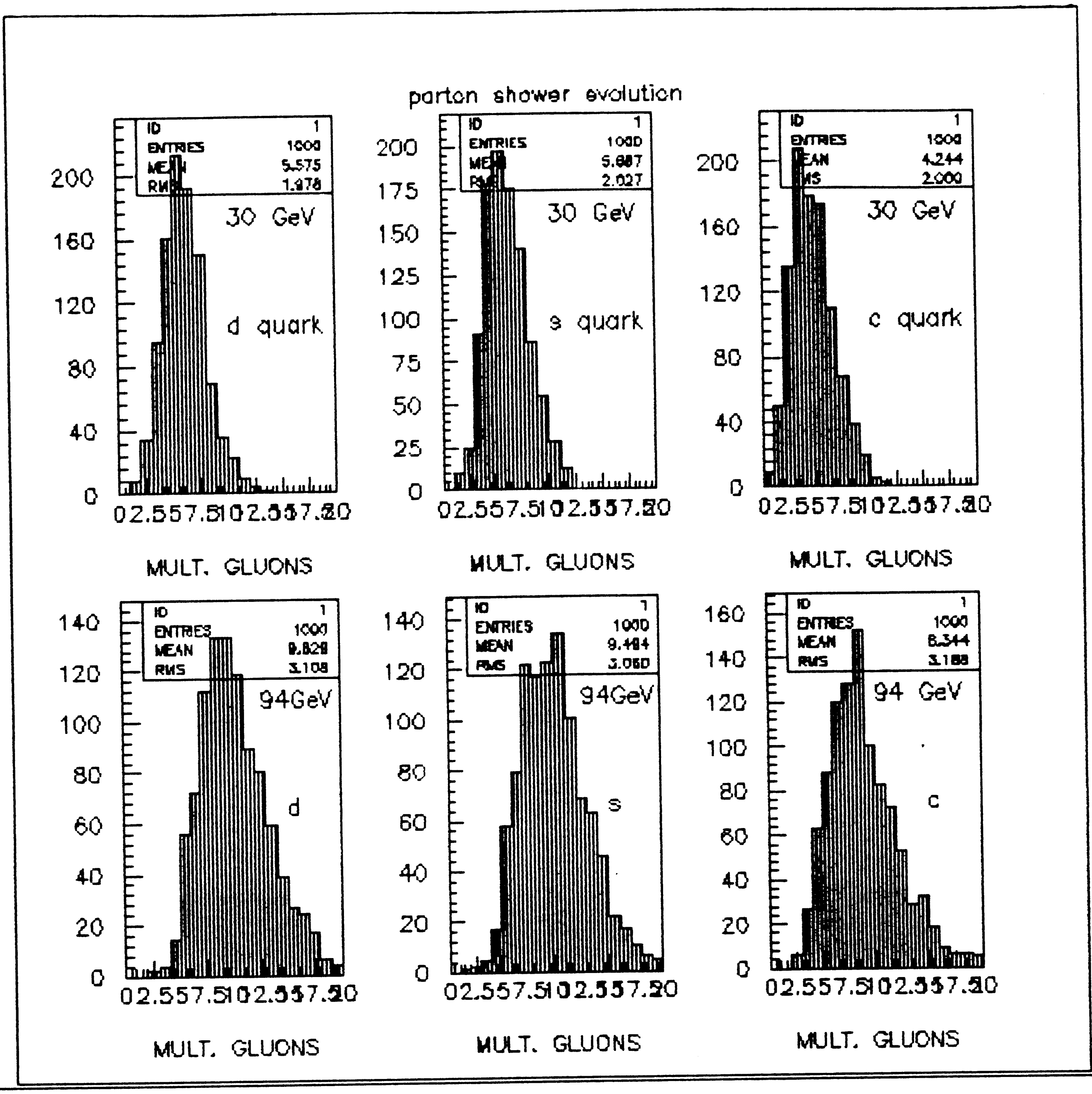


**figure 6.a :**
*LLA option.*
*Gluon multiplicity at $\sqrt{s}$ = 30 and 94 GeV, for d, s, c quarks.*

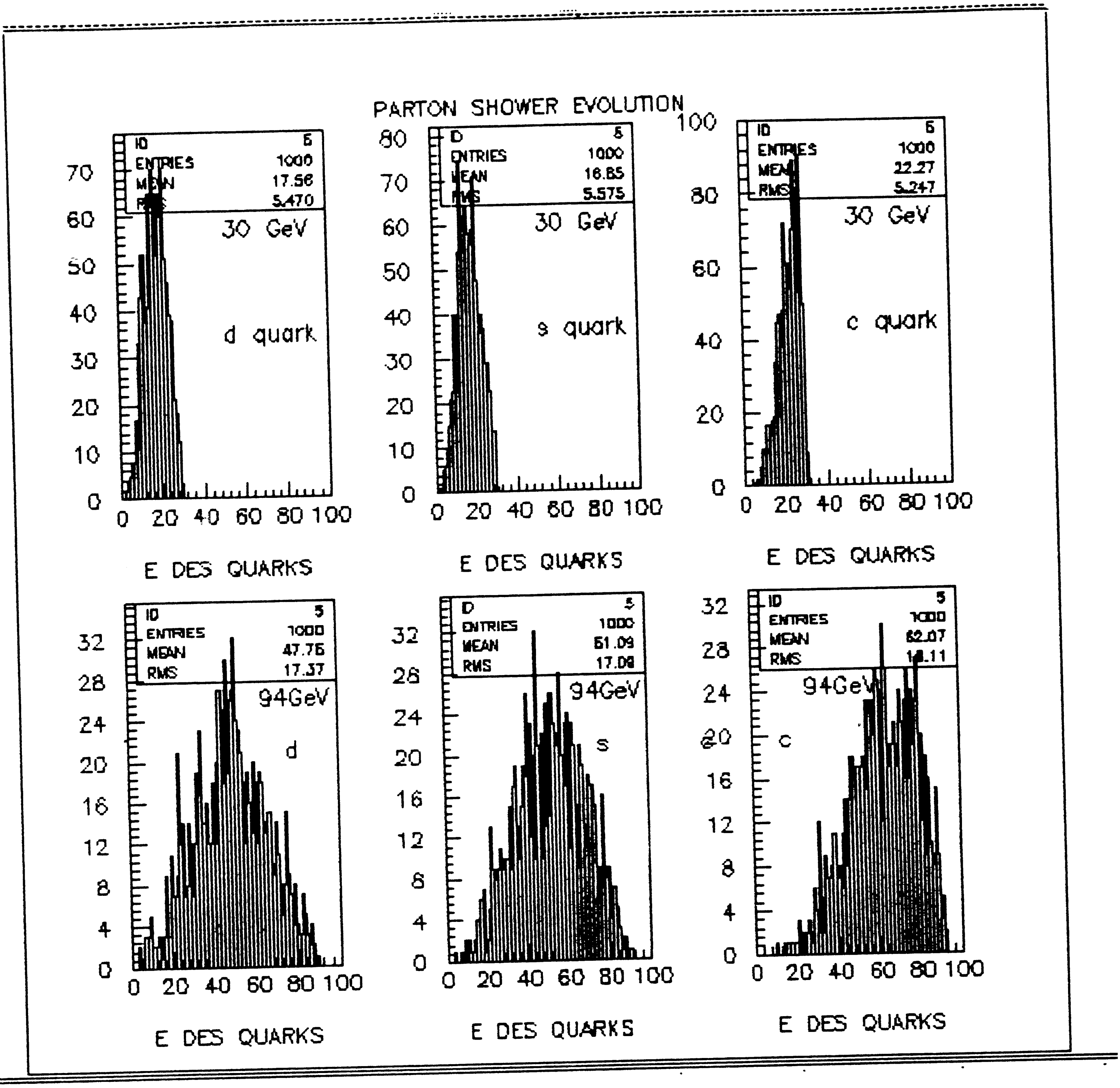


**figure 6.b :**
*LLA option.*
*Total energy from quarks in the event, at* $\sqrt{s}$ = 30 *and* 94 *GeV, for d, s, c quarks.*

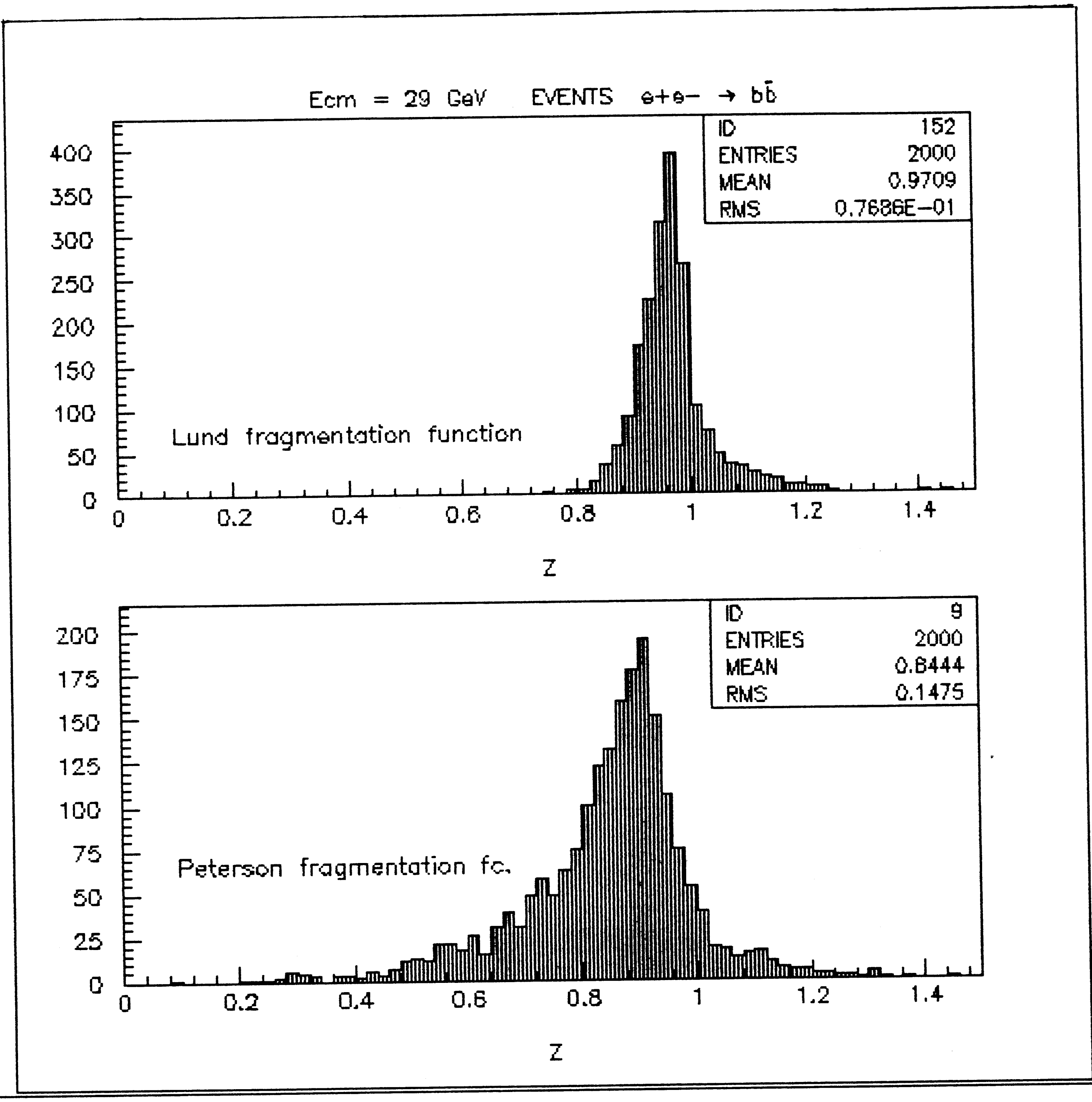


**figure 7.a :**

*LLA option.*

*events $e^+e^- \to b\bar{b}$ at $\sqrt{s}$ = 29 GeV*

*Comparison between the Lund and the Peterson fragmentation function* ($\epsilon_b$ = 0.015).

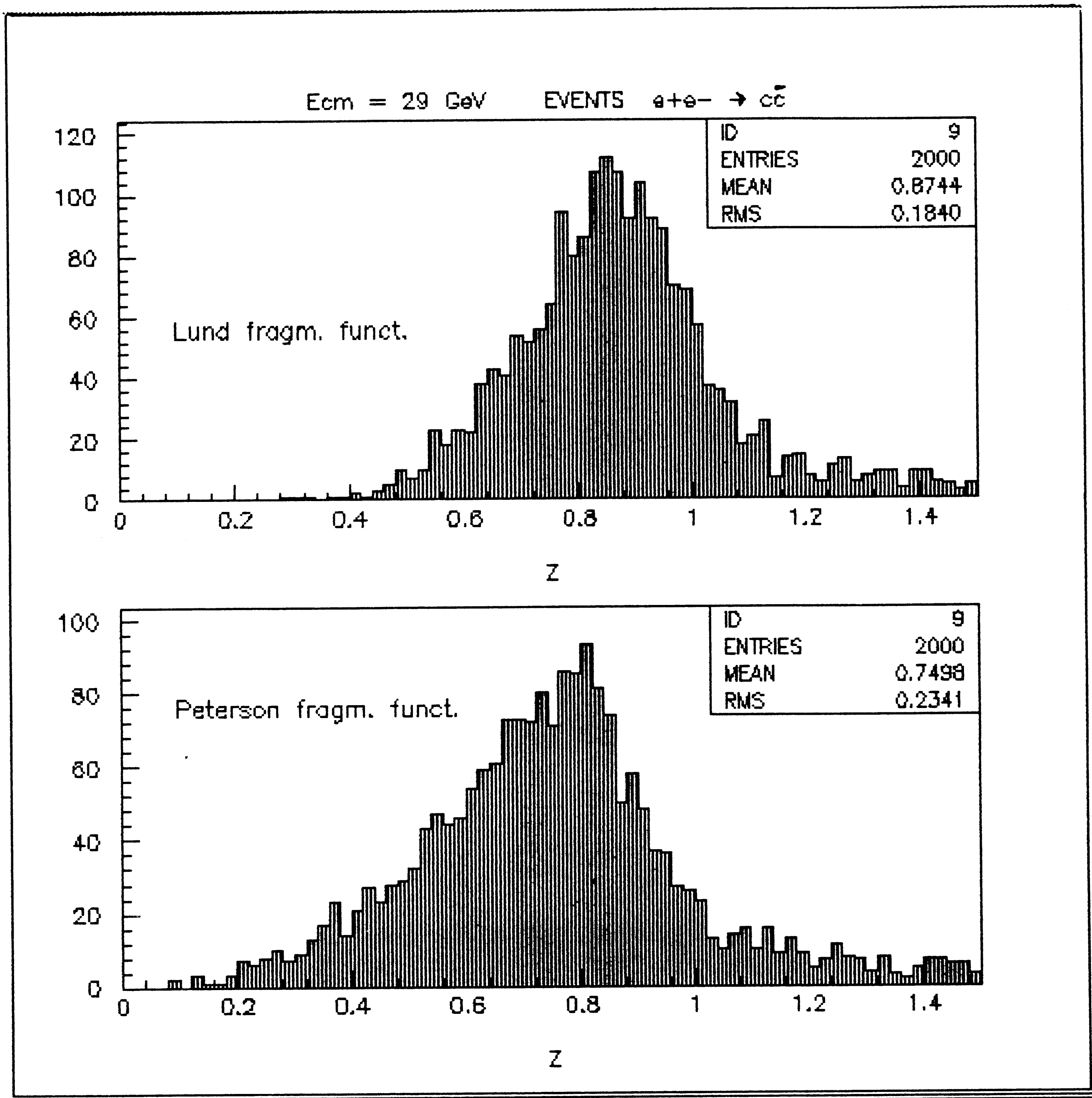


**figure 7.b :**

*LLA option.*

*events* $e^+e^- \to c\bar{c}$ *at* $\sqrt{s} = 29\ GeV$

*Comparison between the Lund and the Peterson fragmentation function* ($\epsilon_c = 0.10$).

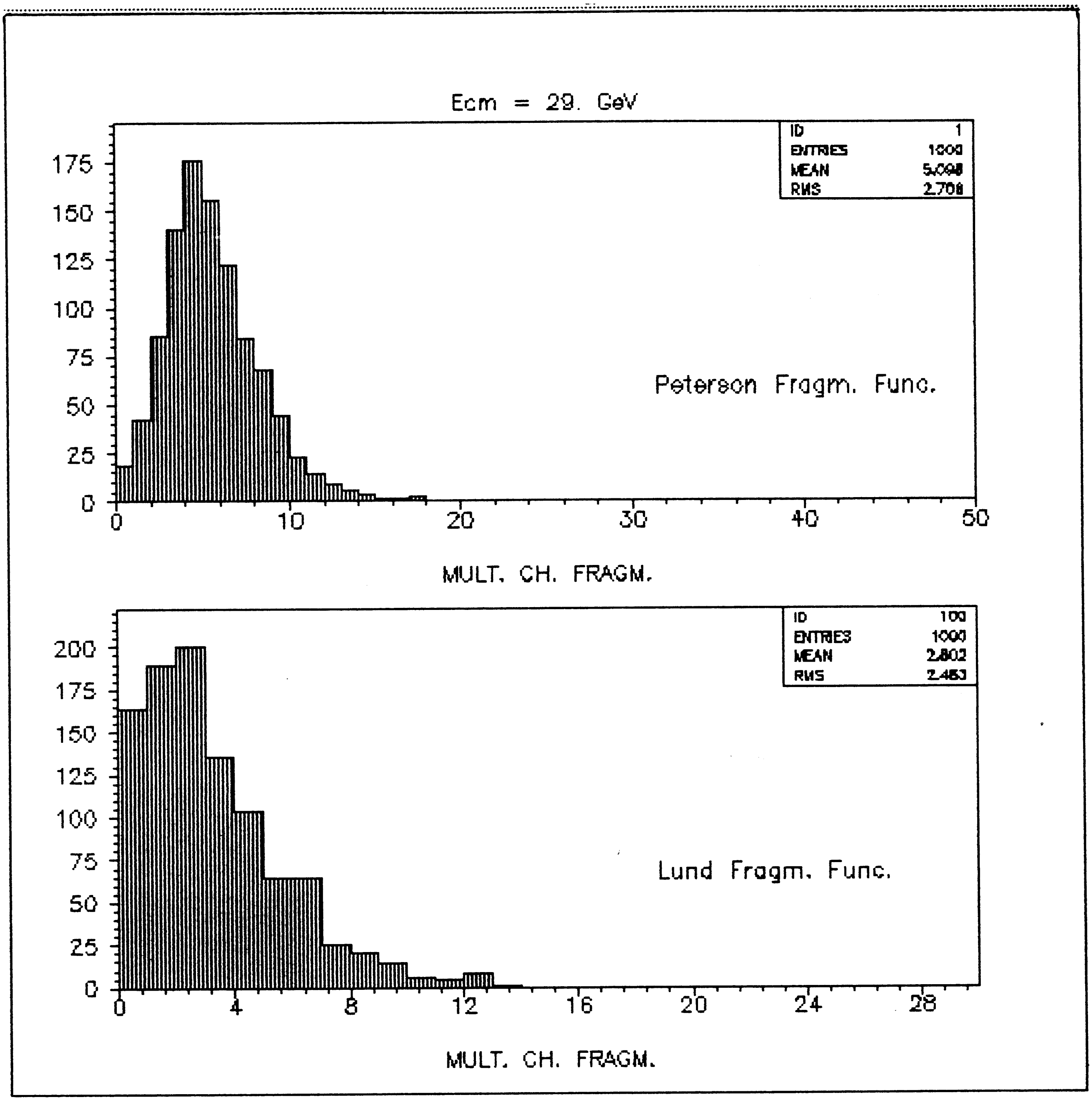


**figure 8.a :**
*LLA option.*
*events $e^+e^- \rightarrow b\bar{b}$ at $\sqrt{s}$ = 29 GeV*
*Charged multiplicity from fragmentation using the Lund and the Peterson fragm. fc.* ($\epsilon_b$ = 0.015).

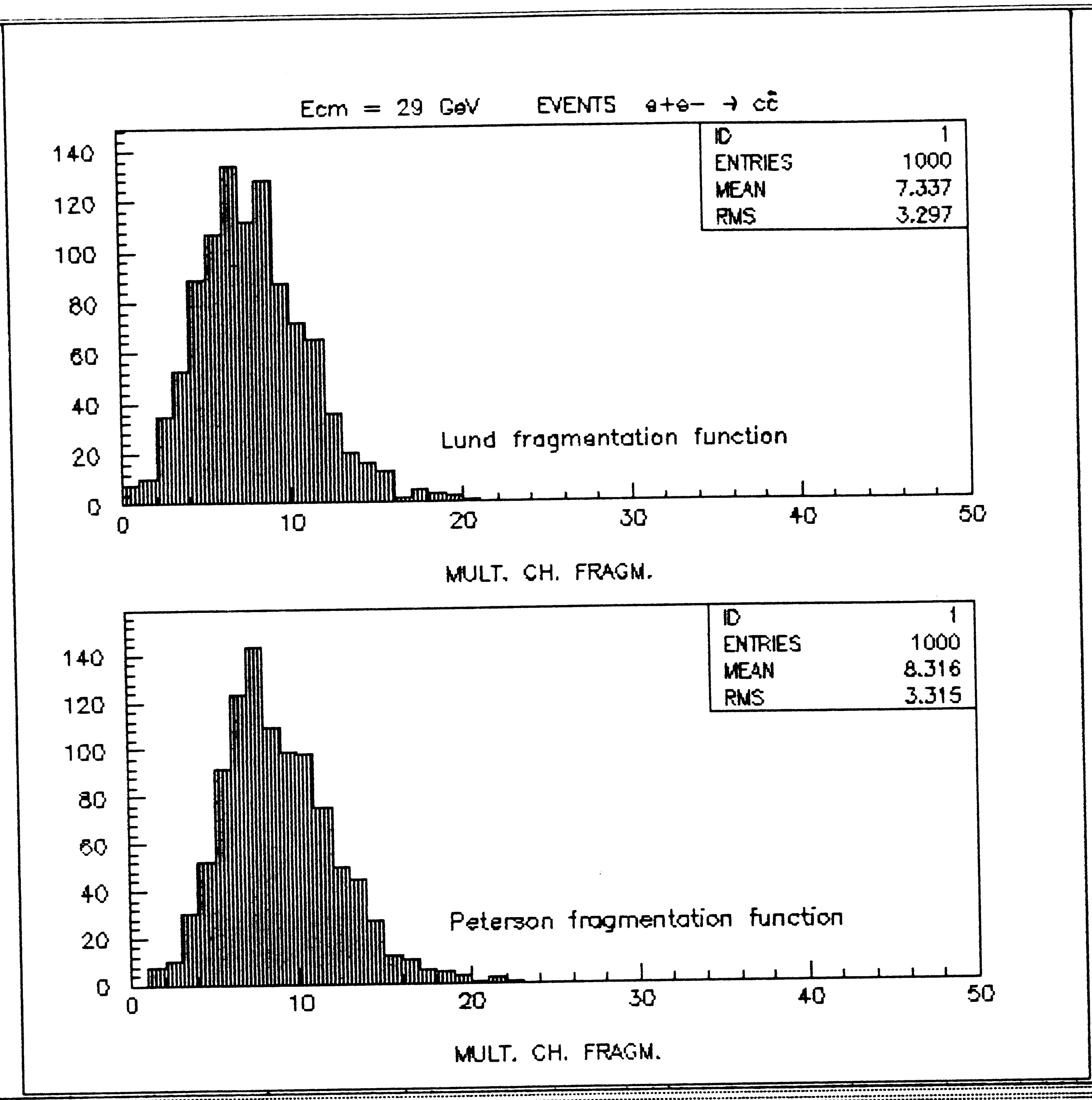


**figure 8.b :**
*LLA option.*
*events* $e^+e^- \rightarrow c\bar{c}$ *at* $\sqrt{s} = 29$ *GeV*
*Charged multiplicity from fragmentation using the Lund and the Peterson fragm. fc.* ($\epsilon_c = 0.10$).

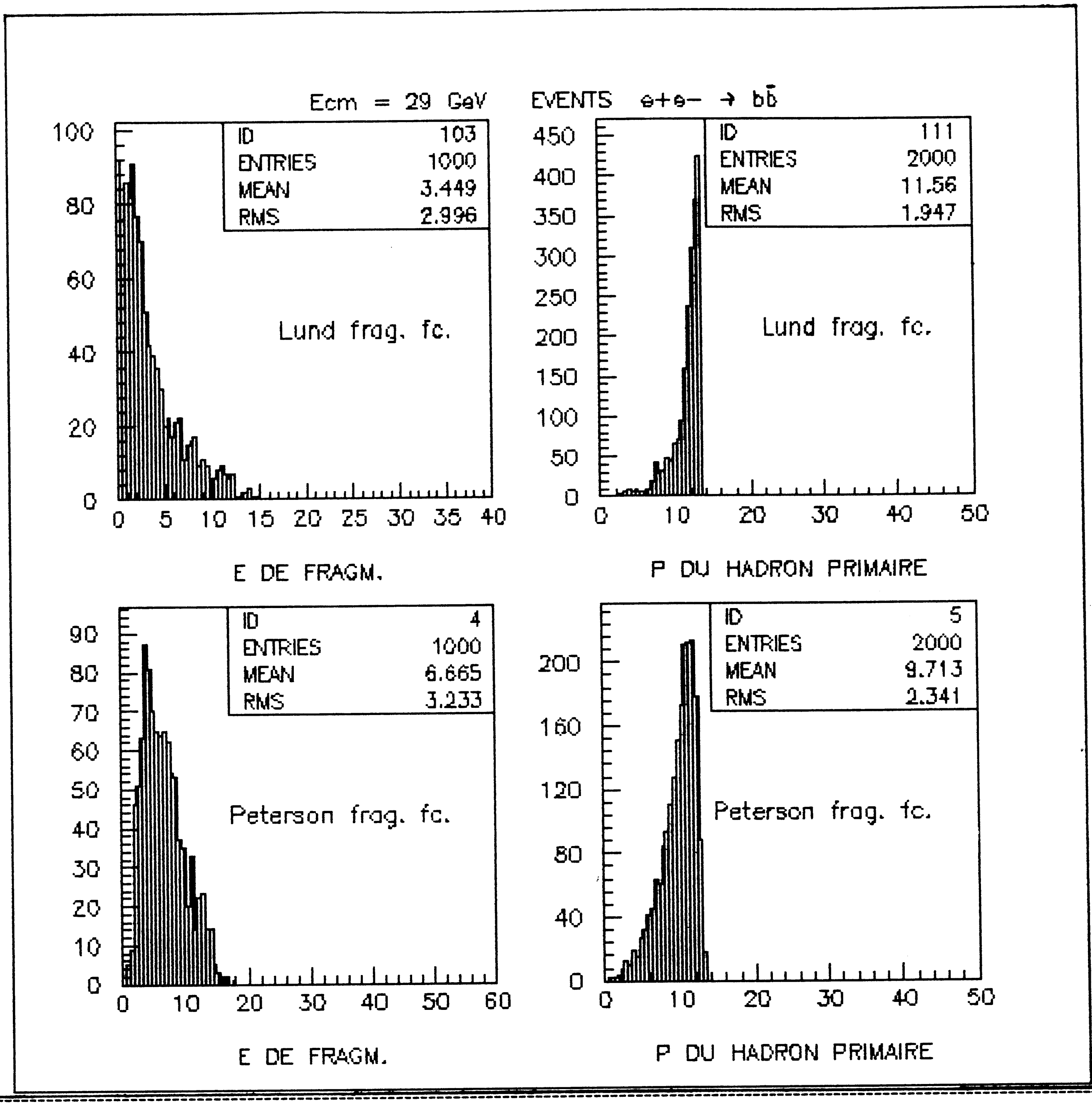


**figure 9.a :**
*LLA option.*
*events* $e^+e^- \rightarrow b\bar{b}$ *at* $\sqrt{s} = 29$ *GeV*
*Energy from fragmentation and Momentum of the primary hadron using the Lund and the Peterson fragm. fc.* ($\epsilon_b = 0.015$

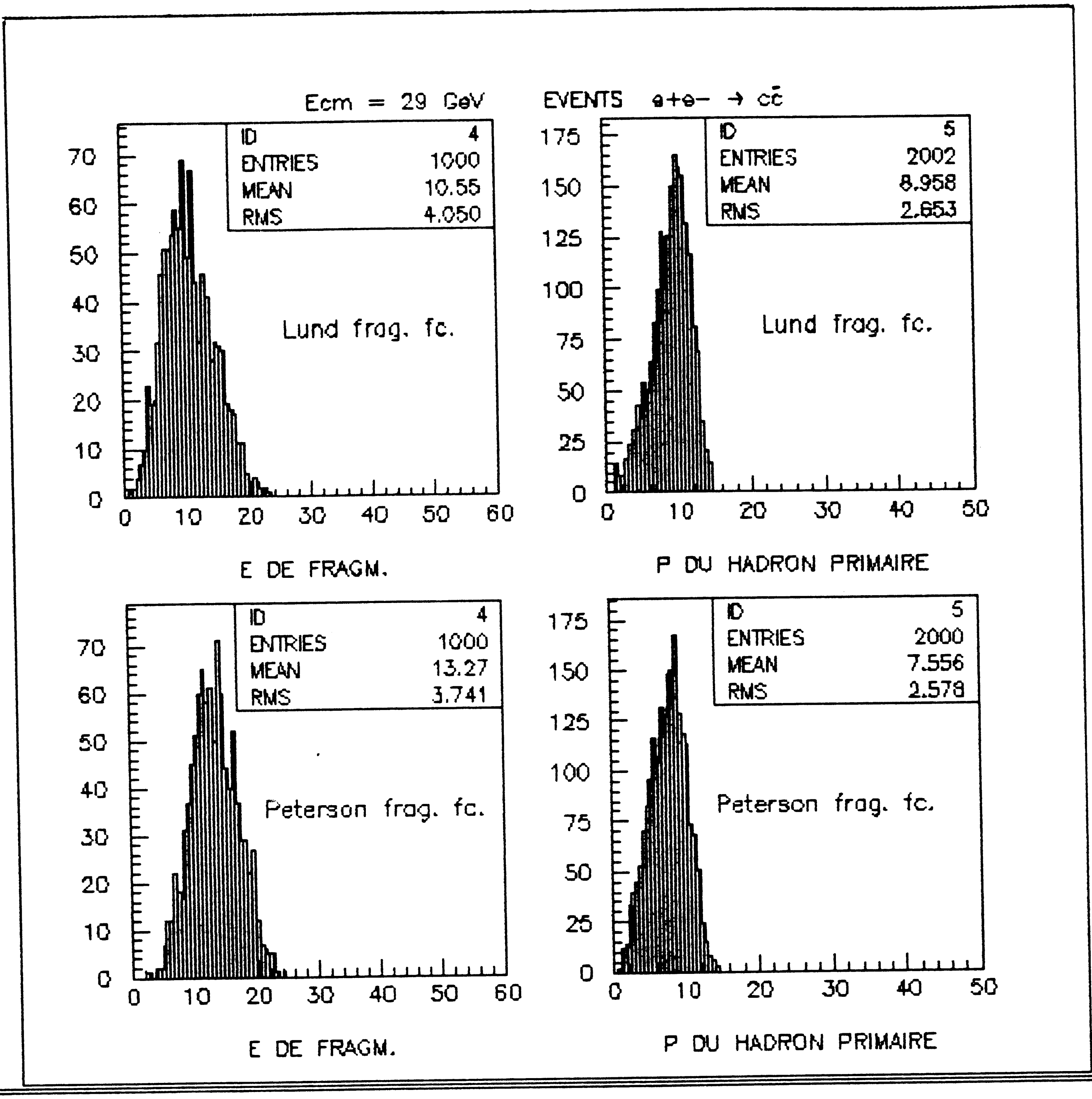


**figure 9.b :**

*LLA option.*

*events* $e^+e^- \to c\bar{c}$ *at* $\sqrt{s}\ =\ 29\ GeV$

*Energy from fragmentation and Momentum of the primary hadron using the Lund and the Peterson fragm. fc.* ($\epsilon_c\ =\ 0.1$(