Massive graviton as a testable cold dark matter candidate

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We construct a consistent model of gravity where the tensor graviton mode is massive, while linearized equations for scalar and vector metric perturbations are not modified. The Friedmann equation acquires an extra dark-energy component leading to accelerated expansion. The mass of the graviton can be as large as $\sim (10^{15}\text{cm})^{-1}$, being constrained by the pulsar timing measurements. We argue that non-relativistic gravitational waves can comprise the cold dark matter and may be detected by the future gravitational wave searches.

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1. Introduction. The current cosmological model is in a beautiful agreement with the data [1]. However, it requires introduction of exotic density components (dark matter, dark energy) with abundances highly tuned to the baryonic matter. This motivates interest in modified theories of gravity deviating from the Einstein theory at large distance scales. Generically, in such theories the graviton has a non-zero mass. The common lore is that the inverse graviton masses significantly smaller than the current Hubble scale are not phenomenologically allowed. In this paper we demonstrate that the inverse graviton mass can be not only significantly smaller than the current size of the Universe, but even many orders of magnitude smaller than the galactic scales. We argue that massive graviton provides specific signatures for gravitational wave experiments and may even account for the cold dark matter (CDM) in the Universe.

Recent studies of the Fierz–Pauli theory of massive gravity [2] and brane world scenarios where the four-dimensional graviton has a non-zero mass [3, 4] strongly suggest [5–11] that Lorentz-invariant models of massive gravity suffer either from the presence of ghosts (fields with a wrong sign of the kinetic term), or from the vDVZ discontinuity due to extra graviton polarizations [12, 13] and strong coupling at the low energy scale. It is possible that the account for the effects of local curvature may solve these problems in some models [14–17]. Another possibility which attracted attention very recently [18–22] is to allow for a violation of Lorentz invariance. In particular, a class of models was found [22] where tensor graviton mode is massive, vDVZ discontinuity and strong coupling problems are absent, while the absence of ghosts and rapid classical instabilities is ensured by the residual reparametrization symmetry

$$x^i \to x^i + \xi^i(t), \tag{1}$$

x^i being the spatial coordinates. These models are the focus of the current paper.

2. The model. In the covariant formalism of Ref. [22] (see also Refs. [5, 19]), the action for the theory of massive gravity contains the metric $g_{\mu\nu}$ and four scalar Goldstone fields ϕ^0, ϕ^i $(i = 1, \ldots, 3)$. In the presence of the residual symmetry (1) it reads

$$S = \int d^4x\sqrt{-g}\left[-M_{Pl}^2 R + \Lambda^4 F(X, W^{ij}, \ldots)\right], \tag{2}$$

where X and W^{ij} are the scalar quantities constructed from the Goldstone fields and the metric,

$$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0,$$

$$W^{ij} = g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - \frac{g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^i \cdot g^{\lambda\rho}\partial_\lambda\phi^0\partial_\rho\phi^j}{X}, \tag{3}$$

and F is a function to be constrained later. We assume that the Goldstone sector is characterized by a single energy scale Λ. Dots in Eq. (2) stand for higher-derivative terms. Latin indices i, j are contracted using δ_{ij}.

We require that the model admits a background solution with the metric $g_{\mu\nu}$ equal to the Minkowski metric $\eta_{\mu\nu}$ and the scalar fields taking the form

$$\phi^0 = a\Lambda^2 t\ , \quad \phi^i = b\Lambda^2 x^i \tag{4}$$

for some constants a and b. For a generic function F such a solution always exists. In the "unitary gauge" where the Goldstone fields are fixed to their vacuum values (4), the second term in the action (2) gives rise to the following mass term for the metric perturbation $h_{\mu\nu}$,

$$\mathcal{L}_m = \frac{M_{Pl}^2}{2}\left(m_0^2 h_{00}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00}h_{ii}\right), \tag{5}$$

where the values of the mass parameters m_a are determined by the first and the second derivatives of the function $F(X, W^{ij})$ at the vacuum values of its arguments as defined by eqs. (3) and (4). The overall scale m of the graviton masses is related to Λ as $m \sim \Lambda^2/M_{Pl}$. The analysis of Ref. [22] implies that Λ plays the role of the cutoff scale of the theory with the action (2).

The residual reparametrization symmetry (1) arises in the unitary gauge as a consequence of the global symmetry $\phi^i \to \phi^i + \xi^i(\phi^0)$ of the covariant action (2). This symmetry implies, in particular, that there is no graviton mass term proportional to h_{0i}^2.

As is usual in the linearized theory, it is convenient to consider separately tensor, vector and scalar metric perturbations (cf. Refs. [20, 22]). The tensor modes — transverse traceless gravitational waves h_{ij}^{TT} — have non-zero mass equal to m_2 [20]. There are no propagating degrees of freedom in the vector sector [22]. Moreover, the contribution of the mass term (5) in the vector sector has the form of a gauge fixing. Consequently, no modification of gravity arises in the vector sector at the order we are working. Finally, the energy-momentum tensor $\delta T_{\mu\nu}$ induces the following perturbations in the scalar sector,

$$\Psi = \Psi_E, \tag{6}$$

$$\Phi = \Phi_E + \frac{m_2^2[3m_4^4 - m_0^2\left(3m_3^2 - m_2^2\right)]}{m_4^4 - m_0^2(m_3^2 - m_2^2)} \frac{1}{\partial_i^4} \frac{\delta T_{00}}{M_{Pl}^2}, \tag{7}$$

where Ψ, Φ are the gauge-invariant scalar potentials defined in a standard way [23], and Ψ_E, Φ_E are their values in the Einstein theory. The modification of gravity manifests itself in the last term in Eq. (7). There is no vDVZ discontinuity as this term vanishes in the limit when all graviton masses uniformly go to zero.

The extra term in Eq. (7) grows linearly with the distance from the source, indicating the breakdown of the linearized theory. This growth cannot be eliminated by a proper choice of the gauge as Φ is the gauge-invariant quantity. However, the Riemann curvature associated with the extra term goes to zero as $1/r$ at large r, so the space-time becomes flat far from the source. (This breakdown of perturbation theory is very different in nature from the seemingly similar problem in the Fierz–Pauli theory [14], where it happens in the vicinity of the source. The close analogue of the phenomenon discussed here is the breakdown of perturbation theory far from the source in the three-dimensional classical Yang–Mills theory.) In the region where the non-standard term in Eq. (7) is still small it produces the r-independent force, imitating the effect of a halo with the density profile $\propto r^{-1}$.

The analysis of Eq. (7) in the region where it enters the non-linear regime goes beyond the scope of this paper. Instead, we chose the masses m_a in such a way that the second term in Eq. (7) vanishes. It is important that this can be achieved by imposing, in addition to (1), the following dilatation symmetry,

$$t \to \lambda t\ ,\quad x^i \to \lambda^{-\gamma} x^i, \tag{8}$$

where γ is a real constant. At the linearized level this symmetry implies the following relations among masses,

$$m_0^2 = -3\gamma m_4^2\ ,\quad m_2^2 - 3m_3^2 = \gamma^{-1} m_4^2, \tag{9}$$

which lead to the cancellation of the second term in Eq. (7) for any γ. Thus, when the symmetry (8) is imposed, the only modification of gravity at the linearized level is the non-zero mass of the graviton.

The inclusion of higher-derivative terms in the action (2) leads in general to the appearance of the dynamical degree of freedom in the scalar sector [22]. This degree of freedom is similar to that present in the ghost condensate model [19]. It has a healthy kinetic term provided the following inequality holds [22],

$$m_0^2 - \frac{m_4^4}{(m_3^2 - m_2^2)} > 0. \tag{10}$$

The latter condition is compatible with Eqs. (9) and the requirement that the graviton mass is not tachyonic, $m_2^2 > 0$. The effects related to this degree of freedom are characterized by the huge retardation time $\sim m^{-1}(M_{Pl}/\Lambda)$ [19, 24, 25]. This time is larger than the current age of the Universe for the values of the graviton mass m specified below, so we can consistently neglect these effects.

In the covariant formalism the residual symmetry (8) translates into the following global symmetry of the Goldstone sector, $\phi^0 \to \lambda\phi^0$, $\phi^i \to \lambda^{-\gamma}\phi^i$. The action invariant under the symmetries (1), (8) has the form (2) with the function F depending on the single combination $X^\gamma W^{ij}$. The case of the ghost condensate [19] emerges in the limit $\gamma \to \infty$ and requires a fine-tuning of F to obtain the Minkowski vacuum. The Minkowski vacuum with the scalar vev's of the form (4) exists for a general function F if $\gamma = 1/d$, where $d = 3$ is a number of spatial dimensions. For definiteness, in what follows we consider the case $F = F(X^{1/3}W^{ij})$.

3. Cosmological solutions. The spatially flat homogeneous cosmological ansatz is

$$ds^2 = a^2(\eta)\left(d\eta^2 - dx_i^2\right), \tag{11}$$

$$\phi^0 = \phi(\eta)\ ,\quad \phi^i = \Lambda^2 x^i. \tag{12}$$

In what follows we assume that the rate of the expansion is much smaller than the energy scale Λ, so one can neglect higher derivative terms in the action (2). For simplicity, let us also assume that the function F depends only on the combination $Z \equiv X^{1/3}W^{ij}\delta_{ij}$. The Einstein equations are reduced to the Friedmann equation

$$\left(\frac{\dot a}{a^2}\right)^2 = \frac{1}{3M_{Pl}^2}\left(\rho_m + \frac{2}{3}\Lambda^4 F'(Z)Z - \Lambda^4 F(Z)\right), \tag{13}$$

where ρ_m is the energy density of matter, and the field equation for ϕ^0,

$$\partial_\eta\left(a^3 F'(Z) W X^{-1/6}\right) = 0. \tag{14}$$

Eq. (14) implies $Z = const$ or, equivalently, $\phi^0 \propto \int d\eta a^4(\eta)$. Then Eq. (13) takes the form of the standard Friedmann equation with the value of the cosmological constant determined by the value of Z, *i.e.* by the initial conditions in the Goldstone sector. Note that these initial conditions may be different in different regions of space. Therefore, this model is an example of the setup where de Sitter solutions with different expansion rates exist for any value of the vacuum energy. This property is a welcome feature for the application of the weak anthropic principle [26] to the cosmological constant problem.

To summarize, we have constructed a consistent model where gravitational waves are massive, while linearized equations for the metric perturbations in the scalar and vector sectors, as well as spatially flat cosmological solutions, are the same as in the Einstein theory. In this model, the tests of (linear) gravity based on the solar system and Cavendish-type experiments [27] are automatically satisfied, while the main constraints are coming from emission and/or propagation of gravitational waves.

4. Relic gravitational waves. Observations of the slow down of the orbital motion in binary pulsar systems [28] imply that the mass of the gravitational waves cannot be larger than the frequency of the waves emitted by these systems. The latter is determined by the period of the orbital motion which is of order 10 hours, implying the following limit on the graviton mass,

$$\frac{m_2}{2\pi} \equiv \nu_2 \lesssim 3 \cdot 10^{-5} \text{ Hz} \approx \left(10^{15} \text{ cm}\right)^{-1}. \qquad (15)$$

Let us estimate the cosmological abundance of relic gravitons. For this purpose we consider the transverse traceless perturbation of the metric h_{ij}. The quadratic action for h_{ij} in the expanding Universe takes the following form,

$$M_{Pl}^2 \int d^3k d\eta a^2(\eta) \left(\dot{h}_{ij}^2 - (\partial_k h_{ij})^2 - m_2^2 a^2(\eta) h_{ij}^2\right). \qquad (16)$$

This has a form of the action for a minimally coupled massive scalar field. Therefore, gravitons in our model are produced efficiently during inflation (cf. Ref. [29]).

To be concrete, consider a scenario where the Hubble parameter H_i is constant during inflation. This scenario may be realized, for instance, in hybrid models of inflation [30]. First, we need to check that the phenomenologically relevant values of parameters correspond to the regime below the cutoff scale of the effective theory, i.e. $H_i \lesssim \Lambda$. For the energy scale of inflation $E_i \sim \sqrt{H_i M_{Pl}}$ this implies

$$E_i < m_2^{1/4} M_{Pl}^{3/4} \approx 10^7 \text{ GeV} \left(m_2 \cdot 10^{15} \text{ cm}\right)^{1/4}. \qquad (17)$$

This value is high enough to allow for a successful baryogenesis even for graviton masses of the order of the current Hubble scale.

Consider now the production of massive gravitons. Assuming the above scenario of inflation, the perturbation spectrum for the massive gravitons is that for the minimally coupled massive scalar field in the de Sitter space [31],

$$\langle h_{ij}^2 \rangle \simeq \frac{1}{4\pi^2} \left(\frac{H_i}{M_{Pl}}\right)^2 \int \frac{dk}{k} \left(\frac{k}{H_i}\right)^{\frac{2m_2^2}{3H^2}}. \qquad (18)$$

Superhorizon metric fluctuations remain frozen until the Hubble factor becomes smaller than the graviton mass, when they start to oscillate with the amplitude decreasing as $a^{-3/2}$. The energy density in massive gravitons at the beginning of oscillations is of order

$$\rho_o \sim M_{Pl}^2 m_2^2 \langle h_{ij}^2 \rangle \simeq \frac{3H_i^4}{8\pi^2}, \qquad (19)$$

where we integrated in Eq. (18) over the modes longer than the horizon. Today the fraction of the energy density in the massive gravitational waves is

$$\Omega_g = \frac{\rho_o}{z_o^3 \rho_c} = \frac{\rho_o}{z_e^3 \rho_c} \left(\frac{H_e}{H_o}\right)^{3/2}, \qquad (20)$$

where z_o is the redshift at the start of oscillations, $H_o \sim m_2$ is the Hubble parameter at that time, $H_e \approx 0.4 \cdot 10^{-12}$ s^{-1} is the Hubble parameter at the matter/radiation equality, and $z_e \approx 3200$ is the corresponding redshift. Combining all the factors together one gets

$$\Omega_g \sim 3 \cdot 10^3 (m_2 \cdot 10^{15}\text{cm})^{1/2} \left(\frac{H_i}{\Lambda}\right)^4. \qquad (21)$$

This estimate assumes that the number of e-foldings during inflation is large, $\ln N_e > H^2/m^2$, which is quite natural in the model of inflation considered here.

According to Eq. (21), the massive gravitons are produced efficiently enough to comprise all of the cold dark matter, provided the value of the Hubble parameter during inflation is about one order of magnitude below the scale Λ. We find it encouraging that one obtains $\Omega_g \sim 1$ when the initial energy density in the metric perturbations is close to the cutoff scale, $\rho_o^{1/4} \sim \Lambda$. This suggests that other mechanisms of production unrelated to inflation (e.g., similar to those invoked for the axion or Polony fields) may naturally lead to the same result, $\Omega_g \sim 1$.

The produced gravitons may cluster in galaxies. To account for the dark matter in galactic halos the graviton mass should satisfy $(mv)^{-1} \lesssim 1$ kpc $\sim 3 \cdot 10^{21}$ cm, where $v \sim 10^{-3}$ is a typical velocity in the halo.

5. Detection. Let us now briefly describe potential observational signatures of the above scenario. Note first that at distances shorter than the wavelength, the effect of a transverse traceless gravitational wave on test massive particles in Newtonian approximation is described by the acceleration $\ddot{h}_{ij} x^j/2$ (see, e.g., Ref. [32] for a review). The same is true for massive gravitational waves, the only difference being that the wavelengths are longer in the non-relativistic case, so the Newtonian description works for the larger range of distances. Thus, the non-relativistic waves act on the detector in the same way as massless waves of the same frequency.

Let us estimate the amplitude of the gravitational waves assuming that they comprise all of the dark matter in the halo of our Galaxy. The energy density in non-relativistic gravitational waves is of order $M_{Pl}^2 m_2^2 h_{ij}^2$. Equating this to the local halo density one gets

$$\langle h_{ij} \rangle \sim 10^{-10} \left(\frac{3 \cdot 10^{-5}\text{Hz}}{\nu_2}\right). \qquad (22)$$

At the frequencies $10^{-6} \div 10^{-5}$ Hz this value is well above the expected sensitivity of the LISA detector [33]. Note that in the close frequency range $10^{-9} \div 10^{-7}$ Hz there is a restrictive bound [34] at the level $\Omega_g < 10^{-9}$ on the stochastic background of the gravitational waves coming from the timing of the millisecond pulsars [35]. So, it is possible that our scenario can be tested by the reanalysis of the already existing data on the pulsar timing.

The relic abundance of gravitons may depend on both the specific inflationary model and the details of the (unknown) UV completion of massive gravity. In general, massive gravitons may not comprise the whole of the CDM in the galaxy halos. It is important that the expected LISA sensitivity allows to detect the presence of massive gravitons at the significantly lower level than in Eq. (22).

6. Concluding remarks. In this paper we limited ourselves to a specific choice of the parameters (graviton masses and the constant γ entering Eq. (8)) such that there is no modification of the Newton potential at the linear level, and the cosmological evolution remains standard. We also did not consider possible non-linear effects, which may become a necessity at different choice of the parameters. A number of interesting questions is related to these effects, including the limits on graviton masses, clustering of massive gravitons in haloes and proper modifications of Eqs. (13) and (14) to account for the direct coupling between Goldstone fields and gravitons. We expect, however, that our main conclusions — that gravitons may have large masses and may be produced with cosmologically significant abundance — are generic in this class of models. In the relevant range of parameters, a specific signature of the gravitons with non-zero mass is a strong monochromatic signal in the detectors of gravitational waves. An independent measurement of the graviton mass may be performed at future gravitational wave detectors (for a review, see, e.g. [36]) operating at higher frequencies by testing the delay between the electromagnetic and gravitational signals from a distant supernova explosion.

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