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# THRESHOLD CORRECTIONS IN QED⊗QCD AT THE LHC

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In some processes at the LHC, theoretical precisions of 1% are desired. With an eye toward such precisions, we introduce the theory of the simultaneous YFS resummation of QED and QCD to compute the size of the expected resummed soft radiative threshold effects in precision studies of heavy particle production at the LHC. Our results, that the soft QED threshold effects are at the level of 0.3% whereas the soft QCD threshold effects enter at the level of 20%, show that both must be controlled to be on the conservative side to achieve such precision goals.

## 1 Introduction

In high energy collider processes, such as $t\bar{t}$ production at FNAL, precision predictions for soft multiple gluon (n(g)) effects are already needed: the uncertainty on $m_t$ [1], $\delta m_t = 4.3$ GeV, receives a soft n(g) uncertainty $\sim$ 2-3 GeV, for example. At the LHC/ILC, the requirements will be even more demanding and soft n(g) MC exponentiation results will be an important part of the necessary theory – YFS exponentiated $\mathcal{O}(\alpha_s^2)L$ calculations, in the presence of parton showers, on an event-by-event basis.

How relevant are QED higher order corrections when QCD is controlled at $\sim$ 1% precision? Many authors [2] are preparing the necessary results that would lead to such a precision on QCD for LHC processes. Estimates by Refs. [3–7] show that one gets few per mille effects from QED corrections to structure function evolution. The well-known possible enhancement of QED corrections at threshold, especially in resonance production, leads us to estimate how big are these effects at the LHC.

We treat QED and QCD simultaneously in the respective YFS [8,9] exponentiation to estimate the role of the QED threshold effects at the LHC in the representative processes $pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X$, where $V = W^{\pm}, Z,$ and $\ell = e, \mu$, $\ell' = \nu_e, \nu_\mu(e, \mu)$ respectively for $V = W^+(Z)$, and $\ell = \nu_e, \nu_\mu$, $\ell' = e, \mu$ respectively for $V = W^-$. Precision studies of these processes have been proposed for luminometry at the LHC [10] and at FNAL [11], where 2-3% is the target precision tag for the LHC, for example. The latter would indeed require a theoretical precision tag of $\sim$ 1% in order that the theory error not figure too prominently in the over-all precision.

Our discussion is organized as follows. After giving a brief review of the YFS theory and its extension to QCD in the next section, in Section 3 we introduce QED⊗QCD YFS exponentiation. In Section 4, we apply

the new development to the threshold corrections in single V production at the LHC and at FNAL. Section 5 contains some concluding remarks.

## 2 Review of the YFS Theory and its Extension to QCD

As realized in Refs. [9] by Monte Carlo methods, for $e^+(p_1)e^-(q_1) \rightarrow \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$, renormalization group improved YFS theory [12] gives,

$$d\sigma_{exp} = e^{2\alpha\,Re\,B + 2\alpha\,\tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0}$$
$$\int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j) + D}$$
$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0} \qquad (1)$$

where the YFS infrared functions $\tilde{B}$, $B$ and $D$ are known. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in the first paper in Ref. [9] and realize the YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1).

In Refs. [13, 14] we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{exp} = \sum_n d\hat{\sigma}^n = e^{\text{SUM}_{\text{IR}}(\text{QCD})} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j}$$
$$\int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (P_1 + P_2 - Q_1 - Q_2 - \sum k_j) + D_{\text{QCD}}}$$
$$* \bar{\tilde{\beta}}_n(k_1, \ldots, k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0}$$
$$\qquad (2)$$

where gluon residuals $\bar{\tilde{\beta}}_n(k_1, \ldots, k_n)$, defined by Ref. [13], are free of all infrared divergences to all orders in $\alpha_s(Q)$. The functions $SUM_{IR}(QCD), D_{\text{QCD}}$, together with the basic infrared functions $B_{QCD}^{nls}, \tilde{B}_{QCD}^{nls}, \tilde{S}_{QCD}^{nls}$ are specified in Ref. [13]. We call attention to the essential compensation between the left over genuine non-Abelian IR virtual and real singularities between $\int dPh\bar{\beta}_n$ and $\int dPh\bar{\beta}_{n+1}$ respectively that really allows us

to isolate $\bar{\tilde{\beta}}_j$ and distinguishes QCD from QED, where no such compensation occurs.

We stress that the YFS resummation which we exhibit here is fully consistent with that of Refs. [15, 16]. We refer the reader to Ref. [17] for more discussion of this point.

## 3 Extension to QED⊗QCD and QCED

Simultaneous exponentiation of QED and QCD higher order effects [17] gives

$$B_{QCD}^{nls} \rightarrow B_{QCD}^{nls} + B_{QED}^{nls} \equiv B_{QCED}^{nls},$$
$$\tilde{B}_{QCD}^{nls} \rightarrow \tilde{B}_{QCD}^{nls} + \tilde{B}_{QED}^{nls} \equiv \tilde{B}_{QCED}^{nls},$$
$$\tilde{S}_{QCD}^{nls} \rightarrow \tilde{S}_{QCD}^{nls} + \tilde{S}_{QED}^{nls} \equiv \tilde{S}_{QCED}^{nls} \qquad (3)$$

which leads to

$$d\hat{\sigma}_{\text{exp}} = e^{\text{SUM}_{\text{IR}}(\text{QCED})}$$
$$\sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4}$$
$$e^{iy \cdot (p_1+q_1-p_2-q_2-\sum k_{j_1} - \sum k'_{j_2}) + D_{\text{QCED}}}$$
$$\bar{\tilde{\beta}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0},$$
$$\qquad (4)$$

where the new YFS residuals, defined in Ref. [17], $\bar{\tilde{\beta}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)$, with $n$ hard gluons and $m$ hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED. The functions $\text{SUM}_{\text{IR}}(\text{QCED}), D_{\text{QCED}}$ are determined from their analoga $\text{SUM}_{\text{IR}}(\text{QCD}), D_{\text{QCD}}$ via the substitutions in (3) everywhere in expressions for the latter functions given in Refs. [13].

Infrared Algebra(QCED): the average Bjorken $x$ values

$$x_{avg}(QED) \cong \gamma(QED)/(1 + \gamma(QED))$$
$$x_{avg}(QCD) \cong \gamma(QCD)/(1 + \gamma(QCD))$$

where $\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$, with $\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$ and the big log $L_s$, imply that QCD dominant corrections happen an order of magnitude earlier than those for

QED.This means that that the leading $\bar{\bar{\beta}}^{(0,0)}_{0,0}$-level gives a good estimate of the size of the effects we study.

## 4  QED⊗QCD Threshold Corrections at the LHC

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the single Z production with leptonic decay at the LHC ( and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Refs. [18–20] for exact $\mathcal{O}(\alpha)$ results and Refs. [21–23] for exact $\mathcal{O}(\alpha_s^2)$ results.

For the basic formula (we use the standard notation here [17])

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') =$$
$$\sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \quad (5)$$

we use the result in (4) here with semi-analytical methods and structure functions from Ref. [24]. A Monte Carlo realization will appear elsewhere [25].

We **do not** attempt to replace HER-WIG [26] and/or PYTHIA [27] – we intend to combine our exact YFS calculus with HER-WIG and/or PYTHIA **by using the latter in lieu of the** $\{F_i\}$. This combination of theoretical constructs can be systematically improved with exact results order-by-order in $\alpha_s$, where currently the state of the art in such a calculation is the work of Frixione and Webber in Ref. [28] which accomplishes the combination of an exact $\mathcal{O}(\alpha_s)$ correction with HERWIG. We note that, even in this latter result, the gluon azimuthal angle is averaged in the combination. We note that the recent alternative parton shower algorithm by Jadach and Skrzypek in Ref. [29] can also be used in our theoretical construction here. Due to its lack of the appropriate color coherence [30], we do not consider ISAJET [31] here.

We compute , with and without QED, the ratio $r_{exp} = \sigma_{exp}/\sigma_{Born}$ to get the results

(We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & , \text{QCED} \equiv \text{QCD+QED},\ \text{LHC} \\ 1.1872 & , \text{QCD},\ \text{LHC} \\ 1.1911 & , \text{QCED} \equiv \text{QCD+QED},\ \text{Tevatron} \\ 1.1879 & , \text{QCD},\ \text{Tevatron}. \end{cases}$$
$$(6)$$

We see that QED is at the level of .3% at both LHC and FNAL. This is stable under scale variations [17]. We agree with the results in Refs. [18–22] on both of the respective sizes of the QED and QCD effects. The QED effect is similar in size to structure function results in Refs. [3–7].

## 5  Conclusions

YFS theory (EEX and CEEX) extends to non-Abelian gauge theory and allows simultaneous exponentiation of QED and QCD. For QED⊗QCD we find that full MC event generator realization is possible in a way that combines our calculus with Herwig and Pythia in principle. Semi-analytical results for QED (and QCD) threshold effects agree with literature on Z production. As QED is at the .3% level, it is needed for 1% LHC theory predictions. A firm basis for the complete $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ results needed for the FNAL/LHC/RHIC/TESLA/ILC physics has been demonstrated and all of the latter are in progress.

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