The scaling dimension of low lying Dirac eigenmodes and of the topological charge density

MILC Collaboration: C. Aubin[a] C. Bernard[a], Steven Gottlieb[b], E.B. Gregory[c], Urs M. Heller[d], J.E. Hetrick[e*], J. Osborn[f], R. Sugar[g], D. Toussaint[c] with: Ph. de Forcrand[hi], and O. Jahn[j]

[a]Department of Physics, Washington University, St. Louis, MO 63130, USA

[b]Department of Physics, Indiana Univerity, Bloomington, IN 47405, USA

[c]Department of Physics, University of Arizona, Tucson, AZ 85721, USA

[d]American Physical Society, One Research Road, Box 9000, Ridge, NY 11961, USA

[e]Physics Department, University of the Pacific, Stockton, CA 95211, USA

[f]Physics Department, University of Utah, Salt Lake City, UT 84112, USA

[g]Department of Physics, University of California, Santa Barbara, CA 93106, USA

[h]Institute for Theoretical Physics, ETH Zürich, CH-8093 Zürich, Switzerland

[i]CERN, Theory Division, CH-1211 Geneva 23, Switzerland

[j]Center for Theoretical Physics, MIT, Cambridge, MA 02139, USA

As a quantitative measure of localization, the inverse participation ratio of low lying Dirac eigenmodes and topological charge density is calculated on quenched lattices over a wide range of lattice spacings and volumes. Since different topological objects (instantons, vortices, monopoles, and artifacts) have different co-dimension, scaling analysis provides information on the amount of each present and their correlation with the localization of low lying eigenmodes.

1. INTRODUCTION

With modern computational power has come the ability to examine the low lying eigenvectors of the Dirac operator and hence their spatial correlation with instantons and other related objects thought to be involved in chiral symmetry breaking and confinement [1,2]. While these studies focused primarily on the local relationship between instantons and low-lying Dirac eigenmodes (LDEs), other models of confinement and chiral symmetry breaking involving objects of lower co-dimension are popular, based on monopoles, vortices, and hybrid objects [3]. Presumably these objects would have a rather different effect on the LDEs than 4-dimensional instantons, due to their different co-dimension. Furthermore, a recent study [4] has suggested a dense layered 3-dimensional structure to the LDEs.

One difficulty is the quantitative characterization of localization of the LDEs or related quantities such as the topological charge density. In [2] localization of the LDEs was studied using the inverse participation ratio (IPR) which yields a number characterizing the localization of an eigenmode.

By studying the scaling dimension of the IPR, we can find the co-dimension of the structures which localize the LDEs, thus giving some insight as to the possible confining objects and mecha-

*Presented by J.E. Hetrick

nism.

2. INVERSE PARTICIPATION RATIO (IPR)

The IPR of a normalized field $\rho_i(x)$ is defined as

$$I = N \sum_x \rho_i^2(x) \quad (1)$$

where N is the number of lattice sites x. Here we use $\rho_i(x) = \psi_i^\dagger \psi_i(x)$ and $\psi_i(x)$ is the i-th, normalized ($\sum_x \rho_i(x) = 1$), lowest eigenvector of the Dirac operator.

With this definition, I characterizes the inverse "fraction" of sites contributing significantly to the support of $\rho(x)$ (we now drop the subscript i). A simple calculation shows that the IPR takes the following values for these simple situations:

$$\begin{aligned} \text{Unlocalized}: &\ \rho(x) = \text{const.} & I = 1 \\ \delta-\text{function}: &\ \rho(x) = \delta(x_o) & I = N \\ \text{localized on fraction } f \text{ of sites}: & & I = 1/f \end{aligned}$$

Suppose that the objects responsible for confinement, or indeed any physics governing the lowest Dirac eigenmodes, localize the LDEs. As the lattice spacing is reduced, the fraction of sites contributing to the IPR scales as a^d/a^4. Thus the IPR indicates the co-dimension of these objects: $d = 4$ for instantons, $d = 3$ for monopoles, and $d = 2$ for vortices. Gauge dislocations should contribute as $d = 0$ objects, however their density diverges as a^{-4} so that they should give a ∼constant contribution: $a^0/(a^4a^{-4})$).

Since the IPR $\sim 1/f$, if we reduce the lattice spacing at fixed physical volume, we have

$$a \to 0 \ \text{ at fixed volume}: \quad I \sim a^{4-d} \quad (2)$$

On the other hand, increasing the volume at fixed lattice spacing includes proportionately more of the confining objects, whatever their dimension. Thus we expect the IPR to remain constant,

$$L \to \infty \ \text{ at fixed } a: \quad I \sim \text{constant} \quad (3)$$

3. RESULTS

We have explored these two regimes using quenched lattices generated with the tadpole improved Symanzik gauge action, and the parameter set shown in Table 1. On each lattice we computed the lowest eight eigenvectors of the Asqtad Dirac matrix.

Table 1
Lattices analyzed

a	L	vol	β	no. configs.
$a \to 0$:				
0.20 fm	12	(2.4 fm)^4	7.56	100
0.15	16	.	7.847	97
0.12	20	.	8.109	93
0.095	24	(2.3 fm)^4	8.456	118
$L \to \infty$:				
0.12 fm	12	(1.4 fm)^4	8.109	100
.	16	(1.9 fm)^4	.	100
.	20	(2.4 fm)^4	.	93
.	24	(2.9 fm)^4	.	100

We see clear evidence for lower dimensional scaling as $a \to 0$ with co-dimension between 2 and 3. Figure 1 shows both the distribution of the IPRs and the scaling of the averages. The

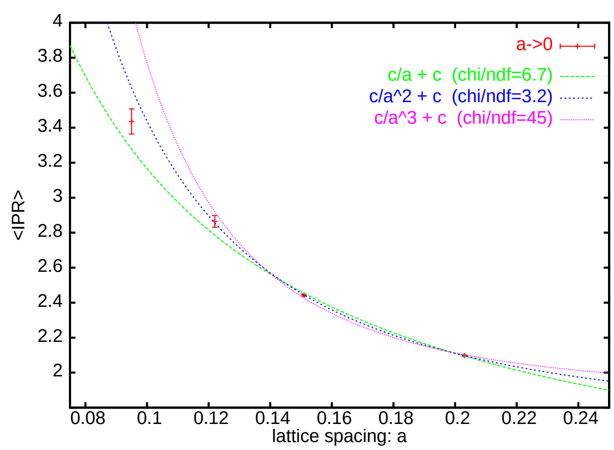

Figure 1. Scaling of the average IPR as $a \to 0$

data points are fit to $c_1/a^n + c_2$, where c_1 and c_2 are constants and $n = 1, 2, 3$. The reduced chi squared values[2] for the fits are 6.7, 3.2, and 45 for n =1, 2, and 3, respectively.

[2]The error bars shown here are corrected from those (much larger) shown at the conference.

In figure 2 we show the behaviour of the IPR as we increase the volume at fixed lattice spacing; it is rather unaffected, as we expected above.

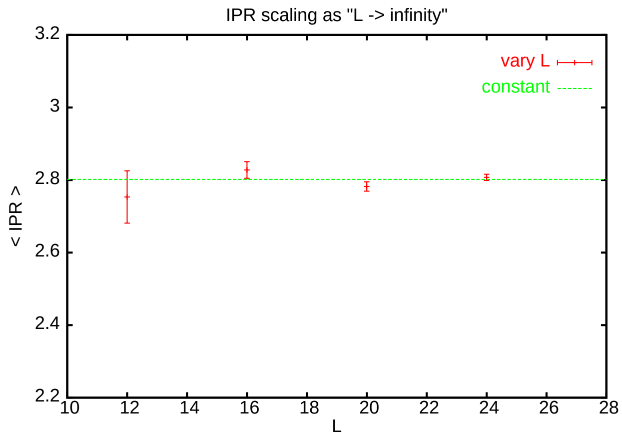

Figure 2. Scaling of the average IPR as $L \to \infty$ with $a = 0.12$ fm

4. TOPOLOGICAL CHARGE

We can also investigate the localization of the topological charge density by computing the IPR from $q(x) = F_{\mu\nu}\tilde{F}^{\mu\nu}$, where we have normalized $\sum_x |q(x)| = 1$. We have computed $q(x)$ by successive HYP smearing sweeps [5] on the $a \to 0$ series, and show the results in Figure 3 (note that only 5 HYP smearing steps were performed on the $a = 0.12$ fm $L = 20$ lattice set). While this plot does not show us new information on the localization of the LDEs, it is nonetheless instructive.

First, we see that all lattices without smoothing have an IPR $= \pi/2$. This is the value expected if the field is a gaussian fluctuation at each site, regardless of its width. We further see the approach to a stable localization of topological charge versus HYP smearing as the lattice spacing is decreased (at fixed volume). We note that <IPR> is not large, meaning that $q(x)$ is not strongly localized. Also, it increases as $a \to 0$ as for the LDEs.

5. CONCLUSIONS

The main result of this study is the indication of a localization of the low-lying Dirac eigenmodes on surfaces of co-dimension between 2 and

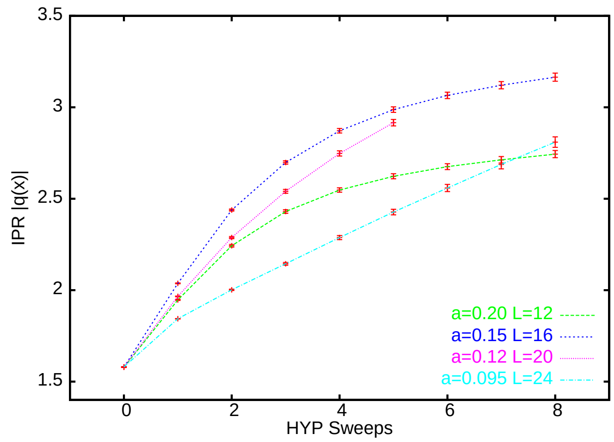

Figure 3. Avereage topological IPR vs. number of HYP smearing sweeps, as $a \to 0$

3, qualitatively supporting the center vortex or monopole pictures of confinement. Note, however, that the singularities of *thin* objects (vortices or monopoles) are expected to be smoothed out by the QCD interactions and become *thick*, with a size $\sim 1/\Lambda_{\rm QCD}$. Thick objects fill a fixed fraction of space, not a divergent one. The indication we have, via the divergence of <IPR> as $a \to 0$, of localization on singular manifolds, is remarkable, whatever these manifolds are.

REFERENCES

1. Ph. de Forcrand et. al., Nucl. Phys. Proc. Suppl. **73** (1999) 578; T. DeGrand and A. Hasenfratz, Phys. Rev. **D64** (2001) 034512; T.L. Ivanenko and J.W. Negele, Nucl. Phys. Proc. Suppl. **63** (1998) 504
2. C. Gattringer et. al., Nucl. Phys. **B617** (2001) 101; T. Kovacs, Phys. Rev. **D67** (2003) 094501
3. M. Engelhardt, these proceedings.
4. I. Horvath et. al., Phys. Rev. **D68** (2003) 114505; H.B. Thacker, these proceedings.
5. T. DeGrand, A. Hasenfratz, T.G. Kovacs Nucl. Phys. **B505** (1997) 417