89-74

Stochastic Cooling

by

K. Bongardt und M. Conte

XV EHF Workshop Lecce 9-14 October 1989

Stochastic cooling is required for pushing the Tails of particle distribution(s) into the CORE(S), where e-cool is more effective.

Betatron Cooling

$$\frac{1}{\tau} = \frac{\cancel{2}W^{(*)}}{N}\left[2g\left(1-\frac{1}{\tilde{M}^2}\right)-g^2(M+U)\right]$$

$g \leq 1$ gain

N = Number of fragments/pulse = 6.07×10^5

W = Amplifier Band-Width = 250 MHz

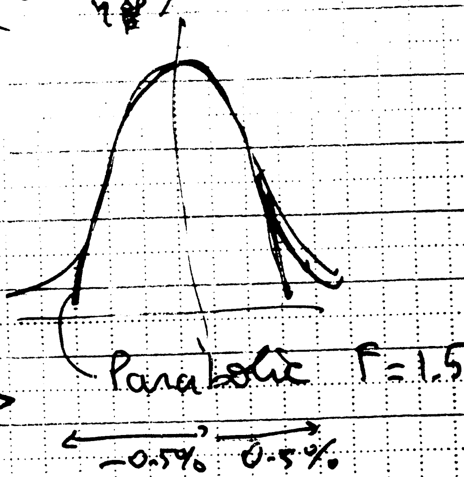
(*)

$$\varepsilon_{H,V}(t) = \varepsilon_{H,V}(0)\, e^{-\frac{t}{\tau}}$$

$$x(t)\left(\propto \sqrt{\varepsilon_H}\right) = x(0)\, e^{-\frac{t}{2\tau}}$$

Overlapping frequency = 500 MHz $\left(\propto \frac{1}{\eta \frac{\Delta p}{p}}\right)$

$$M = \frac{F}{2W|\eta|\frac{\Delta p}{p}\tau_{rev}} \ln\frac{f_{max}}{f_{min}}$$



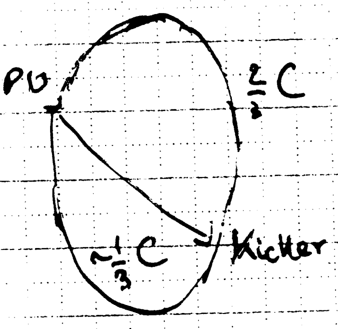
$\eta = 0.18$ $\qquad \ln\frac{f_M}{f_{min}} \simeq \ln 2 \simeq 0.69$ $\qquad$ (f_M = 500 MHz, f_{min} = 250 MHz)

$\frac{\Delta p}{p} = 1\%$ $\longrightarrow$

$\tau_{rev} = 1.02\ \mu s$

$M = 1.102 \simeq 1$ ($\sim$ LEAR-like)

$C = 266.67$ m



$$\frac{M}{\tilde{M}} = \frac{\frac{1}{3}C}{\frac{2}{3}C}$$

$$\tilde{M} = 3M$$

$$1-\frac{1}{\tilde{M}^2} = 1-\frac{1}{9} \simeq 0.9$$

$$U = \frac{P_{thermal}}{P_{Schottky}}$$

$$P_{th} = K_B(T_{PU} + T_A)\, g_A^2\, W$$

$$P_s = \frac{1}{2} N\, n_{PU}\, \beta_{PU}\left(\frac{?}{h}\right)^2 Z_{PU}\, Z^2 e^2 f_{rev}\, \varepsilon_{r.m.s.}\, g_A^2\, W$$

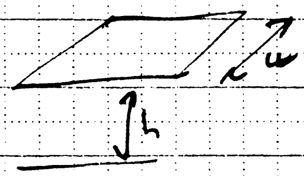
$$U = \frac{2 K_B (T_{PU} + T_A)}{N\, n_{PU}\, \beta_{PU} \left(\frac{\sigma}{h}\right)^2 Z_{PU}\, Z^2 e^2 f_{rev}\, \varepsilon_{rms}} \simeq 4.4 \quad !!!$$

$T_{R} = 300\,°K$

$T_A = 100\,°K$

$K_B = 1.38 \times 10^{-23}\ J/°K$

P.U.



$h = w = 15\ mm$

$\sigma = 2 \tanh\left(\frac{\pi w}{2h}\right) = 1.834$

$\lambda = \frac{c}{\langle f \rangle} = 0.8\ m \rightarrow \ell_{PU} = \frac{\lambda}{4} = 20\ cm$

$\langle f \rangle = \frac{500\,MHz + 250\,MHz}{2} = 375\ MHz$

$\|$

USEFUL ROOM $\simeq 2\ m$

REASONABLE $n_{PU} = 8$

$\beta_{PU} = 10\ m$

$Z_{PU} = 50\ \Omega$

$Z = 50$ (Fragments Charge State)

$e = 1.6 \times 10^{-19}\ C$

$f_{rev} = 0.98\ MHz$

$\varepsilon_{rms} \simeq 1\pi \times 10^{-6}\ rad\ m$

$$\frac{1}{\tau} = \frac{W}{N}\left[2g\left(1-\frac{1}{M^2}\right) - g^2(M+U)\right] \simeq \frac{W}{N}\left[2g - g^2(M+U)\right]$$

$$\left(\frac{1}{\tau}\right)_{MAX} = \frac{1}{\tau_{min}} = \frac{W}{N}\left[\frac{2}{M+U} - \frac{\cancel{M+U}}{(M+U)^{\cancel{2}}}\right]$$

$$\tau_{min} = \frac{N}{W}(M+U) \simeq 10\ ms$$

$N = 6.07 \times 10^5$ fragmen

$W = 250$ MHz

for $g = \frac{1}{M+U} = 0.2$

BUT

$$g_A = \frac{2g}{N}\,\frac{AV_0}{\sqrt{n_{PU}\, n_K\, \beta_{PU}\beta_K}\left(\frac{\sigma}{h}\right)^2 Z_{PU} Z\, l_{PU}\, e f_{rev}}\,\frac{\beta^2\gamma}{1+\beta}$$

$n_{Kicker} \simeq n_{PU} = 8$, $\beta_K = \beta_{PU} \simeq 10$ m

$$g_A = 2.3 \times 10^9$$

and then . . . . .

$$P_{th} = \left(k_B T_A + k_B T_A\right) g_A^2 W = 7.5\ MW \ !!!$$

$$P_{Schottky} = \frac{P_{thermal}}{U} \simeq 2\ MW \quad !!!!$$

$$U = 4$$

$$P_{total} \simeq 10\ MW$$

$\Downarrow$ {increase Number of PU K}

reduced by 5 order of magnitude

$\Downarrow$

$$\boxed{P_{total} \simeq 100\ Watt}$$

[illegible] $\rightarrow$ $g'^2_A = \frac{g_A^2}{10^5} \rightarrow g'_A \simeq \frac{g_A}{\sqrt{10^5}} \rightarrow g' = \frac{g}{\sqrt{10^5}}$

then

$$\frac{1}{\tau} \simeq \frac{W}{N}\left(\sqrt{2} \times 10^{-3} - 10^{-5} \ldots\right) \simeq 3 \times 10^{-3} \frac{W}{N}$$

6 $\boxed{\tau \simeq 3\ s}$ $\left(4\pi \rightarrow 1\pi\right) \sim \pi\, 10^{-6}$ rad m

Limiting Emittance

$$\mathcal{E}(t) = \mathcal{E}(\infty)\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\mathcal{E}(\infty) = B\tau$$

$$B = 3(kT_{PU} + kT_A)\,\frac{g^2}{N^2}\,\frac{W}{Z_{PU}\,n_{PU}\,\beta_{PU}}\,\frac{1}{\left(\frac{\sigma}{\omega}\right)^2 e^2 f_{rev} Z^2} \simeq 1.33\times10^{-3}\ m/s$$

for $g \simeq 1$

$$\tau = 2\times10^{-3}$$

$$\mathcal{E}(\infty) = 2.66\times10^{-6}\ \pi\ rad\ m$$

New (small) gain

$$B \rightarrow \frac{B}{10^6}$$

$$\tau \rightarrow \tau\times10^3$$

new $\mathcal{E}(\infty) \simeq 2.66\times10^{-9}\ \pi\ rad\ m$

practically ZERO